Forgetting the Past: Habit or Not?

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Abstract

It has been well documented in the data that agents put more weight on recent events or information compared to more distant events when making decisions. In this paper we show that external habit preferences can potentially explain this overweighing of recent information. In the model we assume a three period model with two types of agents: Young and Old both with "Catching up with the Joneses" preferences. Both types of agents observe information from their birth onwards and hence Young agents have observed less information. Due to the external habit preferences Older agents take into account investment decisions (i.e. consumption) of Young agents when making their own decisions. Since Young agents have only observed the more recent information, Older agents overweight this recent information in their own investment decisions. Furthermore, our theoretical model predicts that in areas with a lower median age, the weights put on recent information is higher compared to areas with a higher median age.

1 Introduction

It has been well documented in empirical literature that agents, when making decisions, put more weight on current events/information than on events occurred further away in the past (inflation, GDP forecasts and etc.). For instance, when making investment decisions, recent stock returns matter more than stock returns in the far past. Malmendier and Nagel (2009) show that household's risk-taking is strongly related to "experienced returns" (weighted average of past returns observed during the lifetime of an agent). they also show that more recent stock market realizations receive higher weights in belief formation (higher weights when the experienced returns are calculated) compared to stock returns in the far past. In this paper we would like to explore the question WHY agents overweight recent information.

One possible rational behind that fact is structural changes in the environment. I.e. the "world" changes over time, and thus past returns contain less "useful" information than recent returns. Or the structural changes change the risk aversion of the agents (Guiso, Sapienza, Zingales, 2004). The model with structural changes can generate higher weights for the recent information than for the past returns. But as empirical literature show, the weighting function for the experienced returns requires large number of structure shocks. This number of structural shocks has not been detected in literature before. Another possible explanation of the higher weights allocation to more recent information can be the changes in beliefs of agents about future returns over time. Malmendier and Nagel (2009) proposes this explanation for their result.

In this paper we would like to propose another possible rational behind that fact. We want to show that external habit preferences can be considered as the channel explaining overweighting of recent information compared to the past. In the model described in details below we assume there are two types of agents: Young and Older. Both types of agents have "Catching up with Joneses" type of preferences. Both types of agents observe information from their birth till recent times. As a result Young agents by construction observe short period of more recent information. The external habit makes Older agents care about investment decisions (consumption) of Young agents. This results in the Older agents putting more weight on recent information, since Young agents condition their portfolio choices (and thus affecting their future consumption) only on that information. Herding behavior of Older agents leads to overweighting of the recent information comparing with information further in the past.

The paper is organized as follows: In Section I we discuss the related literature. We introduce the simple "one-time decision" example of the model with Young and Old types of agents in the Section II. Also in the Section II we show how the habit component and population age distribution affects portfolio allocation of agents with KUJ type of preferences. Section III presents the data and the methodology. The preliminary empirical results are reported in Section IV. In Section V we discuss potential problems and solutions for them. Section VI offers some conclusions.

2 Literature Review

Our paper on the one hand is related to empirical literature. As part of the motivation for our paper we use Malmendier and Nagel (2009) paper. In this paper, know as "Depression Babies", authors highlight the following paradox. Standard models in economics assume that individuals are endowed with stable risk preferences, which are unaltered by economic experience. At the same time psychology literature argues that personal experiences, especially recent ones, have greater influence on personal decisions than information obtained from books or via education. Malmendier and Nagel test hypothesis that difference in willingness of individuals to take financial risks (stock market participation, for example) is related to the macroeconomic history they have experience (proxied by experiences returns). Authors show that individuals are influenced more strongly by recent returns than distant returns. Recent experience receives higher weights from individuals compared to past experience. In the paper authors construct the measure of the "experienced returns", defined as a weighted sum of all stock returns observed by an agent during her lifetime.

Another strand of literature that this paper can be related to is *social utility* literature. This literature is well summed up in the paper by Burstyn et. al (2014). In this paper authors explore two channels of social influence in financial decisions. One of channels is so-called social utility, when someone purchases an asset, his/her peers may also want to purchase it because his/her possession of the asset directly affects others' utility of owning the same asset. One of the examples of social utility explored in Bursztyn et. al (2014) is "keeping up with the Joneses" (KUJ) (Abel, 1990; Gali, 1994; and Campbell and Cochrane, 1999; Gomez, Priestley and Zapatero, 2009) utility function. In that case investors may be concerned with their incomes or consumption levels, relative to their peers'.

From the theoretical point of view Gali (1994) shows that in the absence of frictions, utility based on KUJ preferences yields the symmetric equilibrium, in which all agents hold the same global portfolio, and Joneses behavior translates into a lower price of risk on incorporate frictions. In general equilibrium setting according to Gali no portfolio allocation bias occurs. Gomez (2006) in his turn shows that there is a set of particular conditions on utility function under which result of Gali (1994) holds. In our paper we concentrate on partial equilibrium setting and fixed preferences parameters of agents.

From the literature we can see that there are empirical evidence of the overweighting of the recent information by individuals while making investment decisions. In this paper we want to show that higher sensitivity to current information can be explained by the desire of older agents (result of habit formation presence) to be closer in terms of consumption to younger agents.

3 Theoretical Model with External-Habit Preferences

We consider a simple 3 time-period partial equilibrium economy, $t = -2, -1, 0$. There are 3 generations of risk-averse agents in the economy: "*Newborn*" (NB), "*Young*" (Y), and "*Old*" (Old). Periods $t = -2, -1$ describe the history of realized stock market returns potentially observed by the agents.

Newborn agents do not make any consumption or investment decisions. When they are born they observe a realization of stock market returns and update their prior about risk premium.

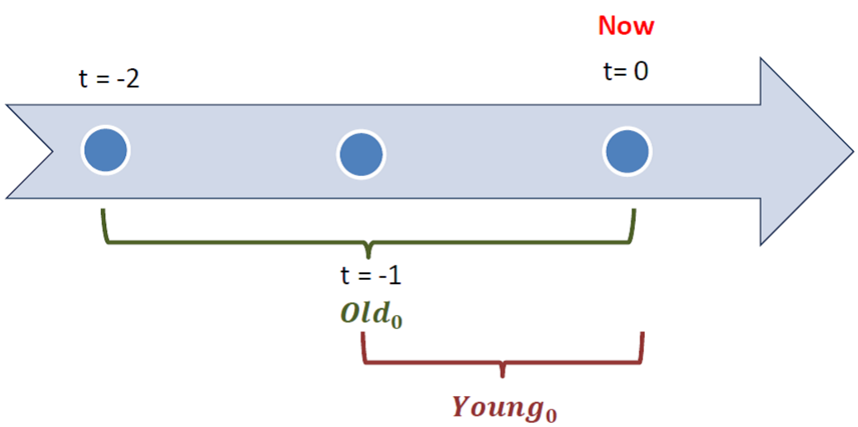
Agents of types *Young* and *Old* make their investment decision at the beginning of the period $t = 0$. At the end of the period all portfolio payoffs are consumed. Every period agents have a positive initial endowment w_{NB}, w_Y, and w_O respectively.

Agents can invest in two types of assets: risk-free assets (bond) and a portfolio of risky assets (stock market portfolio). The gross return on risk-free asset, R^f, is positive. For simplicity later on we assume that return on risk-free asset is equal to 1. The number of risky projects in the portfolio is normalized to one. In period t the risky portfolio has a random gross return, R_t.

In this economy agents are price takers, i.e. they observe the realization of returns and make their investment decision based on that information. All types of agents have a common prior at birth for the mean return of the risky portfolio, $E[R_t] = 1$, i.e. $E\,[risk\ premium] = 0$[1]. Agents start learning from birth (similar as in Malmendier and Nagel (2009)), thus older agents have observed more realizations of risky portfolio returns. As a result the economy is characterized by this information asymmetry.

3.1 Timeline

The timeline of the model is describe on the Figure 1.



The Model Timeline

From the timeline above we see the following:

In period $\boldsymbol{t = 0}$ **- Now** economy is populated by 3 types of agents: $Newborn_0$, $Young_0$, and Old_0 . Agents make their investment decisions in the beginning of the period $t = 0$ and consume portfolio payoffs in the form of the single consumption good in the end of the period $t = 0$. Agent type Old_0 has observed realizations of risky asset returns in the end of periods $t = -2$ and $t = -1$. Agent type $Young_0$ has observed realizations of risky asset returns only in the end of period $t = -1$.

In the period $\boldsymbol{t = -1}$ there are 3 types of agents present: $Newborn_{-1}$, $Young_{-1}$, and Old_{-1}. By assumption of the evolution of the model we have that $Newborn_{-1}$

[1] Agents observe historical variance of stock market returns, σ^2, and do not update their belief about its value

in the next period becomes $Young_0$ and $Young_{-1}$ in the next period becomes Old_0.

In the period $t = -2$ the economy is populated by 3 types of agents: $Newborn_{-2}$, $Young_{-2}$, and Old_{-2}. We know that in the next period agent type $Newborn_{-2}$ will become agent type $Young_{-1}$. In that way we can track the transformation (aging) of an agent from Newborn to Old in 2 periods. While getting older agent observe/experience more realizations of risky asset returns. Thus agent born in period $t = -2$ has longer span of information about risky asset returns than agents born in period $t = -1$ and $t = 0$.

3.2 Effect of Consumption Externalities on Portfolio Choice.

The economy is populated by three types of agents. *Newborn* agents do not make any decisions in the economy, so we can disregard them at this stage.

Young and *Old* agents have KUJ (external habit) type of preferences. The population is constant and is normalized to unity. Let μ_{NB} denote the fraction of the Newborn type of agents in the economy in period $t = 0$, μ_Y denote the fraction of the Young type agents in period. The fraction of Old type of agents is equal to $\mu_O = (1 - \mu_{NB} - \mu_Y)$ in period $t = 0$. We assume that the discount factor is equal to one for all type of agents.

At time $t = 0$ agent type i solves the following problem (Gali, 1994):

$$\begin{aligned} &\max_{\lambda} E\left[U_i\left(c_i, X\right)\right] \\ &s.t.\ c_i = w_i\left(R^f + \lambda_i rp\right) \\ &rp \equiv R_t - R^f \end{aligned} \tag{1}$$

where c_i denotes the household own consumption level at the end of period t, and X is the reference consumption level in the economy. The reference consumption level is equal to average consumption level in the economy, i.e.

$$X = \theta * c_Y + (1 - \theta) * c_O \tag{2}$$

w_i denotes initial wealth of an agent i, λ_i is the "risky share" (the fraction of wealth allocated to risk asset investment), θ denotes the fraction of each type of agents in the population, and rp is the (ex-post) risk premium, difference between risky asset and bond returns. We assume that rp is an exogenous random variable with distribution function $F_i\left(rp\right)$. The posterior distribution of the risk premium is different for each type of agent since they observe different time span of information.

We specialize the utility function for agent of type i to be of the form

$$U_i(c, X) = \frac{c^{1-\alpha}}{1-\alpha} X^{\gamma_i \alpha}, \ \alpha > 0, \ \gamma_i < 1 \tag{3}$$

where α is a measure of relative risk aversion, the same for all types of agents, and γ_i is a "habit" parameter for agent $i = Y, O$.

The sign of γ_i plays a crucial role in determining the effects of consumption externalities. When $\gamma_i > 0$, an increase in average consumption raises the marginal utility of an individual household's own consumption: any given addition to his current level of consumption becomes more valuable, for it is need to "keep up with the Joneses". In the literature this type of externalities is referred as *positive consumption externality.* Alternatively, if $\gamma_i < 0$, other households' consumption behaves as a substitute for each household's own consumption: increases in X lower the marginal utility of own consumption.

In this paper we assume that agents are homogeneous in there type of consumption externality. *Young* and *Old* types of agents express positive consumption externality, i.e. KUJ type of preferences: $\gamma > 0$. At the same time we observe information asymmetry. Agents of type *Old* have "lived" for two periods. They have observed stock market returns realizations and they have acted like *Young* type in the period before. Thus agents of type *Old* know exact value of "risky" share allocated by *Young* type of agents (knowledge is limited up to the moments of posterior distribution).

At the same time, agents of type *Young* have access to stock market returns information revealed only in one period. They do not observe past actions of *Old* type of agents. To calculate the average consumption in the economy *Young* agents have to make a guess about consumption level of agents of type *Old*.

The problem for the Newborn and Young types of agents can be simplified to

$$\max_{\lambda} E\left[U_Y(c, X_Y)\right] \tag{4}$$

$$s.t. \ c_Y = w_Y \left(R^f + \lambda_Y rp\right)$$

$$rp \equiv R_t - R^f \tag{5}$$

where $U_Y(c, X_Y) = \frac{c^{1-\alpha}}{1-\alpha} X_Y^{\gamma\alpha}$ and

$$X_Y = \tau_Y * c_Y + \tau_O * c_O \tag{6}$$

where the size of population is normalized to $(1 - \mu_{NB})$ and $\tau_Y = \frac{\mu_Y}{1-\mu_{NB}}$ and $\tau_O = 1 - \frac{\mu_Y}{1-\mu_{NB}}$.

To define the reference consumption level, X_Y, *Young* agents have to make a guess about consumption level of agents of type *Old*, c_O. We assume that the best guess that *Young* agents can make is that consumption of an *Old* agent is the same as consumption of a *Young* agent, i.e. $c_O = c_Y$. Thus the average consumption in the economy for an agent of type Young is equal to his/her own level of consumption, $X_Y = \tau_Y * c_Y + \tau_O * c_Y = c_Y$.

As a result the risky share of agent of type *Young* is equal to (calculations are provided in the Appendix 1.A)

$$\lambda_Y^* = \frac{E_Y(rp)}{\alpha(1-\gamma)\sigma^2}, \tag{7}$$

where $E_Y(rp)$ and σ^2 are moments of $F_Y(rp)$, posterior distribution of *Young* agent. We can see that the share of wealth invested in risky asset by *Young* type of agents depends only on moments of posterior distribution of returns, i.e. on information they observe, and their preferences parameters.

The agent of type Old instead solves the following problem described in eq. (1) with utility function

$$U_O(c, X) = \frac{c_O^{1-\alpha}}{1-\alpha} X^{\gamma\alpha},\ \alpha > 0,\ \gamma_O < 1 \tag{8}$$

To simplify derivations we assume that all types of agents receive the same initial endowment, $w_{NB} = w_Y = w_O = w$ and that Old type of agents care only about consumption of Young type of agent, i.e. $X_{old} = \tau_Y * c_Y + \tau_O * c_O$. The first-order condition for the problem for the Old type of agent is

$$E\left(\left[U_c\left(w\left(R^f + \lambda_O * rp\right)\right), \tau_Y * c_Y + \tau_O * c_O\right] * rp\right) = 0 \tag{9}$$

Solving the problem (Appendix 1.B) we get that the optimal fraction of wealth allocated to risky asset for an agent of type Old is

$$\lambda_O^* = \frac{E_O\left(rp\right)}{\sigma^2}\left(\frac{1-\tau_y+\gamma\alpha\frac{\tau_y c_Y^*}{w}}{\alpha\left(1-\gamma\right)+\tau_Y}\right) \tag{10}$$

An Old agent knows the level of consumption of a Young agent. I.e. s/he knows that

$$\frac{c_Y^*}{w} = R^f + \lambda_Y^* * E_Y\left[rp\right] = R^f + \frac{E_Y\left(rp\right)}{\alpha\left(1-\gamma\right)\sigma^2} * E_Y\left[rp\right] \tag{11}$$

We can show that $\frac{\partial\lambda_O^*}{\partial\gamma} > 0$, i.e. keeping everything else constant the more agent of type Old care about consumption of an agent of type Young (γ goes up), the higher is the fraction of wealth s/he allocates to risky asset.

Based on the result above we can also say that if an agent of type Young expects higher risk premium, $E_Y\left(rp\right)\ \uparrow$, i.e. "risky" share of a Young agent increases, $\lambda_Y^*\ \uparrow$, share of wealth invested by an Old agent in risky asset goes up as well, $\lambda_O^*\ \uparrow$.

3.3 Portfolio Choice With and Without Habit

In the presence of consumption externality agents tend to make similar decisions. In other words, Old type of agents after observing consumption level of a Young type adjusts her/his "risky" share, by making it more similar to the one of Young. Using the model described above this result can be described as the following inequality

$$\left|\lambda_O^H - \lambda_Y^H\right| < \left|\lambda_O^{WH} - \lambda_Y^{WH}\right| \tag{12}$$

where λ_O^H and λ_Y^H define optimal "risky" shares for Old and Young agents, respectively, in the case when both types of agents have KUJ type of preference (have "external habit"); and λ_O^{WH} and λ_Y^{WH} define optimal "risky" shares for Old and Young agents, respectively, when both types of agents do not have external habit preferences.

Based on the solution of the optimal portfolio choice for an agent with CRRA preferences (Appendix 1.C) we can rewrite the left-hand side of the inequality (12) as

$$\lambda_O^H - \lambda_Y^H = \frac{E_O\left(rp\right)}{\alpha\sigma^2} - \frac{E_Y\left(rp\right)}{\alpha\sigma^2} = \frac{1}{\alpha\sigma^2}\left(E_O\left(rp\right) - E_Y\left(rp\right)\right) \tag{13}$$

Using results described above we can also rewrite the right-hand side of the expression (12):

$$\lambda_O^{WH} - \lambda_Y^{WH} = \frac{E_O\left(rp\right)}{\sigma^2}\left(\frac{1-\tau_y+\gamma\alpha\frac{\tau_y c_Y^*}{w}}{\alpha\left(1-\gamma\right)+\tau_Y}\right) - \frac{E_Y\left(rp\right)}{\alpha\left(1-\gamma\right)\sigma^2} \tag{14}$$

Both left-hand side and right-hand side of the equation (12) describe gaps between portfolio choices of Old and Young agents in the situation when both of agents have external habit preferences and when both don't, respectively Let us define the $GAP^H = \left|\lambda_O^H - \lambda_Y^H\right|$ and $GAP^{WH} = \left|\lambda_O^{WH} - \lambda_Y^{WH}\right|$. In this paper we want to show that the stronger is the habit in the economy, the higher is the difference between gap "*with habit*" and gap "*without habit*", i.e. we want to show that

$$\frac{\partial\left(GAP^H - GAP^{WH}\right)}{\partial\gamma} > 0 \tag{15}$$

Let us define difference in gaps as $dGAP = GAP^H - GAP^{WH}$. After some algebra this function can be written in the following way

$$\begin{aligned} dGap &= \frac{E_O\left(rp\right)}{\alpha\sigma^2} * \mathbf{A} + \frac{E_Y\left(rp\right)}{\alpha\sigma^2} * \mathbf{B}, \\ \mathbf{A} &= \frac{\gamma}{1-\gamma}, \\ \mathbf{B} &= \frac{\tau\left(\frac{1}{\alpha}+1\right) - \gamma\left(1+\tau\alpha\frac{c_Y}{w}\right)}{\left(1-\gamma\right)+\frac{\tau}{\alpha}} \end{aligned} \tag{16}$$

So the condition (15) can be rewritten as

$$\frac{\partial\left(dGap\right)}{\partial\gamma} = \frac{E_O\left(rp\right)}{\alpha\sigma^2} * \frac{\partial\mathbf{A}}{\partial\gamma} + \frac{E_Y\left(rp\right)}{\alpha\sigma^2} * \frac{\partial\mathbf{B}}{\partial\gamma} \tag{17}$$

Difference in gaps is a non-linear function of habit parameter, γ, as well as such parameter as population distribution among Young an Old in the economy, τ. we describe $\frac{\partial(dGap)}{\partial\gamma}$ on the figure below.

Conclusion

Due to the external habit preferences Older agents take into account investment decisions (i.e. consumption) of Young agents when making their own decisions. Since Young agents have only observed the more recent information, Older agents overweight this recent information in their own investment decisions. Furthermore, our theoretical model predicts that in areas with a lower median age, the weights put on recent information is higher compared to areas with a higher median age.

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Appendix 1.A: Young Type of Agent with "habit" parameter

The household without external habit formation solves the following problem:

$$\begin{aligned}&\max_{\lambda} E\left[U_Y\left(c, X_Y\right)\right]\\&s.t.\ c_Y = w\left(R^f + \lambda_Y rp\right)\\&rp = R_t - R^f\\&X_Y = c_Y\end{aligned}$$

where $U_Y\left(c\right) = \frac{c_Y^{1-\alpha}}{1-\alpha} X_Y^{\gamma\alpha}$. Since $X_Y = c_Y$, we have

$$U_Y\left(c\right) = \frac{c_Y^{1-\alpha(1-\gamma)}}{1-\alpha}$$

The first-order condition for the problem above is

$$E\left[U_c\left(w\left(R^f + \lambda_Y rp\right)\right) * rp\right] = 0$$

Note that $U_c\left(c\right) = c^{-\alpha(1-\gamma)}$ and $\frac{-U_{cc}(c)}{U_c(c)} c = \alpha\left(1-\gamma\right)$. Using Taylor expansion around initial wealth point we get

$$\begin{aligned}U_c\left(c\right) &\cong U_c\left(w\right) + U_{cc}\left(w\right)\left(c - w\right)\\&= U_c\left(w\right) + U_{cc}\left(w\right) w\left(R^f - 1 + \lambda rp\right)\\&= U_c\left(w\right)\left(1 - \alpha\left(1-\gamma\right)\left(R^f - 1 + \lambda rp\right)\right)\end{aligned}$$

An approximation to the first-order condition is then obtained by multiplying the above expression by rp, taking expectations, and equating the resulting expression to zero.

$$E\left[U_c\left(w\right)\left(1 - \alpha\left(R^f - 1 + \lambda rp\right)\right) * rp\right] = 0$$

We are solving the above equation for λ, using the fact that $\sigma^2 \cong E\left[rp^2\right]$ and $\left(R^f - 1\right) E\left[rp\right] \cong 0$ for small values of $E\left[rp\right]$ and the riskless rate. As a result we get the expression for the optimal "risky" share for an agent without "habit" preferences:

$$\lambda_Y^* = \frac{E_Y(rp)}{\alpha(1-\gamma)\sigma^2},$$

Appendix 1.B: Old Type of Agent with "habit" parameter

The household of type Old solves the following problem:

$$\max_{\lambda} E\left[U\left(c, X_{old}\right)\right]$$
$$s.t.\ c = w\left(R^f + \lambda rp\right)$$
$$rp \equiv R_t - R^f$$

where $U(c,X) = \frac{c^{1-\alpha}}{1-\alpha}X_{old}^{\gamma\alpha}$ and $X_{old} = \tau * \widetilde{c} + (1-\tau) * c$ and $\widetilde{c}$ defines consumption level of a reference group.

The first-order condition for the problem above is

$$E\left[U_c\left(w\left(R^f + \lambda rp\right), \tau * \widetilde{c} + (1-\tau) * w\left(R^f + \lambda rp\right)\right) * rp\right] = 0$$

Note that

$$U_c(c,X) = c^{-\alpha}X^{\gamma\alpha}$$
$$U_c(c,X) = -\alpha c^{-\alpha-1}X^{\gamma\alpha}$$
$$U_{cX}(c,X) = \gamma\alpha c^{-\alpha}X^{\gamma\alpha-1}$$
$$\frac{-U_{cc}(c)}{U_c(c)}c = \alpha \ and \ \frac{-U_{cX}(c)}{U_c(c)}c = \gamma\alpha$$

Using Taylor expansion around initial wealth point and the assumption that all types of agents receive the same amount of initial wealth we get

$$\begin{aligned} U_c(c,X) &\cong U_c(w,w) + U_{cc}(w,w)(c-w) + U_{cX}(w,w)(X-w) \\ &= U_c(w,w) + U_{cc}(w,w)\,w\left(R^f - 1 + \lambda rp\right) + +U_{cX}(w,w)(X-w) \\ &= U_c(w,w)\left(1 - \alpha\left(R^f - 1 + \lambda rp\right) + \gamma\alpha\frac{X-w}{w}\right) \end{aligned}$$

An approximation to the first-order condition is then obtained by multiplying the above expression by rp, taking expectations, and equating the resulting expression to zero.

$$E\left[w^{-\alpha}w^{\gamma\alpha}\left(1-\alpha\left(R^{f}-1+\lambda rp\right)+\gamma\alpha\frac{X-w}{w}\right)*rp\right]=0$$
$$E\left[\left(1-\alpha\left(R^{f}-1+\lambda rp\right)+\gamma\alpha\left[\left(R^{f}-1+\lambda*rp\right)-\tau\left(R^{f}+\lambda*rp\right)\right]+\gamma\alpha\frac{\tau\widetilde{c}}{w}\right)*rp\right]=0$$

We are solving the above equation for λ, using the fact that $\sigma^2 \cong E\left[rp^2\right]$ and $\left(R^f-1\right)E\left[rp\right]\cong 0$ for small values of $E\left[rp\right]$ and the riskless rate. As a result we get the expression for the optimal "risky" share for an agent with "habit" preferences:

$$\lambda^{*}=\frac{E\left(rp\right)}{\sigma^{2}}\left(\frac{1-\tau+\gamma\alpha\frac{\tau\widetilde{c}}{w}}{\alpha\left(1-\gamma\right)+\tau}\right)$$

Appendix 1.C: Agents without "habit" parameter

The household without external habit formation solves the following problem:

$$\max_{\lambda} E\left[U\left(c\right)\right]$$
$$s.t.\ c=w\left(R^{f}+\lambda rp\right)$$

where $U\left(c\right)=\frac{c^{1-\alpha}}{1-\alpha}$.

The first-order condition for the problem above is

$$E\left[U_{c}\left(w\left(R^{f}+\lambda rp\right)\right)*rp\right]=0$$

Note that $U_c\left(c\right)=c^{-\alpha}$ and $\frac{-U_{cc}(c)}{U_c(c)}c=\alpha$. Using Taylor expansion around initial wealth point we get

$$\begin{aligned}U_{c}\left(c\right) &\cong U_{c}\left(w\right)+U_{cc}\left(w\right)\left(c-w\right)\\ &= U_{c}\left(w\right)+U_{cc}\left(w\right)w\left(R^{f}-1+\lambda rp\right)\\ &= U_{c}\left(w\right)\left(1-\alpha\left(R^{f}-1+\lambda rp\right)\right)\end{aligned}$$

An approximation to the first-order condition is then obtained by multiplying the above expression by rp, taking expectations, and equating the resulting expression to zero.

$$E\left[U_c\left(w\right)\left(1-\alpha\left(R^f-1+\lambda rp\right)\right)*rp\right]=0$$

We are solving the above equation for λ, using the fact that $\sigma^2 \cong E\left[rp^2\right]$ and $\left(R^f-1\right)E\left[rp\right] \cong 0$ for small values of $E\left[rp\right]$ and the riskless rate. As a result we get the expression for the optimal "risky" share for an agent without "habit" preferences:

$$\lambda^* = \frac{E\left(rp\right)}{\alpha\sigma^2},$$