Credit rationing, debt issuance costs and corporate investment*

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Abstract

This paper examines the impact of credit rationing (exogenous debt capacity) on corporate investment in the setting with costly debt financing. Our main finding is the inverted U-shaped relationship between the scale of investment and debt capacity of a firm which faces binding credit constraints. We show that this non-monotonicity of the investment scale on debt capacity is entirely due to the effect of the lump-sum debt issuance costs in the dynamic context of investment.

Keywords: credit rationing, investment, debt issuance costs

JEL classification: G31, G32, G33

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1 Introduction

The corporate finance literature examining the impact of financing constraints on corporate investment decisions allows for two alternative approaches for identifying financing constraints. Under the first approach, credit constraints are interpreted as the limited ability to borrow, caused by the cost wedge between internal and external financing. The extensive empirical literature relying on this definition examines the relation between investment-to-cash flow sensitivities and the tightness of financial constraints. The seminal paper of *Fazzari et al. (1988)* reports that investment-cash flow sensitivities have to be much stronger for the firms that are likely to be financially constrained. However, *Kaplan and Zingales (1997)* provide conflicting evidence showing that less financially constrained firms exhibit higher investment cash-flow sensitivities. One of the possible explanations of such a discrepancy in the results might be related to the fact that *Fazzari et al. (1988)* implicitly assume that firms are able to raise external funds as long as they can pay for external financing. In contrast, the classification approach, used by *Kaplan and Zingales (1997)* to identify the degree of financial constraints, accounts for the difficulties the firms have in obtaining access to external financing.[1]

Indeed, it is possible that firms limit their investment because of the low availability of credit, rather than because of the high costs of external financing *(Greenwald et al. (1984))*. Thus, the alternative approach to deal with credit constraints is to consider them in the form of *credit quantity rationing*. Quantity rationing often results in response to information asymmetry problems *(Jaffe and Russel (1999))*. It can also arise in the context of collaterized lending, given that the fluctuations of collateral value induce the changes of the firm's debt capacity.[2] Empirical evidence suggests the existence of a negative relationship between the degree of credit rationing and the scale of investment (see, for example, *Gan (2007)*, *Gelos and Werner (2002)*, *Almeida and Campello (2007))*. On the theoretical side, however, the literature is still scarce and inconclusive. In line with their empirical findings, *Almeida and Campello (2007)* build a theoretical model showing that investment-cash flow sensitivity increases with asset tangibility for financially constrained firms, but remains neutral for unconstrained firms. The similar result is reported by *Chaney et al. (2008)*. However, *Wong (2009)* shows that the scale of investment is not affected by the degree of credit rationing, because a positive effect of higher project

[1] *Moyen (2004)* shows that access to external financing matters in the investment-cash flow sensitivity analysis. Her empirical analysis reveals that the investment-cash flow sensitivities of the firms with no access to external financing turn out to be lower as compared to that of the firms facing no restrictions in access to external financing.

[2] Here, reference can be made to the macroeconomic literature which identifies a so-called "collateral channel" to explain the transmissions of negative shocks on asset prices to the real sector (see *Kiyotaki and Moore (1997)*, *Bernanke and Gertler (1989)*, *(1990))*.

return as a result of larger investment would be offset by the negative effect of higher default risk.

In the present paper, we attempt to get a better understanding of how the credit quantity rationing (hereafter termed "debt capacity") affects corporate investment. We build our model on *Belhaj and Djimbissi (2007)*[3] extending their framework by the possibility of choosing the optimal investment scale. We consider a firm endowed with an option to set-up an investment project, which can be partially financed by debt. There are two sources of financial frictions related to debt financing: (i) credit quantity rationing and (ii) the explicit costs of debt issuance, both proportional and lump-sum. In practice, debt issuance costs typically involve underwriting, registration and legal fees. These costs can be quite important, especially, in the case of public debt offerings.[4] Given that financing and investment decisions have to be jointly determined in the described framework, we analyze the impact of financial frictions on the optimal choice of investment timing and, especially, on the optimal choice of investment scale.

Two alternative cases are examined. In the first case, the firm can finance the entire investment expenditure without binding credit constraints. Here, investment decisions are found to be unaffected by the degree of credit rationing. Yet, they are sensitive to the magnitude of debt issuance costs. In particular, we find that, faced with higher lump-sum debt issuance costs, the firm will delay investment but will set up a larger investment project to compensate the higher sunk costs of debt financing by higher return. The higher proportional costs of debt issuance induce the firm to delay investment as well, however, without producing any effect on the optimal investment scale.

In the second case, the firm raises a maximum feasible amount of debt allowed by credit constraints. Here, we observe a strong impact of financial constraints on investment decisions. Consistent with the existing theoretical findings (see, for example, *Belhaj and Djimbissi (2007)*, *Shibata and Nishihara (2011))*, the optimal investment trigger is found to be U-shaped when plotted against debt capacity. In contrast, there is an *inverted* U-shaped relationship between the optimal investment scale and debt capacity. Therefore, the firms with intermediate debt capacity will invest earlier and establish larger investment projects, whereas the firms with relatively low/high debt capacity will delay investment and set up smaller projects. Interestingly, the *inverted* U-shaped relationship between investment scale and debt capacity emerges only in the presence of non-zero *lump-sum* debt issuance costs. This observation allows us to explain our result: given that earlier investment implies a higher expected value of lump-sum debt issuance costs,

[3] *Belhaj and Djimbissi (2007)* study the impact of debt capacity on the optimal choice of investment timing and financing structure.

[4] For example, *Datta, Iscandar-Datta and Patel (1997)* document that total expenses of debt issuance might vary between 0.53% and 7.38%, with an average of 2.96% of the total debt issuance.

investors will slightly increase project scale to compensate higher expected sunk costs of debt financing by higher expected return.

The divergence of our main result with the existing empirical and theoretical findings may have several explanations. Firstly, the empirical studies generally do not allow for any distinctions between the initial investment (building a new plant) and realization of growth option opportunities (improving production capacity of an existing plant). In this context, it might be difficult to disentangle the pure effect of credit quantity rationing in the case where investment decisions are made by already existing firms. The availability of external funds for such firms can also depend on their financial performance, internal funds, as well as on their current level of debt. Secondly, existing empirical studies typically do not consider the impact of financial constraints on the timing of lumpy investment, whereas the investment timing and investment scale should be jointly determined.[5] Finally, the literature on the topic we are aware of does not account for the explicit costs of external financing, when analyzing the impact of credit quantity rationing on corporate investment decisions. Yet, these costs can be quite important, which would induce a firm to adjust its investment policy. It is actually the impact of lump-sum debt issuance costs in the dynamic context of investment, which explains the contrasting divergence of our result with that obtained by *Wong (2009)*.

The reminder of the paper is organized as follows. Section 2 presents the model. Section 3 describes the optimal investment decisions of an all-equity financed firm. In Section 4 we analyze the optimal financing and investment decisions of a levered firm. Section 5 concludes. The results of numerical analysis are reported in Appendix B.

2 The model

We consider the setting where investors are risk neutral and discount cash flows at a constant rate r. An owner-managed firm is endowed with a perpetual investment option. The investment project can be undertaken at irreversible costs $I(q)$, which are increasing with the project scale $q > 0$. Moreover, we assume that $I(0) \geq 0$ and $I''(q) > 0$. Project scale affects the scale of stochastic earnings before interest and taxes (EBIT), qX_t, where X_t evolves according to:

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \tag{1}$$

where $\mu < r$ and σ are constant parameters and $W = \{W_t, \mathcal{F}_t, 0 \leq t < \infty\}$ is a standard Brownian motion on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Operating cash flows are taxed at

[5] The only exception is the study by *Whited (2006)* that examines the impact of financial constraints on the timing of undertaking large investment projects. However, *Whited (2006)* interprets financial constraints as the additional costs of adjusting capital stock, rather than as credit quantity rationing.

a constant rate $\theta < 1$.

A firm has access to debt financing. To finance a part of investment costs, equity holders can raise an amount $b \leq \psi I(q)$ of debt at the investment date τ. Here, the parameter ψ reflects the exogenously given degree of credit quantity rationing. The remaining fraction of investment costs, $I(q) - b > 0$, has to be financed by equity capital. Issuing debt is costly.[6] As in *Belhaj and Djembissi (2007)*, we consider a linear form of debt issuance costs, $kb + K$, with both variable and lump-sum components.

Debt is perpetual, so that a constant coupon C is continuously paid to debt holders until the firm goes bankrupt. The optimal default trigger maximizing the firm's equity value is given by the standard formula:

$$x_L(q, C) = \frac{\beta_1}{\beta_1 - 1} \frac{r - \mu}{r} \frac{C}{q} \equiv \delta \frac{C}{q}, \tag{2}$$

where $\beta_1 < 0$ is a root of $\sigma^2/2\beta(\beta - 1) + \mu\beta = r$.

In the case of a default, debt holders recover a fraction $(1-\gamma)$ of the firm's liquidation value, where γ reflects bankruptcy costs. After the default, the firm will be run by new owners as an unlevered concern. Thus, the firm's liquidation value is equal to the total expected value of EBIT evaluated at the default trigger.

Equity holders optimally decide about the timing, the scale and the financing structure of investment, maximizing the ex-ante equity value under the exogenously given credit constraints. For any coupon C and investment scale q, let $D(X_\tau, q, C)$ denote the market value of debt evaluated at the investment time τ. By the absence of arbitrage, we should have $b = D(X_\tau, q, C)$. Then, the equity holders' maximization program can be formalized as follows:

$$\begin{aligned} Sup_{\tau,q,C} \ \mathbb{E}\left[e^{-r\tau}(V(X_\tau, q, C) - I(q) - kD(X_\tau, q, C) - K)\right] \\ \text{s.t. } D(X_\tau, q, C) \leq \psi I(q), \end{aligned} \tag{3}$$

where $V(X_\tau, q, C)$ denotes a value of the firm at the investment time τ (see Appendix A.1 for the expressions of contingent claims).

To have a benchmark, we start by analyzing the optimal investment decisions of an all-equity financed firm. Then, we study the interaction between financial and investment decisions of a levered firm.

[6] Issuing equity might be costly as well. However, as shown in Appendix A, this will not impact our main finding, so that we let equity issuance costs be zero.

3 Investment decisions of an all-equity financed firm: a benchmark

Consider first a benchmark case where $\psi = 0$. In this case the investment project is entirely financed by equity and the firm never goes bankrupt. Let x_I^e denote the investment trigger such that $\tau = \inf\{t \geq 0 : X_t = x_I^e\}$. Then, for any current value X_0, the maximization problem of equity holders can be rewritten as follows:

$$Sup_{x_I^e, q^e} \left[((1-\theta)\nu q^e x_I^e - I(q)) \left(\frac{X_0}{x_I^e}\right)^{\beta_2} \right], \tag{4}$$

where $\nu = (r-\mu)^{-1}$ and $\beta_2 > 1$ is a positive root of $\sigma^2/2\beta(\beta-1) + \mu\beta = r$.

The optimal investment scale then satisfies the equation:

$$qI'(q^e) = \frac{\beta_2}{\beta_2 - 1} I(q^e), \tag{5}$$

whereas the optimal investment trigger is given as follows:

$$x_I^e = \frac{I'(q^e)}{(1-\theta)\nu}. \tag{6}$$

Therefore, in the case of the all-equity financed firm, the choice of the optimal investment scale will be made independently of the investment timing. However, the choice of the optimal investment timing will be affected by the investment scale. Since $I''(q) > 0$, we can easily conclude that larger investment projects will be delayed.

Using the fact that the firm's value at the investment date amounts to $V(x_I^e, q^e) = (1-\theta)\nu q^e x_I^e$, we can restate the optimal investment rule in terms of Tobin's Q_T ratio computed at the investment date:[7]

$$Q_T \equiv \frac{V(x_I^e, q^e)}{I(q^e)} = \frac{\beta_2}{\beta_2 - 1}. \tag{7}$$

Note that Q_T is increasing on both μ and σ, whereas the optimal investment scale q^e is increasing on Q_T. Thus, the increase in the expected earnings' rate or earnings' volatility will delay investment but, at the same time, will induce investors to set up larger investment projects.[8]

[7] It follows from (5) that the elasticity of investment expenditure to the investment scale also amounts to $\beta_2/(\beta_2 - 1)$.

[8] The effect produced by higher volatility is in line with *Bar-Ilan and Strange (1999)*, who find that the increase of output price uncertainty will delay investment, simultaneously increasing its scale.

4 Investment and financing decisions of a levered firm

4.1 The case of non-binding credit constraints

Let us now turn to the analysis of the optimal financing and investment decisions of a levered firm. First, we are going to discuss a solution to the maximization problem in the case when ψ is sufficiently high, so that the firm can raise the required amount of debt without binding the credit constraints. In this case, credit constraints in (3) can be omitted and the program can be resolved in two steps: (i) first, for any fixed investment parameters q and x_I, we define the optimal coupon $C^*(q, x_I)$; (ii) then, the problem is solved for the remaining parameters of investment policy, and the optimal coupon is recovered.

For any given q and x_I, the optimal coupon $C^*(q, x_I)$ maximizing the firm's value net of debt issuance costs is given by:

$$C^*(q, x_I) = \frac{h(k)}{\delta} q x_I, \tag{8}$$

where $h(k)$ denotes

$$h(k) = \left[\frac{\theta - k}{(1-\beta_1)(\theta - k) - \beta_1(1-\theta)(\gamma + k(1-\gamma))} \right]^{-\frac{1}{\beta_1}}. \tag{9}$$

Replacing the optimal coupon $C^*(q, x_I)$ into the maximization problem of equity holders yields:

$$Sup_{x_I,q} \quad ((1-\theta)\nu q x_I + (\theta - k)h(k)\nu q x_I - I(q) - K) \left(\frac{X_0}{x_I} \right)^{\beta_2}. \tag{10}$$

The solution of the above problem is given by the optimal investment scale $q^f(K)$, satisfying the equation

$$q^f I'(q^f) = \frac{\beta_2}{\beta_2 - 1}(I(q^f) + K), \tag{11}$$

and the optimal investment trigger x_I^f, such that

$$x_I^f = \frac{I'(q^f)}{(1-\theta)\nu + (\theta - k)h(k)\nu}. \tag{12}$$

Lemma 1 *The optimal investment scale $q^f(K)$ is increasing on K.*

$\Box$ Let $f_1(q)$ and $f_2(q, K)$ denote the left and the right side of (11) respectively. Note that both $f_1(q)$ and $f_2(q, K)$ monotonically increase on q and $f_2(0, K) > f_1(0)$ (see Fig.

1). Since $qf_1'(q) - f_1(q) = q^2 I''(q)$ and $I''(q) > 0$, the function $f_1(q)$ is convex. Thus, we have $f_1''(q) > 0$. Then, (11) has at most one solution, $q^f(K)$. Since $\frac{\partial f_2(q,K)}{\partial K} > 0$, we have $\frac{\partial q^f(K)}{\partial K} > 0$.■

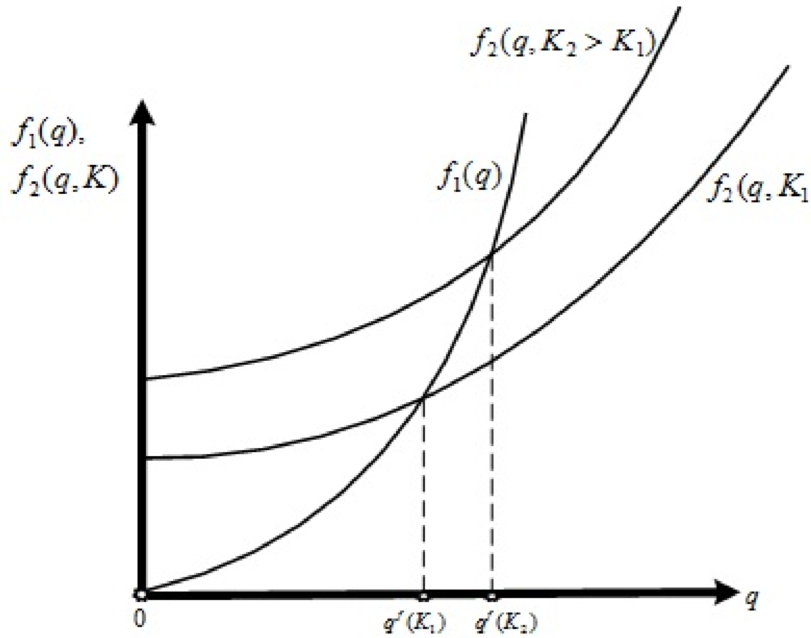

Figure 1: The impact of K on $q^f(K)$.

Thus, similar to the benchmark case with all-equity financing, the optimal investment scale of a financially unconstrained levered firm is set independently of the optimal investment time. Moreover, it does not depend on the financing structure. However, under non-zero lump-sum debt issuance costs, a levered firm will realize larger investment as compared to an all-equity financed firm, since $q^f(K)$ increases on K (see Lemma 1). Indeed, higher K will induce the firm to raise more debt and to set a larger project, in order to compensate higher sunk costs of investment by higher project return. At the same time, proportional debt issuance costs will have no impact on the optimal choice of investment scale: higher k will reduce the value of tax shields associated with debt financing, and this impact will be entirely captured by the optimal choice of C^f and x_I^f.

As in the benchmark case, the optimal investment trigger, x_I^f, is positively related with the chosen investment scale. Moreover, x_I^f is increasing on both k and K, so that investment will be postponed under higher debt issuance costs. However, due to the beneficial effect of tax shields, the investment trigger of a levered firm would be lower than that of an all-equity financed firm, when debt issuance costs are relatively small.

To examine the impact of debt issuance costs on the optimal financing structure, we perform numerical analysis. Its results are reported in Table 1.a-1.b of Appendix B. We find that the optimal coupon C^f and, thus, the firm's leverage are increasing on

K, since the firm will tend to reduce the average cost of debt issuance by raising more debt. However, the amount of debt decreases on k, since proportional debt issuance costs destroy the value of tax shields. As a result, the firm's leverage ratio $D(x_I^f, q^f, C^f)/I(q)^f$ tends to zero when $k \to \theta$. We also observe that the net present value of the levered project calculated at the investment date is higher than that of the unlevered project. Thus, due to the positive effect of tax shields and the ability to realize larger investment, investors will benefit from the access to debt financing, even though issuing debt is costly.

4.2 The case of binding credit constraints

Consider now the optimal investment and financing decisions of the firm with limited debt capacity, so that $D(X_\tau, q, C) = \psi I(q)$. For any given investment scale q and coupon C, binding credit constraints provide the following investment trigger:

$$x_I(q,C) = \left[\frac{1 - \psi r I(q)/C}{1 - (1-\gamma)(1-\theta)\beta_1/(\beta_1 - 1)}\right]^{\frac{1}{\beta_1}} x_L(q,C), \tag{13}$$

where $x_L(q,C)$ is given by (2).

Then, the equity holders' problem (3) can be rewritten as follows:

$$Sup_{q,C}\ \left(V(q,C) - I(q)(1 + k\psi) - K\right)\left(\frac{X_0}{x_I(q,C)}\right)^{\beta_2}, \tag{14}$$

where $V(q,C)$ is firm value defined under the investment threshold $x_I(q,C)$.

Let q^* and C^* denote the solution of the maximization problem (14). Then, replacing q^* and C^* in (13) will deliver the optimal investment trigger.

It is worth noting that, under non-zero lump-sum debt issuance costs, the optimal investment scale q^* will be affected by both investment timing and debt capacity. To illustrate this, we use the Lagrangian method to derive the following optimality condition (see Appendix A.2):

$$qI'(q) = \frac{\beta_2}{\beta_2 - 1}\left[I(q) + K\left(1 - \lambda\psi\left(\frac{x_I}{X_0}\right)^{\beta_2}\right)^{-1}\right], \tag{15}$$

where X_0 is a current value of the state variable and λ is the optimal Lagrange multiplier.

Proposition 1 *When credit constraints are binding, the optimal investment decisions of the firm are affected by the financing structure of investment.*

Numerical simulations (see Table 2.c in Appendix B) suggest the existence of the *inverted U*-shaped relationship between the optimal investment scale q^* and the firm's debt capacity ψ. At the same time, in line with the theoretical findings of *Belhaj and Djimbissi (2007)* and *Shibata and Nishihara (2011)*, the optimal investment trigger appears to be U-shaped when plotted against the debt capacity ψ.[9] Thus, the firms with relatively low or relatively high levels of debt capacity will undertake smaller projects and delay investment, whereas the firms with intermediate levels of debt capacity will set up larger projects and invest earlier (see Figure 2).

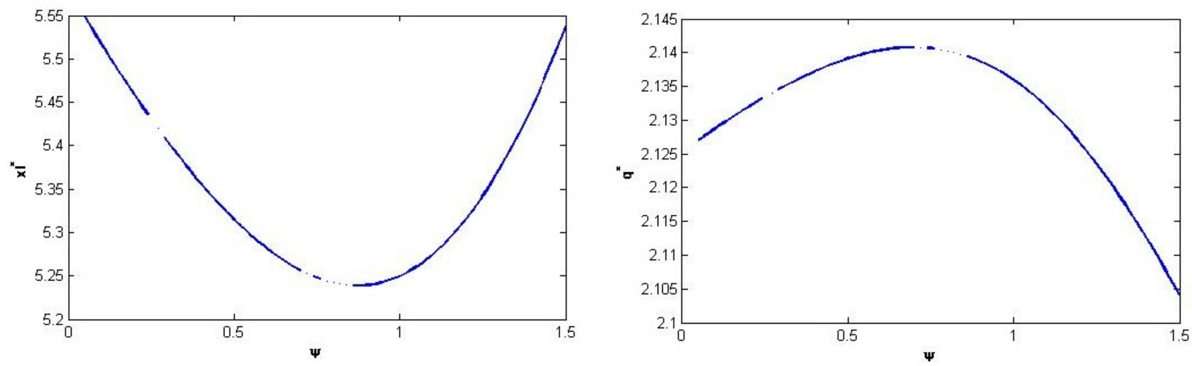

Figure 2: The impact of debt capacity on the optimal investment decisions.

To explain this phenomenon, counterintuitive at first glance, consider the setting of *Wong (2009)*, where there are no debt issuance costs. In the absence of debt issuance costs, the choice of investment scale made by the firm facing binding credit constraints is unaffected by the financing structure of investment. *Wong (2009)* shows that, when credit constraints are binding, the marginal costs of larger investment would offset its marginal benefits. As a result, the financially-constrained firm will choose the same investment scale as it would have chosen, if there was no access to debt financing. However, in the considered setting, the presence of lump-sum debt issuance costs breaks this balance. Note that, for the firms with the intermediate levels of debt capacity, it is optimal to accelerate investment. However, earlier investment implies a higher expected value of lump-sum debt issuance costs. Thus, in order to compensate higher expected sunk costs of debt financing associated with earlier investment, the firm with the intermediate level of debt capacity will slightly increase investment scale. Numerical experiments show that the changes of the optimal investment scale with respect to the changes of debt capacity are relatively small. This allows us to conclude that the non-monotonicity of investment on the degree of credit constraints is uniquely due to the effect of lump-sum debt issuance costs in the dynamic context of investment.

Expression (15) also shows that, in contrast to the case where credit constraints

[9] This pattern results from the trade-off between tax-shield benefits and bankruptcy costs of debt.

are not binding, the optimal choice of investment scale is affected by proportional debt issuance costs k as well. This result arises due to the fact that, under the binding credit constraints, the choice of investment scale becomes inseparable from the choice of investment trigger, whereas the investment trigger is affected by the proportional debt issuance costs through the link with the optimal coupon. Numerical experiments show that the optimal investment scale, as well as the optimal volume of debt, is decreasing with k.

5 Concluding remarks

Intuitively, it would be natural to assume the existence of a positive monotonic relationship between the optimal scale of investment and the debt capacity of the firm which faces binding credit constraints. Yet, we show that this is not the case, if the firm incurs the lump-sum costs of debt financing. Thus, the main practical implication ensuing from our analysis is that the usage of the investment scale as an implicit measure of credit constraints might be misleading.

It is also worth noting that a practical interpretation of the inverted U-shaped relationship between the investment scale and debt capacity may vary depending on the nature of the parameter ψ.

First, ψ may reflect the tangibility of investment project, meaning that creditors lend against the value of tangible assets *(Almeida and Campello (2007))*. This interpretation fits well with the setting where, for any given project scale q, the allocation between tangible and intangible assets is relatively rigid, so that investors can not voluntary choose the degree of asset tangibility.[10] In this case, our model predicts that the projects with relatively low and relatively high degrees of asset tangibility would attract less investment, as compared to the projects with a better balance between tangible and intangible assets.

Second, ψ may reflect market the expectations about the market value of investment project. In this light, our main result might be interpreted as the non-monotonicity of investment on the phase of the economic cycle. A surprising conclusion, which can be drawn in such a case, is that, during the "boom" phase of economic cycle characterized by overoptimistic expectations, firms will invest less than in the "moderate" phase of economic cycle characterized by lower expectations.

Finally, as mentioned by many existing studies (see, for example *Whited (1992)*, *Jaffe and Russel (1999))* quantity rationing can arise as a consequence of information asym-

[10]For example, the construction of energy and power generation plants mostly implies investment in tangible assets. In contrast, for alcohol and tobacco businesses, the largest part of investment consists of the costs of manufacturing and distribution permits (intangible assets).

metry problems. In this case, $(1-\psi)$ can be viewed as the minimum rate of shareholders' contribution required by a bank to prevent moral hazard. In such a context, our model suggests that less stringent requirements on the minimum amount of capital contributions from shareholders would *not* necessarily encourage larger investment.

6 Appendix A. Mathematical proofs

A.1. Valuation of contingent claims

Equity value

The firm's equity value represents the expected present value of EBIT net of coupon payments and taxes:

$$E(X_t, q, C) = \mathbb{E}\left[\int_t^{\tau_L} e^{-r(s-t)}(1-\theta)(qX_s - C)ds\right], \tag{A1}$$

where $\tau_L = \inf\{t \geq 0 : X_t = x_L\}$ denote liquidation time. Solving a corresponding ODE

$$1/2\sigma^2 X^2 E''(X) + \mu X E'(X) - rE(X) + (1-\theta)(qX - C) = 0, \tag{A2}$$

subject to boundary condition

$$E(x_L, q, C) = 0, \tag{A3}$$

yields:

$$E(X_t, q, C) = (1-\theta)\left[q\nu X_t - \frac{C}{r} + \left(\frac{C}{r} - q\nu x_L(q,C)\right)\left(\frac{X_t}{x_L(q,C)}\right)^{\beta_1}\right], \tag{A4}$$

where $\nu = (r-\mu)^{-1}$, $\beta_1 < 0$ is a root of characteristic equation $\sigma^2/2\beta(\beta-1) + \mu\beta = r$ and the optimal liquidation rule $x_L(q,C)$, such that $E'_{x_L}(X_t, q, C) = 0$, is given by:

$$x_L(q,C) = \frac{\beta_1}{\beta_1 - 1}\frac{1}{q\nu}\frac{C}{r} \equiv \delta\frac{C}{q}. \tag{A5}$$

Debt value

Let $V_L(x_L, q, C)$ denote the firm's liquidation value:

$$V_L(x_L, q, C) = E\left[\int_{\tau_L}^{+\infty} e^{-r(s-\tau_L)}(1-\theta)qX_s ds\right] = (1-\theta)q\nu x_L(q,C), \tag{A6}$$

where $\tau_L = \inf\{t \geq 0 : X_t = x_L(q,C)\}$.

The value of the firm's debt represents the expected present value of coupon payments up to liquidation plus the firm's liquidation value net of bankruptcy costs.

$$D(X_t, q, C) = \mathbb{E}\left[\int_t^{\tau_L} e^{-r(s-t)}C ds + e^{-r(\tau_L - t)}(1-\gamma)V_L(x_L, q, C)\right]. \tag{A7}$$

The value of the firm's debt is given by:

$$D(X_t, q, C) = \frac{C}{r} - \left(\frac{C}{r} - (1-\gamma)(1-\theta)q\nu x_L(q, C)\right)\left(\frac{X_t}{x_L(q, C)}\right)^{\beta_1}. \tag{A8}$$

A total value of the firm

A total value of the firm is given by the sum of equity and debt values:

$$V(X_t, q, C) = (1-\theta)q\nu X_t + \frac{\theta C}{r} - \left(\theta + \gamma(1-\theta)\frac{\beta_1}{\beta_1 - 1}\right)(q\nu x_I)^{\beta_1}\left(\frac{\beta_1 - 1}{\beta_1}\right)^{\beta_1}\left(\frac{r}{C}\right)^{\beta_1 - 1}. \tag{A9}$$

A.2. Proof of Proposition 1

Consider the optimization problem of equity holders:

$$\begin{gathered} Sup_{\tau, q, C<\infty}\ \mathbb{E}\left[e^{-r\tau}(V(X_\tau, q, C) - I(q) - kD(X_\tau, q, C) - K)\right] \\ \text{s.t. } D(X_\tau, q, C) \leq \psi I(q) \end{gathered} \tag{A10}$$

To simplify the exposition of calculus, we introduce the following notation:

$$H(x_I, q, C) = (q\nu x_I)^{\beta_1}\left(\frac{\beta_1 - 1}{\beta_1}\right)^{\beta_1}\left(\frac{r}{C}\right)^{\beta_1 - 1}, \tag{A11}$$

$$A_1 = 1 + (1-\gamma)(1-\theta)\frac{\beta_1}{\beta_1 - 1}, \tag{A12}$$

$$A_2 = \theta + \gamma(1-\theta)\frac{\beta_1}{\beta_1 - 1}. \tag{A13}$$

Then, the Lagrangian of the above problem, $\mathcal{L}(x_I, q, C)$, can be written as follows:

$$\begin{aligned} \mathcal{L}(x_I, q, C) = &\left\{(1-\theta)q\nu x_I - I(q) - K + (\theta - k)\frac{C}{r} + (kA_1 - A_2)\,H(x_I, q, C)\right\}\left(\frac{x_0}{x_I}\right)^{\beta_2} + \\ &+ \lambda\left\{\psi I(q) - \frac{C}{r} + A_1 H(x_I, q, C)\right\}, \end{aligned} \tag{A14}$$

where $\beta_2 > 0$ is the roots of $\sigma^2/2\beta(\beta - 1) + \mu\beta = r$ and λ is the Lagrange multiplier.

The Kuhn-Tucker conditions for the above maximization program are given by:

$$\frac{\partial L(x_I, q, C)}{\partial q}q = 0, \tag{A15}$$

$$\frac{\partial L(x_I, q, C)}{\partial C}C = 0, \tag{A16}$$

$$\frac{\partial L(x_I, q, C)}{\partial x_I} x_I = 0, \tag{A17}$$

$$\lambda \left\{ \psi I(q) - \frac{C}{r} + A_1 H(x_I, q, C) \right\} = 0. \tag{A18}$$

Equation (A15) can be rewritten as follows:

$$\{(1-\theta)q\nu x_I - I'(q)q + \beta_1(kA_1 - A_2)H(x_I, q, C)\} \left(\frac{x_0}{x_I}\right)^{\beta_2} + \lambda \{\psi I'(q)q + \beta_1 A_1 H(x_I, q, C)\} = 0. \tag{A19}$$

Equation (A16) can be rewritten as follows:

$$\left\{(\theta - k)\frac{C}{r} - (\beta_1 - 1)(kA_1 - A_2)H(x_I, q, C)\right\} \left(\frac{x_I}{x_0}\right)^{\beta_2} - \lambda \left\{\frac{C}{r} + A_1(\beta_1 - 1)H(x_I, q, C)\right\} = 0. \tag{A20}$$

Equation (A17) can be rewritten as follows:

$$\begin{aligned} \left\{(1-\theta)q\nu x_I - \frac{\beta_2}{\beta_2 - 1}\left(I(q) + K - (\theta - k)\frac{C}{r}\right) + (kA_1 - A_2)\frac{\beta_2 - \beta_1}{\beta_2 - 1} H(x_I, q, C)\right\} \left(\frac{x_0}{x_I}\right)^{\beta_2} - \\ - \lambda A_1 \frac{\beta_1}{\beta_2 - 1} H(x_I, q, C) = 0. \end{aligned} \tag{A21}$$

The sum of (A19) and (A20) yields:

$$\begin{aligned} \left\{(1-\theta)q\nu x_I - I'(q)q + (\theta - k)\frac{C}{r} + (kA_1 - A_2)H(x_I, q, C)\right\} \left(\frac{x_0}{x_I}\right)^{\beta_2} \\ + \lambda \left\{\psi I'(q)q - \frac{C}{r} + A_1 H(x_I, q, C)\right\} = 0. \end{aligned} \tag{A22}$$

Using (A9) and (A18), we can rewrite the above equation as follows:

$$\{V(x_I, q, C) - I'(q)q - k\psi I(q)\} \left(\frac{x_0}{x_I}\right)^{\beta_2} + \lambda\psi \{I'(q)q - I(q)\} = 0. \tag{A23}$$

From (A21) $-$ (A20)/$(\beta_2 - 1)$ we get:

$$\begin{aligned} \left\{(1-\theta)q\nu x_I - \frac{\beta_2}{\beta_2 - 1}(I(q) + K) + (\theta - k)\frac{C}{r} + (kA_1 - A_2)H(x_I, q, C)\right\} \left(\frac{x_0}{x_I}\right)^{\beta_2} \\ + \frac{\lambda}{\beta_2 - 1}\left\{\frac{C}{r} - A_1 H(x_I, q, C)\right\} = 0. \end{aligned} \tag{A24}$$

Using (A9) and (A18), we can rewrite the above equation as follows:

$$\left\{V(x_I,q,C)-\frac{\beta_2}{\beta_2-1}(I(q)+K)-k\psi I(q)\right\}\left(\frac{x_0}{x_I}\right)^{\beta_2}+\frac{\lambda}{\beta_2-1}\psi I(q)=0. \tag{A25}$$

Subtracting (A23) from (A25) and rearranging terms, we obtain:

$$\left(1-\lambda\psi\left(\frac{x_I}{x_0}\right)^{\beta_2}\right)\left(\frac{\beta_2-1}{\beta_2}I'(q)q-I(q)\right)=K. \tag{A26}$$

When the firm is financially constrained, we have $\lambda > 0$. Thus, in the presence of lump-sum debt issuance costs K, the optimal investment decisions (in particular, the optimal scale of investment) will be affected by financing decisions.[11]

Robustness check: costly equity issuance

Assume that financing by equity also involves proportional and lump-sum issuance costs, which we denote ξ_1 and ξ_0 respectively. Then, the equity holders' maximization problem takes the following form:

$$\begin{gathered} Sup_{\tau,q,C<\infty}\ \mathbb{E}\left[e^{-r\tau}(V(X_\tau,q,C)-(1+\xi_1)I(q)-(k-\xi_1)D(X_\tau,q,C)-(K+\xi_0))\right] \\ \text{s.t. } D(X_\tau,q,C)\leq\psi I(q) \end{gathered} \tag{A27}$$

Performing the similar computations as in Appendix A.2, we can obtain:

$$\left(1+\xi_1-\lambda\psi\left(\frac{x_I}{x_0}\right)^{\beta_2}\right)\left(\frac{\beta_2-1}{\beta_2}I'(q)q-I(q)\right)=K+\xi_0. \tag{A28}$$

Thus, a financing structure will affect the optimal investment decisions in the presence of lump-sum equity issuance costs.

[11] However, if $K = 0$, the debt neutrality on the investment scale established in *Wong (2009)* would hold even under the variable debt issuance costs.

7 Appendix B. Numerical simulations

We use the following parameter values: investors' discount factor $r = 8\%$, the expected growth rate of operating profit $\mu = 1\%$, operating profit volatility $\sigma = 30\%$, the default cost coefficient $\gamma = 0.3$, the tax rate $\theta = 0.15$, the investment function $I(q) = 10 + 5q^3$. A current value of the state variable is taken as $X_0 = 1$. The optimal investment strategy in the benchmark case (all-equity financing) is given by $q^e = 1.8566$ and $x_I^e = 4.2582$.

B.1. Optimal investment and financing decisions when credit constraints are not binding

Table 1.a displays simulation results for $k = 0.1$ and different values of K_0. **Table 1.b** displays the results obtained for $K = 5$ and different values of $k < \theta$. We denote F the current value of the firm investment option, whereas $\rho = D(x_I, q, C)/I(q)$ represents the firm's leverage ratio.

Table 1.a

	$k = 0.1$						
K	q^f	x_I^f	C^f	$I(q^f)$	$D(x_I^f, q^f, C^f)$	ρ	F
0	**1,8566**	4,2282	2,1644	42	24,7629	0,5896	4,1613
1	1,9166	4,5056	2,3809	45,2	27,2392	0,6026	4,0885
2	1,9730	4,7746	2,5973	48,4	29,7155	0,6140	4,0232
3	2,0263	5,0363	2,8138	51,6	32,1917	0,6239	3,9640
4	2,0770	5,2914	3,0302	54,8	34,6680	0,6326	3,9100
5	2,1253	5,5405	3,2467	58	37,1443	0,6404	3,8603

Table 1.b

	$K = 5$						
k	q^f	x_I^f	C^f	$I(q^f)$	$D(x_I^f, q^f, C^f)$	ρ	F
0	2,1253	5,3258	6,9924	58	70,8100	1,2209	4,1413
0,02	2,1253	5,3769	6,4445	58	66,5943	1,1482	4,0716
0,04	2,1253	5,4250	5,8118	58	61,3926	1,0585	4,0077
0,06	2,1253	5,4691	5,0799	58	54,9698	0,9478	3,9504
0,08	2,1253	5,5080	4,2318	58	47,0209	0,8107	3,9008
0,1	2,1253	5,5405	3,2467	58	37,1443	0,6404	3,8603
0,12	2,1253	5,5645	2,0987	58	24,8034	0,4276	3,8307
0,14	2,1253	5,5780	0,7563	58	9,26800	0,1598	3,8143

B.2. Optimal investment and financing decisions when credit constraints are binding

Table 2.a displays simulation results for $\psi = 0.7$, $k = 0.1$ and different values of K. **Table 2.b** displays the results obtained for $\psi = 0.7$, $K = 5$ and different values of $k < \theta$. **Table 2.c** reports simulation results for $K = 5$, $k = 0$ and different values of $\psi \in [0, 1.5]$.

Table 2.a

	$\psi = 0.7,\ k = 0.1$					
K	q	x_I	C	$I(q)$	$D(x_I, q, C)$	F
0	**1,8566**	4,2507	2,6192	42	29,4003	4,1591
1	1,9161	4,5242	2,8097	45,1751	31,6225	4,0869
2	1,9723	4,7903	3,0011	48,3585	33,8510	4,0220
3	2,0254	5,0494	3,1928	51,5445	36,0812	3,9631
4	2,0760	5,3025	3,3852	54,7372	38,3161	3,9093
5	2,1243	5,5499	3,5778	57,9305	40,5514	3,8598

Table 2.b

	$\psi = 0.7,\ K = 5$					
k	q	x_I	C	$I(q)$	$D(x_I, q, C)$	F
0	2,1408	5,2565	3,6762	59,0559	41,3391	4,0640
0,02	2,1373	5,3151	3,6554	58,8190	41,1733	4,0212
0,04	2,1340	5,3737	3,6352	58,5892	41,0125	3,9795
0,06	2,1307	5,4323	3,6155	58,3642	40,8549	3,9387
0,08	2,1274	5,4911	3,5964	58,1443	40,7010	3,8988
0,1	2,1243	5,5499	3,5778	57,9305	40,5514	3,8598
0,12	2,1212	5,6087	3,5599	57,7229	40,4061	3,8217
0,14	2,1182	5,6676	3,5423	57,5193	40,2635	3,7844

Table 2.c

	$k=0, K=5$					
ψ	q	x_I	C	$I(q)$	$D(x_I, q, C)$	F
0	**1,8566**	**4,2582**		**42**		**0,7387**
0,1	2,1287	5,5165	0,4717	58,2309	5,8231	3,8527
0,2	2,1319	5,4576	0,9595	58,4466	11,6893	3,8924
0,3	2,1348	5,4041	1,4646	58,6451	17,5935	3,9309
0,4	2,1372	5,3561	1,9881	58,8124	23,525	3,9678
0,5	2,1391	5,3147	2,5308	58,9418	29,4709	4,0026
0,6	2,1404	5,2811	3,0934	59,026	35,4156	4,0349
0,7	2,1408	5,2565	3,6763	59,0561	41,3393	4,064
0,8	2,1403	5,2421	4,2789	59,021	47,2168	4,0893
0,9	2,1387	5,2393	4,9003	58,9125	53,0213	4,1103
1	2,1359	5,2497	5,5386	58,7221	58,7221	4,1263
1,1	2,1319	5,2747	6,1908	58,4464	64,2911	4,1367
1,2	2,1266	5,3152	6,8533	58,0852	69,7023	4,1412
1,3	2,1201	5,3723	7,5218	57,6454	74,939	4,1393
1,4	2,1125	5,4463	8,1921	57,1374	79,9923	4,1311
1,5	2,1041	5,5368	8,8602	56,5764	84,8646	4,1165

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