Endogenous Business Cycles in Two-Country OLG Economy

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Abstract: *This paper analyzes the co-movement of endogenous business cycle of large economies in a Heckscher-Ohlin free trade equilibrium. A two-country two-good two-factor overlapping generations model is developed. We first identify the determinants of each country's accumulation pattern in autarky equilibrium, and secondly we prove how a country's business cycle may spread throughout the world once trade opens. We show that market integration through international trade has a stabilizing effect for each country. Indeed, the scope of endogenous business cycle in the world market is smaller than in autarky regime. Finally, we prove that some fiscal policy rules can prevent the existence of business-cycle fluctuations in the world economy by driving it to the optimal steady state as soon as it is announced.*

Keywords: *two-sector OLG model, two-country, local indeterminacy, endogenous fluctuations.*

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1 Introduction

European countries experienced a large market integration. This process has speeded up dramatically in the last two decades, e.g. the single market in the 1990s in European country and the adoption of a single currency. These developments raise the possibility of the contagion of macroeconomic volatility across countries. European market integration is known for more synchronized national business cycle[1].

In the present study, we analyze the co-movement of endogenous business cycle in two-large countries in a Heckscher-Ohlin world where countries only differ with respect to their rate of time preference. Endogenous business cycle are driven by changes in expectations about the fundamental. This change in expectaton is based on the concept of sunspot equilibria[2]. As shown by Woodford [35], the existence of sunspot equilibria is related to the indeterminacy of the equilibrium under perfect foresight, i.e. the existence of a continuum of equilibrium paths converging towards one steady state from the same initial value of the state variable

The infinitely-lived agent model is the standard framework to study such issue. However, the steady state rental rate of capital is uniquely characterized by the rate of time preference. Therefore, in a world with heterogeneous rates of time preference across countries, there exist different steady state rental rates of capital. It follows that the world steady state rental rate of capital is not uniquely determined and that at least one country is specialized at the steady state[3]. The major part of the literature has focused on the introduction of market failure to investigate the interlinkage of endogenous business cycle in two-large diversified economies[4].

A more convenient framework to study the co-movement of endogenous business cycle in two-large diversified economies is the overlapping generations

[1] Artis and Zhang [2] shows that after the European Exchange Rate Mechanism, european countries experience more synchronized national business cycle.

[2] See Shell [31], Azariadis [3] and Cass and Shell [9]

[3] See Baxter [4] and Stiglitz [33].

[4] See Nishimura et al. [23, 24, 25] introduces decreasing returns to scale, Nishimura and Shimomura [21] introduces sector-specific externalities, and Ghiglino [16] introduces technological externalities.

model (OLG). Indeed, an OLG economy can simultaneously have heterogeneous rate of time preference and a common world steady state rental rate of capital implying that both countries may be diversified at steady state[5]. Several papers are devoted to open economy. Galor and Lin [15] and Bianconi [6] discuss trade pattern with differences in rate of time preference or taxation. Cremers [11] is concerned with the dynamic gains from trade where countries differ only on their rate of time preference[6]. Unfortunately none of these studies consider the interlinkage of endogenous business cycle in two-large country trade model[7].

The existence of endogenous fluctuations has been demonstrated for two-sector OLG closed economy with one consumption good since Galor [14], Reichlin [30] and Venditti [34]. Reichlin [30] exhibits the possibility of endogenous fluctuations in a two-sector model under Leontief technology while Galor [14] and Venditti [34] consider a two-sector model upon standard sectoral technologies. The occurrence of endogenous fluctuations and local indeterminacy fundamentally depend on the capital intensity hypothesis (a capital intensive consumption good) and the allocations of factor between sectors. Indeed, this allocation causes fluctuations of the capital accumulation path which may be important enough to be spread in the economy.

In the present paper, a two-sector two-country OLG model with two consumption goods based upon generic sectoral technologies is considered based on Le Riche et al. [18]. There are one consumption good and one mixed good which can be either consumed or invested. We assume a unitary elasticity of intratemporal substitution between the two consumption goods and a homogenous life cycle utility. The two countries are identical in all aspects except for their rate of time preference. Furthermore, factors are internationally immobile. Our main objective is to identify whether or not market integration has a stabilizing or a

[5]See Bianconi [6], Buiter [8], Cremers [10] and Galor and Lin [15].

[6]See Naito and Zhao [20] for a discussion of gain from trade with difference in population growth.

[7]In a two-country one-sector OLG model, Aloi and Lloyd-Braga [1] assume that one country is characterized by labor market rigidits and the other country is characterized by a competitive labor market. They show that market integration may bring macroeconomic stability and efficiency if the competitive country operating at full employment becomes a net importer of workers.

destabilizing effect. In autarky regime, building on the homogeneity of the utility function we provide a simple condition for the existence of a steady state. We then show that each country exhibits endogenous business cycle when the consumption good is capital intensive. In trade regime, we derive the free trade equilibrium and the steady state equilibrium. First, we determine a simple condition where each country will produce both goods at steady state. Second we characterize the trade patterns, and show that the patient country is a net exporter of the capital intensive good. Third we characterize the international endogenous business cycle. We provide a numerical exercice which highlights the stabilizing effect of the trade regime, i.e. market integration will reduce the scope of endogenous fluctuations. Finally we conclude that a fiscal policy rules can prevent the existence of business-cycle fluctuations in the world economy by driving it to the optimal steady state as soon as it is announced.

Our main conclusion can be compared to one contribution dealing with the existence of endogenous fluctuations in large country trade model where country remains diversify. Nishimura and Shimomura [21] consider the infinitely lived agent framework in which sector-specific externalities are introduced in a continuous-time version of the Heckscher-Ohlin model. Assuming that technologies are cobb-douglas and identical across country they show that if both countries are characterized by endogenous fluctuations in autarky regime then the world market is also characterized by endogenous fluctuations once trade opens[8].

This paper is organized as follows. Section 2 describes the autarky equilibrium of the two countries and the autarky endogenous business cycle. Section 3 characterizes the free trade equilibrium along with the dynamic efficiency properties and the pattern of trade. Section 4 provides the existence of endogenous business cycle under trade regime. Section 5 contains a numerical exercise to highlight the trade effect on the stability properties of the countries. Section 6 discusses a fiscal policy rule. Concluding remarks are in Section 7 and the proofs are gathered in a final Appendix.

[8]Sim and Ho [32] consider a similar framework in which one country is characterized by sector-specific externality and the other is not. Assuming that one country is characterized by endogenous fluctuations in autarky, and not the other, they show that the world market is stabilized.

2 Autarky

This section describes a single closed two-sector overlapping generations economy $i = \{N, S\}$ with two consumptions goods and general formulation with non-zero factor substitution in production presented in Le Riche et al. [18].

2.1 Technology

Consider a competitive economy in which there are two sectors, one representative firm for each sector and each firm producing one final good. In this economy there exist two goods: one consumption good Y_0^i and one mixed good Y^i which can be either consumed or invested. The consumption good is taken as a numeraire. Each sector uses two factors, capital K^i and labor L^i, both factors are mobile between sectors. Depreciation of capital is complete within one period[9]: $K_{t+1}^i = Y_t^i - Z_t^i$ with K_{t+1}^i the total amount of capital in period $t+1$ and Z_t^i the consumption share of the mixed good in period t. A constant returns to scale technology is used for each sector: $Y_0^i = F^{i0}(K^{i0}, L^{i0})$, $Y = F^{i1}(K^{i1}, L^{i1})$ with $K^{i0} + K^{i1} \leq K^i$, and $L^{i0} + L^{i1} \leq L^i$, and satisfy the following properties:

Assumption 1. *The production function $F^{ij} : \mathbb{R}_+ \to \mathbb{R}_+$, is C^2, increasing, concave, homogeneous of degree one and satisfy the Inada conditions such that for any $\chi > 0$, $F_1^{ij}(0,\chi) = F_2^{ij}(\chi, 0) = \infty$, $F_1^{ij}(\infty,\chi) = F_2^{ij}(\chi,\infty) = 0$, $j = \{0, 1\}$ and $i = \{N, S\}$.*

Let define the production possibility frontier by the social production function $T^i\left(K^i, Y^i, L^i\right)$:

$$T^i\left(K^i, Y^i, L^i\right) = \max_{K^{ij},L^{ij}} \left\{Y_0^i \mid Y^i \leq F^{i1}(K^{i1}, L^{i1}),\ K^{i0} + K^{i1} \leq K^i,\ L^{i0} + L^{i1} \leq L^i\right\} \tag{1}$$

Under Assumption 1, the function $T^i\left(K^i, Y^i, L^i\right)$ is homogeneous of degree one, concave and C^2. Denoting r^i the rental rate of capital, p^i the price of the mixed

[9]In a two-periods OLG model, full depreciation of capital is justified by the fact that one period is about thirty years.

good and w^i the wage rate, all in terms of the price of the consumption good, using the envelope theorem the following three relationships hold:

$$\begin{aligned} r^i\left(K^i, Y^i, L^i\right) &= T_1^i\left(K^i, Y^i, L^i\right) \\ p^i\left(K^i, Y^i, L^i\right) &= -T_2^i\left(K^i, Y^i, L^i\right) \\ w^i\left(K^i, Y^i, L^i\right) &= T_3^i\left(K^i, Y^i, L^i\right) \end{aligned} \tag{2}$$

The relative capital intensity difference is derived from the factor-price frontier

$$b^i = \frac{L^{i1}}{Y^i}\left(\frac{K^{i1}}{L^{i1}} - \frac{K^{i0}}{L^{i0}}\right) \tag{3}$$

The sign of b^i is positive (negative) if and only if the consumption good is labor (capital) intensive. The Stolper-Samuelson effect and the Rybczynski effect are determined by the factor-price frontier and the full employment condition and explained by the relative capital intensity difference. Indeed, under a consumption good labor intensive, the Stolper-Samuelson states that an increase of the relative price decreases the rental rate of capital whereas the Rybczynski effect specifies that an increase of the capital-labor ratio decreases the production of the consumption good and increases the production of the mixed good. Furthermore, from the GDP function $T^i\left(K^i, Y^i, L^i\right) + p^iY^i = w^iL^i + r^iK^i$, we get the share of capital in the economy:

$$s^i\left(K^i, Y^i, L^i\right) = \frac{r^iK^i}{T^i\left(K^i, Y^i, L^i\right) + p^iY^i} \tag{4}$$

2.2 Preferences

Consider an infinite-horizon discrete time economy that is populated by overlapping generations who live for two periods: young and old. At each period, a new generation N_t^i is born. The population is constant over time and normalized to one. In the first period, young agents inelastically supply one unit of labor and receive an income. They assign this income between saving ϕ_t^i and the consumption of the composite good C_t^i over the bundle of goods $\left(c_{0,t}^i, c_{1,t}^i\right)$. In the second period, old agents are retired. The return on saving, $R_{t+1}^i\phi_t^i$, give their income which they spend entirely in the consumption of the composite good D_{t+1}^i over the bundle of goods $\left(d_{0,t+1}^i, d_{1,t+1}^i\right)$. An agent born in period t has preferences defined over consumption of the composite good C_t^i and D_{t+1}^i. Intertemporal preferences of agents are described by the following CES utility function[10]

$$U\left(C_t^i, \frac{D_{t+1}^i}{\Gamma^i}\right) = \left[\left(C_t^i\right)^{\frac{\gamma^i-1}{\gamma^i}} + \delta^i \left(\frac{D_{t+1}^i}{\Gamma^i}\right)^{\frac{\gamma^i-1}{\gamma}{}^{i}}\right]^{\frac{\gamma^i}{\gamma^i-1}} \tag{5}$$

Moreover, each agent has an intratemporal preference which differ across age:

$$C_t^i = \left(c_{0,t}^i\right)^{\theta_c^i} \left(c_{1,t}^i\right)^{1-\theta_c^i}, \; D_{t+1}^i = \left(d_{0,t+1}^i\right)^{\theta_d^i} \left(d_{1,t+1}^i\right)^{1-\theta_d^i} \tag{6}$$

where δ^i is the discount factor, θ_c^i (resp. θ_d^i) is the share of good 0 in the composite C_t^i (resp. D_{t+1}^i), γ^i is the elasticity of intertemporal substitution in consumption and Γ^i is a scaling constant parameter. Agent born in period t has to solve the optimal composition of the composite goods C_t^i and D_{t+1}^i which are derived from the following static optimizations:

$$\max_{c_{0,t}^i, c_{1,t}^i} \left\{\left(c_{0,t}^i\right)^{\theta_c^i} \left(c_{1,t}^i\right)^{1-\theta_c^i} \mid \pi_{c,t}^i C_t^i = c_{0,t}^i + p_t^i c_{1,t}^i\right\} \tag{7}$$

$$\max_{d_{0,t+1}^i, d_{1,t+1}^i} \left\{\left(d_{0,t+1}^i\right)^{\theta_d^i} \left(d_{1,t+1}^i\right)^{1-\theta_d^i} \mid \pi_{d,t+1}^i D_{t+1}^i = d_{0,t+1}^i + p_{t+1}^i d_{1,t+1}^i\right\} \tag{8}$$

where $\pi_{g,s}^i$ is the price of the composite good with $g = \{c, d\}$; $s = \{t, t+1\}$. Solving the first order conditions gives:

$$c_{0,t}^i = \theta_c^i \pi_{c,t}^i C_t^i, \; c_{1,t}^i = \frac{\left(1-\theta_c^i\right)\pi_{c,t}^i C_t^i}{p_t^i}, \; \pi_{c,t}^i = \left(\frac{p_t^i}{1-\theta_c^i}\right)^{1-\theta_c^i} \left(\theta_c^i\right)^{-\theta_c^i}. \tag{9}$$

$$d_{0,t+1}^i = \theta_d^i \pi_{d,t+1}^i D_{t+1}^i, \; d_{1,t+1}^i = \frac{\left(1-\theta_d^i\right)\pi_{d,t+1}^i D_{t+1}^i}{p_{t+1}^i}, \; \pi_{d,t+1}^i = \left(\frac{p_{t+1}^i}{1-\theta_d^i}\right)^{1-\theta_d^i} \left(\theta_d^i\right)^{-\theta_d^i}. \tag{10}$$

Under perfect foresight w_t^i, R_{t+1}^i and $\pi_{g,t}^i$ are considered as given. A young agent born at period t solves the following dynamic program:

$$\max_{C_t^i, D_{t+1}^i} \left\{U\left(C_t^i, \frac{D_{t+1}^i}{\Gamma^i}\right) \mid w_t^i = \pi_{c,t}^i C_t^i + \phi_t^i, \pi_{d,t+1}^i D_{t+1}^i = R_{t+1}^i \phi_t^i\right\} \tag{11}$$

[10]All the conclusion of this paper can be obtained with a general concave and homothetic utility function $U\left(C_t^i, \frac{D_{t+1}^i}{\Gamma^i}\right)$.

Solving the first order condition gives:

$$\pi^i_{c,t}C^i_t = \alpha^i\left(\frac{R^i_{t+1}\pi^i_{c,t}}{\Gamma^i\pi^i_{d,t+1}}\right)w^i_t \equiv \frac{w^i_t}{1+\left(\delta^i\right)^\gamma\left(\frac{R^i_{t+1}\pi^i_{c,t}}{\Gamma^i\pi^i_{d,t+1}}\right)^{\gamma-1}} \tag{12}$$

where $\alpha^i\left(\frac{R^i_{t+1}\pi^i_{c,t}}{\Gamma^i\pi^i_{d,t+1}}\right)^i \in (0,1)$ is the propensity to consume of young agent. From the budget constraint (38), the saving function is obtained $\phi^i_t = 1-\alpha^i\left(\frac{R^i_{t+1}\pi^i_{c,t}}{\Gamma^i\pi^i_{d,t+1}}\right)$. In the following, we assume that the saving function is increasing with respect to the gross rate of return R^i.

Assumption 2. $\gamma^i > 1$.

2.3 Dynamic equilibrium

The dynamics of the economy is described by the evolution of the capital stock and the market clearing condition for good 1:

$$k^i_{t+1} - \frac{w^i\left(k^i_t,y^i_t\right)}{p^i\left(k^i_t,y^i_t\right)}\left\{1-\alpha^i\left[\frac{\Theta^i r^i\left(k^i_{t+1},y^i_{t+1}\right)}{\Gamma^i p^i\left(k^i_t,y^i_t\right)^{\theta^i_c} p^i\left(k^i_{t+1},y^i_{t+1}\right)^{1-\theta^i_d}}\right]\right\} = 0 \tag{13}$$

$$\theta^i_c k^i_{t+1} - y^i_t + \left(1-\theta^i_d\right)k^i_t\frac{r^i\left(k^i_t,y^i_t\right)}{p^i\left(k^i_t,y^i_t\right)} + \left(1-\theta^i_c\right)\frac{w^i\left(k^i_t,y^i_t\right)}{p^i\left(k^i_t,y^i_t\right)} = 0 \tag{14}$$

with a constant term $\Theta^i = \frac{\left(1-\theta^i_d\right)^{1-\theta^i_d}\left(\theta^i_d\right)^{\theta^i_d}}{\left(1-\theta^i_c\right)^{1-\theta^i_c}\left(\theta^i_c\right)^{\theta^i_c}}$. The set of admissible paths is defined as follows:

$$\omega^i = \left\{(k^i_t,y^i_t)\in\mathbb{R}^2_+ \mid k^i_t \leq \frac{\overline{K^i}(L^i)}{L^i} \equiv \overline{k}^i,\ y^i_t \leq F^{i1}\left(k^i_t,1\right)\right\} \tag{15}$$

where $\overline{k}^i$ is solution of $k^i - F^{i1}\left(k^i,1\right) = 0$. A perfect-foresight competitive equilibrium, defined as a sequence of allocations $\{k^i_t,y^i_t\}^\infty_{t=0}$, satisfies the two differences equations (13)-(14) for every period t, with the pair $\left(k^i_0,y^i_0\right)$ given.

2.4 Steady state and efficiency properties

A steady state $\left(k^i_t,y^i_t\right) = \left(k^{i*},y^{i*}\right)$ is defined by:

$$k^{i*} - \frac{w^i\left(k^{i*},y^{i*}\right)}{p^i\left(k^{i*},y^{i*}\right)}\left\{1-\alpha^i\left[\frac{\Theta^i r^i\left(k^{i*},y^{i*}\right)}{\Gamma^i p^i\left(k^{i*},y^{i*}\right)^{1+\theta^i_c-\theta^i_d}}\right]\right\} = 0 \tag{16}$$

$$\theta_c^i k^{i*} - y^{i*} + \left(1-\theta_d^i\right) k^{i*} \frac{r^i\left(k^{i*},y^{i*}\right)}{p^i\left(k^{i*},y^{i*}\right)} + \left(1-\theta_c^i\right) \frac{w^i\left(k^{i*},y^{i*}\right)}{p^i\left(k^{i*},y^{i*}\right)} = 0 \qquad (17)$$

The steady state $\left(k^{i*}, y^{i*}\right)$ depends on consumption and production behavior, e.g. the propensite to consume of young agent α^i and the capital-labor ratio, and is parameterized by the elasticity of intertemporal substitution in consumption γ^i. Indeed, any variation of γ^i will induce a change of the propensity to consume of young agent α^i and the capital-labor ratio k^i .

To guarantee that the steady state remains unaltered when γ^i vary, the steady state $\left(k^{i*}, y^{i*}\right)$ is normalized by using two scaling parameters $\left(\Gamma^i, \theta_d^i\right)$.

Let define $\nu^i = \frac{y^i}{k^i}$, the output-capital ratio, the inverse of ν^i represents the share of the mixed good which is invested. From (17), the scaling parameter θ_d^i lies between 0 and 1 if ν lies between $\underline{\nu}^i$ and $\overline{\nu}^i$, with $\underline{\nu}^i < \overline{\nu}^i$:

$$\underline{\nu}^i = \frac{1-\alpha^i\theta_c^i}{1-\alpha^i}, \; \overline{\nu}^i = \frac{1-\left(1-s^i\right)\alpha^i\theta_c^i}{\left(1-\alpha^i\right)\left(1-s^i\right)} \qquad (18)$$

Then, the following Proposition holds:

Proposition 1 . *Under Assumptions 1-2,* $\left(k^{i*}, y^{i*}\right)$ *is a normalized steady state if and only if* $\Gamma^i = \Gamma^i\left(k^{i*}, y^{i*}, \gamma^i\right) > 0$ *and* $\theta_d^i = \theta_d^i\left(k^{i*}, y^{i*}\right) \in (0,1)$.

Proof: See Appendix 7.1. ■

Assumption 3. $\Gamma^i = \Gamma\left(k^{i*}, y^{i*}, \gamma^i\right)$ *and* $\theta_d^i = \theta_d^i\left(k^{i*}, y^{i*}\right)$.

Let us define R^i the stationnary gross rate of return at steady state

$$R^i\left(k^{i*}, y^{i*}\right) = \frac{s^i\left(k^{i*},y^{i*}\right)}{\left(1-\alpha^i\left(k^{i*},y^{i*}\right)\right)\left(1-s^i\left(k^{i*},y^{i*}\right)\right)} \qquad (19)$$

The Golden-Rule level is given by $R^i = 1$. If $R^i > 1$, the steady state ν^{i*} is lower than the Golden-Rule level, i.e. an efficient normalized steady state. From Drugeon et al. [13], the following lemma holds:

Lemma 1 . *Under Assumptions 1-3, let* $\underline{\alpha}^i = \frac{1-2s^i}{1-s^i}$. *Then an intertemporal competitive equilibrium converging toward the NSS is dynamically efficient if* $\alpha^i \in \left(\underline{\alpha}^i, 1\right)$*, and dynamically inefficient if* $\alpha^i \in \left(0, \underline{\alpha}^i\right)$.

To get positive value for $\underline{\alpha}^i$ and to consider empirical plausible value for the share of capital in the economy s^i, the following assumption is made:

Assumption 4. $s^i \in \left(\frac{1}{3}, \frac{1}{2}\right)$.

2.5 Autarky business cycle

This section states the result on autarky business cycle on which this study is built. Local indeterminacy is defined as the existence of a continuum of equilibrium paths converging around one steady state from the same initial value of the state variable, it allows the introduction of sunspot shocks that is an impulse to shock in economic fundamentals. To proceed to the analysis of the occurrence of local indeterminacy, let introduce the share of capital in total income and the elasticity of the rental rate of capital:

$$\varepsilon^i_{rk} = -\frac{T^i_{11}\left(k^{i*}, y^{i*}\right)k^{i*}}{T^i_1\left(k^{i*}, y^{i*}\right)} \tag{20}$$

In two-sector OLG model under gross substituability, local indeterminacy occurs when the consumption is capital intensive. The following Proposition presents the occurrence of endogenous fluctuations, and directly follows from result of Le Riche et al. [18]:

Proposition 2 . *Under Assumptions 1-4, if the consumption is capital intensive, there exists* $\underline{\varepsilon}^i_{rk}$, $\overline{\varepsilon}^i_{rk}$, $\overline{b}^i$ *and* $\overline{\theta}^i$, *such that any steady state is locally indeterminate if one of the following condition is satisfied:*
i⌋ $\alpha^i \in \left(\frac{1}{2}, \frac{s^i}{1-s^i}\right)$, $\theta^i_c < \overline{\theta}^i$, $\nu^i \in \left(\tilde{\nu}^i, \nu^i_0\right)$, $b^i > \underline{b}^i$, $\varepsilon^i_{rk} \in \left(\underline{\varepsilon}^i_{rk}, \overline{\varepsilon}^i_{rk}\right)$ *and* $\gamma^i \in \left(\gamma^{i,\mathcal{T}}, \gamma^{i,\mathcal{H}}\right)$;
ii⌋ $\alpha^i < \underline{\alpha}^i$, $\nu^i \in \left(\nu^i_0, \nu^i_3\right)$, $b^i \in \left(b^i_3, b^i_2\right)$, $\varepsilon^i_{rk} < \varepsilon^i_{rk2}$ *and* $\gamma^i > \gamma^{\rangle,\mathcal{F}}$.

Proof: See Appendix 7.2. ■

Remark 1 *It has to be noting that in Proposition* 2 – i⌋ $\gamma^{i,\mathcal{H}}$ *correspond to a Hopf bifurcation value leading to quasi-periodic cycle which are locally indeterminate* *in a right neigborhood of $\gamma^{i,\mathcal{H}}$, while $\gamma^{i,\mathcal{T}}$ is a Transcritical bifurcation value inducing the existence of a second steady state which is locally unstable in a right neighborhood of $\gamma^{i,\mathcal{T}}$. Moreover, in Proposition $2-ii\rfloor$ $\gamma^{i,\mathcal{F}}$ correspond to a Flip bifurcation value brought about to period-two cycle which are locally indeterminate in a right neigborhood of $\gamma^{i,\mathcal{F}}$*

To illustrate all the various case of Proposition 2, let assume the following characterization of the technology:

$$\begin{array}{r} F^{i0}\left(K^{i0},L^{i0}\right)=\left[\mu^i\left(K^{i0}\right)^{-\rho^i}+\left(1-\mu^i\right)\left(L^{i0}\right)^{-\rho^i}\right]^{-\frac{1}{\rho^i}} \\ F^{i1}\left(K^{i1},L^{i1}\right)=\min\left\{\frac{K^{i1}}{\eta^i},L^{i1}\right\} \end{array} \tag{21}$$

with $\eta^i > 0$, $\mu^i \in (0,1)$ and $\rho^i > -1$. The sectoral elasticities of capital-labor substitution are given by $\sigma^i = \frac{1}{1+\rho^i}$. Then the social production function is defined as:

$$T^i\left(K^i,Y^i,L^i\right)=\left[\mu^i\left(K^i-\eta^iY^i\right)^{-\rho^i}+\left(1-\mu^i\right)\left(L^i-Y^i\right)^{-\rho^i}\right]^{-\frac{1}{\rho^i}} \tag{22}$$

To show that efficient endogenous fluctuations are empirically plausible, we consider the following set of parameters: $\mu^i = 0.999865, \eta^i = 0.022, \theta^i_c = 0.125, \rho^i = 3, k^i = 0.45, y^i = 0.9, \nu^i = 2, \gamma^i = 2$. We get an efficient steady state with $\alpha^i \simeq 0.53$ and $\underline{\alpha}^i \simeq 0.03$. The corresponding share of capital is $s^i \approx 0.49$. We find the share of the consumption good 0 in the composite good D^i_{t+1}, $\theta^i_d \simeq 0.99$, and the relative capital intensity difference $b^i \simeq -4.28$. We obtain the elasticity of the rental rate of capital $\varepsilon^i_{rk} \simeq 1.68$. Then the efficient normalized steady state is locally indeterminate for any $\gamma^i \in \left(\gamma^{i,\mathcal{T}},\gamma^{i,\mathcal{H}}\right)$. Proposition 2 is guaranteed for any $b^i \in (-6.99,-4.18)$, $\nu^i \in (1.98, 2.12)$, $\varepsilon^i_{rk} \in (0.38, 1.68)$ and $\gamma^i \in (1.69, 2.55)$.

To show that inefficient endogenous fluctuations are empirically plausible, we consider the following set of parameters: $\mu^i = 0.5, \eta^i = 0.28, \theta^i_c = 0.99, \rho^i = -0.5, k^i = 0.65, y^i = 0.735, \nu^i = 1.13, \gamma^i = 265$. We get an inefficient steady state with $\alpha^i \simeq 0.21$ and $\underline{\alpha}^i \simeq 0.49$. The corresponding share of capital is $s^i \approx 0.33$. We find the share of the consumption good 0 in the composite good D^i_{t+1}, $\theta^i_d \simeq 0.79$, and the relative capital intensity difference $b^i \simeq -1.39$. We obtain the elasticity of the rental rate of capital $\varepsilon^i_{rk} \simeq 0.31$. Then the inefficient normalized steady state is locally indeterminate for any $\gamma > \gamma^{i,\mathcal{F}}$. Proposition 2 is guaranteed for any $b^i \in (-1.99, -1.16)$, $\nu^i \in (1, 1.26)$, $\varepsilon^i_{rk} < 1.39$ and $\gamma^i > 264$.

3 Free trade

In this section, a world that consists of two countries, North and South, which are identical in all aspects except for their normalized rate of time preference, i.e. $\Gamma^N \delta^N \neq \Gamma^S \delta^S$, is introduced. Country $i = \{N, S\}$ is described by the economy presented in Chapter 1, namely a two-sector Overlapping Generations model with one consumption good and one mixed good. The factors of production, capital and labor, are immobile across countries. The two consumption goods are freely tradable between the two countries. Thus the relative price of the mixed good will be equalized across North and South. International lending and borrowing are not permitted and the population in both countries are normalized to one. Let denote $x^i_{1,t}$ ($x^i_{0,t}$) the net import of the mixed (consumption) good. Each period trade is balanced , i.e. $x^i_{0,t} + p_t x^i_{1,t} = 0$. Through this paper, it is assumed that North and South produce both goods.

3.1 World dynamic equilibrium

Producers in North and South have access to the same homogeneous technology functions $T\left(K^i_t, Y^i_t, L^i_t\right)$, $i = \{N, S\}$, so that the existence of a world social production function $\tau\left(K^w_t, Y^w_t, L^w_t\right)$ is given by

$$\tau\left(K^w_t, Y^w_t, L^w_t\right) = T\left(K^N_t, Y^N_t, L^N_t\right) = T\left(K^S_t, Y^S_t, L^S_t\right) \tag{23}$$

where $K^w = K^N + K^S$ the aggregate capital stock, $Y^w = Y^N + Y^S$ the aggregate production of good 1 and $L^w = L^N + L^S$ the aggregate labor force. From the GDP function $\tau\left(K^w_t, Y^w_t, L^w_t\right) + pY^w = wL^w + rK^w$, we get the share of capital in the economy:

$$s^w\left(K^w_t, Y^w_t, L^w_t\right) = \frac{rK^w}{\tau\left(K^w_t, Y^w_t, L^w_t\right) + pY^w} \tag{24}$$

The world maximization problem is:

$$\max_{K^N,K^S,L^N,L^S} \left\{\tau\left(K_t^w, Y_t^w, L_t^w\right) \mid K_t^N + K_t^S \leq K_t^w, Y_t^N + Y_t^S \leq Y_t^w, L_t^N + L_t^S \leq L_t^w\right\} \tag{25}$$

With similar technology and free trade in goods the maximization problem (25) implies that factor prices are equalized (envelope theorem):

$$\begin{aligned} r\left(K_t^w, Y_t^w, L_t^w\right) &= \tau_1\left(K_t^w, Y_t^w, L_t^w\right), \\ p\left(K_t^w, Y_t^w, L_t^w\right) &= -\tau_2\left(K_t^w, Y_t^w, L_t^w\right), \\ w\left(K_t^w, Y_t^w, L_t^w\right) &= \tau_3\left(K_t^w, Y_t^w, L_t^w\right). \end{aligned} \tag{26}$$

Let define the $\kappa = \frac{K^w}{L^w}$ and $\zeta = \frac{Y^w}{L^w}$, the dynamics of the world economy is described by the evolution of aggregate capital stock and the world market clearing condition for good 1:

$$\kappa_{t+1} - \frac{w(\kappa_t,\zeta_t)}{2p(\kappa_t,\zeta_t)}\left\{2 - \alpha^N\left[\frac{\Theta r(\kappa_{t+1},\zeta_{t+1})}{\Gamma^N p(\kappa_t,\zeta_t)^{\theta_c} p(\kappa_{t+1},\zeta_{t+1})^{1-\theta_d}}\right] - \alpha^S\left[\frac{\Theta r(\kappa_{t+1},\zeta_{t+1})}{\Gamma^S p(\kappa_t,\zeta_t)^{\theta_c} p(\kappa_{t+1},\zeta_{t+1})^{1-\theta_d}}\right]\right\} = 0 \tag{27}$$

$$\theta_c\kappa_{t+1} - \zeta_t + (1-\theta_d)\,\kappa_t\frac{r(\kappa_t,\zeta_t)}{p(\kappa_t,\zeta_t)} + (1-\theta_c)\,\frac{w(\kappa_t,\zeta_t)}{p(\kappa_t,\zeta_t)} = 0 \tag{28}$$

The set of admissible paths is defined as follows:

$$\Omega = \left\{(\kappa_t,\zeta_t) \in \mathbb{R}_+^2 \mid \kappa_t \leq \overline{\kappa},\ \zeta_t \leq \tfrac{1}{2}\left[F^1\left(k_t^N, 1\right) + F^1\left(k_t^S, 1\right)\right]\right\} \tag{29}$$

where $\overline{\kappa}$ is solution of $\kappa - \frac{1}{2}\left[F^1\left(k^N, 1\right) + F^1\left(k_t^S, 1\right)\right] = 0$. A perfect-foresight competitive equilibrium, defined as a sequence of allocations $\{\kappa_t, \zeta_t\}_{t=0}^{\infty}$, satisfies the two differences equations (27)-(28) for every period t, with the pair (κ_0, ζ_0) given.

3.2 Efficiency properties

A steady state $(\kappa_t, \zeta_t) = (\kappa^*, \zeta^*)$ is defined by:

$$\kappa^* - \frac{w(\kappa^*,\zeta^*)}{2p(\kappa^*,\zeta^*)}\left\{2 - \alpha^N\left[\frac{\Theta r(\kappa^*,\zeta^*)}{\Gamma^N p(\kappa^*,\zeta^*)^{1+\theta_c-\theta_d}}\right] - \alpha^S\left[\frac{\Theta r(\kappa^*,\zeta^*)}{\Gamma^S p(\kappa^*,\zeta^*)^{1+\theta_c-\theta_d}}\right]\right\} = 0 \tag{30}$$

$$\theta_c\kappa^* - \zeta^* + (1-\theta_d)\,\kappa^*\frac{r(\kappa^*,\zeta^*)}{p(\kappa^*,\zeta^*)} + (1-\theta_c)\,\frac{w(\kappa^*,\zeta^*)}{p(\kappa^*,\zeta^*)} = 0 \tag{31}$$

Again, the normalization procedure of Proposition 1 is used to keep the same steady state when the preference and technology parameters change. Let define $\nu = \frac{\zeta}{\kappa}$, the output-capital ratio at the world level, the inverse of ν represents the share of the mixed good which is invested. From (17), the scaling parameter θ_d lies between 0 and 1 if ν lies between $\underline{\nu}$ and $\overline{\nu}$, with $\underline{\nu} < \overline{\nu}$:

$$\underline{\nu} = \frac{2-\left(\alpha^N+\alpha^S\right)\theta_c}{\left(2-\alpha^N-\alpha^S\right)}, \ \overline{\nu} = \frac{2-(1-s^w)\left(\alpha^N+\alpha^S\right)\theta_c}{\left(2-\alpha^N-\alpha^S\right)(1-s^w)} \tag{32}$$

Then, the following Corollary holds:

Corollary 1 . *Under Assumptions 1-2,* (κ^*, ζ^*) *is a normalized steady state if and only if* $\Gamma^N = \Gamma(\kappa^*, \zeta^*, \gamma)$, $\Gamma^S = 1$ *and* $\theta_d = \theta_d(\kappa^*, \zeta^*)$.

Proof: See Appendix 7.3. ■

Assumption 5. $\Gamma^N = \Gamma(\kappa^*, \zeta^*, \gamma)$, $\Gamma^S = 1$ *and* $\theta_d = \theta_d(\kappa^*, \zeta^*)$.

Let define α^w the propensity to consume of young agent in the world economy and R the stationary gross rate of return at steady state

$$\alpha^w = \frac{1}{2}\left(\alpha^N + \alpha^S\right), \ R = \frac{2s^w}{(2-\alpha^N-\alpha^S)(1-s^w)} \tag{33}$$

The Golden-Rule level is given by $R = 1$. If $R > 1$, the steady state (κ^*, ζ^*) is lower than the Golden-Rule level, i.e. an efficient steady state. From Drugeon et al. [13], the following lemma holds:

Lemma 2 . *Under Assumptions 1-2 and 5, let* $\underline{\alpha}^w = \frac{1-2s^w}{1-s^w}$. *An intertemporal dynamic equilibrium converging towards a NSS is dynamically efficient if* $\alpha^w \in \left(\underline{\alpha}^w, 1\right)$, *and dynamically inefficient if* $\alpha^w \in \left(0, \underline{\alpha}^w\right)$.

In order to get positive value for $\underline{\alpha}^w$, the following assumption is made:

Assumption 6. $s^w \in \left(\frac{1}{3}, \frac{1}{2}\right)$.

3.3 Pattern of trade

Let consider the long-run trade properties of the model. In infinitely-lived agent model the steady state gross rental rate of capital is uniquely determined by the rate of time preference. Therefore, in a world with two countries which differ only on their rate of time preferences a world steady state can not be characterized by a common rental rate of capital[11]. It follows that at least one country is specialized in the steady state. Contrarily, in the OLG model with two countries which differ only on their rate of time preferences a world steady state be characterized by a common rental rate of capital. It follows that at the steady state both countries can be diversified[12]. However, North and South can trade and produce both goods during the dynamic transition while in the long-run equilibrium one of the two countries can be specialized in one of the two goods.

Let suppose that $\Gamma^N \delta^N > \Gamma^S \delta^S$ such that the North is the more patient country. As a result the comparative advantage are driven by the difference in the rate of time preference, thus North has a comparative advantage in the production of the capital intensive good:

Proposition 3 *Consider a locally saddle path stable world trade equilibrium and a steady state trade equilibrium in which North and South are identical in every aspect except for the rate of time preference, i.e.* $\Gamma^N \delta^N \neq \Gamma^S \delta^S$. *Then, the more patient country exports the capital intensive good.*

Proof: See Appendix 7.4. ■

Consequently, the patient country will be a net exporter of the capital intensive good while South will be a net importer of the capital intensive good. Thus if the consumption is labor intensive, the patient country has a higher steady state level of capital labor ratio and a higher price of the mixed good than the impatient country.

[11] See Baxter [4] and Stiglitz [33].

[12] See Bianconi [6], Buiter [8], Cremers [11] and Galor and Lin [15].

4 International endogenous business cycle

We focus on local stability of equilibria when the consumption good is capital intensive, i.e. $b < 0$. As in autarky regime such capital-intensity configuration may be related with endogenous fluctuations.

Proposition 4 . *Under Assumptions 1-2 and 5-7, if the consumption is capital intensive, there exists* $\underline{\varepsilon}_{rk}$*,* $\overline{\varepsilon}_{rk}$*,* $\overline{b}$ *and* $\overline{\theta}$*, such that any steady state is locally indeterminate if one of the following condition is satisfied:*
i] $\alpha \in \left(\frac{1}{2}, \overline{\alpha}\right)$, $\nu \in \left(\underline{\nu}, \nu_2\right)$, $b > b_3$, $\theta < \overline{\theta}$, $\varepsilon_{rk} \in \left(\underline{\varepsilon}_{rk}, \overline{\varepsilon}_{rk}\right)$ *and* $\gamma \in \left(\gamma^{\mathcal{T}}, \gamma^{\mathcal{H}}\right)$*;*
ii] $\alpha < \underline{\alpha}$, $\nu \in \left(\underline{\nu}, \nu_0\right)$, $b \in (b_3, b_2)$, $\varepsilon_{rk} < \varepsilon_{rk2}$ *and* $\gamma > \gamma^{\mathcal{F}}$.

Proof: See Appendix 7.5. ■

It has to be noting that in Proposition $4 - i]$ $\gamma^{\mathcal{H}}$ correspond to a Hopf bifurcation value leading to quasi-periodic cycle which are locally indeterminate in a right neigborhood of $\gamma^{\mathcal{H}}$, while $\gamma^{\mathcal{T}}$ is a Transcritical bifurcation value inducing the existence of a second steady state which is locally unstable in a right neighborhood of $\gamma^{\mathcal{T}}$. Moreover, in Proposition $4 - ii]$ $\gamma^{\mathcal{F}}$ correspond to a Flip bifurcation value brought about to period-two cycle which are locally indeterminate in a right neigborhood of $\gamma^{\mathcal{F}}$. To conclude whether or not that trade can have a stabilizing or a destabilizing effect, it is necessary to compare the simulation result of Proposition 2 and 4.

4.1 A CES-Leontief economy

The theoretical result is discussed by a numerical exercise to emphasize the effect of a common market under free trade regime. To illustrate the two cases of Proposition 4, let assume that North and South are characterized by the following technology (21)-(22). The world social production is given by

$$\tau(\kappa_t, \zeta_t, l_t) = T\left(K^N, Y^N, L^N\right) = T\left(K^S, Y^S, L^S\right) \tag{34}$$

Under Assumptions 1 and 7, similar technology and free trade in goods, factor price equalization holds at steady state:

$$\begin{aligned} r(\kappa,\zeta,l) &= \tau_1(\kappa,\zeta,l) = T_1\left(K^N,Y^N,L^N\right) = T_1\left(K^S,Y^S,L^S\right), \\ p(\kappa,\zeta,l) &= -\tau_2(\kappa,\zeta,l) = -T_2\left(K^N,Y^N,L^N\right) = -T_2\left(K^S,Y^S,L^S\right), \\ w(\kappa,\zeta,l) &= \tau_3(\kappa,\zeta,l) = T_3\left(K^N,Y^N,L^N\right) = T_3\left(K^S,Y^S,L^S\right). \end{aligned} \tag{35}$$

Furthermore, as technology are identical between North and South, the relative capital intensity difference is the same across countries:

$$b = b^N = b^S \tag{36}$$

Finally let derive the propensity to consume of young agent in the world economy α^w and the share of capital in the world economy s^w:

$$\alpha^w = 1 - \frac{p\kappa}{w},\; s^w = \frac{r\kappa}{\tau(\kappa,\zeta)+py} \tag{37}$$

The critical bounds of Proposition 4 are derived by the parameter value used in the simulation of the autarky regime, in order to know the effect of trade in consumption goods on the occurence of endogenous fluctuations of North and South.

4.2 An efficient endogenous fluctuations

Efficient endogenous fluctuations occurs in North and South under the following set of parameters $\mu = 0.999865, \eta = 0.022, \theta_c = 0.125, \rho = 3, k = 0.45, y = 0.9, \nu = 2, \gamma = 2$. We use the same set of parameters to simulate the world market business cycle, i.e. the case $i\rfloor$ in Proposition 4. In doing so, we obtain similar numerical result between autarky and trade regime and a different interval for the bifurcation parameter: the elasticity of the intertemporal substitution in the consumption γ. Indeed, in autarky regime, the admissible range is $\gamma \in \left(1.69 \equiv \gamma^{\mathcal{T}}_{Aut}, 2.55 \equiv \gamma^{\mathcal{H}}_{Aut}\right)$ while in the trade regime $\gamma \in \left(1.7 \equiv \gamma^{\mathcal{T}}_{Free}, 2.54 \equiv \gamma^{\mathcal{H}}_{Free}\right)$. Then the following Proposition holds:

Proposition 5 . *Under Assumptions 1-7 and for given parameters* $\alpha^N, \alpha^S, s^w, \varepsilon_{rk}, \gamma^{\mathcal{T}}_{Aut}, \gamma^{\mathcal{T}}_{Free}, \gamma^{\mathcal{H}}_{Aut}$ *and* $\gamma^{\mathcal{H}}_{Free}$*, North and South are in a free trade* *regime, then the following result holds:*

i⌋ if $\gamma \in \left(1, \gamma^{\mathcal{T}}_{Aut}\right) \cup \left(\gamma^{\mathcal{H}}_{Aut}, \infty\right)$ *North and South are locally determinate;*

ii⌋ if $\gamma \in \left(\gamma^{\mathcal{T}}_{Aut}, \gamma^{\mathcal{T}}_{Free}\right) \cup \left(\gamma^{\mathcal{H}}_{Free}, \gamma^{\mathcal{H}}_{Aut}\right)$*, North and South are subjected to efficient endogenous fluctuations in autarky but are stabilized in free trade regime;*

iii⌋ if $\gamma \in \left(\gamma^{\mathcal{T}}_{Free}, \gamma^{\mathcal{H}}_{Free}\right)$ *North and South are subjected to efficient endogenous fluctuations in autarky and in free trade regime.*

This Proposition emphasizes two elements. First, it shows that if North and South are subjected to local uniqueness of the equilibrium in autarky they have the same behavior in the trade regime. Second, it shows that if North and South are subjected to efficient endogenous fluctuations in autarky they could either stabilized or still affected by efficient endogenous fluctuations in the trade regime. Trade regime has a stabilizing effect for both countries. This result is similar to the main conclusion of Nishimura and Shimomura [21]. Considering a continuous Heckscher-Ohlin model with cobb-douglas technology and sector-specific externality. They show that if both countries are characterized by endogenous fluctuations in autarky regime then the world market is also characterized by endogenous fluctuations.

4.3 An inefficient endogenous fluctuations

Inefficient endogenous fluctuations occurs in North and South under the following set of parameters $\mu = 0.7, \eta = 0.2575, \theta_c = 0.795, \rho_0 = 1, k = 0.65, y = 0.735, \nu = 1.1308, \gamma = 460$. We use the same set of parameters to simulate the world market business cycle, i.e. the case *ii*⌋ in Proposition 4. In doing so, we obtain similar numerical result between autarky and trade regime and a different interval for the bifurcation parameter: the elasticity of the intertemporal substitution in the consumption γ. Indeed, in autarky regime, the admissible range is $\gamma > 264 \equiv \gamma^{\mathcal{F}}_{Aut}$ while in the trade regime $\gamma > 309 \equiv \gamma^{\mathcal{F}}_{Free}$. Then the following Proposition holds:

Proposition 6 . *Under Assumptions 1-7 and for given parameters* $\alpha^N, \alpha^S, s^w, \varepsilon_{rk}, \gamma^{\mathcal{F}}_{Aut}$ *and* $\gamma^{\mathcal{F}}_{Free}$*, North and South are in a free trade regime,* *then the following result holds:*

i⌋ if $\gamma \in \left(1, \gamma^{\mathcal{F}}_{Aut}\right)$ *North and South are locally determinate;*

ii⌋ if $\gamma \in \left(\gamma^{\mathcal{F}}_{Aut}, \gamma^{\mathcal{F}}_{Free}\right)$*, North and South are subjected to inefficient endogenous fluctuations in autarky but are stabilized in free trade regime;*

iii⌋ if $\gamma > \gamma^{\mathcal{F}}_{Free}$ *North and South are subjected to inefficient endogenous fluctuations in autarky and in free trade regime.*

As soon as inefficient endogenous fluctuations are considered, similar conclusions are obtained.

5 Stabilization policy

The result of the previous section emphasizes that the world economy may exhibits efficient endogenous fluctuations, it raises the necessity of a stabilization policy directed by the policymarker that would stabilize the economy. In a context of an efficient endogenous fluctuations, the introduction of a fiscal policy based on transfers and taxes could simultaneously stabilize the economy and move the equilibrium to the optimal steady state which provide an equal level of utility to all generations[13]. In the present section, it is proven that a fiscal policy exists under the assumption that agents and policymaker do not make forecasting mistakes.

Consider that the policymaker buy goods, levies taxes and makes transfers under balanced budget rule. Let G_t be the flows of consumption goods which is bought, $\beta_{g,t} > 0 (< 0)$ the taxes (transfers) on the income of period of life g, $g = \{c, d\}$, for generation t. In country i, $i = \{N, S\}$, the intertemporal maximization problem (38) turns to:

$$\max_{C^i_t, D^i_{t+1}} \left\{ U\left(C^i_t, \frac{D^i_{t+1}}{\Gamma}\right) \mid w_t + \beta_{c,t} = \pi_{c,t} C^i_t + \phi^i_t, \pi_{d,t+1} D^i_{t+1} = R_{t+1}\phi_t + \beta_{d,t+1} \right\} \tag{38}$$

The optimal saving function becomes:

$$\phi^i_t = \frac{\Gamma^i\left(w_t + \beta_{c,t}\right)\left(\frac{\delta^i R_{t+1}\Pi_{c,d}}{\Gamma^i}\right)^{\gamma} - \beta_{d,t+1}\Pi_{c,d}}{R_{t+1}\Pi_{c,d} + \Gamma^i\left(\frac{\delta^i R_{t+1}\Pi_{c,d}}{\Gamma^i}\right)^{\gamma}} \tag{39}$$

[13]See Reichlin [29].

where $\Pi_{c,d} = \frac{\pi_{c,t}}{\pi_{d,t+1}}$ Consider Proposition 1, the scaling parameter Γ^i is written as:

$$\Gamma^i(k,y) = R(k,y)\Pi_{c,d}\left(\frac{kp(k,y)}{\delta^\gamma\left[w(k,y)-k(k,y)p(k,y)\right]}\right)^{\frac{1}{1-\gamma}} \tag{40}$$

where $r(k,y) = T_1(k,y),\ p(k,y) = -T_2(k,y),\ w(k,y) = T_3(k,y)$. Let,

$$\begin{aligned}\beta^o_{t+1} = kp(k,y)&\left[R(k,y)\Pi_{c,d} + \Gamma^i(k,y)\left(\frac{\delta^i R(k,y)\Pi_{c,d}}{\Gamma^i}\right)^\gamma\right]\left(\frac{R_{t+1}}{\Pi_{c,d}R(k,y)}\right)^\gamma \\ &-k_{t+1}p_t\left[R_{t+1}\Pi_{c,d} + \Gamma^i(k,y)\left(\frac{\delta^i R_{t+1}\Pi_{c,d}}{\Gamma^i}\right)^\gamma\right]\end{aligned} \tag{41}$$

$$\beta_{c,t} = w(k,y) - w_t \tag{42}$$

where $R(k,y) = \frac{r(k,y)}{p(k,y)}$

Plugging $\beta_{c,t} = \widehat{\beta}_{c,t},\ \beta_{d,t+1} = \widehat{\beta}_{d,t+1},\ \beta_{c,t+1} = \beta_{d,t+2} = 0$ into (39) gives

$$\phi^i_t = \frac{\Gamma^i w(k,y)\left(\frac{\delta^i R(k,y)\Pi_{c,d}}{\Gamma^i}\right)^\gamma}{R(k,y)\Pi_{c,d} + \Gamma^i\left(\frac{\delta^i R(k,y)\Pi_{c,d}}{\Gamma^i}\right)^\gamma} \tag{43}$$

It follows that if agents believe the announced policy rule, they will expect the optimal steady state to hold in the future. This expectation in turn drives the system to the steady state and keeps it there forever.

6 Concluding remarks

This paper considers the effect of international trade on the co-movement of endogenous business cycle in two-large countries Heckscher-Ohlin free trade equilibrium. Our main result shows that market integration can be a source of stability at the world level. Indeed, considering a common market, in which patient country are net exporters of the capital intensive good, we have proved that endogenous fluctuations can occur at the world level once trade opens when endogenous fluctuations occur in autarky for each countries. In this case we have shown that the scope of endogenous fluctuations in the trade regime is smaller than autarky regime, thus market integration has a stabilizing effect. Finally, we prove that some fiscal policy rules can prevent the existence of business-cycle fluctuations in the world economy by driving it to the optimal steady state as soon as it is announced.

7 Appendix

7.1 Proof of Proposition 1

Let $\left(k^{i*}, y^{i*}\right) \in \left(0, \overline{k}^i\right) \times \left(0, \overline{k}^i\right)$ and $\Pi^i_{c,d} = \Theta^i \frac{p^{i*\theta_d}}{p^{i*\theta_c}}$. Solving the equation (16) with respect to Γ yields to:

$$\Gamma^i\left(k^{i*}, y^{i*}, \gamma^i\right) = \Pi^i_{c,d}\left[\frac{\left(\delta^i\right)^{\gamma}\left(s^i\right)^{\gamma-1}\alpha^i}{\left(1-s^i\right)^{\gamma-1}\left(1-\alpha^i\right)^{\gamma}}\right]^{\frac{1}{\gamma-1}} > 0 \tag{44}$$

Solve the equation (17) with respect to θ_d yields to:

$$\theta^i_d\left(k^{i*}, y^{i*}\right) = 1 + \frac{\underline{\nu}^i - \nu^{i*}}{R^i} \tag{45}$$

$\left(k^{i*}, y^{i*}\right)$ is a normalized steady state if and only if $\Gamma^i = \Gamma^i\left(k^{i*}, y^{i*}, \gamma^i\right)$ and $\theta^i_d = \theta^i_d\left(k^{i*}, y^{i*}\right)$. ■

7.2 Proof of Proposition 2

The two difference equations (13)-(14) are linearized in the neighborhood of the normalized steady state. In our setting, the existence of local indeterminacy occurs if the characteristics roots associated with the linearization around the normalized steady state are less than 1 in absolute value. Let us denote two critical bounds on ν^i which appears to be important for the stability properties of the normalized steady state $\left(k^{i*}, y^{i*}\right)$:

$$\nu^i_0 = \frac{1}{1-\alpha^i}, \ \nu^i_1 = \frac{1-\alpha^i\theta_c}{\left(1-\alpha^i\right)\left(1-s^i\right)}. \tag{46}$$

Under Assumptions 1-3, the characteristic polynomial is defined by $\mathscr{P}(\lambda) = \lambda^2 - \lambda\mathscr{T} + \mathscr{D}$ where:

$$\mathscr{T}^i = \frac{1+\alpha^i(\gamma-1)\varepsilon^i_{rk}\left\{\left[1-b^i\left(\nu^i-\underline{\nu}^i\right)\right]^2+\theta_c^2 b^{i2} R^i\right\}+\varepsilon^i_{rk} b^i\left(\nu^i_1-\nu^i+R^i b^i\alpha^i\theta_c\nu^i\right)}{\left[1-b^i\left(\nu^i-\underline{\nu}^i\right)\right]\alpha^i(\gamma-1)b^i\theta_c\varepsilon^i_{rk}} \tag{47}$$

$$\mathscr{D}^i = \frac{s^i\left\{1-b^i\left[\nu-{}^i\nu^i_0-\frac{\theta_c\alpha^i(\gamma-1)}{1-\alpha^i}\right]\right\}}{\left(1-s^i\right)b^i\theta_c\alpha^i(\gamma-1)} \tag{48}$$

Following Grandmont et al. [17], we study the local dynamics properties by analysing the trace $\mathscr{T}^i$ and the determinant $\mathscr{D}^i$. This methodology allows to analyse the variation of the trace $\mathscr{T}^i$ and the determinant $\mathscr{D}^i$ by choosing a bifurcation parameter. In this setting, the bifurcation parameter choosen is the elasticity of intertemporal substitution in consumption γ^i. Then the variation of the trace $\mathscr{T}^i$ and the determinant $\mathscr{D}^i$ in the $(\mathscr{T}^i, \mathscr{D}^i)$ plane will be studied as γ^i evolves continuously within $(1, +\infty)$. The relationship between $\mathscr{T}^i$ and $\mathscr{D}^i$ is given by a half-line $\Delta^i\left(\mathscr{T}^i\right)$ which is characterized from the consideration of its extremities (Figure 1). The starting point is the couple $(\lim_{\gamma^i\to+\infty} \mathscr{T}^i \equiv \mathscr{T}^i_\infty, \lim_{\gamma^i\to+\infty} \mathscr{D}^i \equiv \mathscr{D}^i_\infty)$, while the end point is the couple $(\lim_{\gamma^i\to 1} \mathscr{T}^i \equiv \mathscr{T}^i_1, \lim_{\gamma^i\to 1} \mathscr{D}^i \equiv \mathscr{D}^i_1)$.

$\Delta^i\left(\mathscr{T}^i\right)$ is obtained from the two difference equations (13)-(14), solving $\mathscr{T}^i$ and $\mathscr{D}^i$ with respect to $\alpha\left(\gamma^i - 1\right)$ yields to the following relationship:

$$\Delta^i(\mathscr{T}^i) = \mathscr{D}^i_\infty + \mathscr{S}^i(\mathscr{T}^i - \mathscr{T}^i_\infty) \tag{49}$$

where the slope $\mathscr{S}^i$, $\mathscr{D}^i_\infty$ and $\mathscr{T}^i_\infty$ are:

$$\mathscr{S}^i = \frac{s\varepsilon_{rk}[1-b(\nu-\nu_0)]\left[1-b\left(\nu-\underline{\nu}\right)\right]}{(1-s)\{1+\varepsilon_{rk}b(\nu_1-\nu+Rb\alpha\theta_c\nu)\}},\ \mathscr{D}^i_\infty = \frac{s}{(1-\alpha)(1-s)},\ \mathscr{T}^i_\infty = \frac{\left[1-b\left(\nu-\underline{\nu}\right)\right]^2+\theta_c^2b^2R}{\left[1-b\left(\nu-\underline{\nu}\right)\right]b\theta_c} \tag{50}$$

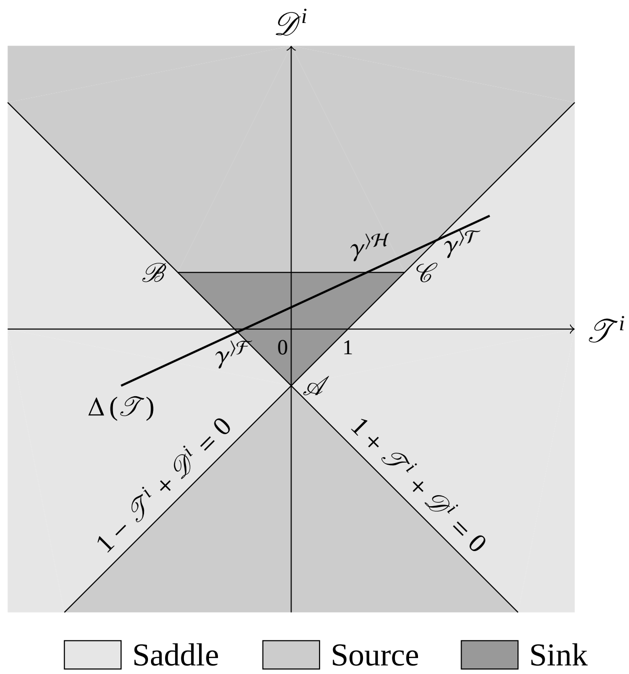


Figure 1: Stability triangle and $\Delta^i\left(\mathscr{T}^i\right)$ segment.

Let us compute $\mathscr{P}^i_\infty(-1)$:

$$\mathscr{P}^i_\infty(-1) = 1 + \mathscr{T}^i_\infty + \mathscr{D}^i_\infty = \frac{\left[1-b^i\left(v^i-v^i_2\right)\right]\left[1-b^i\left(v^i-v^i_3\right)\right]}{\left[1-b^i\left(v^i-\underline{v}^i\right)\right]b^i\theta_c} > 0 \tag{51}$$

where:

$$v^i_2 = \frac{1+\left(1-2\alpha^i\right)\theta_c}{\left(1-\alpha^i\right)},\ v^i_3 = \frac{1-\left(\alpha^i-\overline{\alpha}^i\right)\theta_c}{\left(1-\alpha^i\right)},\ \overline{\alpha}^i = \frac{s^i}{1-s^i}. \tag{52}$$

Let consider the two cases of Proposition 2:

$i\rfloor$ Consider that $b^i < 0$ and $\alpha^i \in \left(\frac{1}{2}, \frac{s^i}{1-s^i}\right)$ so that $\mathscr{D}^i_\infty > 1$, $\mathscr{T}^i_\infty < 0$ and $\mathscr{P}^i_\infty(1) > 0$. Let assume that $\theta_c < \overline{\theta}$ implying that $v^i_0 < v^i_1$ and $b^i_1 < b^i_0$, where:

$$\underline{\theta} = \min\left\{\frac{s^i}{\alpha+\left(1-2\alpha^i\right)\left(1-s^i\right)}, \frac{s^i}{\alpha^i+s^i-\alpha^i\left(1-s^i\right)}\right\},\ b^i_1 = -\frac{v^i_1-v^i}{R^i\alpha^i v^i\theta_c},\ b^i_0 = \frac{1}{v^i-v^i_0} \tag{53}$$

Under $v^i \in \left(\underline{v}^i, v^i_0\right)$ and $b^i > \tilde{b}^i$ we get $\mathscr{D}^i_1 = -\infty$ and $\mathscr{P}^i_\infty(-1) = 1+\mathscr{T}^i_\infty+\mathscr{D}^i_\infty <$ 0.

$$\tilde{b}^i = \max\left\{-\frac{1-\alpha^i}{\alpha^i+\left(1-2\alpha^i\right)\theta_c}, -\frac{1-\alpha^i}{\alpha^i+\left(\alpha^i+\frac{s^i}{1-s^i}\right)\theta_c}\right\} \tag{54}$$

Finally, $\mathscr{T}^i_1 = +\infty$ if and only if $v^i < v^i_1$, $b^i > b^i_1$ and $\varepsilon^i_{rk1} > \varepsilon^i_{rk1}$ with

$$\varepsilon^i_{rk1} = -\frac{1}{R^i b^i\alpha^i\theta_c v^i\left(b^i-b^i_1\right)} \tag{55}$$

Using the expression of $\mathscr{T}^i$ and $\mathscr{D}^i$ allows to show that when $\mathscr{D}^i = 1$, $\mathscr{T} > -2$ if and only if:

$$1 + \varepsilon^i_{rk}\underbrace{\left\{\left[\alpha^i(\gamma-1)\right]|_{\mathscr{D}=1}\ \mathscr{P}^i_{-2}\left(b^i\right) + b^i\left(v^i_1 - v^i + R^i b^i\alpha^i\theta_c v^i\right)\right\}}_{=\Xi} \leq 0 \tag{56}$$

where:

$$\mathscr{P}^i_{-2}\left(b^i\right) = \theta_c^2 R^i b^{i,2} + 2\theta_c b^i\left[1-b^i\left(v^i-\underline{v}^i\right)\right] + \left[1-b^i\left(v^i-\underline{v}^i\right)\right]^2 > 0 \tag{57}$$

and

$$\left[\alpha^i(\gamma-1)\right]|_{\mathscr{D}^i=1} = \frac{s^i\left(v^i-v^i_0\right)\left(b^i_0-b^i\right)}{b^i\theta_c\left(\underline{\alpha}^i-\alpha^i\right)} \tag{58}$$

Assume that $v^i = v^i_0 - dv^i$ with $dv^i > 0$ small, it follows that Ξ in (56) is negative. As a result $\mathscr{T} \geq -2$ if and only if:

$$\underline{\varepsilon}^i_{rk} = -\frac{1}{\left[\alpha^i(\gamma-1)\right]|_{\mathscr{D}^i=1}\mathscr{P}^i_{-2}\left(b^i\right)+R^i b^i\alpha^i\theta_c v^i\left(b^i-b^i_1\right)} \tag{59}$$

Moreover there exist $\underline{\varepsilon}^i_{rk} < \overline{\varepsilon^i}_{rk}$ such that $\mathcal{T}^i = 2$ when $\varepsilon^i_{rk} = \overline{\varepsilon}^i_{rk}$. Therefore, $\mathcal{T}^i \in (-2,2)$ as long as $\varepsilon^i_{rk}\left(\underline{\varepsilon}^i_{rk}, \overline{\varepsilon}^i_{rk}\right)$. Denote $d\tilde{\nu}^i$ the value of $d\nu^i$ such that the denominator of (59) is equal to zero. The maximal admissible value of $d\nu^i$ is such that $d\overline{\nu}^i = \min\left\{d\tilde{\nu}^i, \nu^i_0 - \underline{\nu}^i\right\}$. It follows that when $\nu^i \in \left(\tilde{\nu}^i, \nu^i_0\right)$ with $\tilde{\nu}^i = \nu^i_0 - d\nu^i$, $\theta_c < \overline{\theta}$, $b^i \in \left(\underline{b}^i, \overline{b}^i\right)$ and $\epsilon^i_{rk} \in \left(\underline{\varepsilon}^i_{rk}, \overline{\varepsilon}^i_{rk}\right)$, $\Delta^i(\mathcal{T}^i)$ is between $\mathcal{T}^i \in (-2,2)$ when $\mathcal{D}^i = 1$. Result follows.

$ii\rfloor$ $\alpha < \underline{\alpha}^i$ and $b^i < 0$ so that $\mathcal{D}^i_\infty \in (0,1)$, $\mathcal{T}^i_\infty < 0$ and $\mathcal{P}^i_\infty(1) > 0$. Under $\nu^i \in \left(\nu^i_0, \nu^i_3\right)$ and $b^i \in \left(b^i_3, b^i_2\right)$, we get $\mathcal{P}^i_\infty(-1) > 0$, $\mathcal{T}^i_1 = -\infty$ and $\mathcal{D}^i_1 = -\infty$. A Flip bifurcation occurs if $\mathcal{T}^i < 0$ when $\mathcal{D}^i = -1$. Under these conditions, one get from $\Delta^i\left(\mathcal{T}^i\right)$ (47)-(48) that:

$$\mathcal{T}^i = -\frac{1+\mathcal{D}^i_\infty}{\mathcal{S}^i} + \mathcal{T}^i_\infty \tag{60}$$

Direct computation shows that $\mathcal{T}^i < 0$. The half-line $\Delta^i\left(\mathcal{T}^i\right)$ crosses the line $1 + \mathcal{T}^i_\infty + \mathcal{D}^i_\infty$ if and only if $\mathcal{S}^i > 1$ which implies that $\varepsilon^i_{rk} > \varepsilon^i_{rk2}$.

7.3 Proof of Corollary 1

This proof is based upon similar arguments of Proof of Proposition 1. Using the equations (27)-(28), the three scaling parameters are:

$$\Gamma^{N*} = R\left\{\frac{\delta^{N\gamma}\left[\left(2-\frac{kp}{w}\right)\delta^{S\gamma}R^{\gamma-1}-1\right]}{\left(1+\delta^{S\gamma}\right)\frac{kp}{w}-\delta^{S\gamma}R^{\gamma-1}}\right\}^{\frac{1}{\gamma-1}} \tag{61}$$

$$\Gamma^{S*} = 1 \tag{62}$$

$$\theta_d = 1 + \frac{1-s^w}{2s^w}\left[2 - \left(\alpha^N + \alpha^S\right)\theta_c - \left(2 - \alpha^N - \alpha^S\right)\nu\right] \tag{63}$$

(κ^*, ζ^*) is a normalized steady state if and only if $\Gamma^N = \Gamma^N(\kappa^*, \zeta^*, \gamma)$, $\Gamma^S = 1$ and $\theta_d = \theta_d(\kappa^*, \zeta^*)$. ■

7.4 Proof of Proposition 3

The proof is based upon arguments similar to Galor and Lin [15]. Let $\delta^N > \delta^S$, as a result the North country is more patient than the South country, i.e. $\phi^S < \phi^N$.

Let express k^S and y^S in terms of k^N and y^N so that

$$k^S = k^N + dk, y^S = y^N + dy, \tag{64}$$

with $dk, dy > 0$.

The world market condition for mixed good imply the following relationship:

$$2y^N + \frac{dy}{dk}dk = \xi + 2k^N + dk \tag{65}$$

with $\xi = z^N + z^S$. From the Rybczynscki effect, (65) can be express as:

$$\frac{1-b}{b}dk = \xi + 2\left(k^N - y^N\right) \gtrless 0 \Leftrightarrow b \gtrless 0 \tag{66}$$

Then the more patient country exports the capital intensive good. ∎

7.5 Proof of Proposition 4

From (12), one gets:

$$\alpha^{i'} = -\frac{\alpha^i(\gamma-1)\left(1-\alpha^i\right)\Gamma^i}{R\Pi_{c,d}}, i = N, S \tag{67}$$

Under Assumption 1, the first order conditions of firm's profit maxmization problem (25) yield to[14]: $\tau_{12} = -\tau_{11}b$, $\tau_{22} = \tau_{11}b^2$, $\tau_{31} = -\tau_{11}a$ and $\tau_{32} = \tau_{11}ab$. $\mathscr{T}$ and $\mathscr{D}$ are obtained from the linearization of the two difference equations (27)-(28) in the neighborhood of the steady state. Let define $A^N = \frac{1-\alpha^N}{2-\alpha^N-\alpha^S}$ and $A^S = \frac{1-\alpha^S}{2-\alpha^N-\alpha^S}$. To pursue the analysis, we linearize the two difference equations (27)-(28) in the neighborhood of the steady state. Let us define two critical bounds on ν which appears to be important for the sign of the trace $\mathscr{T}$ and the determinant $\mathscr{D}$:

$$\nu_0 = \frac{2}{2-\alpha^N-\alpha^S}, \ \nu_1 = \frac{2-\left(\alpha^N+\alpha^S\right)\theta_c}{\left(2-\alpha^N-\alpha^S\right)(1-s^w)}. \tag{68}$$

Under Assumptions 1-2 and 5-7, the characteristic polynomial is defined by $\mathscr{P}(\lambda) = \lambda^2 - \lambda\mathscr{T} + \mathscr{D}$ where:

$$\mathscr{T} = \frac{2+2\varepsilon_{rk}\left(\alpha^N A^N+\alpha^S A^S\right)(\gamma-1)\left\{\left[1-b\left(\nu-\underline{\nu}\right)\right]^2+\theta_c^2 Rb^2\right\}+b\varepsilon_{rk}\left[\nu_1-2\nu+Rb\left(\alpha^N+\alpha^S\right)\theta_c\nu^w\right]}{2b\theta_c\varepsilon_{rk}\left(\alpha^N A^N+\alpha^S A^S\right)\left[1-b\left(\nu-\underline{\nu}\right)\right](\gamma-1)} \tag{69}$$

[14]See Benhabib and Nishimura [5] and Bosi et al. [7].

$$\mathcal{D} = \frac{s^w\left\{1-b\left[\nu-\nu_0+\frac{2\theta_c\left(\alpha^N A^N+\alpha^S A^S\right)(\gamma-1)}{2-\alpha^N-\alpha^S}\right]\right\}}{b\theta_c(1-s^w)\left(\alpha^N A^N+\alpha^S A^S\right)(\gamma-1)} \tag{70}$$

Let consider the two cases of Proposition 4:

$i\rfloor$ Consider that $b<0$ and $\alpha \in \left(\frac{1}{2}, \overline{\alpha}\right)$ so that $\mathcal{D}_\infty > 1$, $\mathcal{T}_\infty < 0$ and $\mathcal{P}_\infty(1) > 0$. Let assume that $\theta_c < \overline{\theta} = \frac{1}{1+\alpha}$ implying that

$$\begin{array}{c} \underline{\nu} < \nu_0 < \nu_2 < \nu_1 \\ b_1 < \overline{b} < \underline{b} < b_3 \end{array} \tag{71}$$

where

$$\begin{array}{c} \nu_2 = \frac{2+2\left(1-\alpha^N-\alpha^S\right)\theta_c}{\left(2-\alpha^N-\alpha^S\right)}, b_1 = -\frac{\nu_1-2\nu}{R\alpha\nu\theta_c} \\ \underline{b} = \frac{1}{1-\nu_0}, \quad \overline{b} = \frac{1}{1-\nu_2} \end{array} \tag{72}$$

Under $\nu \in \left(\underline{\nu}, \nu_2\right)$ and $b > b_3$ we get $\mathcal{D}_1 = -\infty$ and $\mathcal{P}_\infty(-1) < 0$. Finally, $\mathcal{T}_1 = +\infty$ if and only if $\nu < \nu_1$, $b > b_1$ and $\varepsilon_{rk1} > \varepsilon_{rk1}$ with

$$\varepsilon_{rk1} = -\frac{2}{Rb\left(\alpha^N+\alpha^S\right)\theta_c\nu(b-b_1)} \tag{73}$$

Using the expression of $\mathcal{T}$ and $\mathcal{D}$ allows to show that when $\mathcal{D} = 1$, $\mathcal{T} > -2$ if and only if:

$$1 + \varepsilon_{rk}\underbrace{\left\{[\alpha(\gamma-1)]\,|_{\mathcal{D}=1}\ \mathcal{P}_{-2}(b) + b\left(\nu_1 - 2\nu + Rb\left(\alpha^N+\alpha^S\right)\theta_c\nu\right)\right\}}_{=\Xi} \leq 0 \tag{74}$$

where:

$$\mathcal{P}_{-2}(b) = \theta_c^2 R b^2 + 2\theta_c b\left[1 - b\left(\nu - \underline{\nu}\right)\right] + \left[1 - b\left(\nu - \underline{\nu}\right)\right]^2 > 0 \tag{75}$$

and

$$[\alpha(\gamma-1)]\,|_{\mathcal{D}=1} = \frac{s^w(\nu-\nu_0)(b_0-b)}{b\theta_c\left(\underline{\alpha}^w-\alpha^w\right)} \tag{76}$$

Assume that $\nu = \nu_0 - d\nu$ with $d\nu > 0$ small, it follows that Ξ in (74) is negative. As a result $\mathcal{T} \geq -2$ if and only if:

$$\underline{\varepsilon}_{rk} = -\frac{1}{\left[\alpha(\gamma-1)\right]|_{\mathcal{D}=1}\mathcal{P}_{-2}(b)+Rb\left(\alpha^N+\alpha^S\right)\theta_c\nu(b-b_1)} \tag{77}$$

Moreover there exist $\underline{\varepsilon}_{rk} < \overline{\varepsilon}_{rk}$ such that $\mathcal{T} = 2$ when $\varepsilon_{rk} = \overline{\varepsilon}_{rk}$ with:

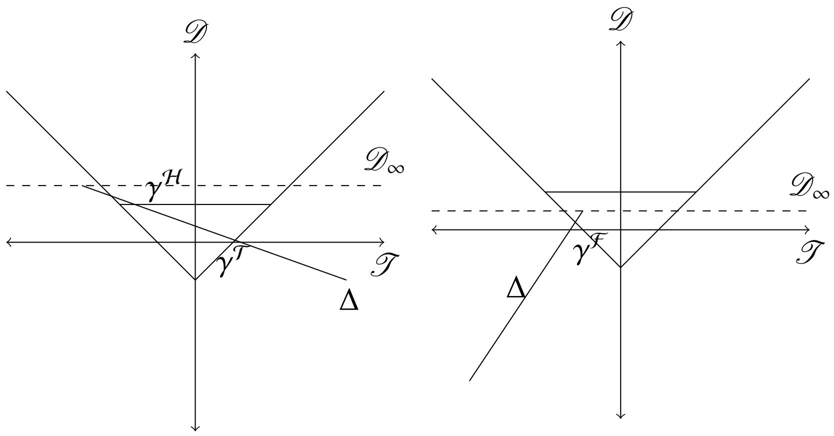
$$\begin{array}{c} \underline{\varepsilon}_{rk} = -\frac{1}{\left[\alpha(\gamma-1)\right]|_{\mathcal{D}=1}\mathcal{P}_{2}(b)+Rb\left(\alpha^N+\alpha^S\right)\theta_c\nu(b-b_1)} \\ \mathcal{P}_2(b) = \theta_c^2 R b^2 - 2\theta_c b\left[1 - b\left(\nu - \underline{\nu}\right)\right] + \left[1 - b\left(\nu - \underline{\nu}\right)\right]^2 > 0 \end{array} \tag{78}$$

Therefore, $\mathscr{T} \in (-2,2)$ as long as $\varepsilon_{rk}\left(\underline{\varepsilon}_{rk}, \overline{\varepsilon}_{rk}\right)$. Denote $d\tilde{\nu}$ the value of $d\nu$ such that the denominator of (77) is equal to zero. The maximal admissible value of $d\nu$ is such that $d\overline{\nu} = \min\left\{d\tilde{\nu}, \nu_0 - \underline{\nu}\right\}$. It follows that when $\nu \in (\tilde{\nu}, \nu_0)$ with $\tilde{\nu} = \nu_0 - d\nu$, $\theta_c < \overline{\theta}$, $b \in \left(\underline{b}, \overline{b}\right)$ and $\epsilon_{rk} \in \left(\underline{\varepsilon}_{rk}, \overline{\varepsilon}_{rk}\right)$, $\Delta(\mathscr{T})$ is between $\mathscr{T} \in (-2,2)$ when $\mathscr{D} = 1$. Result follows.

ii⌋ Let $\alpha < \underline{\alpha}$ and $b < 0$ so that $\mathscr{D}_\infty \in (0,1)$, $\mathscr{T}_\infty < 0$ and $\mathscr{P}_\infty(1) > 0$. Under $\nu \in \left(\underline{\nu}, \nu_0\right)$ and $b \in (b_3, b_2)$, we get $\mathscr{P}_\infty(-1) > 0$, $\mathscr{T}_1 = -\infty$ and $\mathscr{D}_1 = -\infty$. A Flip bifurcation occurs if $\mathscr{T} < 0$ when $\mathscr{D} = -1$. Under these conditions, one get from $\Delta(\mathscr{T})$ (49) that:

$$\mathscr{T} = -\tfrac{1+\mathscr{D}_\infty}{\mathscr{S}} + \mathscr{T}_\infty \tag{79}$$

Direct computation shows that $\mathscr{T} < 0$. The half-line $\Delta(\mathscr{T})$ does not cross the line $1 + \mathscr{T}_\infty + \mathscr{D}_\infty$ if and only if $\mathscr{S} < 1$ which implies that $\varepsilon_{rk} < \varepsilon_{rk2}$. This prove the second part of the proposition. ■



Hopf & Transcritical bifurcations. Flip bifurcation.

Figure 2: Local indeterminacy.

References

[1] Aloi, M. and T Lloyd-Braga (2010): “National Labor Markets, International Factor Mobility and Macroeconomic Instability ", *Economic Theory* 43, 431-456.

[2] Artis, M. J. and W. Zhang (1997): “International Business Cycles and the ERM: Is There a European Business Cycle?", *International Journal of Finance and Economics* 2, 1-16.

[3] Azariadis C. (1981): “Self Fulfilling Prophecies", *Journal of Economic Theory* 25, 380-396.

[4] Baxter, M. (1992): “Fiscal Policy, Specialization, and Trade in Two-Sector Model: The Return of Ricardo?", *Journal of Political Economy* 100, 713-744.

[5] Benhabib, J. and K. Nishimura (1985): “Competitive Equilibrium Cycles", *Journal of Economic Theory* 35, 284-306.

[6] Bianconi, M. (1995): “On Dynamic Real Trade Models", *Economics Letters* 47, 47-52.

[7] Bosi, S. Magris, F. and A. Venditti, (2005): “Competitive Equilibrium Cycles with Endogenous Labor", *Journal of Mathematical Economics* 41, 325-349.

[8] Buiter, W. H. (1981): “Time Preference and International Lending and Borrowing in an Overlapping-Generations Model", *Journal of Political Economy* 89, 769-797.

[9] Cass, D. and K. Shell (1983): “Do Sunspots Matter?", *Journal of Political Economy* 91, 193-227.

[10] Cremers, E. T. (2001): “General Equilibrium with Trade Balance and Real Interest Rate Parity", *Economic Theory* 17, 641-663.

[11] Cremers, E. T. (2005): “International Welfare and Trade", *Macroeconomic Dynamics* 9, 585-611.

[12] Drugeon, J.-P. (2004): “On Consumptions, Inputs and Outputs Substituabilities and the Evanescence of Optimal Cycles", *Journal of Difference Equations and Applications* 10, 473-487.

[13] Drugeon, J.-P. Nourry, C. and A. Venditti, (2010): “On Efficiency and Local Uniqueness in Two-Sector OLG Economies", *Mathematical Social Sciences* 59, 120-144.

[14] Galor, O. (1992): “A Two-Sector Overlapping-Generations Model: A Global Characterization of the Dynamical System", *Econometrica* 60, 1351-1386.

[15] Galor, O. and S. Lin (1997): “Dynamic Foundations for the Factor Endowment Model of International Trade", *Dynamics, Economic Growth, and International Trade* Chapter 5, 151-168.

[16] Ghiglino, C. (2007): “Trade, Redistribution and Indeterminacy", *Journal of Mathematical Economics* 10, 365-389.

[17] Grandmont, J.-M., Pintus, P. and R. de Vilder (1998): “Capital-Labor Substitution and Competitive Nonlinear Endogenous Business Cycles", *Journal of Economic Theory* 80, 14-59.

[18] Le Riche, A., Nourry, C. and A. Venditti (2012):“Efficient Endogenous Fluctuations in Two-Sector OLG Model", .

[19] Mountford, A. (1998): “Trade, Convergence and Overtaking", *Journal of International Economics* 46, 167-182.

[20] Naito, T. and L. Zhao (2009): “Aging, Transitional Dynamics, and Gains from Trade", *Journal of Economic Dynamics & Control* 33, 1531-1542.

[21] Nishimura, K., Shimomura, K. (2002): “Trade and Indeterminacy in a Dynamic General Equilibrium Model", *Journal of Economic Theory* 105, 244-260.

[22] Nishimura, K., Shimomura, K. (2006): “Indeterminacy in a Dynamic Two-Country Model", *Economic Theory* 29, 307-324.

[23] Nishimura, K., Venditti, A. and M. Yano (2006): “Endogenous Fluctuations in Two-Country Models", *Japanese Economic Review* 4, 516-532.

[24] Nishimura, K., Venditti, A. and M. Yano (2009): “Optimal Growth and Competitive Business Cycles under Decreasing Returns in Two-Country Models", *Review of International Economics* 17, 371-391.

[25] Nishimura, K., Venditti, A. and M. Yano (2010): “Expectation-Driven Fluctuations and Welfare Loss under Free Trade in Two-Country Models", *International Journal of Economic Theory* 6, 97-125.

[26] Nishimura, K. and M. Yano (1993): “Interlinkage in the Endogenous Real Business Cycles of International Economies", *Economic Theory* 3, 151-168.

[27] Nishimura, K. and M. Yano (1993): “Endogenous Real Business Cycles and International Specialization", in *Theory, Policy and Dynamics in International Trade* Cambridge University Press, Cambridge, UK.

[28] Nourry, C. and A. Venditti, “Endogenous Business Cycles in OLG Economies with Multiple Consumption Goods", *Macroeconomic Dynamics* forthcoming.

[29] Reichlin, P. (1986): “Equilibrium Cycles in Overlapping Generations Economy with Production", *Journal of Economic Theory* 40, 89-102.

[30] Reichlin, P. (1992): “Endogenous Fluctuations in a Two-Sector Overlapping Generations Economy", *In: J. Benhabib, Cycles and Chaos in Economic Equilibrium.* Princeton University Press.

[31] Shell, K. (1977): “Monnaie et Allocation Intertemporelle", mimeo, Séminaire d’Econométrie Roy-Malinvaud, Centre National de la Recherche Scientifique, Paris.

[32] Sim, N., and K.-W. Ho (2007): “Autarky Indeterminacy and Trade Determinacy", *International Journal of Economic Theory* 3, 151-168.

[33] Stiglitz, J.E. (1970): “Factor Price Equalization in a Dynamic Economy", *Journal of Political Economy* 78, 456-488.

[34] Venditti, A. (2005): “The Two Sector Overlapping Generations Model: A Simple Formulation", *Research in Economics* 59, 164-188.

[35] Woodford, M. (1986): “Stationary Sunspot Equilibria. The Case of Small Fluctuations Around a Deterministic Steady State", Mimeo, University of Chicago and New York University.