Capital inertia and the timing of the energy transition

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Abstract

Energy power plants have an unusually long lifetime. This characteristic might well be one of the reasons behind the slow transition from fossil to renewable energy. I introduce this specific feature through investment-specific capital accumulation *à la* Krusell (1998) into a multi-sector exogenous growth model with climate economics, on a yearly basis. I find that: (i) there is capital inertia due to already engaged fossil energies, (ii) the carbon tax needs to be set at 5% of the capital rent in the "dirty" sector in 2010, and to be increasing over time, (iii) the economy is augmenting its fossil energy stock until 2070 and (iv) clean energy reaches 50% of total energy production in 2060. Theoretical results are sensitive to the rate of technological progress, making it essential even in presence of inertia. To investigate empirically its role in driving these results, I estimate VECM and bayesian VAR with stochastic search variable selection (SSVS) using US quarterly data from 1976 to 2019. The findings suggest that research toward renewable energy is the best option to increase its share in energy production.

Keywords: *Energy transition, investment specific, exogenous growth, climate economics,VAR, macroeconometrics*

JEL Classification Numbers: C61, O41, Q43.

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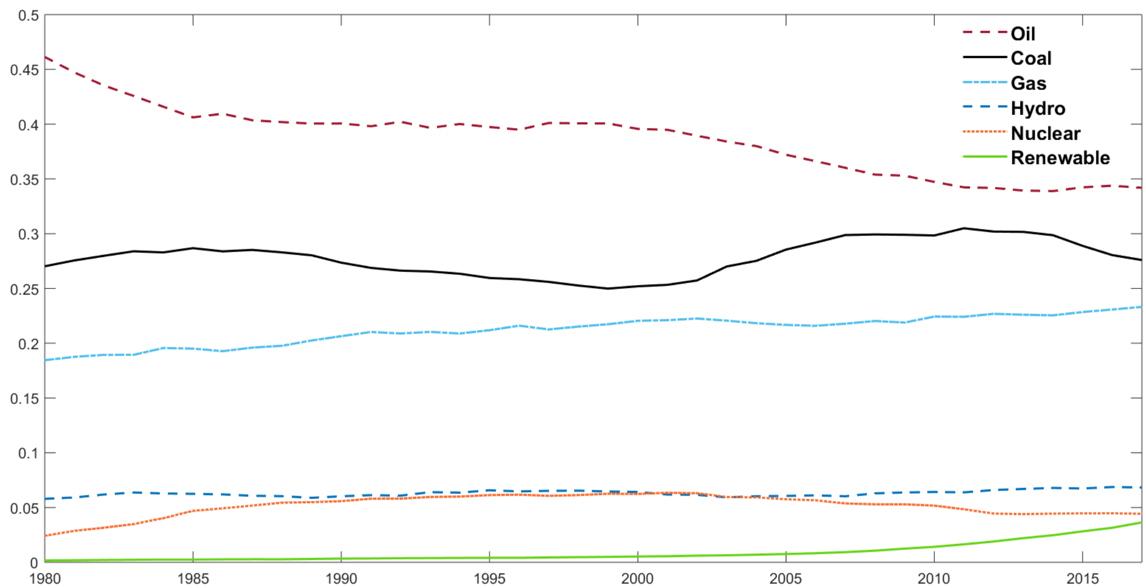
1 Introduction

The latest efforts from European Commission for a sustainable world can be summed up in the “green deal”: “a new growth strategy that aims to transform the EU into a fair and prosperous society, with a modern, resource-efficient and competitive economy where there are no net emissions of greenhouse gases in 2050 and where economic growth is decoupled from resource use”. Achieving the objective of a carbon-free economy requires facing a double challenge: provide energy to the less developed countries, Csereklyei et al. (2016), and stabilize energy security, Matsumoto et al. (2018), without relying intensively on fossil resources. The case of Poland exemplifies the issue at hand, GNI per capita is 2.5 times lower than the EU average while CO_2 emissions per capita are larger (7.9 metric tons per capita in Poland against 6.5 for EU). The Polish economy relies mainly on coal (80% of the energy mix) but is among the least dependent, Manowska et al. (2017). Developing countries are incited to invest in fossil energies and are also subsidizing them, see Clements et al. (2013) and Coady et al. (2015) for a complete investigation of fossil fuels subsidies. This tension between economic development and energy security appears also when looking at global data on the energy mix over the past thirty years (see figure 1). The share of fossil energies (gas,...) has remained roughly stable at 85% between 1990 and 2018. While the share of renewable energies went up from 1 to 4% in the last 10 years. The transition is not as fast as one can wish, especially according to the literature on climate change (Stern et al. (2006), Weitzman (2009), Nordhaus and Boyer (2000) or any IPCC report). However this slow process might found its origin in something different than investment decisions. Power plants are unusual capital units, while macroeconomics literature considers a 10 years lifetime for capital, in the energy market it exhibits higher average. Looking at NEEDS data for US, the average age of coal, biomass and geothermal power plants are respectively 60, 35 and 30 years old (more details in section 2). Each investment decision is made to last more than 2 decades, creating capital inertia toward energy production: new units cannot be scraped in the short-run.



source: BP statistical review of world energy

Figure 1: Share of energies in World energy mix

One specific aspect of the energy market that affects the pace of the transition is the unusually long lifetime of power plants, which I call capital inertia. The second aspect is how technical progress is spread over different technology within energy production process. The last one concerns pollution emissions, renewable and fossil power plants do not exhibits the same level or the same type of pollution. This paper aims at understand, both theoretically and empirically, the role played by capital inertia and technology in the transitional process, when the economy is threatened by pollution. The energy transition is needed due to the risks link to climate change and pollution emissions, see Nordhaus and Boyer (2000). While the literature about directed technical change launched by the seminal paper of Acemoglu et al. (2012) often focuses on the second and the third aspect, this paper tries to add capital lifetime. It generates a new type of rigidity in energy economics, and a possibly new explanation for the slow transition process.

I use a multi-sector model of exogenous growth with climate economics. The final good, which can be consumed or invested, is produced using labor and an aggregate of the two intermediary goods, the latter are produced using a continuous time version of Krusell (1998) which is an investment specific accumulation equation. The representative agents consumes, sells her work and owns the firms, she has a disutility of work. Intermediary sector are called "clean" for renewable energy, and "dirty" for fossil sources, they differ regarding to the rate of technical progress and to pollution emissions. Theoretical innovation of the paper lies in the form of accumulation equations, representative household decides optimally how much of the final good to invest in each sector, according to the current state of technology. Once investment is made the capital unit is created it cannot be transformed or removed, but it depreciates over time. The rate of technological progress is designed to be a relative performance indicator of the "clean" sector, therefore there is no technical change for the "dirty" sector. The final good being the numéraire one unit always creates one unit of "dirty" capital, but in the "clean" one it creates $q > 0$ units of capital. Exogenous growth then concerns this q variable, which starts below 1 and increases over time making renewable energy more attractive. It also features embodied technical change, because each unit is built using the current level of technology and cannot be enhanced ex-post. In a *laissez-faire* equilibrium, the representative household would never invest in the "clean" sector before $q > 1$, but the negative externality associated to the use of fossil energy advocates for the application of a carbon tax in this economy in favor of the "clean" sector. In the paper I also question empirically the role of technology in energy transition, There is a consensus of the literature about first role played by research in renewable technologies, but no formal empirical analysis has been conducted so far. I conduct VECM in level and Bayesian VAR in differences on US quarterly data from 1976 to 2019, looking at the impact of patent publication on the share of renewable energy in the US.

Solving the decentralize equilibrium allows to compute both asymptotic behavior and optimal transition of the model. Main findings are: (i) already engaged capital in the "dirty" sector is implicitly at the origin of capital inertia, (ii) the carbon tax needs to be set at 5% of the "dirty" capital rent in 2010, and to be increasing over time, (iii) the economy is augmenting its fossil energy stock until 2070 and (iv) the 50% threshold of "clean" energy is reached in 2060. The first result is due to model structure, calibration made on the US economy (high share of fossil energy) implicitly assume a low level of relative performance, q, in the "clean" sector. It takes time for renewable to catch up with fossil energy, creating an inertia phenomenon: long-term investments in the "dirty" sector. Third finding is a corollary of the first, if technology is delayed in the "clean" sector it is still more effective to invest in the "dirty" one, even in the presence of a negative externality. Long lifetime of power plants also diminishes the switching rate of "dirty" capital. The second result is similar to recent literature about optimal carbon tax (Golosov et al. (2014), Li et al. (2016) or Adao et al. (2017)). By solving the decentralize equilibrium I explicitly characterize the carbon tax and calibrate its optimal level using 2010 data for US, it evolves proportionally to level of fossil energy and is therefore increasing. Last finding is the relative weight of the "clean" sector, this result needs to be taken carefully because going above the 50% threshold does not mean the environment is doing better. The pollution stock will be increasing as long as the economy accumulates "dirty" capital. Quantitative results are sensitive to the rate of technological progress and the timing changes significantly along its value. This parameter seems crucial in the analysis of energy transition, the second part tries to quantifies empirically this link.

Using both VECM in level and stochastic search variable selection (SSVS) model in differences highlights a 5 quarters delay between publication of patents and its effect on energy production. A positive shock on the number of patents in the renewable energy sector have a persistent effect on its share in the energy mix. To this duration one should add 7 quarters between the moment a patent is submitted and the moment it is granted (according to USPTO). The effective time is then around 3 years for a new technology to be widely used. Following Acemoglu et al. (2016) I use the number of patents as a technology indicator, it captures output of research while budget and investments characterize the inputs, it do not give clear insight of research realization. Using CPC classification for USPTO data I am able to create an indicator for both "clean" and "dirty" technological progress within the US. GDP, oil price, global investments and energy consumption are added to the analysis. A vector Error Correction Model (VECM) in level and Vector Autoregressive (VAR) in differences are used to study possible impact of biased technological progress. I use a bayesian model with stochastic search variable selection (SSVS) prior is used to deal with possibly large number of variables due to the number of lags, such prior aims at avoiding overspecification and helps to shrink non-informative parameters to zero. These methods are widely used in macroeconometrics following papers by Korobilis (2013) or Giannone et al. (2015) and are really efficient to keep informative results. Few papers are using these methods with energy data, and they are often used to study oil price, like Degiannakis et al. (2018) or Alsalman (2016), but to the best of my knowledge this paper is the first one trying to link energy research to en energy production.

Investment in power plants are long-term decisions and refers to the literature about vintage capital, there are several version o the same type of capital that aim at producing the same output, energy, but using different technologies, embedded in the plant vintage. There is a wide range of papers about vintage capital in the following of Solow et al. (1960), see Boucekkine et al. (1997), Benhabib and Rustichini (1991), Greenwood et al. (1997) or Feichtinger et al. (2006) among others. In these articles capital market differs from Solow (1957), frictions arise from the coexistence of different vintages of capital and endogenous scraping associated to it, dynamics of the business cycle are then more complex to analyze. In this paper, the analysis uses a continuous time version of Krusell (1998), an investment-specific framework. It gathers the vintage structure and embodied technical change but abandon endogenous scraping. It allows to account for the main characteristics of the energy capital market and to lower computational complexity of vintage capital literature at the same time. This formulation is embedded in the literature of directed technological change and energy transition, launched by Acemoglu et al. (2012). It studies the reallocation of research toward the "clean" industry using taxation and subvention tools, like Acemoglu et al. (2016), Golosov et al. (2014), Li et al. (2016) or Adao et al. (2017) among many others. An optimal and increasing carbon tax is designed to correct pollution externality and relies more on cleaner alternative. However, to the best of my knowledge, Lennox and Witajewski-Baltvilks (2017) is the only paper trying to deal with both investment-specific and directed technical change. It embedded Krusell (1998) accumulation equation in Acemoglu et al. (2012) model, and their main contribution is to shed light on the obsolescence cost generated by switching from an asset to the other in the transitional process. Compare to their, this paper gives more insights on the transitional process and on the inertia created by investment-specific accumulation equation.

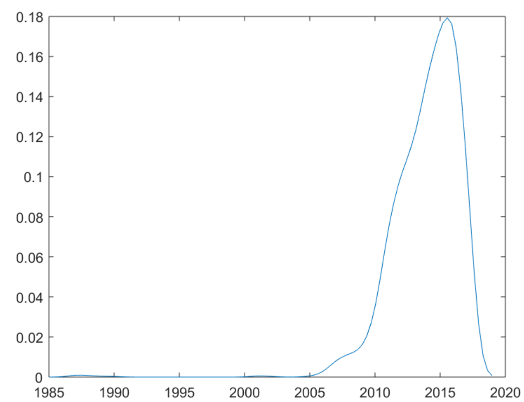
The paper is organized as follow, section 2 details the facts about capital inertia and is importance using US data. Section 3 describes the model while section 4 characterizes optimal growth path and numerical simulations. Section 5 develops possible extensions for the model and section 6 builds and solves the econometric model.

2 Capital Inertia and lifetime of energy plants

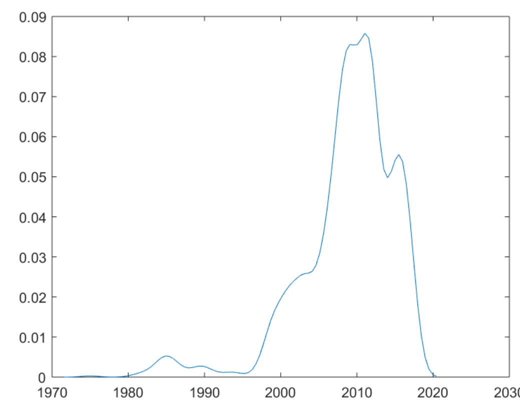
This section aims to document more consistently capital inertia from energy power plants. The motivation of this paper is to look if the lifetime of energy power plants may be an issue for our transition from fossil fuel to renewable energies. Ii is argue that when capital units are long living, with embodied technology, there are frictions in the transition process.

Power plants are using different type of energy sources with their own characteristics in term of production capacity, pollutant emission, geographical preferences,... Each energy source depends on one or several technologies, like solar energy, it can be transformed using photovoltaic (PV) or concentrating solar power (CSP) units. Technological progress is then embedded in each power plant, PV panels are formed of numerous photovoltaic modules which convert sun light into electricity using the photovoltaic effect. Performance of PV panels can be enhanced using more recent modules or coupling the system with an heat pump for example, but it is not possible to apply better modules or provide heat pump association on already engaged PV farms: technology is embedded in each generation (vintage) of panels, and is incompatible with other kind of solar energy like concentrating solar power (CSP). This special feature advocates for models with embodied technical change.

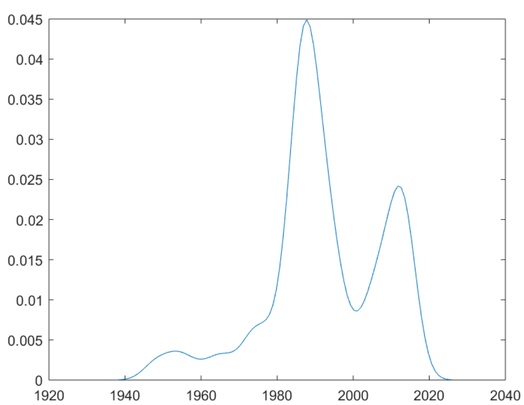
Vintage capital, or embodied technical change, has been covered by a huge literature of paper from Solow et al. (1960) to Benhabib and Rustichini (1991), Boucekkine et al. (1997) or Jovanovic (1998) among other, but this literature has not focused a lot on energy questions. To the best of my knowledge there are only Lennox and Witajewski-Baltvilks (2017) and Hartley and Medlock III (2017) that are dealing with both embodied technical change and energy questions while issue of vintage capital is even greater for energy production when power plants lifetime is taken into account. These units are long lived, to switch from one technology to another is then a long process. Using NEEDS data one can study, at least for US, lifetime of energy power plants. This dataset contains information about the commissioning year of actual power plants according to energy they use. Figures 2 and 3 show kernel density of On Line Year for renewable and fossil energy plants.



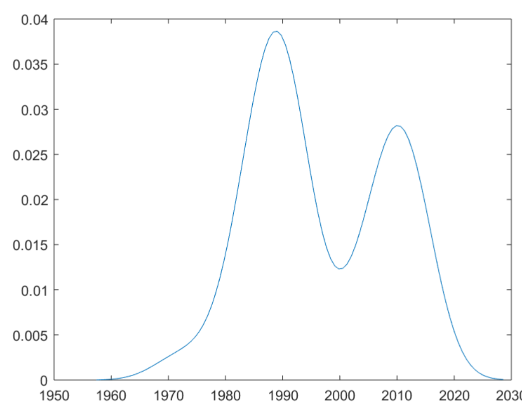
(a) Solar



(b) Wind

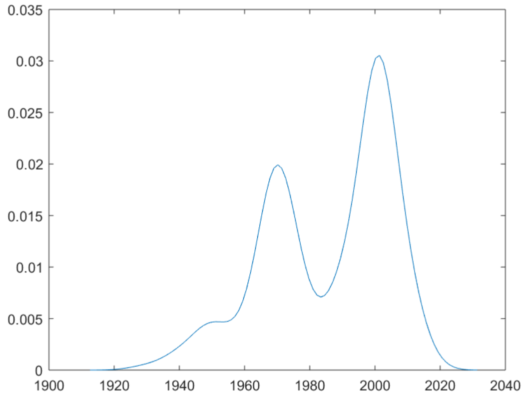


(c) Biomass

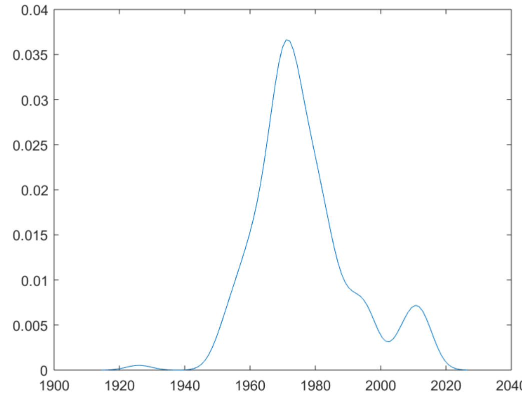


(d) Geothermal

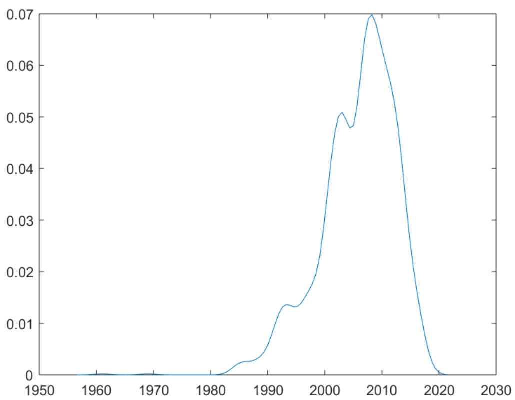
Figure 2: Renewable energy sources - On line year kernel density



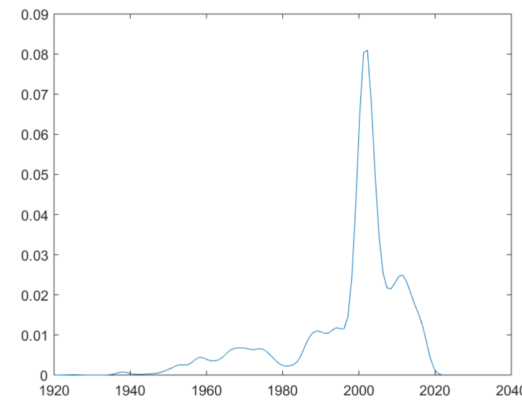
(a) Oil



(b) Coal



(c) Landfill gas



(d) Natural gas

Figure 3: Fossil energy sources - On line year kernel density

We observe that fossil energy plants are older than renewable ones, but the later are still long living. especially for geothermal and biomass, there is a non-negligible share of them which between 40 and 60 years old. For gas power plants a majority of them were built around 2000 but some are a little bit older, for coal and oil power plants a big proportions of them are aged between 40 and 60 years. In conclusion, lifetime of power plants in US can be very long, almost 100 years for some specific units, slowing down the capacity to scrap old plants to build new ones.

Second challenge faced for energy transition might be the power generation delay for renewable technologies, it is more difficult to produce the same power level than a nuclear or a gas power plant with a green one. To verify this an OLS regression using NEEDS data (table 1) is sufficient.

	Capacity (MW)
renewable	-35.22***
lifetime	0.344***
Constant	60.04***
Observations	14305
R^2	0.0291

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1: US power plants regression

Looking at the correlation between generation capacity and energy used, it appears renewable power plants are delivering less power, at 0.1% confidence interval. Additionally, lifetime of power plant has a positive impact on their capacity, meaning elder power plants are more powerful. These phenomena lie in the difficulty for municipalities to shut down biggest unit and fully replace them by renewable power plants. Opportunity cost of shutting-down a powerful power plant, no matter the energy used, is very large because it asks for consequent investments and renewable energies are not able yet to compete with these large scale producers. In conclusion, the energy transition is facing several challenges and this paper deals about one in particular: capital inertia from embodied technical change and long lifetime of power plants.

3 Theoretical model

This section is dedicated to the development of the baseline model. The set-up is similar to the literature launched by Acemoglu et al. (2012), however time is continuous and intermediate energy capital is accumulated through an investment specific accumulation equation, which is a continuous version of Krusell (1998). Structural change comes from the supply side, subsection 3.1 describes the representative household who is well behaved and completely standard regarding to the literature. Subsection 3.2 is about the productive sector, final and intermediate goods with investment specific accumulation equation. Subsection 3.3 introduces an environmental variable and a damage function *à la* Nordhaus. In section 5 new assumptions are introduced like the possibility for a government to apply abatement policies or to subsidy research in the renewable sector.

3.1 Household

Representative household maximizes an instantaneous separable logarithmic utility function by choosing her consumption and labor participation, discounted through time.

$$U = \max \quad \int_0^{\infty} (ln(c_t) - \chi ln(L_t))e^{-\rho t} dt \tag{1}$$

Where ρ is the discount factor, c_t the instantaneous consumption, L_t labor participation and χ is the elasticity of labor. This formulation ensure that household will exhibit standard behavior, preferences are homogenous and compatible with both exogenous and endogenous growth. One possibility excluded from this version of the paper is to consider environment concern into the utlity function, such possibility is let for further research.

The representative household also owns firms and decide on the amount to invest on new machines, therefore she maximizes her lifetime utility subject to the following budget constraint,

$$c(t) = Y(t) - i_c(t) - i_d(t) \tag{2}$$

Where $Y(t)$ is the production of the economy, and i_c and i_d are respectively investments in clean and dirty intermediate goods. Household decide on the amount to invest in the productive economy, in the baseline scenario there are no tax scheme and she do not interact with research outcome, it will be specified in the next section, R&D activities are exogenous.

3.2 Production sector

The final good is produced through a standard Cobb-Douglas function, without aggregate technological progress Labor and capital are used with constant return to scale. In the model the final good is also used as the numéraire.

$$\tilde{Y}(t) = L(t)^{1-\alpha} K(t)^{\alpha}$$

Where $0 < \alpha < 1$ is capital intensity of final good production, $L(t)$ is labor and $K(t)$ is an aggregate of “dirty" and “clean" capital, coming from a CES function.

$$K(t) = \left(K_c(t)^{\sigma} + K_d(t)^{\sigma}\right)^{\frac{1}{\sigma}}$$

$K_c(t)$ and $K_d(t)$ are respectively clean and dirty capital, they are accumulated using an investment specific law of motion *à la* Krusell (1998). σ is the elasticity of substitution between clean and dirty inputs.

Assumption 1 *"Clean" and "dirty" capital are substitutable inputs,* $\sigma > 0$

Proof In the framework of AABH, Papageorgiou et al. (2017) shows that "clean" and "dirty" capital units are substitutable. □

"Dirty" capital will then have an impact on the pollution stock. In the baseline model clean capital has no effect on pollution level, for the sake of simplicity. The possibility to have polluting implementation of "clean" capital is developed in section 5. Production of the economy is then,

$$\tilde{Y}_t = L_t^{1-\alpha} \left(K_{ct}^{\sigma} + K_{dt}^{\sigma}\right)^{\alpha/\sigma} \tag{3}$$

As mentioned above, "clean" and "dirty" capital are accumulated according to an investment specific equation,

$$\dot{K}_d = i_{dt} - \delta K_{dt} \tag{4}$$

$$\dot{K}_c = q_t i_{ct} - \delta K_{ct} \tag{5}$$

Where δ is the depreciation rate of capital and is the same for both type, and q is the relative efficiency of clean sector, determining the amount of capital produced with one unit of investment. The law of motion are similar to Greenwood and Jovanovic (2001), i_j is the amount invested in new machines for sector j=c,d, as seen in previous subsection investment decision are taken by the representative household, she owns intermediate firms.

Assumption 2 *The initial condition of relative efficiency in the clean sector is such that:* $q(0) < 1$

Assumption 2 creates inertia in the dirty sector, it ensures "dirty" capital to be more efficient in a first time creating the actual trade-off of energy investment: fossil sources are cheaper but more polluting on the long term. The investment specific form also creates lasting effect of capital, as seen in section 2 energy power plants are long living and not easy to scrap, decision taken at date t still have economic and polluting effect at date t+T. The existence of 2 intermediate goods introduces also the irreversibility of investment decision, "dirty" capital cannot be turned into "clean" and conversely. Technological progress is exogenous, the relative efficiency evolves at a constant rate γ such that,

$$\dot{q}_t = \gamma q_t \tag{6}$$

Imposing a restriction on q_0 allows existence of both capital at the same time. Assumption 2 ensures capital inertia of energy power plants and a trade-off between cheap and pollution free energy. Section 5 also look on the possibility to get endogenous growth in the model.

3.3 Pollution stock and damage function

Following the literature on energy transition and climate change, see Acemoglu et al. (2012), Golosov et al. (2014), Li et al. (2016), Nordhaus (2014b) or Lennox and Witajewski-Baltvilks (2017) among other, a pollution stock equation is introduced. Carbon accumulation, through use of "dirty" capital has a negative impact on the economy. Justification of this effect can be found in Graff Zivin and Neidell

(2012) or Nordhaus and Boyer (2000). The baseline model assumes “dirty” capital is the only source of new pollution, accumulated in the global carbon stock. The environment is regenerating himself at a constant rate through photosynthesis and other carbon absorption mechanism, $S(t)$ represents the carbon stock of the economy nd is described by,

$$\dot{S}(t) = -\varphi_1 S(t) + \varphi_2 K_d(t) \tag{7}$$

Where φ_1 is the natural rate of absorption and φ_2 the linear transformation rate of dirty capital into carbon. If “dirty" capital stock falls under a sufficiently low level, the environment starts to decarbonize. This assumption about pollution stock might seems too simple but allows to reproduce short term behavior of the economy, which is the objective here compare to asymptotic properties. Nevertheless, section 5 introduces a multi-level pollution stock equation, with permanent and transitory carbon emissions like it is done in Li et al. (2016) or Adao et al. (2017).

In this model, pollution stock has a negative impact on GDP, the damage function is an exponential version of Nordhaus mapping and is the same than Golosov et al. (2014), such that GDP is given by $Y(t) = (1-d(S(t)))\tilde{Y}(t)$ where $d(S(t))$ is the fraction of GDP lost because of pollution. The damage function is then characterized by $1 - d(S(t)) = exp(\theta(S(t) - \bar{S}))$ with $\bar{S}$ the pre-industrial level of pollution. Everything being considered, the expression for GDP is:

$$Y_t = L^{1-\alpha} \left(K_c^\sigma + K_d^\sigma\right)^{\alpha/\sigma} e^{-\theta(S_t - \bar{S})} \tag{8}$$

Such model aims at computing the timing of the energy transition according to 3 phenomena. First, exogenous growth in the “clean" capital sector should increase its share with certainty. Second, the dirty capital sector exhibits an higher relative efficiency in a first time, characterizing the advancement of fossil technologies and its inertia in the energy market. Third, the accumulation of dirty capital increases the damages to GDP, there will be a trade-off between growth and environment preservation. These three effects should characterize the optimal growth of the economy and evolution of the energy transition, each effect will be decomposed and analyzed into the simulation section.

4 Optimal energy transition

4.1 The planner problem

The baseline model has been fully characterized and will be solved using a social planner, who maximizes utility of the representative household.

$$\begin{aligned} \max_{l_t, i_{ct}, i_{dt}} \quad & \int_0^{+\infty} \left(ln(c_t) - \chi ln(l_t)\right) e^{-\rho t} dt \\ s.t. \quad & (3) - (7) \text{ and} \\ & c_t = y_t - i_{ct} - i_{dt} \end{aligned} \tag{9}$$

The central planner solves the Hamiltonian in current value,

$$\begin{aligned} \mathcal{H} &= ln(Y - i_c - i_d) - \chi ln(L) + P[L^{1-\alpha} \left(K_c^{\sigma} + K_d^{\sigma}\right)^{\alpha/\sigma} e^{-\theta(S_t - \bar{S})} - Y] \\ &+ P_d[i_d - \delta k_d] + P_c[q i_c - \delta k_c] + Q_t[-\varphi_1 S_t + \varphi_2 K_{dt}] \end{aligned}$$

Giving first order conditions,

$$
\begin{aligned}
Y &= \frac{\chi}{(1-\alpha)P} && (10)\\
P &= P_d = qP_c = \frac{1}{Y - i_c - i_d} && (11)
\end{aligned}
$$

Labor variable $L(t)$ has disappear of first order conditions and is now implicit, equation (11) shows shadow price of production and dirty capital are the same and they equal the product $P_c q$, at t_0 $q(0) < 1$ means $P_c(0) > P_d(0)$ validating actual empirical facts of cheapest fossil energy compared to renewable. Equation (10) show the direct relationship between the shadow price of the final good and production itself in a straightforward equation.

Dynamic equations of state variables are the following,

$$
\begin{aligned}
\frac{\dot{P}_c}{P_c} &= \rho + \delta - \alpha q K_c^{\sigma-1} \frac{Y}{K_c^{\sigma} + K_d^{\sigma}}\\
\frac{\dot{P}_d}{P_d} &= \rho + \delta - \alpha K_d^{\sigma-1} \frac{Y}{K_c^{\sigma} + K_d^{\sigma}} - \varphi_2 \frac{Q}{P_d}\\
\frac{\dot{Q}}{Q} &= \rho + \varphi_1 + \frac{\theta Y P_d}{Q}
\end{aligned}
$$

As expected evolution of both shadow prices are similar but differ in the presence of q for the clean sector and the term $-\varphi_2 \frac{Q}{P_d}$, the decentralized equilibrium will show that the latter is equivalent to the carbon tax. Further in the paper we will see $Q < 0$, ensuring the dynamic equation for the shadow price of “dirty" capital to be bigger with carbon emissions than without.

In order to go further in the analysis the ratio κ is introduced, such that

$$\kappa \equiv \frac{K_c^\sigma}{K_c^\sigma + K_d^\sigma}$$

This ratio will be the proxy for energy transition, the closer from 1 κ is, the higher the share of "clean" energy. Calibration of the model will match $\kappa(0)$ with renewable share in energy mix in 2010.

This proxy allows to rewrite dynamic equation of both P_d and P_c,

$$\frac{\dot{P}_c}{P_c} = \rho + \delta - \alpha q \kappa \frac{Y}{K_c} \tag{12}$$

$$\frac{\dot{P}_d}{P_d} = \rho + \delta - \alpha(1-\kappa)\frac{Y}{K_d} - \varphi_2 \frac{Q}{P_d} \tag{13}$$

Shadow price of both type of capital depends on clean energy ratio. As one might expect, the closer from 1 κ is, the higher the growth difference will be. Next section will characterize the steady growth path and transitional patterns of the model, κ will be the central element of the analysis as it drives energy transition and all other variables.

4.2 Steady growth path

This section aims at computing the asymptotic behavior of the model, to derive steady growth rate is the first step before characterizing the transitional path of the economy. Model behavior is in line with some papers of structural changes like Acemoglu and Guerrieri (2008) or Genna et al. (2019), economic transition and structural change occurs along the growth path of other variables. In this paper, the energy transition takes place along with constant growth of prices, labor and

GDP. As mentioned above, κ is the proxy for energy transition, therefore the final goal of this section is to derive the asymptotic and transitional growth rate of the "clean" capital ratio. By differentiating its definition one obtains,

$$\frac{\dot{\kappa}}{\kappa} = \sigma(1-\kappa)(g_{K_c} - g_{K_d})$$

In the following the term g_x refers to growth rate of variable x. κ growth rate depends on its own value and on the difference between "clean" and "dirty" capital growth. If "clean" capital grows faster (slower) than "dirty" one, κ is increasing (decreasing), and there are no inconsistent behavior because if κ is equal to one, the growth rate is equal to 0. To derive the complete characterization of κ's growth rate one uses the following statement: growth rate of shadow prices are assumed to be constant, such that $\dot{g_{P_c}} = \dot{g_{P_d}} = 0$. Using this property on equations (12) and (13) gives,

$$\begin{aligned} g_{K_c} &= \gamma + \frac{\dot{\kappa}}{\kappa} + g_Y && (14) \\ g_{K_d} &= g_Y - \frac{\dot{\kappa}}{\kappa}\frac{\kappa}{1-\kappa} + \frac{\varphi_2 Q K_d}{\alpha(1-\kappa)\chi}(g_Q - g_{P_d}) && (15) \end{aligned}$$

g_{K_d} can be simplified by differentiating (10) and using the following proposition,

Proposition 1 *The shadow price of pollution, Q, is always at its steady-state value and is negative.*

Proof. Using (10) and (11), the shadow price of pollution growth can be rewritten as $\frac{\dot{Q}}{Q} = \rho + \varphi_1 + \frac{\theta\chi}{(1-\alpha)Q}$, it depends on the value of Q and on model's parameter. As for every variable of the model, asymptotically this growth rate should be constant, $\dot{g}_Q = 0 \Leftrightarrow g_Q = 0$. One is able to derive $Q^* = -\frac{\theta\chi}{(1-\alpha)(\rho+\varphi_1)}$, the steady state value of the shadow price of pollution, and this value is negative. In the long-run, Q must converge to its steady-state level, however it appears that if Q deviates from this value, its trend is explosive and cannot converge. The conclusion is there exist only one value for the shadow price of pollution leading to a stable steady-state, Q is a jump variable and is always at its steady-state level. □

At first, proposition 1 seems counter-intuitive, one should expects the constraint on pollution stock to variate with the level of pollution, to capture the constraint induced, by definition, by a shadow price. However, each new unit of pollutant emitted in the atmosphere has the same impact on this economy because of (10) and (11), the marginal impact of pollution is expected to co-move with the level of GDP($\frac{\partial Y(t)}{\partial S(t)} = -\theta Y(t)$), but the equivalence between $Y(t)$ and $P(t)$ is cutting this co-movement. The increasing impact of one unit of pollution is compensate by a drop in prices, such that the constraint is always the same, $Q(t)$ is then constant.

Equation (15) can be rewritten using proposition 1 and differentiating (10)

$$g_{K_d} = g_Y\left(1 - \frac{\varphi_2\theta K_d}{\alpha(1-\kappa)(1-\alpha)(\rho+\varphi_1)}\right) - \frac{\dot{\kappa}}{\kappa}\frac{\kappa}{1-\kappa} \tag{15*}$$

Output growth, dirty capital level and evolution of the clean share are the three variables defining “dirty" capital growth rate. Because of equation (10) and the assumption made on g_{P_d} the output growth rate is constant, then the only variables at play are K_d and κ. Combining this result with equation (14) in κ's growth rate and rearranging it characterizes the rhythm of energy transition,

$$\frac{\dot{\kappa}}{\kappa} = \frac{\sigma}{1-\sigma}\gamma(1-\kappa) + \frac{\sigma\varphi_2\theta K_d}{(1-\sigma)\alpha(\rho+\varphi_1)}g_Y \tag{16}$$

This expression can be divided in 2 parts, $\frac{\sigma}{1-\sigma}\gamma(1-\kappa)$ represents the transition implied by technology, it relies on γ, the efficiency differential between the two intermediates, and on the imperfect substitution reflected by the parameter σ. Without any environmental damages, growth rate of κ is only defined by this first part. $\frac{\sigma\varphi_2\theta K_d}{(1-\sigma)\alpha(\rho+\varphi_1)}g_Y$ represents the second part in which transition is implied by damages from pollution. The term $\theta\varphi_2 K_d$ represents the damages induced by "dirty" capital on GDP, the higher it is the faster transition is, Pollution has an acceleration effect on the energy transition. There are also 2 straightforward remarks, i) it appears that the bigger κ is, the slower the transition, which is due to scale effect ; ii) $\frac{\sigma}{1-\sigma}$ is present in each part, it represents the substituability effect implied by the CES function for capital aggregation.

κ growth rate depends on $(1-\kappa)$ and on K_d, it is straightforward that asymptotically κ will tend to 1. Growth rate of the clean ratio proxy is always larger than 0 and κ cannot be higher than 1 by construction. We have $\kappa^* = 1$ as asymptotic condition. When $t \to \infty$ the clean technology will be the dominant on the energy market, it does not mean that dirty technologies will totally disappear but its level will be non significant in the energy mix. Their existence will be discussed below and is of first importance when it comes to the energy sector. Structural change occurs, production becomes relatively more green but if we continue to buy new dirty inputs, it will not reduce greenhouse gas emissions.

Using the clean capital ratio result, I am able to derive the other growth rates of the economy. When $t \to \infty$ economy tends toward the Non Balanced Growth Path (NBGP) detailed in following theorem.

Theorem 1 *Under assumptions 1 and 2, the set of Non Balanced Growth Rates (NBGR) of this model are as follow:*

$$\begin{aligned} g_Y &= \frac{\alpha\gamma}{1-\alpha} \quad ; \qquad g_{K_c} = \gamma + g_Y \quad ; \qquad g_{K_d} = g_Y - \frac{\sigma\gamma}{1-\sigma} \\ g_{P_c} &= -g_Y - \gamma \quad ; \qquad g_{P_d} = -g_Y \quad ; \qquad g_{i_c} = g_Y \\ g_{i_d} &= g_{K_d} \end{aligned}$$

Proof: see Appendix (in construction) □

Theorem 1 shows the set of non-balanced growth rates, when energy transition has been completed $\kappa \to 1$, some features can be derived from the asymptotic behavior of the model. In the long run, “dirty" capital might still be increasing even if its share becomes non significant, if $\alpha > \sigma$ one obtains $g_{K_d} > 0$ which has no consequences for the energy transition but has some disastrous effect on environment quality, through the carbon stock. In the illustrative calibration in section **4.4**, $\alpha = 0.4$ and $\sigma = 0.44$, it coincides with an asymptotically decreasing growth of “dirty" capital. Nevertheless, it seems difficult to imagine an infinitely increasing fossil energy due to resources limitations, however this paper omits intentionally to include a resource stock because it is not the problematic here. Jaffe et al. (2011) survey about world oil reserves lets one think resource constraint will not be the major problematic of tomorrow. The idea of a negligible Hotelling effect in the short run is also present in Hart and Spiro (2011), in which they argue that scarcity rents do not dominate prices of fossil resources . Therefore, the main limitation of the non-balanced growth path is the lack of an Hotelling rule, but in the short and middle-run this absence seems less problematic.

4.3 Decentralized equilibrium

Previous section aimed at solving the social planner problem, correcting for pollution damages from use of a “dirty" technology. To solve the decentralized equilibrium will help at characterizing the optimal tax rate required for this economy to reach the optimal growth path. In this model there is only one externality, pollution from use of dirty capital, which needs one instrument to be corrected, the tax rate. The final good is used as a numéraire, its price is normalized to 1.

4.3.1 Household

The representative household owns the intermediate firms and lend his work to the final good producer, he exhibits the same utility function than for the social planner but his budget constraint is: $c(t) = L(t)w(t) + r_c(t)K_c(t) + r_d(t)K_d(t)$. The representative household maximizes its utility with respect to consumption, labor and capital investment,

$$\begin{aligned} U = \max_{L(t),c(t),i_c(t),i_d(t)} \quad & \int_0^{+\infty} (ln(c_t) - \chi ln(L_t))\, e^{-\rho t} dt \\ s.t. \quad & Y(t) = L(t)w(t) + r_c(t)K_c(t) + r_d(t)K_d(t) \\ & c(t) = Y(t) - i_c(t) - i_d(t) \\ & \dot{K_c} = q(t)i_c(t) - \delta K_c(t) \\ & \dot{K_d} = i_d(t) - \delta K_d(t) \end{aligned} \tag{17}$$

First order conditions state:

$$\frac{\chi}{L(t)} = w(t)P(t) \qquad ; \qquad P_c(t)q(t) = P_d(t) = P(t)$$

Where $P(t), P_d, P_c$ are the multiplier of respectively the budget constraint, the "dirty" capital accumulation and the "clean" capital accumulation. FOC conditions in the decentralized equilibrium are similar to social planner.

Dynamic equations are also similar to what one can found in the centralized equilibrium,

$$\begin{aligned} \frac{\dot{P}_c}{P_c} &= \rho + \delta - q r_c \\ \frac{\dot{P}_d}{P_d} &= \rho + \delta - r_d \end{aligned}$$

Household do not take into account damages from pollution to the global production, the externality will need to be corrected by a tax rate as it will be shown below. The main difference here is for dynamic equation for the shadow price of the "dirty" capital because in the suboptimal equilibrium adverse pollution effects are not taken into account.

4.3.2 Final good

The final good firm maximizes its profit, it sells its production, buy work of the household and rent capital units,

$$\max_{L(t),K_c(t),K_d(t)} \pi = L^{1-\alpha}\left(K_c^{\sigma} + K_d^{\sigma}\right)^{\alpha/\sigma} - w(t)L(t) - r_c(t)K_c(t) - r_d K_d(t) \tag{18}$$

Deriving first order conditions leads to,

$$
\begin{aligned}
w(t) &= (1-\alpha)\frac{Y(t)}{L(t)} && (19)\\
r_c(t) &= \alpha\kappa(t)\frac{Y(t)}{K_c(t)} && (20)\\
r_d(t) &= \alpha(1-\kappa(t))\frac{Y(t)}{K_d(t)} && (21)
\end{aligned}
$$

$w(t)$, $r_c(t)$ and $r_d(t)$ represent, respectively, wages, rental price of “clean" capital and rental price of “dirty" capital. The next section will look at the optimal tax rate needed to coincide with the socail planner equilibrium and correct for the externality.

4.3.3 Optimal tax rate

Pollution accumulation due to use of “dirty" capital destroys a share, $d(t)$, of the production such that $d(t) = 1 - e^{-\theta(S(t)-\bar{S})}$ is the damage function. In order to correct for this externality a government needs to introduce a tax on “dirty" capital for decentralized equilibrium to coincide with central planner scheme.

The tax will apply on the rental price of dirty capital, a slower rate of return for each “dirty" unit slow-down the investment in this kind of capital. The government modifies the household maximization (17) such that the budget constraint becomes

$$Y(t) = L(t)w(t) + r_c(t)K_c(t) + (r_d(t) - \tau(t))K_d(t)$$

Using (21) and solving the new maximization for dynamic equation it appears,

$$\frac{\dot{P}_d}{P_d} = \rho + \delta - \alpha(1-\kappa(t))\frac{Y(t)}{K_d(t)} + \tau(t)$$

Comparing to the central planner results for the dynamic equation of the shadow price of "dirty" capital it appears clearly that the tax rate is such that,

$$\tau(t) = \frac{-\varphi_2 Q(t)}{P_d(t)}$$

Proposition 1 states $Q(t)$ is constant and negative, and it still hold in the decentralized equilibrium making the tax rate dependent of the shadow price of "dirty" capital. The lower $P_d(t)$, the higher the tax rate. $\tau(t)$ is inversely proportional to $P_d(t)$, and so is proportional to $Y(t)$ because of (10) and (11). Using this and proposition 1 the expression for the tax rate can be rewritten,

$$\tau(t) = \frac{\varphi_2 \theta}{\rho + \varphi_1} Y(t) \tag{22}$$

To reach the optimal growth rate asks for an increasing tax rate on "dirty" capital, proportional to production in the economy, such a result is in line with recent literature on the topic of taxation of fossil energies, like Golosov et al. (2014), Lennox and Witajewski-Baltvilks (2017) , Li et al. (2016) or Adao et al. (2017). The taxation weight has to be bigger as the price of "dirty" input is decreasing (see theorem 1), due to capital deepening, to maintain the increasing attractivity of the "clean" backstop. The next section calibrates the model using US data to provide a numerical analysis of the model.

4.4 Calibration

Compare to a Ramsey model, this paper differs in its double capital market with investment-specific accumulation equations and the presence of a damage function, linked to the emissions of pollutants. The model is then characterized by the parameters summarized in table 1 and by 3 initial conditions, $Y(0)$, $K_c(0)$ and $K_d(0)$. Technological progress is computed using IEA technology R&D budget, delivering detailed budget for each type of source. γ is a proxy of technology differential in the energy sector, its value is extracted computing growth differential between fossil and renewable technology in public budget. On the considered period (1980-2015), R&D budget for “clean" energy grows 2.5% faster than for “dirty" energy sources, growth differential is then calibrated such as $\gamma = 0.025$. The value for α, the labor share is given directly by the bureau of labor and statistics, its average on the considered period is such as $\alpha = 0.4$. Depreciation rate of capital and discount factor, respectively δ and ρ are calibrated following Barro and Sala-i Martin (2004), the values $\delta = 0.05$ and $\rho = 0.02$ are widely use in macroeconomic literature and calibration of Ramsey models, this paper does not innovate in this regard. The value for the elasticity of substitution is chosen following Papageorgiou et al. (2017), they estimate its value in AABH framework which is close to the one in this paper, regarding to their computations the elasticity of substitution is calibrated as $\sigma = 0.44$. The frisch labor supply, χ, is calibrated using Peterman (2016) in which the author tries to redefine the macroeconomic value of this parameter, leading to $\chi = 3$. And lastly, the 3 parameters associated to environment are calibrated using the last IAM model used by Nordhaus, such that $\varphi_1 = 0.1$, $\varphi_2 = 0.0228$ and $\theta = 0.02$.

This part tries to calibrate initial values for GDP, “clean" and “dirty" capital in 2010. At this date, according to world bank data, GDP per capita was 48 kUS$, using this and equation (2) one can compute the level of “clean" and “dirty" capital. According to “US primary energy production by major sources", in 2010 renewable

Parameter	Value	Data
γ	0.025	IEA technology R&D budget (2000-2018)
α	0.4	Bureau of Labor statistics
σ	0.33	Papageorgiou et al. (2017)
δ	0.05	Barro and Sala-i Martin (2004)
ρ	0.02	Barro and Sala-i Martin (2004)
χ	3	Peterman (2016)
φ_1	0.1	*Nordhaus (2014a)*
φ_2	0.0228	*Nordhaus (2014a)*
θ	0.02	*Nordhaus (2014a)*

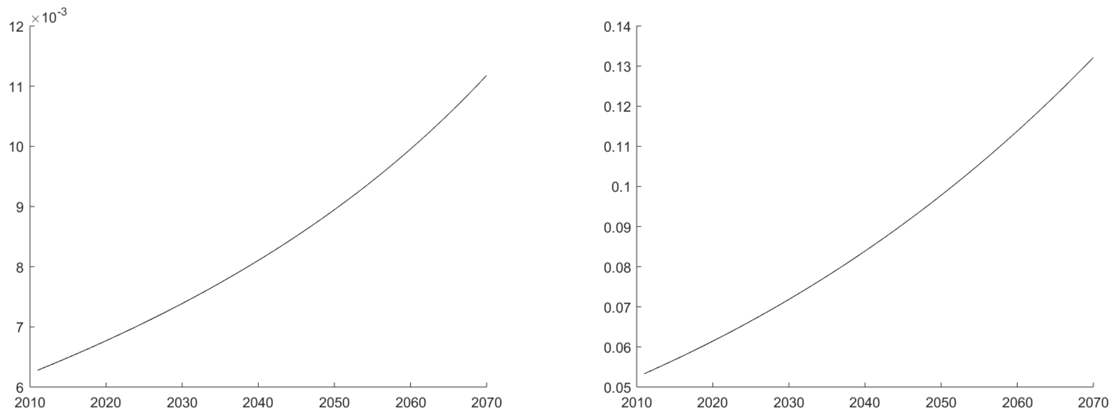
Table 2: Parameters value

energies (without hydroelectricity, which is particular in the energy mix) represents 12.5% of the energy mix, we then solve $y(2010) = \left((\frac{0.125}{0.875}K_d(0))^\sigma + K_d(0)^\sigma\right)^{\alpha/\sigma}$. And then obtain $K_d(0) = 7.24$ and $K_c(0) = 1.03$. We can compute value of $\kappa(0)$ to obtain $\kappa(0) = 0.30$. Parameter values and initial conditions have been calibrated, the next part can now deal with the model simulations.

4.5 Simulations

This sections aims at providing some useful insight on the transitional pattern of this model, the asymptotic behavior of the model have already been described by theorem 1 but nothing was really clear about transition from initial conditions to Non-balanced growth path. Without any doubt the economy will switch from a fossil energy dominance to a renewable world, but in the presence of climate change the major problem is how long this switching will take. This section will first deal with the optimal tax rate of the economy, from whom the transitional timing will come from. And lastly, this model allows to derive the evolution of both “clean" and “dirty" capital level in the economy. A rising level of “dirty" capital would be problematic with respect to pollution emissions and threats of climate change.

Using equation (22), one is able to simulate the value of the tax rate, and it relative weight on the “dirty" capital rent. The latter is used to confront the real impact of the taxation instead of its absolute level. The tax rate diminishes the rent of “dirty" capital, simulating $\frac{\tau(t)}{r_d(t)}$ shows the real impact of taxation on “dirty" capital owners (figure 5).



(a) Tax rate

(b) Weight of the tax rate

Figure 4: Optimal taxation

The tax rate is increasing in absolute and relative terms. To be located on the optimal path, the tax rate should have been set around 5% of the “dirty" capital rent in 2010, 6% in 2020 and reach 14% in 2070 if one wants to follow the optimal path. Such tax schedule is used if one wants to maximize growth in presence of environmental externality, other objectives might be considered, as minimizing the pollution without giving up to much on growth, it would end-up with a totality different tax schedule.

Next simulation (figure 6) displays both the proxy ($\kappa = \frac{K_c^\sigma}{K_c^\sigma + K_d^\sigma}$) and the real share ($\frac{K_c}{K_c+K_d}$) of clean capital into the energy mix. It takes more than 50 years to reach 50% of clean technology, the process is slow as it takes 20 years to exhibits a 10% increase in the the early years. The rhythm of energy transition highly depends on the value of γ, the technology differential, alternative scenarios with different values for γ are provided in the appendix. The inertia implied by investment-specific accumulation equation in the two intermediate capital market tends to delay energy transition. The lasting effect of energy capital units is also present in the transitional process, stopping the possibility of a smooth and fast transition to renewable technologies.

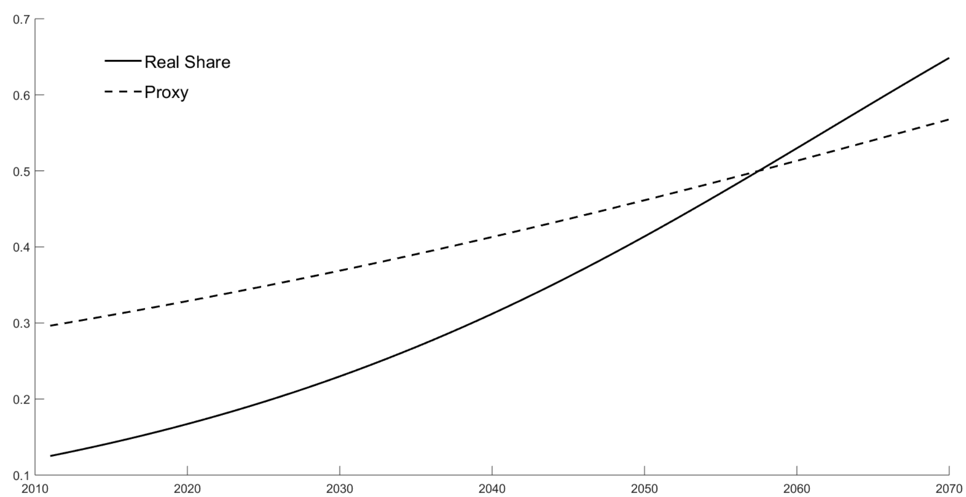


Figure 5: Share of clean technology across time

Figure 6 displays energy transition in a relative way, but is quiet about the impact on environment and pollution stock. The next plot (figure 7) is about absolute level of both “clean" and “dirty" capital and their evolution through time. An higher level of “dirty" capital is linked to an increase of pollutant emissions in the atmosphere

According to this simulated model, the level of dirty capital will continue to grow during the century to reach its maximum around 2060, its level will then slowly decrease. Comparing to 2010 level, the stock of “dirty" capital will be aug-

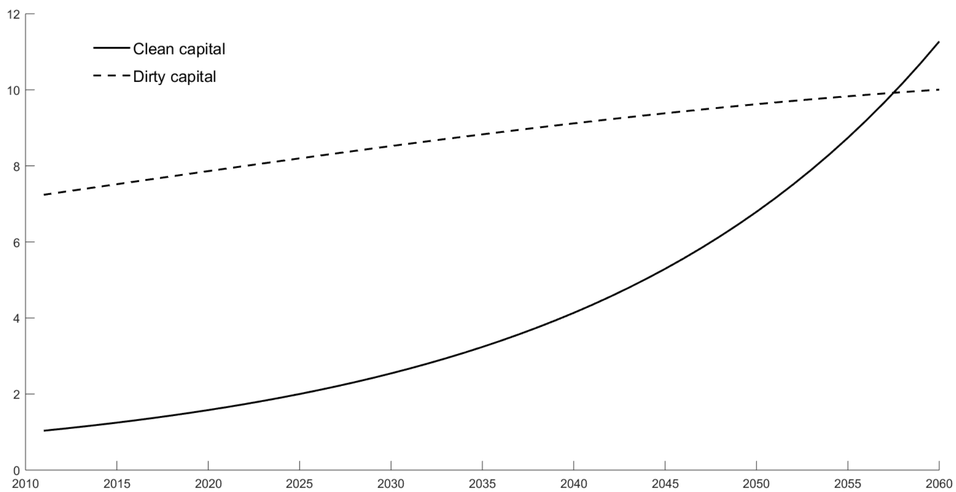


Figure 6: Level of clean and dirty capital in absolute value

mented by nearly 50%, emissions will grow at the same time. The second thing that one should notice is the “snowball" effect of the clean capital. The beginning of the period is characterized by a very low level of clean technology that increases exponentially, higher level of clean capital means higher growth of its level (see equation 16), additionally to the exogenously growing technology differential that makes “clean" sector more and more attractive.

The quantitative result that can be kept from these simulation is the very slow transitional process. Even in this simple framework with completely exogenous growth and expected outcome, there is an inertia effect from investment specific capital accumulation and initial conditions, calibrated on actual and historical data. Going back to the carbon lock-in argument of Unruh (2000), the pessimistic view would say that in practice we cannot escape carbon lock-in easily due to capital inertia. Looking at the actual discussion around carbon taxation and the work by Clements et al. (2013) and Coady et al. (2015), the economy is not so close for such a taxation scheme. However, the optimistic view would say this paper do not capture the complete inertia of the energy sector, nor political decisions, nor consumers behavior with respect to climate change, nor uncertainties linked to the energy sector. Also, there are a couple of parameters of first importance in this model, like technology differential and damages to GDP, a government would be able to affect and change their value. Increasing public research, carbon capture storage or geoengineering technology may have significant consequences on the short and middle-run (on this topic, Acemoglu and Rafey (2018) shows that relying on geoengineering technology is suboptimal). But this is out of the scope of this paper, for now it just tries to highlight the importance of lasting effect on energy capital market. Policy recommendations are the purpose of the next sections, using time series data this paper will try to exhibit correlation and causality on US energy market, using VAR and Bayesian VAR approach.

5 Model extensions

Following European commission green deal, the final objective would be to speed-up energy transition, enhance environment quality without giving-up on too much growth. The work done before will be used as a benchmark, in this section the objective is to enhance the model looking at policy measures that might achieve European goal. Different strategies will be analyzed in the upcoming pages like possibilities for abatement policies, endogenous technical change financed by households, pollution from clean technology installation or a multi-level pollution function with permanent and transitory pollution stocks.

5.1 Introduction of abatement policy

One can imagine a situation in which a government would raise a tax on households to applies an abatement policy that helps atmosphere to recover faster than previously. Household budget constraint is now :

$$Y(t) = c(t) + i_c(t) + i_d(t) + T(t)$$

With $T(t)$ the tax applied on household. This tax would served to enhanced atmosphere recovery, such that

$$\dot{S} = -\varphi_1(1 + T(t)) + \varphi_2 K_d(t)$$

Such new hypothesis has a double effect on GDP and "dirty" capital growth rates:

$$\begin{aligned} g_{K_d} &= g_Y \left(1 - \frac{\varphi_2 \theta K_d}{\alpha(1-\kappa)(1-\alpha)(\rho+\varphi_1)}\right) - \frac{\dot{\kappa}}{\kappa}\frac{\kappa}{1-\kappa} \\ \text{vs} \quad g_{K_d} &= g_Y \left(1 - \frac{\varphi_2 \theta K_d}{\alpha(1-\kappa)(1-\alpha)(\rho+\varphi_1(1+T))}\right) - \frac{\dot{\kappa}}{\kappa}\frac{\kappa}{1-\kappa} \end{aligned}$$

$$\begin{aligned} g_Y &= \alpha g_{K_c}\kappa + \alpha g_{K_d}(1-\kappa) + \theta\varphi_1 - \varphi_2 K_d \\ \text{vs} \quad g_Y &= \alpha g_{K_c}\kappa + \alpha g_{K_d}(1-\kappa) + \theta\varphi_1(1+T) - \varphi_2 K_d \end{aligned}$$

The overall effect of $T(t)$ is uncertain, it looks like it decreases g_{K_d} and increases g_Y but these 2 are interconnected, an increase of g_Y increases g_{K_d} and a decrease of g_{K_d} decreases g_Y. Numerical applications are needed to identify eventual threshold effects and direction of the change, direction of the change affects transition speed of this model because $\frac{\dot{\kappa}}{\kappa} = \sigma(1-\kappa)(g_{K_c} - g_{K_d})$

However such policy should have a positive impact on environmental quality because it enhances environment recovery, but if it increases dirty capital gross rate the effect on environment quality might be negligible. It needs to be solved entirely to find optimal level for $T(t)$ and then simulated to compare with the baseline model.

5.2 Tax to finance research

The starting idea is the same, set up household taxation to help financing research.

$$Y(t) = c(t) + i_c(t) + i_d(t) + T(t)$$

This tax serves then to finance research, technological progress becomes endogenous. A simple formulation would be to write technological progress as:

$$\dot{q} = \gamma(1 + T(t))q(t)$$

The effect of such policy is less ambiguous than abatement policy, it clearly speed-up energy transition because we obtain:

$$g_{K_c} = \gamma(1 + T(t)) + \frac{\dot{\kappa}}{\kappa} + g_Y$$

Impact on g_{K_c} will increase transition speed and should logically decrease growth rate of “dirty" capital. But once again, the effect is uncertain because such policy should have a positive impact on GDP which then impacts positively "dirty" capital growth rate.

Numerical analysis is needed to compare these different policies and maybe make a welfare analysis.

5.3 Pollution of clean technology installation

Compare to "dirty" technologies the functioning of "clean" ones do not produce any pollution, however plant production (and transportation, like PV panel) does. If we consider possible pollutant emission from clean technology it would come from their installation rather than from their utilization. We then set-up the following pollution accumulation equation:

$$\dot{S} = -\varphi_1 S(t) + \varphi_2 K_d(t) + \varphi_3 i_c(t)$$

Changes operating by this assumption are deeper than the previous ones, introducing such a pattern impact the FOC of prices such that

$$qP_c = \frac{1}{Y - i_c - i_d} - \varphi_3 Q \qquad (10^*)$$

In that situation $P_d \neq qP_c$ and every other conditions are changing, the model needs to be completely revised with this new assumption. The expected impact is a greater price for clean technology an a lower level of GDP, transition speed might be slow down, the negative impact of both technology should increase the time during which both technology are used simultaneously. However, I do not have enough results yet to conclude on this part.

5.4 Multi level pollution equation

Literature on climate change and directed technical change uses a more advanced pollution function like in Adao et al. (2017) or Li et al. (2016), the idea is to divide pollution in two parts: one transitory and one permanent. Analysis of such model is more complex and there is no specification nor results for the moment.

6 US energy market, an econometric approach

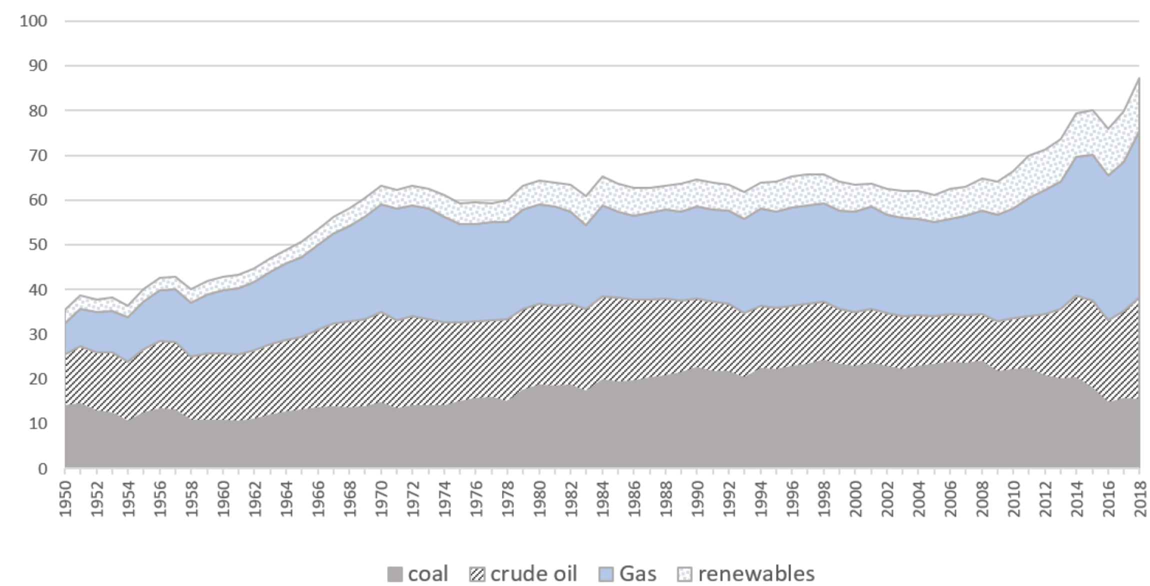
The theoretical part of the paper have paved the way for some policy recommendations in case of capital inertia, this section aims at providing an empirical analysis for the efficiency of such policies in the case of US, using time series approach. Two testable policies are investment in research and development, and increase of pollution abatement. The US economy has been chosen for its variety of energy production, their energy mix is equivalent to what one is able to find in the world economy, and data availability allows for robust econometric estimations. First part will describes the data, secondly model selection will be discussed and a last part shows the results.

6.1 The data

The dataset used is unique and developed for the need of this analysis. It uses data on energy production, GDP, technological progress through patent data, real and foreign investment.

6.1.1 Energy production

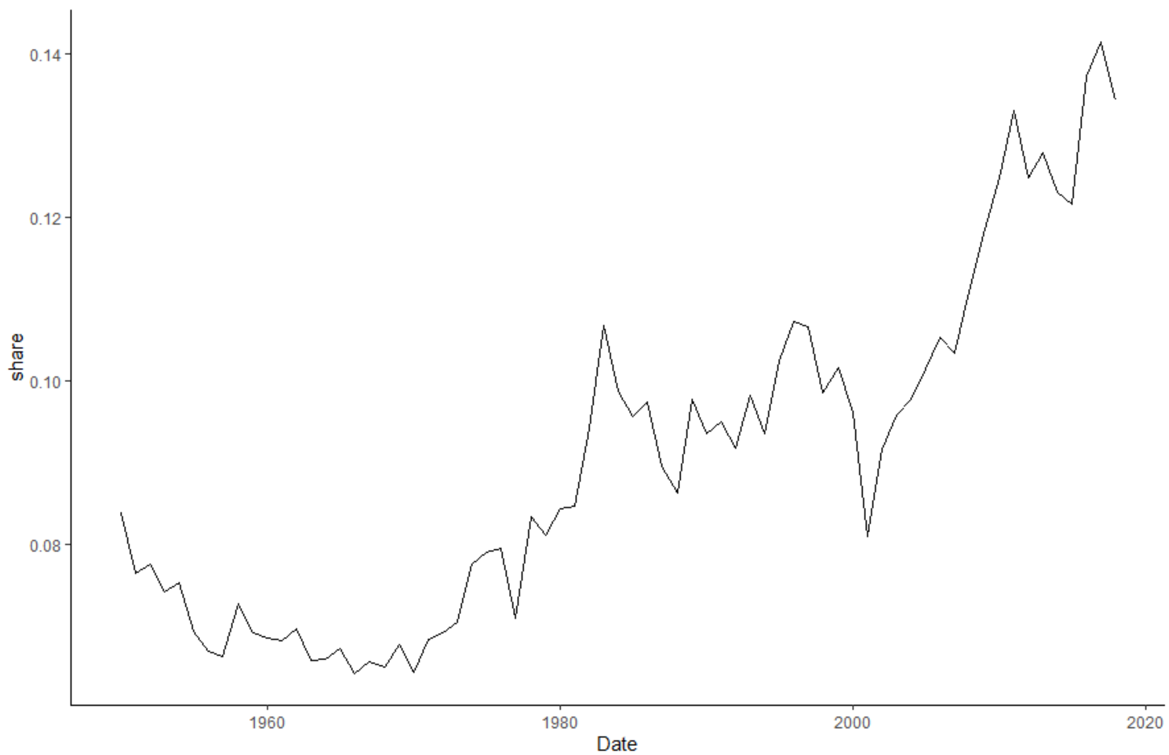
The US are producing energy from all sources, production data are extracted from “US primary energy production by major sources" dataset, provided by US Energy information Administration. Energy production from Hydroelectricity and nuclear power plants have been drop, like for the simulation part. Overview for US energy mix is represented on figure 8, data are expressed in quadrillion of British thermal units, it allows to normalize data on the same unit. As one can see, coal and oil production are increasing from 1950 to 2000, and then are slightly decreasing. Gas is always increasing and especially in the last 20 years, and renewable energy is slowly increasing. These data are available on an annual or a monthly basis, the latter are used to develop “dirty" and “clean" production data on a quarterly basis. The choice to abandon annual data is motivated by the lack of long term data, the maximum observations one can obtain with this kind of data is around 60, which is not completely satisfactory in the view of econometric robustness. Monthly data are avoided because energy supply market is characterized by inertia, a monthly basis is too short for change to appear and be revealed efficiently. Working with quarterly data seems to be the best option regarding these two considerations.



source: US energy information administration

Figure 7: US primary energy production by major sources (in quadrillion Btu)

US energy production is increasing from 1950 to 1970 and is constant until 2009, since then the production is growing, especially for gas. It coincides with the “shale revolution” initiated in US around 2008, making the country one of the



source: US energy information administration

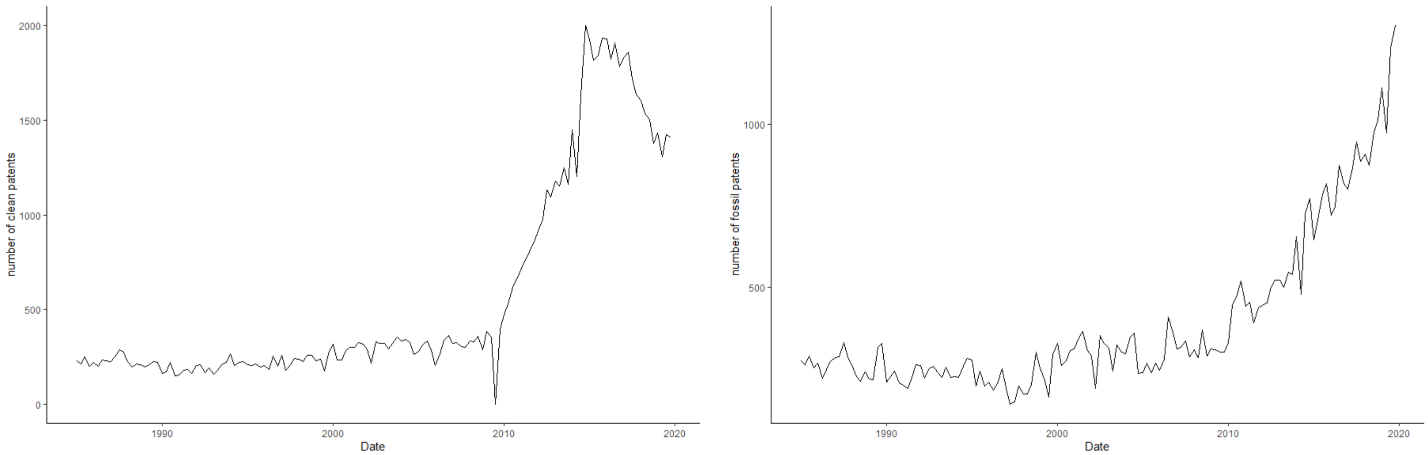
Figure 8: Share of clean energy

biggest producer of oil and gas in the world. Coal, crude oil and gas are assembly together to form the “dirty" sector, while renewable energies (without hydroelectricity) are considered as the “clean" sector. Such classification allows to compute the equivalent of κ in the data. Relative importance of renewable energies in US energy mix is represented in figure 8.

This share is decreasing in the 50’s and 60’s and globally increasing from 1970 to nowadays, with some drops of this ratio. Closing and opening of large scale power plants have significant impact on this ratio. These different values refer to K_c and K_d in the theoretical model, this dataset is use to reproduce these 2 variables.

6.1.2 Technological progress

Usually technological progress is derived from TFP growth, some data are available on this topic. However, there is no TFP index for “clean" and “dirty" capital or sectors. Based on Acemoglu et al. (2016), this paper uses patent data to proxy the level of technological progress in US economy, this approach aims at capturing research output instead of input (research and development budget). USPTO data are then used to create two technology variables: clean and dirty research. it is constructed looking at the CPC (Cooperative Patent Classification) system for US patents. Clean technology variable is build using 5 classes of patent: Y02E technologies or applications for mitigation or adaptation against climate change ; H02S generation of electric power by conversion of infra-red radiation, visible light or ultraviolet light, e.g. using photovoltaic modules ; F03D wind motors ; F24S solar heat collectors, solar heat systems ; F24T geothermal collector, geothermal systems. The number of patents granted each year in all these categories are used as a proxy of clean technological progress. For the dirty technology variable the methodology is the same, categories used are: E21B earth drilling e.g. deep drilling, obtaining oil, gas, water, soluble or meltable materials or a slurry of minerals from wells ; F02C gas-turbine plants, air intake for jet-propulsion plants, controlling fuel supply in air breathing jet-propulsion plants ; F17C vessels for containing or storing compressed, liquefied or solidified gases, fixed-capacity gas-holders, filling vessels with, or discharging from vessels, compressed, liquefied or solidified gases ; F23D Burners ; F23C methods or apparatus for combustion using fluid fuel or solid fuels suspended in (a carrier gas or) air. Evolution of the number of patents for clean and dirty categories can be seen in figure 9.



(a) Renewable (b) Fossil

Figure 9: Evolution of research

First thing to note is the sharp increase of the number of patents starting around 2009, it coincides with the production jump observe previously. This pattern may have two explanations, first one is the peak in US energy R&D budget in 2009 following 2008 crisis. According to "IEA Energy Technology RD&D Budgets", US administration spent 5.14 billions USD in 2008 for R&D in the energy sector, against 11.41 billions USD in 2009. Delay between finacement of a project and its realization being quite long, this government spending shock can explain this increasing number of patents from 2010. Second explanation lies in the "US shale revolution" which is considered to have started around 2009, transforming the US fossil energies productive apparel. This argument is in favor of an impulse from private sector and private research. To work with patent data allows to get information from both the public and the private sector, while expenses in R&D from firms is very difficult to estimate.

6.1.3 GDP, real investment and oil price

The model is augmented with data about GDP, as it is present in the equation (16). Robustness checks are provided by including data about oil prices, it is often assume oil prices are driving world energy prices. Data for gross domestic production are coming from the Bureau of Economic Analysis and are seasonally adjusted, this dataset captures business cycle movements. Oil prices data are extracted from “EIA Short-Term Energy Outlook", using real imported crude oil prices. Inclusion of an investment variable is also considered as a robustness check, because installation of energy power plants can be considered as an investment. Data come from federal reserve of economic data.

Finally, the dataset created is composed by energy production, technological progress, GDP and oil prices at a quarterly frequency from 1976 to 2019. Econometric approach and model selection is the topic of next section, multi variate time series approach will be used to analyze the correlations and causality between all these variables.

6.2 Methodology and model selection

To identify the effect of biased technological progress on the level of clean energy in the energy mix a VAR approach is used, first part is dealing with Vector error correction model (VECM) in level and AR in first differences, to ensure stationarity of variables. The second part uses new bayesian approach with parameter shrinkage, the Stochastic Search Variable Selection (SSVS) model, allowing to deal with overspecification of a VAR with numerous lags, as it is expected with energy data.

6.2.1 VAR and cointegration

A. Variables

This section aims at understanding the empirical relationship between technological progress and energy production in the clean sector, for this purpose data previously described are accessible. For time series analysis, VAR approach is the first technique used and this section will characterize variables used and data transformation for the econometric methodology to be valid. Looking back to the theoretical model, the following variable are considered in the empirical analysis:

- tech_diff: the technology differential, created by subtracting number of patents in the "dirty" sector to the number of patents in the "clean" one. It captures the higher/lower level of R&D in the "clean" sector, knowing patents are a flux and not a stock.
- r_energy: absolute level of renewable energy
- f_energy: absolute level of fossil energy
- kappa: the share of "clean" energy in global production. The same definition as for the theoretical part, capturing the advancement of energy transition. Formally, $\text{kappa}=\frac{\text{r\_energy}}{\text{r\_energy}+\text{f\_energy}}$
- GDP
- oil_price
- real_invest

Looking at the plots and applying Dickey-Fuller test on these variables, it appears that none of them are stationary. Classical VAR in level would lead to incorrect statistic test and false interpretations of the data. This paper looks at two alternatives to correct for non-stationarity: Vector Error Correction Model (VECM) and VAR in first differences. These two approaches are used and described below.

B. Vector Error Correction Model (VECM)

The use of VECM is recommended when there is cointegration between data, one can look if there exist equilibrium relations between some variables in level, making them stationary without taking the differences.

Let's consider the vector y_t, containing 3 variables of interest: kappa, tech_diff and GDP, such that

$$y_t = \begin{pmatrix} \text{kappa}_t \\ \text{tech\_diff}_t \\ \text{GDP}_t \end{pmatrix}$$

The VECM can be written as follow,

$$\Delta y_t = \Pi y_{t-1} + \Sigma_{l=1}^{p-1} \Psi_l \Delta y_{t-l} + C d_t + \varepsilon_t$$

Δy_t is the first differences of variables in vector y, $\Psi$is a coefficient matrix of the lags of difference variables, d is a vector of of deterministic terms and C corresponds to its coefficient matrix. VECM and VAR have the same formulation but differs in the value of Π, the coefficient matrix of cointegrating relationships, if this matrix is equal to 0 we observe a VAR, if it is different from zero, Πy_{t-1} is therefore the error correction term.

Estimation will be conduct using the maximum likelihood estimator of Johansen (1995), also called the reduced rank model. This estimator gives efficient estimates of the adjustment parameters (α in the following) and of the cointegrating vectors (β). Following Lütkepohl (2005) it is not mandatory to assume β is normalized, one starts with an AR(k) model,

$$\Delta y_t = \mu + \Psi_1 \Delta y_{t-1} + ... + \Psi_{k-1} \Delta y_{t-k+1} + \Pi y_{t-k} + \varepsilon_t$$

And make the assumption rk(Π) = r, which implies that the matrix can be represented as $\Pi = \alpha\beta'$, cointegration assumption then transform AR(k) into,

$$\Delta y_t = \mu + \Psi_1 \Delta y_{t-1} + ... + \Psi_{k-1} \Delta y_{t-k+1} + \alpha\beta' y_{t-k} + \varepsilon_t$$

where α and β are (p x k) and rk(α)= rk(β) = r. In the unrestricted model the number of parameters is $p+kp^2+p(p+1)/2$, and let $Z_{0t} = \Delta y_t$, $Z_{1t} = (\Delta y'_{t-1}, ..., \Delta y'_{t-k+1}, 1)'$ and $Z_{kt} = y_{t-k}$, the moment matrices are then defined as,

$$M_{ij} = T^{-1} \Sigma_{t=1}^{T} Z_{it} Z'_{jt} \qquad (i, j = 0, 1, k),$$

One first regress Z_{it} on Z_{1t} and get residuals R_{it}, and then denotes the residual sum of squares from regressing Z_0 and Z_k on Z_1 as S_{ij}:

$$S_{ij} = \frac{1}{T} \Sigma_{t=1}^{T} R_{it} R'_{jt}$$

Following Johansen (1988), these residuals are then used to determine the maximum likelihood estimator of α and β. One needs now to identify the lag order k and the rank r of the model to proceed to estimation and impulse response function.

First, one applies the Akaike Information criterion(AIC) to the basic VAR

$$y_t = D_1 y_{t-1} + ... + D_p y_{t-p} + \varepsilon_t$$

rewritten as

$$y = ZD + \varepsilon_t$$

where

$$y = \begin{pmatrix} \text{kappa}_t \\ \text{tech\_diff}_t \\ \text{GDP}_t \end{pmatrix} Z = \begin{pmatrix} \text{kappa}'_p & ... & \text{kappa}'_1 \\ \text{tech\_diff}'_p & ... & \text{tech\_diff}'_1 \\ \text{GDP}'_p & ... & \text{GDP}'_1 \end{pmatrix} \varepsilon = \begin{pmatrix} \varepsilon'_\kappa \\ \varepsilon'_{tech} \\ \varepsilon'_{GDP} \end{pmatrix} D = \begin{pmatrix} D'_\kappa \\ D'_{tech} \\ D'_{GDP} \end{pmatrix}$$

Once the VAR has been set-up the lag order is chosen apply AIC, it appears that $p = 9$. The optimal number of lags for this model is 9, this information will be provided in the estimation procedure for maximum likelihood estimator to be consistent. Information prior to the 9^{th} quarter will not be considered, meaning this VAR model has $27 + 1$ regressors per variables, 9 lags for each variable and the trend, leading to a total of 84 regressors for the full model. Applying $p = 9$ into Johansen procedure gives the rank of the cointegration of the matrix, r, we have a double cointegration here with $r = 2$. The time series are cointegrated, the VECM should provide some consistent results. The following tables are giving an example for first lag of the α, the loading matrix describing convergence speed of a dependent variable, and β the cointegration matrix. The vector $\beta' y_{t-1}$ might be interpreted as distance from equilibrium value.

Using all these information, one is able to perform impulse response function of the model using the vec2var command contained in the vars package on R. the

	kappa.l1	tech_diff.l1	GDP.l1
kappa.l1	1	1	1
tech_diff.l1	−0.007	−0.0005	−0.001
GDP.l1	−0.0002	−0.001	−0.0001

Table 3: α

	kappa.l1	tech_diff.l1	GDP.l1
kappa.l1	−0.026	−0.073	−0.027
tech_diff.l1	22.779	−7.462	1.157
GDP.l1	−4.472	−10.312	8.710

Table 4: β

main impulse is to look at the impact of tech_diff on kappa, the ratio of renewable power in the energy mix. The impact of this variable on GDP will also be tested and robustness check will be provided then.

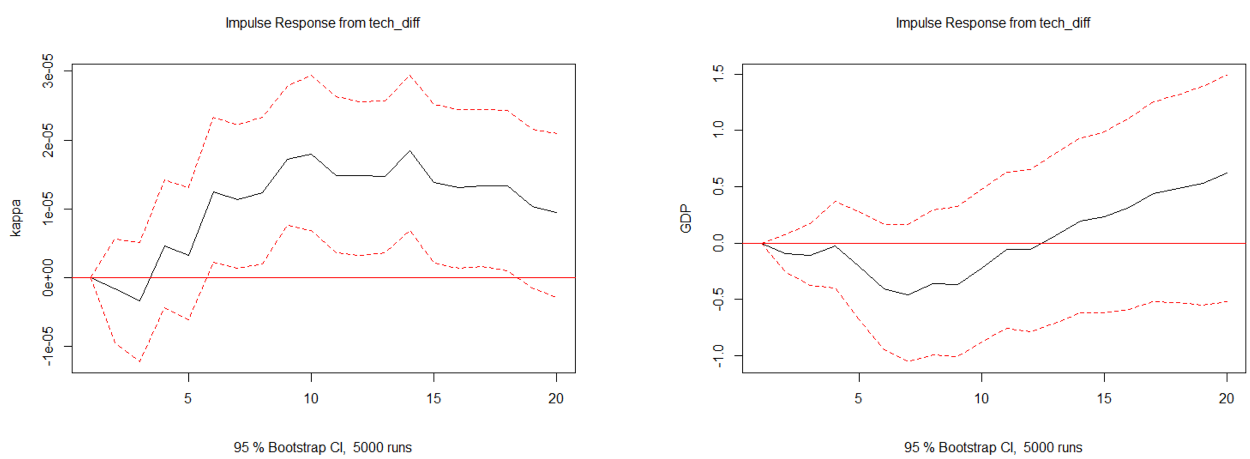


Figure 10: IRF for technology differential shock

A positive shock on technology differential has a positive effect on the share of clean energy after 5 quarters (because the impulse response start at 1, not 0) and last 12 quarters before to become non-significant. As one can expect, the effect of research is not effective right away, it takes more than a year between the moment more research output are created in favor of renewable technology and its significant impact on the energy mix. 5 quarters seems short, but usually research is linked to industry, especially in the energy. If a research group develop a new cell for PV panel, their improvement is known by some industries before the patent is granted. The issuing procedure of a patent is also quite long, looking at USPTO website they indicate that prioritized patent can be reviewed in less than 12months while non-prioritized it takes 21month in average for the patent to be published. Research process is very long and might take up to 3 years and a half to see significant improvement in the energy mix. However, increasing the number of patents in favor of the "clean" sector has no significant impact on GDP, it seems to decrease and then increase but without statistical significance.

Robustness check are now provided, to do so the model is augmented with one endogenous variable: Real Gross Private Domestic Investment, in Billions of Chained 2012 Dollars ; and three exogenous variables : real oil price (WTI index), Foreign Direct Investment in U.S., in millions of dollars, and a dummy variable for recession periods. The addition of real investment is straightforward knowing new energy power plants need big investments to be created, it aims at capturing investing possibilities. Price of barrels may have a direct impact on energy production, especially for an oil producer country like the US, an high level of oil prices might encourage US producers to increase their production while low level would have the converse effect by increasing research in renewable energies. Direct foreign investment is add for the same reason than real investment but as exogenous variable, because the country cannot control completely this amount. Finally, a dummy variable to account for recession captures the global depression faced by the country, not only by its GDP level. Impulse responses for the augmented model are the following,

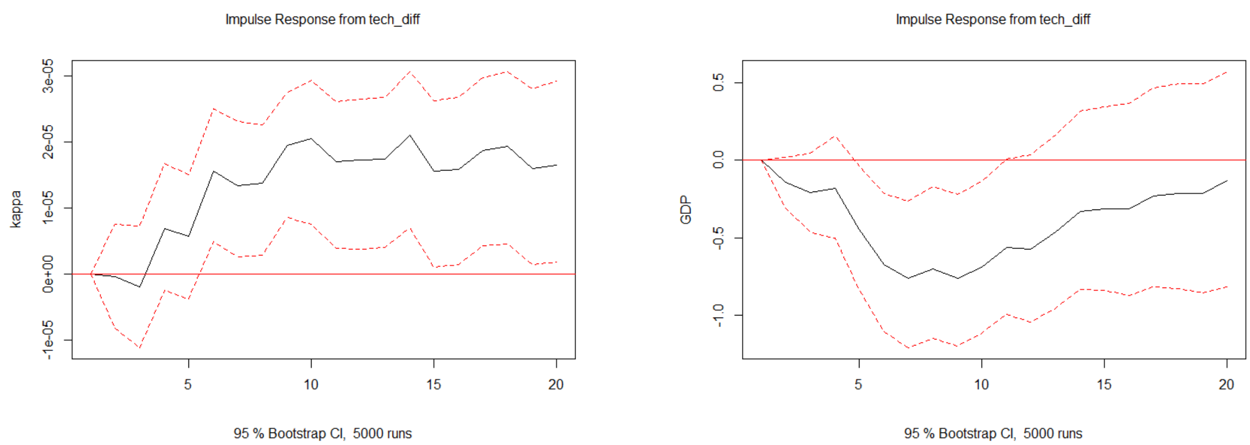


Figure 11: IRF for technology differential shock - augmented model

The augmented model has globally the same pattern than the previous one, the main differences are i) technology differential shock is more persistent, ii) GDP is decreasing from the 4^{th}lag to the 10^{th}. In conclusion the technology differential has a positive and persistent effect, increasing relatively the budget for R&D in renewable energies seems to be an efficient way to speed-up energy transition. However, the VECM approach do not allows for computing value and consistency of estimators, the next part will look at a VAR model in first differences to compute estimators of variables and impulse response functions, it will be used as a second robustness check too.

C. VAR in first differences

In the previous section, use of VAR was discard by the non stationarity of the variables, usually a way to avoid this issue is to work with first differences of the variables. Applying augmented Dickey-Fuller test for stationarity on first differences it appears that the 3 variables are stationary. Estimation of the VAR procedure is then consistent in this second model. An adjustment of variables is also done for this purpose, instead of working with first difference variables for kappa and tech_diff, I differentiate r_energy and f_energy and I make the subtraction of the 2 to create k_energy such that,

- k_energy$_t$ = (r_energy$_t$ - r_energy$_{t-1}$) - (f_energy$_t$ - f_energy$_{t-1}$)

This variable capture the fact that research in the "clean" sector has progressed more or less faster than the fossil one, it details net augmentation. This new variable is stationary according to the augmented Dickey-Fuller test. For technology I have made the same operation to create the second variable k_tech.

- k_tech$_t$ = (r_tech$_t$ - r_tech$_{t-1}$) - (f_tech$_t$ - f_tech$_{t-1}$)

Where r_tech and f_tech are respectively the number of patents granted in the renewable and in the fossil sector.

- dGDP$_t$ = GDP$_t$ - GDP$_{t-1}$

- dreal_invest$_t$ = real_invest$_t$ - real_invest$_{t-1}$

We have then the following vector containing all the information of our 3 stationary variables in first differences,

$$x_t = \begin{pmatrix} \text{k\_energy}_t \\ \text{k\_tech}_t \\ \text{dGDP}_t \\ \text{oil\_price}_t \end{pmatrix}$$

Var coefficient for this model are in the next table,

	k_energy	dGDP
doil_price.l1	−0.007	2.356**
	(0.007)	(1.124)
k_energy.l2	−0.280***	−17.288
	(0.095)	(15.414)
dGDP.l3	0.0003	0.133*
	(0.0005)	(0.079)
k_energy.l4	0.346***	−13.486
	(0.100)	(16.226)
k_tech.l4	0.001	−0.145**
	(0.0004)	(0.066)
k_tech.l5	0.001***	−0.190***
	(0.0004)	(0.064)
doil_price.l5	−0.015**	0.208
	(0.008)	(1.249)
denergy_cons.l5	0.037**	−2.219
	(0.017)	(2.720)
k_tech.l6	−0.00003	−0.156**
	(0.0004)	(0.067)
const	−3.361**	744.093***
	(1.330)	(216.704)
dreal_invest	−0.0003	0.712***
	(0.001)	(0.096)
recession	0.114	−35.669*
	(0.128)	(20.822)
inertia	3.509**	−735.242***
	(1.392)	(226.860)
Observations	130	130
Adjusted R^2	0.439	0.720

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Only the significative variables and technology lags have been kept in this table for simplicity and clarity. The number of lags determined by AIC is 6, we have 3 less lags here.

The Impulse response functions are validating the coefficients in the table, there is still a positive and significant impact of the fifth lag.

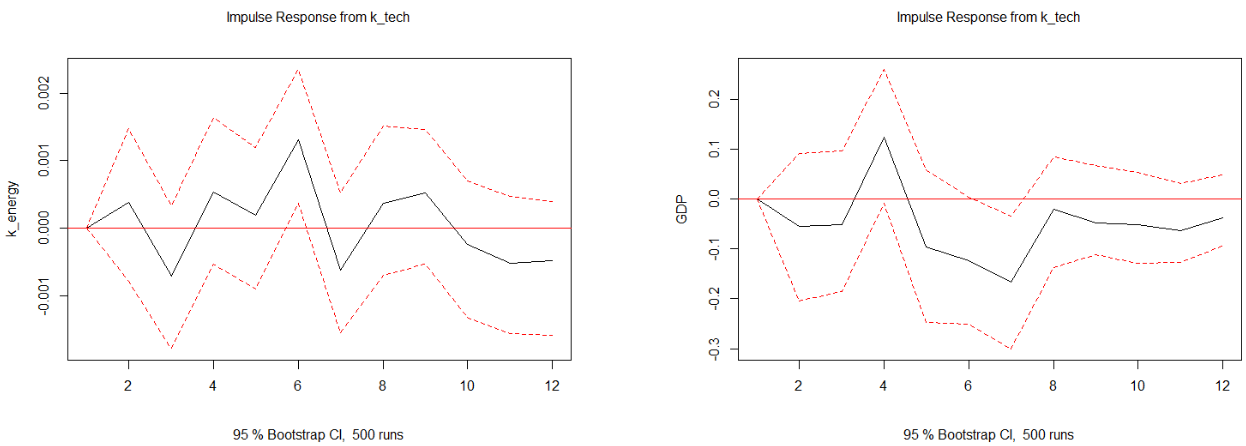


Figure 12: IRF for technology differential shock - first difference model

As one can see, there is a significant impact (below 5% threshold) of "clean" research for the fifth lag even in this VAR analysis, augmented with a dummy variable for recession periods and one other endogenous variable. However, it has a slightly negative effect on GDP at the seventh lag. Results found for the VECM framework are still valid with a VAR in first differences, 5 quarters later the research is effective and this effect is persistent, even if the difference is just a temporary peak.

6.2.2 Bayesian approach and Stochastic Search Variable Selection (SSVS)

As previously mentioned, using a VAR with numerous lags creates a large number of estimators, 84 in the case of VECM with 9x3 lags and 1 trend for each of the variables. Such composition might be noisy in the estimation of the model and in the interpretation of IRF and estimators. Especially since one variable has been added, there are 4 variables with at least 6 lags, a minimum of 100 regressors for 166 observations, overspecification becomes an important issue. One possibility is to use parameters shrinkage methods like LASSO or SSVS. In this paper the latter is used in a Bayesian framework for macroeconomy. Stochastic Search variable Selection (SSVS) prior helps to shrink "useless" parameters to 0 in order to keep the analysis as clean as possible.

The basic idea of SSVS is to assign commonly used prior variances to parameters, which should be included in a model, and prior variances close to zero to irrelevant parameters. By that, relevant parameters are estimated in the usual way and posterior draws of irrelevant variables are close to zero so that they have no significant effect on forecasts and impulse responses. This is achieved by adding a hierarchial prior to the model. The prior variances of the parameters are set in accordance with the semiautomatic approach described in George et al. (2008). Hence, the prior variance of the ith parameter is set to $\tau_{1,i}^2 = (10\hat{\sigma}_i)^2$ if this parameter should be included in the model and to $\tau_{0,i}^2 = (0.1\sigma_i)^2$ if it should be excluded. σ^i is the standard error associated with the unconstrained least squares estimate of parameter i. For all variables the prior inclusion probabilities are set to 0.5. The prior of the error variance-covariance matrix is uninformative. The SSVS prior is coupled to a Gibbs-sampler, a Markov chain Monte Carlo (MCMC) algorithm to obtain a sequence of observations approximated from a specified multivariate probability distribution. As mentioned, it helps to select informative variable and obtain density plots for parameter values as it is presented in figure 13.

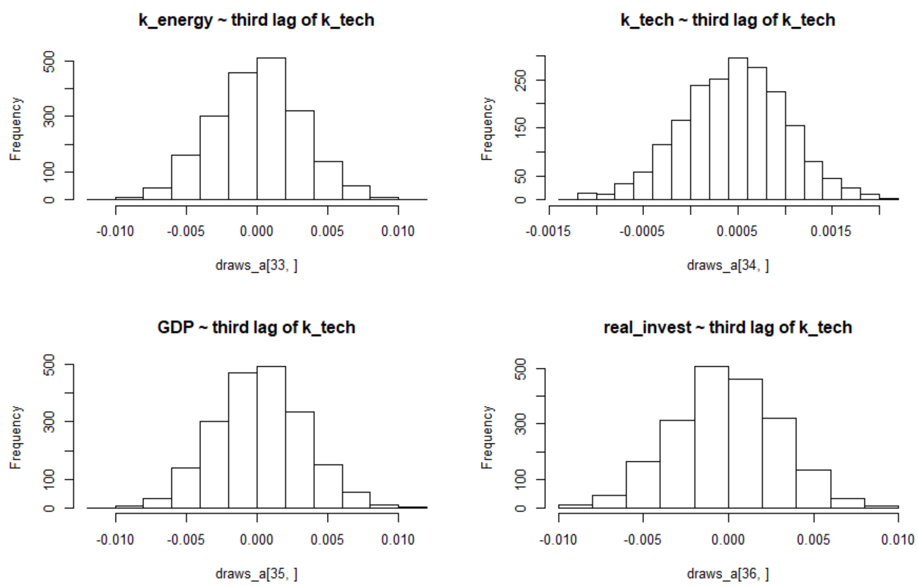


Figure 13: SSVS prior on k_tech

These 4 histogram are summarizing the impact of the third lag of k_tech on the 4 endogenous variables. As one can see, this lag is informative only to estimate its own value, the parameter takes the value 0 for k_energy,GDP and real_invest and is then considered uninformative for these 3 variables. This how SSVS prior is selecting relevant regressors for the model to be the more parsimonious as possible. Once the selection variable has been implemented one is able to provide impulse response functions for the model in first differences. Figure 14 contains IRF for a technology shock on k_energy and GDP.

Impulse response function are in line with what we found previously and seem validate previous assessments: an increase of research differential in favor of the renewable sector favors production of renewable energies with a delay of 5 quar-

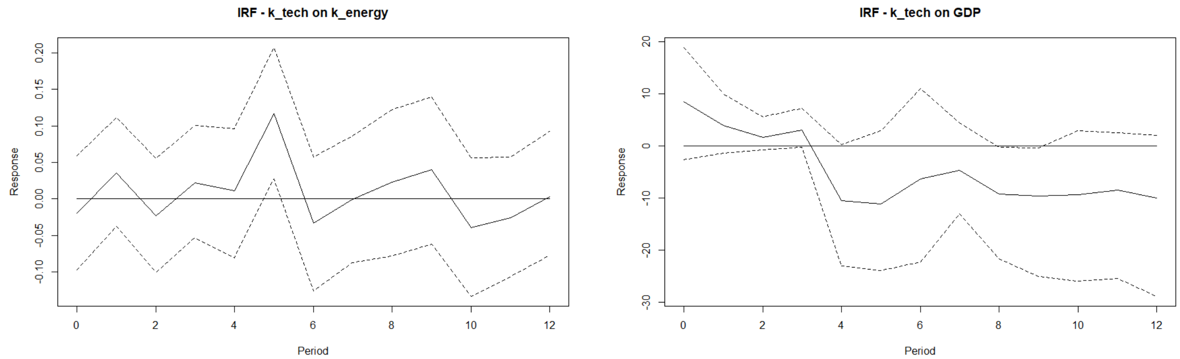


Figure 14: IRF for technology differential shock - SSVS prior

ters. The story behind the fifth lag seems robust to numerous models, specification and variable addition. For the GDP, the IRF is a little bit different, there is a small decrease on the eighth lag but nothing really statistically significant.

The econometric exercise applied above advocates in favor of a significant and persistent effect of technology on energy mix composition. The transitional pattern is quite slow regarding to climate change emergency but such a path exist, to subvention green research is the first step to build a greener world but one should not expect to harvest success of such politics before 3 years.

7 Conclusion

This paper, constructed around the fact that energy capital is long living, is showing capital inertia have stickiness impact on the energy transition. In such framework, a carbon tax accompany the economy on the optimal growth path but without decreasing the stock of “dirty” capital, leading to higher carbon emissions in the future. Growth is exogenous and comes from the relative performance of the “clean” sector compare to the “dirty”, its role is central in the analysis of the theoretical model. Due to this centrality, this paper detailed the impact of research differential (through the number of granted patents) in the US on their energy mix. Using VECM and SSVS (bayesian method of paramete shrinkage) this analysis shows that an exogenous shock on US research differential have a positive,and significant, impact on their investment in “clean” energies 5 quarters later. For now on, investing in research and development in renewable energies seems to be the best alternative.

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