On the optimality of budget rules in a fiscal union (preliminary and incomplete)

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Abstract

This paper considers the welfare impact of balanced budget rules within a fiscal union where borrowing occurs at different levels of government. The analysis is conducted in an optimal policy context from a timeless perspective in a fiscal union made up of atomistic locales sharing an overarching federal government. I assume that governments at both levels have access only to distortionary taxes and single period debt (except when budget rules are in place). Fiscal resources can be moved through the union via intergovernmental transfers which bear a quadratic cost as a friction. Policy is considered both on a cooperative and a strategic basis. Under the benchmark calibration, local balanced budget rules have a minimal impact on welfare under cooperation and produce welfare gains in the strategic case by limiting inefficient borrowing. Federal debt is an irrelevant policy instrument under cooperation but is useful to offset inefficient local government behaviour under strategic interactions. (*JEL Codes: E62 F42 H72 H74.)*

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1. Introduction

The recent, ongoing global financial crisis and the subsequent deterioration in fiscal positions throughout the developed world has served to reignite the debate about the necessity of imposing some form of fiscal restrictions upon governments. In the European Union, the perception that fiscal profligacy lies at the root of financial difficulties in the Eurozone periphery was the core motivation behind the demands for fiscal restrictions amongst member states that were realised in the European Fiscal Compact. On the other side of the Atlantic, the debate to increase the US Federal debt ceiling, in Summer 2011, again resurrected the idea of a Federal balanced budget rule (BBR); while the those already imposed at the state-level have had a major impact in driving contractionary sub-federal fiscal policy in the United States.

From the point of view of many macro-economists the almost ubiquitous discussion point on fiscal rules of this type is their effect on output volatility. Restrictions prevent the operation of counter-cyclical fiscal policy - be it in the form of a classical tax-smoothing motive or discretionary fiscal stimulus in line with Keynesian thinking[2]. Stockman (2001) offers perhaps the most rigourous analysis of balanced budget rules from this theoretical perspective and finds that the cost in terms of policy stabilisation is equivalent to 1% of lifetime utility from consumption, in a closed economy calibrated to match the features of the United States.

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[2] I do not wish to insinuate that the academic debate over the merits of fiscal restrictions is cut and dried. That their are costs in terms of the inability to operate fiscal policy on a counter cyclical basis is not likely to be up for serious dispute, although the size of this effect may be. However, others have countered that the complexity fiscal policy objectives and a lack of transparency over its operation can manifest as an political games that lead to sub-optimal policy choices and deficit bias (Alesina and Perotti (1994), Persson and Tabellini (2002), Debrun and Kumar (2007)). In this regard, fiscal restrictions may limit the fluctuations in output as they prevent unsustainable policy choices and limit the variation of politically motivated fiscal shocks (Fatas and Mihov (2003, 2006)).

The empirical evidence, however, fails to find evidence of this cost; studies on the impact of BBRs generally do not find any significant effect on the volatility of output. Part of the reason maybe that the literature has often focused on budget rules at a level below the highest fiscal authority, primarily because that is where these rules operate in practise and there is statistical variation to exploit. American states, with their rich set of of budget restrictions, have proved a common testing ground (see, for example, Alesina and Bayoumi (1994), Levinson (1998), Canova and Pappa (2006), Krol and Svorny(2007), Bahaj (2011)) but other work has analysed the effect of rules on Swiss Cantons (Pujol and Weber (2003)) and Italian Municipalities (Grembi et al (2011)). The difficulty with these studies is that such sub-national fiscal entities do not operate in isolation. They are often part of an interlocking system of governments through which with resources and financing are mobile; a fiscal union in other words. It is less than clear how a BBR on a specific form of government will effect fiscal policy in such circumstances which may explain the lack of observed effects empirically. Understanding how borrowing restrictions work at different levels of government should be of interest to policymakers. In the US it should serve to underline that the lessons learnt from a long history of BBRs at the state-level may not generalise to federal policy. In the EU, fiscal union between member states may still be a distant prospect, but it is a potential outcome nonetheless; understanding how a transfer policy will interact with the new fiscal compact is of great interest.

In this paper, I set out to consider this issue within the context of optimal policy in a stylised macroeconomic model of a fiscal union. The private sector follows Gali and Monacelli's (2008) model of a closed union made up of continuum of atomistic local economies, linked by trade in a union wide consumption good, complete financial markets and a federal authority. However, I modify the role played by the policy authorities to be more in the spirit Stockman (2001). I dispense with monetary considerations by assuming flexible prices. Instead, the policy trade offs are of a tax-smoothing nature, I assume two levels of fiscal authority, federal and local, which have access only to distortionary taxes and an incomplete market for financial assets in the form of one period risk free debt.

Each local economy has exogenously given, wasteful, public consumption requirement which can be met either by local government spending or a transfer from the federal government. The key assumption here is that transfers are not frictionless and bear a quadratic cost. There is substantial evidence from the literature on fiscal federalism that one level of government cannot substitute seamlessly for another (see Oates (1999)), either because of the classical federal ideal that differing government levels have differing efficiencies over the provision of certain classes of public goods (see Musgrave (1959), Oates (1972))) or because direct transfers between governments are subject to political economy distortions, such as "the fly paper effect" whereby transfers are spent inefficiently (see Hines and Thaler (1995), Leduc and Wilson (2012) offer some recent evidence). I calibrate this friction to match the features of US transfers between federal and state governments. These assumptions generate policy tradeoffs as the union-wide policymaker, in the guise of a federal government, has an incentive not only to smooth inter-temporal fluctuations in distortionary taxes with borrowing but also to stabilise tax raters across the union via transfers. Neither of these objectives can be achieved perfectly due to the transfer friction and incomplete markets for government debt. Introducing BBRs worsens this tradeoff as borrowing is not available as a policy stabilising tool.

Using this framework I characterise an approximation of optimal policy in a co-operative fiscal union, following a *timeless perspective* as in Benigno and Woodford (2012), to serve as a benchmark case from which deviations are then considered. Specifically, it is possible to assess the optimality of different institutional settings regarding balanced budget rules, and ask does it matter which level of government is responsible for borrowing? And, linked to this, how do BBRs at different levels of government influence optimal policy and welfare? The framework is also sufficiently flexible to dispense with cooperative policy and consider how the results change are affected strategic behaviour between governments; that is to say, when the local fiscal authorities use their policy instruments with the objective of maximising local, as opposed to union wide, welfare. To keep the strategic cases tractable within the same framework, it is necessary to restrict attention to Markov perfect equilibria where the federal government acts as a Stackelburg leader. However, these assumptions are not unreasonable in the light of the institutional setup often observed in fiscal unions in practise, where transfers are often decided with a degree of automacy and credible commitments to punishment strategies are in short supply (see Bordo et al (2011)).

The key finding of this work, in line with the empirical evidence above, is that under the benchmark calibration imposing local balanced budget rules has an almost negligible impact on welfare when compared to a cooperative model where borrowing is allowed at all levels of government. Furthermore, by limiting local debt as a strategic policy tool, local BBRs are welfare enhancing relative to the strategic case with unrestricted borrowing. As federal transfers are the key policy tool in offsetting the inflexibility of local BBRs, these results are sensitive to the size of the quadratic cost to transfers. However, for the welfare ordering of policies to be altered the necessary size friction seems excessive when compared to the data. [3]In a cooperative environment federal balanced budget rules are not costly. Indeed, I show that under cooperation an appropriate redistribution of local debt can perfectly imitate the impact of a change in federal borrowing, which implies that federal BBRs are irrelevant. Under strategic interactions local governments will no longer behave in the necessary fashion to offset a lack of federal borrowing. Instead, the federal balance sheet becomes a useful policy instrument to offset inefficient variations in local debt levels; in the event of an idiosyncratic local shock, the federal government uses deficit financed tax cuts and transfers to encourage local governments that would otherwise over-borrow to cut their deficits. This effectively mutualises the debt across the union and limits the cost of variations in distortionary local taxes.

This intra-union tax smoothing motive, coupled with the transfer friction, also implies a unit root process in the transfers assuming no local BBR is present; these unit roots are alongside those present in tax rates and debt levels that are standard in models of this form. In order to limit fluctuations in distortionary taxes, a temporary, idiosyncratic shock is met partly by local borrowing and partly by transfers. Transfers are used to pay for part of this permanent new interest burden to the point where the benefit from smoothing taxes on an intra-union basis is equalised with the quadratic cost to transfers. I highlight this result because it is a common feature of fiscal unions that regions of the union that are net transfer recipients tend to be so almost perpetually. This channel via local borrowing offers a potential mechanism to explain why.

To my knowledge there have been no other attempts to generalise Stockman's (2001) result to open economy or union setting, which represents the main contribution of this paper. However, there are other related works of interest within the new open economy literature. Beetsma and Jensen (2005), Gali and Monacelli (2008), Ferrero (2009) all consider optimal cooperative fiscal policy within unions models with complete markets, their work does not include the role played by a union-wide policy fiscal authority but they include monetary policy features lacking here. Hjortso (2011) extends this field of research to consider the role that fiscal policy plays as a risk sharing tool when private markets are incomplete. Uhlig (2002) and Beetsma et al. (2001) both analyse the strategic interactions between local and union wide policy makers in a stylised linear quadratic framework. These models are not fully micro-founded but are richer than that presented here and thus able to better describe some of the drivers of strategic behaviour. Epifani and Gancia (2009), in a static setup, also discuss how strategic interaction by local fiscal authorities can lead to suboptimal fiscal outcomes; the mechanism is similar to here except it runs through a spending rather than

[3]The size of the friction required such that a cooperative economy where no union was preferable to one with a fiscal union and local BBRs implies that the correlation between idiosyncratic cyclical fluctuations in federal transfers and local output in the simulated economy would be a third of the equivalent correlation seen in the data for the average US state. The required transfer friction to make local borrowing preferable when there is strategic interactions is higher still.

taxation channel.

This paper also draws heavily on the optimal taxation literature: Barro's (1979) seminal tax smoothing paper underpins much of the optimal policy analysis. Generalisations of this tax-smoothing motive and how it responds to richer set fiscal instruments and different degrees of market access can be found in Lucas and Stokey (1983), Aiyagari et al (2002) and Golosov and Sargent (2012). Optimal policy analysis within an equivalent closed economy RBC model to the one presented here can be found in Chari et al (1994) or Benigno and Woodford (2006); the latter also finds that a linear-quadratic approximation (equivalent to the one used here) within such circumstances represents an accurate approximation.

The rest of this paper is organised as follows: section 2 sets the environment and describes the fiscal union model used. Section 3, describes the cooperative and strategic optimal policy problems. It also elaborates more on the tradeoffs faced by policymakers and the drivers behind the differences between strategic and cooperative policy. Section 4 discusses the calibration of the model. Section 5 presents impulse response analysis to provide intuition to how the economy responds to shocks under different institutional setups regarding fiscal rules and cooperative versus strategic interactions. Section 6 presents the quadratic welfare losses for the various optimal policy problems under consideration. The last section concludes.

2. The environment

Consider a union made up of a continuum, $i \in [0, 1]$, of small open economies. I refer to these economies simply as locales to indicate that the model is agnostic as to the exact form of the fiscal union, they could represent municipal governments in towns or cities, states or provinces in a federation or countries in an international union. The atomistic size of the locales imply that local policy decisions do not affect other members in the union such that aggregate, or federal, variables can be considered to be exogenous by local policymakers. There is a complete market for financial securities between locales but they are traded only by private agents.

Utility and Consumption

The representative household in locale i maximises utility given by:

$$V_0^i = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^i, N_t^i) \tag{1}$$

with period utility in a separable form:

$$U = log(C_t^i) - \frac{(N_t^i)^{1+\varphi}}{1+\varphi} \tag{2}$$

Where N_t^i is household labour supply. I assume that household consumption, C_t^i, is aggregated in a similar fashion to Gali and Monacelli (2008); C_t^i is a Cobb-Douglas aggregator over consumption of locale i's own good, $C_{i,t}^i$, and a basket of union-wide goods, $C_{F,t}^i$, combined in CES aggregator from all locales[4]:

$$C_t^i = \frac{(C_{i,t}^i)^{1-\alpha}(C_{F,t}^i)^{\alpha}}{(1-\alpha)\alpha},\ \alpha \in (0,1) \tag{3}$$

$$C_{F,t}^i = [\int_0^1 (C_{f,t}^i)^{\frac{\sigma-1}{\sigma}} df]^{\frac{\sigma}{\sigma-1}},\ \sigma > 1 \tag{4}$$

[4]Gali and Monacelli (2008) assume that $\sigma = 1$ which implies on a logarithmic aggregator. I dispense with this assumption in order for price fluctuations across locales to impact union wide consumption.

Where $C^i_{f,t}$ is locale $i's$ consumption of locale $f's$ goods. This specification is a useful artifice in both circumventing and exploiting the atomistic nature of the locales. Equation 4 implies that locale i in effect imports from itself when constructing its union-wide consumption basket but this quantity is infinitesimal and can be ignored. While the Cobb-Douglas aggregator, as is well known, implies that a fixed proportion of household consumption spending, $1-\alpha$, is spent on the home good. For any $\alpha \in (0,1)$ there is home bias in local consumption as local households consume local goods in a greater proportion than their share in union wide production (which is infinitesimal). This retains the link between local production and local consumption which would otherwise be eroded in the presence of complete financial markets and atomistic locales.

Defining the price of a good produced in locale f at time t as P^f_t, the household budget constraint can be written as:

$$P^i_t C^i_{i,t} + \int_0^1 P^f_t C^i_{f,t} dj df + E_t[Q_{t,t+1} D^i_{t+1}] \leq D^i_t + W^i_t N^i_t + X_t \tag{5}$$

Where the first two left hand side terms are respectively expenditure on locally produced and imported consumption goods. Households have access to a full set of contingent securities that span all states of nature and are traded across the union; accordingly, the third term reflects the price of a portfolio of such assets that have an expected nominal yield $E[D_{t+1}]$. Free trade in financial securities implies the portfolio price $Q_{t,t+1}$ cannot be locale specific. Included in this set of securities are one period, risk-free, nominal bonds issued by local governments to finance their deficits. The term D^i_t refers to the payoff on the complete portfolio securities that have been purchased in period $t-1$ and $W^i_t N^i_t$ is the gross labour income of the households. Firms producing in every locale have diffusely held equity such that the profits (denoted X_t and zero in equilibrium) are taken as exogenous when distributed to households. Using the standard optimisation conditions on equation 4, demand function from locale i for imports from locale f must follow:

$$C^i_{f,t} = \left(\frac{P^f_t}{P_t}\right)^{-\sigma} C^i_{F,t}, \ \forall i, f \in [0,1] \tag{6}$$

Where $P_t = [\int_0^1 P^{f\,1-\sigma}_t df]^{\frac{1}{1-\sigma}}$ is the union-wide price index and correspondingly is the aggregate import price index faced by any given locale i. Equation 3, implies that the CPI for an individual locale i is $P^i_{c,t} = (P^i_t)^{1-\alpha}(P_t)^{\alpha}$ and the optimal allocation between domestic and imported goods is given by:

$$(1-\alpha)P^i_{c,t}C^i_t = P^i_t C^i_{i,t}; \ \alpha P^i_{c,t} C^i_t = P_t C^i_{F,t} \tag{7}$$

Hence, the budget constraint can be rewritten as:

$$P^i_{c,t} C^i_t + E_t[Q_{t,t+1} D^i_{t+1}] \leq D^i_t + W^i_t N^i_t + X^i_t \tag{8}$$

Household optimal behaviour:

The first order conditions to the households problem are:

$$wrt\, N^i_t : \ \frac{W^i_t}{P^i_{c,t}} = (N^i_t)^{\varphi} C_{i,t} \tag{9}$$

$$wrt\, D^i_{t+1} \ \ \beta \left[\frac{P^i_{c,t} C_{i,t}}{P^i_{c,t+1} C_{i,t+1}}\right] = Q_{t,t+1} \tag{10}$$

Equating 10 over two locales i and j yields the standard dynamic risk sharing condition:

$$\left[\frac{P^i_{c,t}C_{i,t}}{P^i_{c,t+1}C_{i,t+1}}\right] = \left[\frac{P^j_{c,t}C_{j,t}}{P^j_{c,t+1}C_{j,t+1}}\right] \tag{11}$$

And the gross yield on a risk free nominal bond (R_t) satisfies:

$$\beta R_t E\left[\frac{P^i_{c,t}C_{i,t}}{P^i_{c,t+1}C_{i,t+1}}\right] = 1, \quad \forall i \in [0,1] \tag{12}$$

Finally, the households have a transversality condition which must satisfy:

$$lim_{T\to\infty}E_t\{Q_{t,T}D^i_T\} = 0 \tag{13}$$

Where $Q_{t,T} = \beta^{T-t}\left[\frac{P^i_{c,t}C_{i,t}}{P^i_{c,T}C_{i,T}}\right]$, the stochastic discount factor between period t and T.

2.1. Relative Variables

The bilateral terms of trade between locale i and locale f can be defined as:

$$S^i_{f,t} = \frac{P^f_t}{P^i_t} \tag{14}$$

And the locale's total terms of trade as:

$$S^i_t = \frac{P_t}{P^i_t} = [\int_0^1 (\frac{P^f_t}{P^i_t})^{1-\sigma}df]^{\frac{1}{1-\sigma}} = [\int_0^1 (S^i_{f,t})^{1-\sigma}df]^{\frac{1}{1-\sigma}} \tag{15}$$

This allows for the redefinition of 7 to obtain the a similar function for local consumer consumer prices:

$$P^i_{c,t} = P^i_t(S^i_t)^{\alpha} = P_t(S^i_t)^{\alpha-1} \tag{16}$$

We can also combine the terms of trade definition with equation 11 to obtain:

$$C_{i,t} = \gamma_i C_{f,t}(S^i_{f,t})^{1-\alpha} \tag{17}$$

The term γ_i is a function of the economy's initial conditions, under initial symmetry this can be set to 1. If we define total expenditure on the union wide basket as:

$$P_tC_{F,t} = [\int_0^1 (P_tC^i_{F,t})di] \tag{18}$$

Substitute in 7 to obtain:

$$P_tC_{F,t} = \alpha[\int_0^1 ((P^i_{c,t}C^i_t)di] \tag{19}$$

Using 11 and initial symmetry one can eliminate the integral:

$$P_tC_{F,t} = \alpha[(P^f_{c,t}C^f_t)] \tag{20}$$

Last, use 16 to obtain the relationship between union-wide imports, the local terms of trade and local consumption:

$$\frac{(S^f_t)^{1-\alpha}C_{F,t}}{\alpha} = C^f_t \tag{21}$$

The union wide good can be used as numeraire setting $P_t = 1$, this implies that $P_t^i = (S_t^i)^{-1}$ and attention only needs to be paid to variations terms of trade in each locale. However, it is worthwhile noting that while the appropriately aggregated union wide terms of trade must be equal to one:

$$[\int_0^1 (S_t^i)^{\sigma-1} di] = 1 \tag{22}$$

It is not necessarily the case that price of union-wide consumption is the same as the numeraire; local variations in the terms of trade are not all canceled in aggregation (with the exception of the case where $\sigma = 1$). [5] This implies that fluctuations in the terms of trade feeds into welfare.

2.2. Firms

This body of work is focused on the interaction between different levels of fiscal authority hence the the production sector is of only limited complexity. The representative firm locale i operates as a perfectly competitive price taking producer of locale $i's$ good and does not discriminate between locale, foreign or government purchasers. Firms hire labour from households and return profits via dividends from equity securities held diffusely by the union-wide household sector. Labour is immobile across locales and labour markets are perfectly segmented [6]. Firms also face an effective sales tax rate equivalent to $\tau^i + \tau^U$ from taxes levied by the local government and federal (union-wide) government respectively.[7] The production technology is linear in labour with locale specific productivity:

$$Y_t^i = A_t^i N_t^i \tag{24}$$

The representative firm maximises period profits of the form:

$$X_t^i = (1 - \tau_t^i - \tau_t^U) S_t^i Y_t^i - W_t^i \frac{Y_t^i}{A_t^i}$$

taking the local relative price level S_t^i and W_t^i as exogenous. This implies the following first order condition:

$$(1 - \tau_t^i - \tau_t^U) S_t^i = \frac{W_t^i}{A_t^i}$$

Substituting in 9, 16, 21 and 24 gives the following supply condition:

[5] To see this formally, define the union wide level of consumption, C_t, as the value that would deliver the utility from consumption for the average locale:

$$log(C_t) = \int_0^1 log(C_t^i) di$$

From the standard optimisation result with a logarithmic aggregator, this implies the union wide consumer price index can be expressed as:

$$P_{c,t} = exp(\int_0^1 log(P_{c,t}^i) di) = exp((\alpha - 1) \int_0^1 log(S_t^i) di) \tag{23}$$

Note that when the terms of trade are perfectly symmetric $P_{c,t} = P_t = 1$ and prices are perfectly equalised across all locales in the union. However, for asymmetric equilibria there is no guarantee that the two aggregate price levels align.

[6] See Woodford (2003) for a discussion of the equivalency between this formulation and other forms of labour market structure under complete markets.

[7] Taxing either labour income or sales (but not both) is equivalent in equilibrium in the setting considered here. Since firms are perfectly competitive, output is chosen in order to set marginal revenue equal to marginal cost; a linear sales tax lowers marginal revenue at the same rate that an income tax would raise marginal cost (this can be verified by inspecting equation 9). This implies that both taxes are equally distortionary. Both taxes also raise the same amount revenue. The perfectly competitive firms return all net (of tax) revenues in wage income; if only labour income taxes are applied all revenues are redistributed in wages are then are taxed at the prevailing labour income tax rate while if only sales taxes are levied gross revenues are taxed in the firm. Either way the total take is identical. The exception is when both tax types are levied at once; in which case the effective rate has a multiplicative term.

$$(Y_t^i)^{\varphi} = \frac{\alpha(A_t^i)^{\varphi+1}(1-\tau_t^i-\tau_t^U)}{C_{F,t}S_t^i} \tag{25}$$

This condition leads to the standard result that tax increases lead to a convex output loss. Supply is also partially determined by the terms of trade; for changes S_t^i the income effect is dominant and so cheaper domestic consumption causes households to substitute to leisure and vice versa. Both these effects are important in determining the objectives for a union-wide policymaker. The latter is a second channel, alongside the consumer price channel discussed in the previous section, by which terms of trade fluctuations are costly from a union-wide welfare perspective. The former is a direct motivator for intra-union and inter-temporal tax-smoothing.

2.3. Policy Authorities

The policy authorities are divided between a union-wide federal government and a continuum individual local governments. As the economy is assumed to be cashless, with the union-wide consumption good ($C_{F,t}$) numeraire for financial transactions, the lack of nominal rigidities imply that the policy is restricted entirely in to the fiscal domain. To preserve meaningful tradeoffs with respect to inter-temporal and intra-union policy choices the fiscal instruments available to governments are restricted in several ways.

First, revenues are raised by a linear combination of two sales taxes; one levied by local governments on local producers and a federal tax levied uniformly across the union on all producers. In this setting, there is a tax-smoothing motive for the union-wide policy maker both over time and across locales. For the latter policy dimension to be of interest, the assumption of a uniform federal tax is necessary. If a union-wide policymaker could set its own locale specific taxes it would offset variations in local government rates and perfectly harmonise the taxes across the union. This removes a key policy dimension worthy of exploration and also precludes any strategic interaction between the two levels of government.[8]

Second, it is assumed individual locales have an exogenously determined level of required government consumption which can be met for either by local government spending or via a transfer from the federal government. However, these transfers are assumed not to be frictionless; there is a quadratic cost from deviating between an optimal ratio between local and federal public consumption. Modeling transfers in this fashion has the advantage of pinning down the relative sizes of the two types of government. Moreover, if federal resources could be moved between locales frictionlessly, the federal government can simply replicate the allocation under which it can set local marginal tax rates. From an optimal policy perspective the local marginal tax rate is set to equalise the marginal efficiency of government revenue to the marginal cost of the tax distortion; if transfers are frictionless, the federal government can choose the former without trading off other policy objectives.

Third governments are assumed to only have incomplete access to financial markets and can issue only a one period bond denominated in the numeraire. Furthermore, I consider additional cases where the government has a balanced budget rule, implying that gross stock of debt is fixed at some initial level. [9] Preserving

[8]The assumption of flat federal taxes seems reasonable as it is how fiscal unions operate in practise. It is very rare for federal governments to set region specific marginal tax rates (see Rodden (2006) and Bordo et al (2011)); indeed, in the US, Article 1, Section 8 of the constitution requires uniform federal taxation. In practice progressive tax systems observed do partially allow average marginal tax rates to vary with regional income differentials but the discrete nature of tax-brackets do not permit perfect tax smoothing.

[9]The assumption that while complete markets exists for the private sector, the public sector is unable to access them is common in the literature and stems from the perception that while in reality the private sector holds a varied equity and derivative securities these instruments make up a negligible share of public financial balance sheets. Ferrero (2009) assumes this structure explicitly in a two country model; however, if one thinks of a representative household sector as equivalent to heterogeneous household with access to complete markets then this assumption about differences in market access underpins Barro (1979) and the wide literature that has followed.

this incomplete market outcome is of particular interest for the objectives of this paper. If governments have access to sufficient instruments to replicate access to complete markets, the cyclical component of fiscal policy is of little interest. As Lucas and Stokey (1983)[10] show, under complete markets, state-contingent securities can be used to insure against future tax distortions leaving tax rates approximately constant, and only responsive to exogenous fluctuations in government consumption. As a result, the role played by government debt as a buffer to smooth fluctuations in distortionary taxes is eliminated and hence, the question of the welfare implications of a balanced budget rule is also moot.[11]

The following is a formal definition of the fiscal policy setup.

Public consumption and transfers

Each individual locale has an exogenously determined level of required government consumption $\bar{G}_t^i$. Government consumption can be provided for via locally produced public consumption goods or via a transfer of public consumption goods produced at the federal level: let h_t^i denote the share of federally produced public consumption goods that are consumed by region i. Local and federal public good consumption are combined using the following aggregator:

$$\bar{G}_t^i = G_t^i + h_t^i G_t^U - \Phi(G_t^i, h(i) G_t^U) \tag{26}$$

$$\int_0^1 h_t^i di = 1 \tag{27}$$

The second term in this expression is a generic function, $\Phi(G_1, G_2) : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$, that captures the cost from deviating from an optimal ratio between the provision of public goods between the federal and local governments, which represents s transfer friction. The function is assumed to have the following properties:

1. $\Phi(G_1, G_2)$ is homogenous of some degree $k \in \mathbb{R}$.
2. There exists a unique $\alpha_G \in (0,1)$ such that $\Phi(\alpha_G \bar{G}, (1-\alpha_G)\bar{G}) = 0$, $\forall \bar{G} \in \mathbb{R}^+$. This implies that for any given level of overall public good provision, it is efficient to have a share α_G provided for by the local government. Correspondingly: $G_1 \neq \frac{\alpha_G}{(1-\alpha_G)} G_2$ then $\Phi(G_1, G_2) > 0 \, \forall G_1, G_2 \in \mathbb{R}^+$.

Define the parameter $\nu(\bar{G}) = \frac{\bar{G}^{k-2}\Phi_{11}(\alpha_G \bar{G}, (1-\alpha_G)\bar{G})}{2(1-\alpha_G)}$ that determines the curvature of the function along the locus of points consistent with an optimal ratio between governments. A second order approximation of the transfer friction around any optimal point $(\alpha_G \bar{G}, (1-\alpha_G)\bar{G})$ is simply: $\Phi(G_1, G_2) \simeq \nu(\bar{G})[(1-\alpha_G)(G_1) - (\alpha_G)(G_2)]^2$. Therefore, for the purposes of a linear-quadratic approximation of optimal policy one only needs to calibrate ν and the remaining features of Φ are left undefined. In this case, the curvature parameter ν determines how costly deviating from the optimal public consumption ratio is: as $\nu \to 0$ implies the two forms of goods become perfect substitutes regardless of α_G and the federal government can transfer resources frictionlessly, while $\nu \to \infty$ implies the aggregator between different forms government consumption is essentially Leontief. The cases where policy is assessed with no fiscal union is equivalent to setting $\alpha_G = 0$ and $\nu \to \infty$.

The literature on fiscal federalism has several justifications for why we should not think that federal government spending can replace wholesale the role played by local governments or that public resources can

[10] See Chari and Kehoe (1999) for a similar analysis.

[11] As Golosov and Sargent 2012 discuss, a specific combination of distortionary taxes and one-period debt is necessary in preserving an incomplete markets economy: *"if the government can trade risk-free debts of different maturities (as in Angeletos (2002) or Buero and Nicolini (2004)) or if it can tax bonds in response to the shocks (as in Chari, Christiano, and Kehoe (1994)), it can effectively replicate equilibrium allocations for a complete market economy."* Furthermore, while the tax structure considered here can equivalently be thought of as either a sales or labour income tax, it is assumed here that the government does not levy both simultaneously as a multiplicative terms also can be used to imitate taxes on debt.

be moved costless throughout the union. It is beyond the scope of this paper to explicitly model all these effects, however they serve motivate the presence of the generic transfer friction. The fundamental principle for the existence of fiscal federations is that different levels of governments have varying efficiencies over the provision of different public goods (Musgrave (1959), Oates (1972, 1999)). For example, it may be the case that economies of scale imply the a federal government is more suited to the provision of defence while local knowledge implies the local governments are better at maintaining basic infrastructure. It has also been argued that electorates have a preferences for localism; because of the smaller scale of local governments an individual voter has more say in the provision of local public goods and is more able to hold the politicians that supply them to account.

Even if federal and local public consumption are not perfect substitutes, the federal government still has the option of financing local government consumption indirectly via grants-in-aid or revenue-sharing. However, inefficiencies lurk here also. The institutions that govern these arrangements often limit the scope of policy that can be financed in such a fashion; in the United States the vast bulk of intergovernmental revenue received by state and local governments is in the form of conditional grants (Oates (1999)) that finance specific programmes. More critically such direct payments are prone to political economy distortions. The disbursement of such funding at the federal level is subject to the pool problem that avails fiscal policy with dispersed interests (see Weingast, Shepsle and Johnsen (1981)); however, it is also well known that federal grants are used inefficiently when they arrive in local coffers. This is referred to as the "fly-paper" effect whereby grants are disproportionately spent where they land and are not redistributed either throughout the local budget or back to local tax payers (see Hines and Thaler (1995)). Implying transferred resources are spent less efficiently than local revenue.

Government expenditure and budget constraints

I assume that the federal government has a technology that generates a (union wide) public consumption good, G_t^U, from the products of all locales in a identical fashion to the numeraire consumption basket:

$$G_t^U = [\int_0^1 (G_{f,t}^U)^{\frac{\sigma-1}{\sigma}} df]^{\frac{\sigma}{\sigma-1}} \tag{28}$$

and that federal government assigns demand across locales on a cost minimisation basis; accordingly, demand from the federal government follows:

$$G_{i,t}^U = (S_t^i)^{\sigma} G_t^U \tag{29}$$

And the price of a unit of the federal public consumption good is equal to one. Financing comes from a one period bond (b_t^U) and a flat sales tax levied equally across the union (τ_t^U). The federal government adheres to the following flow-budget constraint:

$$G_t^U + b_{t-1}^U = R_t b_t^U + \tau_t^U \int_0^1 \frac{Y_t^i}{S_t^i} di \tag{30}$$

The local government has a technology that generates a public consumption good using of domestically produced goods. Financing comes from sales taxes on local producers and revenue raised by the issuance of a one period bond (b_t^i). Both federal and local governments bonds are denominated in the numeraire, which are traded in a union wide market and by arbitrage have gross yield R_t^{-1}. As such that a local government budget constraint can be expressed as:

$$\frac{G_t^i - \tau^i Y_t^i}{S_t^i} + b_{t-1}^i = R_t b_t^i \tag{31}$$

It is useful to also define the level of general government debt as:

$$B_t = b_t^U + \int_0^1 b_t^i di$$

The debt level for both federal and local governments is assumed to be bounded from above. Governments are also prohibited from buying each others debt, an explicit no-Bailout agreement in other words. Coupled with the household transversality condition, these conditions imply that debt levels are also bounded from below. This rules out public sector Ponzi schemes and implies that the the general debt level is also doubly bounded. For the purposes of the perturbed approximation of the economy considered here, these bounds have no meaningful effects on the dynamics or welfare.

The specification of transfers also has implications for how fiscal policy affects demand for local goods. An alternative would be to have federal aid feed directly through the local government budget constraint (including a similar friction to capture the effects above) rather than specifying the relationship via a public consumption requirement. Having funds flowing through the budget constraint would enable the federal transfers to influence the level of taxation and debt at each locale; however, it would not represent a transfer of real resources as all government consumption in a locale would still have to be obtained from local output. The approach taken here allows for transfers to not serve as financing due to reduced expenditure requirements but also as a policy instrument to control the demand, and hence price, of each locale's output.

Balanced budget rules

The flow budget constraints imply that government fiscal balance can be thought of as the change in the debt stock. The deficit of a government $j \in \{U, [0,1]\}$ is given by $def_t^j = b_t^j - b_{t-1}^j$ with a corresponding the general government deficit: $def_t = def_t^U + \int_0^1 def_t^i di = B_t - B_{t-1}$.

Definition 1. *A government, $j \in \{U, [0,1]\}$, operates under a balanced budget rule between periods t and T, if for all $s \in \{t, \ldots, T\}$, $b_s^j = b_{t-1}^j$.*

Definition 1 says nothing more than when the government operates under a balanced budget rule its deficit is equal to zero. I follow Stockman (2001) in requiring a strict equality. One could argue that a weak inequality ($b_s^j \leq b_{t-1}^j$) is more appropriate, permitting surpluses. This is approximately how the 2011 European fiscal compact is structured or the Maastricht criteria that preceded it. On the other hand, the sort of fiscal rules employed among American states often require a strictly balanced budget (Bahaj (2011)). Furthermore, Stockman (2001) argues that the tendency for fiscal policy process to exhibit deficit bias implies that even if the a balanced budget rule permits surpluses it is better to model them as binding as in the definition above. In order to retain a timeless aspect to the optimal policy decision, for the results presented it is assumed that any government that adheres to a balanced budget rule started doing so in the far distant past (with some legacy debt) and is expected to continue to adhere to it for the foreseeable future.

2.4. Market clearing and equilibrium

From equations 6, 7 and 29, market clearing requires:

$$Y_t^i = G_t^i + (S_t^i)^\sigma G_t^U + (1-\alpha)\frac{P_{c,t}^i}{P_t^i}C_t^i + \int_0^1 (S_t^i)^{-\sigma}\alpha P_{c,t}^f C_t^f df) \tag{32}$$

Where the first term is local government demand for local goods, the second is demand from the federal government, the third is local household demand and the fourth is export demand from households in the remainder of the union. Combining equations 14 and 16 with 32 allows the elimination of the local consumer price term:

$$Y_t^i = G_t^i + (S_t^i)^\sigma G_t^U + (1-\alpha)(S_t^i)^\alpha C_t^i + \alpha(S_t^i)^\sigma \int_0^1 (S_t^f)^{\alpha-1} C_t^f df \tag{33}$$

Substituting in 21 allows demand to be expressed only as a function of the local terms of trade, government spending and aggregate demand for the union wide good:-

$$Y_t^i = G_t^i + (S_t^i)^\sigma G_t^U + \frac{(1-\alpha)}{\alpha}(S_t^i)C_{F,t} + (S_t^i)^\sigma C_{F,t} \tag{34}$$

It is also possible to express the gross yield on a government bond as a function of aggregate variables. Recall the household bond Euler equation:

$$\beta R_t E\left[\frac{(P_{c,t}^i)C_{i,t}}{(P_{c,t+1}^i)C_{i,t+1}}\right] = 1$$

Substituting in 16 and 21:

$$\beta R_t E\left[\frac{(S_t^i)^{-(1-\alpha)}(S_t^i)^{1-\alpha}C_{F,t}}{(S_{t+1}^i)^{-(1-\alpha)}(S_{t+1}^i)^{1-\alpha}C_{F,t+1}}\right] = 1$$

$$\beta R_t E_t\left[\frac{C_{F,t}}{C_{F,t+1}}\right] = 1 \tag{35}$$

It is convenient to substitute 35 directly into the government budget constraints in order to eliminate the bond price:

$$([\frac{(G_t^i - \tau_t^i Y_t^i)}{C_{F,t}S_t^i}] + \frac{b_{t-1}^i}{C_{F,t}})) = \beta E\left[\frac{1}{C_{F,t+1}}\right] b_t^i \tag{36}$$

$$\frac{G_t^U - \tau_t^U \int_0^1 \frac{Y_t^i}{S_t^i} di}{C_{F,t}} - \frac{b_{t-1}^U}{C_{F,t}} = \beta E\left[\frac{1}{C_{F,t+1}}\right] b_t^U \tag{37}$$

It is now possible to define an equilibrium for this economy:

Definition 2. *A competitive rational expectations equilibrium is a infinite dimensional sequence of endogenous stochastic processes $X = \{\{Y_t^i, S_t^i, \tau_t^i, G_t^i, h(i), b_t^i\}_i, \tau_t^U, G_t^U, C_{F,t}, b_t^U\}_{t=0}^\infty$, a sequence of exogenous processes $\xi = \{\{A_t^i, \bar{G}_t^i\}_i\}_{t=0}^\infty$ and a set of initial conditions $\mathcal{I}_{-1} = \{\{b_{-1}^i\}_i, b_{-1}^U\}$ that satisfy the constraints from the private economy: the pricing condition, 22, the supply constraint 25 and the demand equation 34; policy is such that the public consumption requirement 26, aggregate transfer condition 27 and government budget constraints 36 and 37 are satisfied $\forall i \in [0,1]$ and $t \geq 0$ and that debt stocks remain suitably bounded.*

3. The Optimal Policy Problem

I consider two cases of optimal policy (Ramsey) problem. In the first case, the union-wide (federal) planner fully controls all policy instruments; equivalent to a cooperative policy setup between the different levels of government. The planner uses its instruments to choose the sequence of endogenous variables with the objective of maximising union wide welfare give by:

$$V_0^U = \int_0^1 V_0^i di$$

This can be rewritten as:

$$V_0^U = E_0 \sum_{t=0}^\infty \beta^t \{ln(C_{F,t}) - ln(\alpha) + (1-\alpha)\int_0^1 ln(S_t^i)di - \frac{1}{1+\varphi}\int_0^1 [\frac{Y_t^i}{A_t^i}]^{1+\varphi} di\} \tag{38}$$

Subject to a set of constraints maintaining an equilibrium and any applicable budget balanced budget conditions. In the second case, the local government i is assumed to set local fiscal policy variables (τ_t^i, b_t^i) with the objective of maximising the inter-temporal of utility their individual locale, V_0^i. While the union-wide planner retains the same objective and acts as a Stackelburg leader. This allows for strategic interaction between the two levels of government. From the perspective from the union-wide planner the strategic behaviour manifests itself as additional constraints in the optimisation problem alongside those apparent in the cooperative case.

The generic formulation of the optimal policy problem to describe these cases in terms of a Lagrangian is laid out in full in AppendixA. The non-linear optimal policy problems considered here lack closed form solutions. Instead, I consider a first order approximation of the solution about a non-stochastic symmetric steady state[12]. As is well known (see, for example, Aiyagari et al (2002)) the incomplete market access of fiscal policymakers implies a unit-root processes in the marginal efficiency of government debt - a generalisation of the Barro (1979) tax smoothing result. The non-stationary behaviour of the economy means that there is no unique steady state, instead there is an infinite number determined by the size of the public debt stock. I prove the existence and characterise of this set of steady states, indexed by the public debt to output ratio (denoted Ω) in AppendixB. Note that regardless of whether the optimal policy is conducted cooperatively or strategically the steady state values of the endogenous choice variables are identical, as is steady state welfare.[13]

As is standard on Ramsey optimal policy of this type, in the absence of additional constraints the planners solution at time $t = 0$ is not time consistent.[14] The solution taken here is to consider the approximated optimal policy plan from a *timeless perspective* using the methodology laid out in Benigno and Woodford (2012). This approach assumes that rather than optimising at time zero, the optimal policymaker has chosen the pattern of behaviour it would have committed itself to if it had optimised in some point in the previous distant past[15]. This implies that, unlike standard Ramsey policy, the initial period has no special properties. Although, it is worth noting that optimal policy from this perspective still requires the union-wide planner to commit to behave in this fashion. It is not time consistent in the sense that a continuously re-optimising policymaker would choose policy of this form.

3.1. The Cooperative Case

The formal definition of cooperative optimal policy is as follows:

Definition 3. *A cooperative optimal policy plan is a infinite dimensional sequence of endogenous stochastic processes* $X = \{\{Y_t^i, S_t^i, \tau_t^i, G_t^i, h(i), b_t^i\}_i, \tau_t^U, G_t^U, C_{F,t}, b_t^U\}_{t=0}^{\infty}$*, a sequence of exogenous processes* $\xi =$

[12]There are two equivalent ways to obtain such an approximation. Judd (1998) suggests calculating the analytical first order conditions to the optimal policy problem which can then be linearised about the steady state to approximate the dynamics of the economy. Benigno and Woodford (2012) suggest approximating the entire optimal policy problem first within a linear quadratic framework: i.e. take an appropriate second order approximation of the objective (transformed to eliminate linear terms) coupled with first order approximations and obtain a set of linear analytical first order conditions. Both methods provide an identical, first-order, approximation of the dynamics of the economy; however, the latter methodology also provides a consistent framework to think about issues of welfare and time consistency which are relied on in this work.

[13]However, as one would expect, the steady values of the Lagrange multipliers do vary.

[14]This is due to the expectations of future policy determining contemporaneous private sector and, in the strategic case, local government behaviour. In the cooperative case this is a result in private sector expectations of consumption determining bond yield; there is an additional channel from transfers in the strategic case. The time inconsistency stems from policymakers failing to take into account the prior expectations of economic agents from earlier periods when optimising at time zero. This is problematic both in the sense that the period zero policy response is altered for a seemingly arbitrary reason and in that there is no reasons to think that the policymaker will not reoptimise again in future.

[15]In the a standard Ramsey problem past values (prior to $t = 0$) of Lagrange multipliers are set to zero in the first order conditions as the policymaker attaches no weight to these periods when optimising. The *timeless perspective* is equivalent to assuming the policymaker has committed not to set these multipliers to zero and adhere to them when constructing current and future policy. This implies that first order conditions are not time dependent and the model can be solved recursively.

$\{\{A^i_t, \bar{G}^i_t\}_i\}_{t=0}^{\infty}$ and a set of initial conditions $\mathcal{I}_{-1} = \{\{b^i_{-1}\}_i, b^U_{-1}\}$ that maximise V^U_0 and satisfy the equilibrium conditions from definition 2, the additional constraints imposed by conducting policy from a timeless perspective and any assumed balanced budget requirements.

For definition 3 to be satisfied one does not need to actually specify the policy instruments; insofar as there is a greater number of choice variables than there are constraints any endogenous variable could be selected as an instrument. However, even if the selection is arbitrary, for intuition it is useful to think about the fiscal variables as instruments in order to describe the dimensions over which policy operates. In this regard, one can separate these dimensions between the policy choices that occur at the union-wide and local level. For every individual locale there are four constraints, respectively, and six endogenous choice variables. This leaves two free policy dimensions: a taxing versus borrowing decision and the decision to transfer resources between locales. At the federal level there is solely a federal taxation versus borrowing decision given the aggregate level of transferred resources. The introduction of a balanced budget rule at either level of government is equivalent to turning off the policy dimension associated with borrowing.

In a cooperative optimal policy the presence of a taxing versus borrowing policy dimension at both levels of government leads to the potential for indeterminacy as changes in the federal debt stock can have an identical impact to changes in the aggregate level of local debt.

Proposition 4. *For a cooperative optimal policy: (i) the relative shares of general government debt held by the federal (b^U_t) and local governments ($\int_0^1 b^i_t di$) are indeterminate. (ii) a federal BBR when coupled with unconstrained local borrowing can achieve an identical welfare level and private sector dynamics to unconstrained borrowing at all levels of government .*

Proof. see AppendixC.1 . □

The intuition behind this result is straightforward: federal debt is financed by tax revenues raised from all locales; redistributing the debt from the federal level to a local level in such a fashion that each locale pays the same tax burden it would have otherwise faced does not impact the real economy since, at the local level, both taxes are equally distortionary. Consequently, any efficiency gain from a change in federal government debt can be equivalently achieved by changing the debt levels of all local governments in a suitable fashion. To deal with this indeterminacy issue it is assumed that the federal government adheres to a BBR when localities borrow as the policymaker has no incentive to issue federal debt. This result does not extend to the strategic case and nor is this assumption extended. Importantly, this result is not symmetric: federal borrowing cannot circumvent local budget rules. Redistributing debt from an individual locale to the federal balance sheet changes the distribution of tax burdens and therefore impacts the real economy. [16]

3.2. The Strategic Case

In this case I dispense with the assumption that the union-wide planner can control policy at all levels of government and instead allow for for local fiscal policy decisions, specifically over taxing versus borrowing, to be set with the objective of maximising the welfare of the locale alone. The atomistic nature of the locale implies that union-wide variables are considered to be exogenous. I further assume that the federal government acts as a Stackelburg leader such that local governments optimise taking federal policy as given and then private sector responds. The specific timing of events within each period follows: *(i)* shocks to the endogenous processes are realised and are observed by the federal and local policymakers and the private

[16] An exception to this occurs when transfers are frictionless, in which case the debt can redistributed costlessly throughout the union by altering federal transfers; this in turn implies that the borrowing of any individual government is indeterminate. This frictionless case is analogous to models of optimal taxation with heterogeneous agents and lump sum payments (Aiyagari et al (2002) and Golosov and Sargent (2012)) where the absolute debt levels of agents and the government are indeterminate, optimal policy is solved by normalising relative to one agent (or the government).

sector; *(ii)* the union-wide planner in the guise of the federal government decides on its transfer policy (h^i_t,G^U_t) and if no federal BBR is present it sets the federal tax rate (τ^U_t)[17]; *(iii)* local governments sets local tax rates; *(iv)* private sector decisions taken and any fiscal deficits are realised.

The local policymakers problem

Under these circumstances local spending is, in effect, federally determined due to the public expenditure constraint; the federal government sets its transfers first and the local government is then obliged to fulfill remaining public consumption requirement. Hence, the local policy problem is reduced to determining what proportion of the (non-federally financed) government consumption requirement is sourced from local taxation or borrowing. The introduction of a local BBR eliminates this free policy dimension and the policy problem reverts to the co-operative case. Therefore, in what follows the assumption that the local government can borrow is maintained. Formally, the local policymakers problem is to choose a stochastic sequence:

$$\{X^i_t\}^{\infty}_{t=t_0} = \{Y^i_t, P^i_t, \tau^i_t, G^i_t, b^i_t\}^{\infty}_{t=t_0}, \quad i \in [0,1]$$

In order to maximise:

$$V^i_0 = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ln(C_{F,t}) - ln(\alpha) + (1-\alpha)ln(S^i_t) - \frac{1}{1+\varphi}[\frac{Y^i_t}{A^i_t}]^{1+\varphi}\} \tag{39}$$

Subject to three constraints:

$$Y^i_t = G^i_t + (S^i_t)^{\sigma}(G^U_t + C_{F,t}) + \frac{(1-\alpha)}{\alpha}(S^i_t)C_{F,t} \tag{40}$$

$$(Y^i_t)^{\varphi} = \frac{\alpha(A^i_t)^{\varphi+1}(1-\tau^i_t-\tau^U_t)}{C_{F,t}S^i_t} \tag{41}$$

$$([\frac{(G^i_t - \tau^i_t Y^i_t)}{C_{F,t}S^i_t}] + \frac{b^i_{t-1}}{C_{F,t}})) = \beta\frac{1}{E(C_{F,t+1})}b^i_t \tag{42}$$

Note that $C_{F,t}$ (and correspondingly the interest rate) is a union wide variable as such is considered exogenous by the local policymaker. For parsimony it is simpler to consider G^i_t as exogenous as well. The policy problem above yields the following first order conditions, let $\beta^t\theta^i_{1t}$, $\beta^t\theta^i_{2t}$ and $\beta^t\theta^i_{3t}$ denote the Lagrange multipliers on constraints 40 to 42:

wrt Y^i_t :

$$\theta^i_{1t} + \varphi(Y^i_t)^{\varphi-1}\theta^i_{2t} = \frac{\tau^i_t}{S^i_t C_{F,t}}\theta^i_{3t} + \frac{1}{Y^i_t}[\frac{Y^i_t}{A^i_t}]^{1+\varphi} \tag{43}$$

wrt S^i_t :

$$\frac{(Y^i_t)^{\varphi}\theta^i_{2t}}{(S^i_t)^2} + \frac{(1-\alpha)}{S^i_t} = \frac{(G^i_t - \tau^i_t Y^i_t)\theta^i_{3t}}{C_{F,t}(S^i_t)^2} + \sigma\theta^i_{1t}(S^i_t)^{\sigma-1}(G^U_t + C_{F,t}) + \frac{(1-\alpha)}{\alpha}C_{F,t}\theta^i_{1t} \tag{44}$$

wrt τ^i_t :

$$\frac{\alpha\theta^i_{2t}(A^i_t)^{\varphi+1}}{C_{F,t}S^i_t} = \frac{\theta^i_{3t}Y^i_t}{C_{F,t}S^i_t} \tag{45}$$

wrt b^i_t

[17] If a BBR is present the tax rate is a function of the level of expenditure.

$$\frac{\theta^i_{3t}}{E(C_{F,t+1})} = E\left[\frac{\theta^i_{3t+1}}{C_{F,t+1}}\right] \tag{46}$$

A few important points emerge from these conditions. Local policy is time-consistent. From the perspective of the private sector the only relevant forward looking variable is the future expectation of aggregate consumption (used to determine the interest rate). For the local policymaker this variable is exogenous and therefore it does not internalise, or exploit, the effect its policy has on these expectations.[18] This is represented in the first order conditions by the lack of lagged Lagrange multipliers that would correspond in, a standard optimal policy problem, to past commitments over future policy behaviour.

The first order condition on debt also implies that local policy variables contain the unit root processes apparent in the co-operative case. As discussed above, the marginal efficiency of past local government borrowing, θ^i_{3t}, follows a unit root process (to the first order). However, the distinction from the co-operative case is that θ^i_{3t} it measured in units of marginal local utility as opposed to union wide welfare. This distinction is key to understanding the differences in the behaviour of local governments in the two cases. The optimal local government policy is to set taxes such that marginal inefficiency of additional borrowing is equal to the marginal cost of the tax distortion, a result which is implied by 45. However, as one would expect, the local government does not fully internalise the impact of local taxation on union-wide welfare. These strategic considerations manifest primarily as two externalities[19]:

1. *A Common Pool Problem on Consumption:* The exogeneity of aggregate consumption basket $C_{F,t}$ in effect operates like a common pool problem. Local policymakers do not internalise the impact local output has on the size of the basket but fully internalises the cost of providing the output in terms of disutility of labour.
2. *Policy spillovers*: As is standard in a strategic optimal policy problem of this form, the local policymaker fails to account for the impact of its taxes on the welfare of other locales; for example, a tax cut that increases the supply of the local good in turn raises the relative price of all other goods and reduces the welfare in other locales. In essence, local policymakers do not take into account the intra-union stabilisation motive of the union wide planner.

Effect 1 is of interest as it implies that local government's have a general tendency to, all else equal, overtax relative to cooperation. Due to Cobb-Douglas preferences, a proportion of any change in consumption from a change of the supply of local goods is internalised. However, since only a $(1 - \alpha)$ share of goods are consumed are domestically the welfare implications from the perspective of local government are strictly less than for the union wide policymaker. If one thinks about a loss of local supply due to an increase in local taxes, the local government effectively exports a proportion of the efficiency loss of taxation to other locales in the union via a reduction in the union wide consumption basket. The net result of this is that, all else equal, the marginal efficiency of local taxation and by extension local borrowing is for the local government than it would be for the union wide policy maker under cooperation. In isolation, this effect implies that a positive cyclical shock would result in smaller surpluses (or indeed deficits) as local governments have less of an incentive to run down debt stocks - policy becomes more pro-cyclical relative to cooperation. Proposition 5 presents this idea formally by comparing static steady states.

[18] This is not to say that the private sector does not form its expectations taking into account the behaviour of local policymakers; but only in aggregate so an individual local government is irrelevant.

[19] There is a third channel in that the local government fails to account for the impact of its policy on resources available to the federal government for transfers. For example, in periods where large transfers are required the union-wide planner would prefer a boost to supply to offset the inefficiency of transfers and vice versa. In general, this is another mechanism whereby local governments fail to internalise the cost of higher taxes, which reduce output and thus federal revenues. The implications for the marginal efficiency of tax revenues are therefore similar to point 1 above. However, this third channel, while present, does not play a significant role in altering strategic behaviour relative to the cooperative case.

Proposition 5. *Consider the set of feasible symmetric steady states indexed by the debt-to-GDP ratio,* Ω*. The following is true: (i) In the steady state where the effective tax burden,* $\tau^l + \tau^U$*, is zero (corresponding to* $\Omega < 0$*)* $\theta_3^i > 0$*. (ii) there exists a* Ω *such that the marginal efficiency of additional government debt from the perspective of the local policymaker,* θ_3^i*, is zero. Denote the corresponding steady state effective tax rate* $(\tau^l + \tau^U)$ *as* τ^**. This tax rate is contained in the unit interval:* $\tau^* \in (0, 1)$*.*

Proof. seeAppendixC.2 . □

What proposition 5 proves is that in the set of symmetric steady states there is a region where the local government does not fully internalise the cost of taxation such that the marginal efficiency of government debt is higher for the local government than it would be under cooperation. Indeed, for suitably low tax rates it has an incentive to increase its debt level in order to increase the tax burden. This is a channel for inefficient over-borrowing on the part of local governments.

The size of this effect depends on the parameters σ and α. The impact of changes in σ, the degree of substitutability between local goods is perhaps contrary to what one would expect. The local government is not behaving monopolistically, in the sense that it exploits its market power over local production to restrict supply and raise their price.[20] The complete nature of private financial markets partly insures the rest of the union against this behaviour and the increase in price of local production lowers local welfare. Indeed, as goods become less substitutable, a decline σ, and market power increases, the response of the local price to any increase in taxation increases which implies a greater cost to restricting supply reducing the incentive for the local government to deviate from the cooperative policy. While if α is relatively small a greater proportion of the cost of taxation is internalised via greater expenditure on local consumption goods and the externality is diminished.[21]

Figure 1 illustrates these effects and proposition 5 graphically by plotting θ_3^i and its cooperative counterpart against the steady state tax rate. From a cooperative perspective the marginal efficiency of additional government borrowing is only zero in the steady state where the effective tax burden is zero. At this point changing the debt level implies a change in the tax rate next period and zero distortionary taxes are optimal under cooperation; as one would expect from the first welfare theorem.

However, from the perspective of the local policymaker this marginal efficiency is positive, more debt and hence higher future taxes increase welfare. This is because at relatively low tax rates increasing the average tax burden causes a small increase in consumer prices which is more than offset by a reduction in the disutility of labour. This is the externality in point 1 at work. From the perspective of the local government the steady state with the optimal tax rate is actually positive, denoted τ^* in the proposition, and corresponds to the point where marginal cost of taxation in terms of higher local prices is equal to the marginal utility gain from increased leisure.[22]

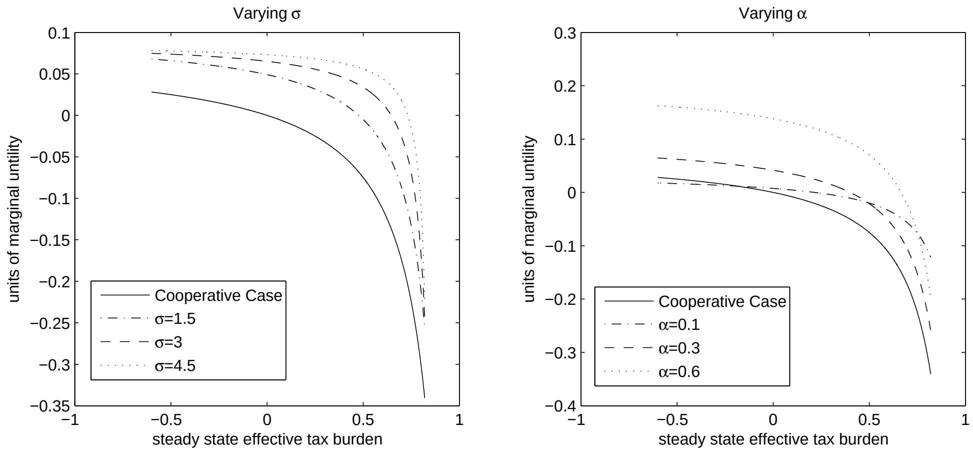
However, to obtain analytical results, proposition 5's implications are necessarily limited. Once one departs from symmetry the general results of proposition 5 that every local government would prefer to increase debt do not continue to hold. Instead, the externality in point 2 also plays a role. This suggests the opposite, that local policy would be more aggressively counter-cyclical - so a positive shock would lead to a larger surplus - as the resulting price increases in other locales would not be internalised. This is reinforced from non-stationary behaviour of the model; an asymmetric shock today will have permanent impact on the

[20] This is in contrast to a standard terms of trade externality (see for example, Epifani and Gancia (2009)) where increasing the substitutability between goods reduces the costs of strategic behaviour.

[21] When supply is severely restricted at very high tax rates, a higher α means there is a larger price response which means it is not possible to obtain the result thatθ_3^i is strictly increasing in α.

[22] For higher tax rates - outside the realistic range of steady states one would traditionally calibrate on - θ_3^i becomes badly behaved as the price level response becomes infinitely large leading to asymptotic behaviour in the marginal efficiency. Hence, it is not possible to say that the marginal efficiency of government borrowing is strictly greater for a local government that is behaving strategically than one that is cooperating in all feasible steady states.

Figure 1: Steady state marginal efficiency of local borrowing



Notes: The figure plots the steady state value of the marginal efficiency of local borrowing in the cooperative policy problem and from the perspective of the local government in the strategic case, across steady states indexed by the government debt to output ratio. All other features of the steady state are held constant. The parametrization follows the calibration described in section 4, with the exception of the parameters α and σ which are allowed to vary to illustrate the points made in the text. The cooperative case is invariant to changes in α and σ.

level of output, prices and consumption in the locale that receives it. Local policymakers have the incentive to use policy in order to converge to the new asymmetric steady state that maximises local welfare subject to the cost of the transition.

The net effect depends on the calibration, particularly α and σ, but also the type of shock. For an aggregate shock the externality 1 is the only driver as there are no asymmetries to exploit. However, as a shock becomes increasingly idiosyncratic the externality 2 plays an becomes a more dominant driver of strategic behaviour.

There is also an inter-temporal aspect to consider as the local government behaves as to equalise the expected marginal efficiency of borrowing over time, equation 46. The future marginal efficiency of local borrowing is not just a function of the current state variables but the future behaviour of federal transfers in determining the required level of public consumption. Thus strategic interactions are not just limited to contemporaneous variables; the federal government can influence local policy today by committing to certain transfer policies in future. Something that the local government does not fail to internalise.

The strategic optimal policy problem

As well as treating the federal government as a Stackelburg leader, I also restrict attention to Markov perfect equilibria whereby federal policy is conditioned only on the exogenous processes, debt levels and its past commitments regarding to its own policy (i.e. the lagged Lagrange multipliers in the federal government's problem). Such equilibria rule out the federal government using punishment strategies conditional on the past history of local government behaviour as a means to ensure, at least partial, co-operation. The assumptions over timing and Markov perfection are useful from a tractability perspective as it implies that the optimal policy problem can be framed as a dynamic Stackelburg game that can be solved recursively.[23] The union-wide planner's problem is simply altered to include a set of additional constraints and variables that correspond

[23] These assumptions over timing and federal behaviour chime with reality. The institutional systems that govern transfers and federal budgeting often have a degree of automacy which would suggest a federal first-mover advantage. Second to this, a federal government is unlikely to have the political wherewithal to be able to punish its localities by with-holding funds, particularly in a democratic system where the federal government is fighting for votes and the political leaders of local governments are potentially short-lived. The lack of credible commitment to no-bailout agreements apparent in many fiscal unions (see Bordo et al (2011) for a discussion) is likely a symptom of this problem.

to the first order conditions for each the local government's optimisation problem, see AppendixA for the formal definition.

It is now possible to define an optimal policy under strategic interaction:

Definition 6. *A Markov perfect strategic optimal policy plan is a infinite dimensional sequence of endogenous stochastic processes* $X = \{\{Y_t^i, S_t^i, \tau_t^i, G_t^i, h(i), b_t^i\}_i, \tau_t^U, G_t^U, C_{F,t}, b_t^U\}_{t=0}^{\infty}$*, a sequence of local Lagrange multipliers* $\Theta = \{\{\theta_{1t}^i, \theta_{2t}^i, \theta_{3t}^i\}_i\}_{t=0}^{\infty}$*, a sequence of exogenous processes* $\xi = \{\{A_t^i, \bar{G}_t^i\}_i\}_{t=0}^{\infty}$*, a set of initial conditions* $\mathcal{I}_{-1} = \{\{b_{-1}^i\}_i, b_{-1}^U\}$ *that maximise* V_0^U*, satisfies the equilibrium conditions from definition 2, the additional constraints imposed by conducting policy from a timeless perspective, the assumed balanced budget requirements and the local policy optimality conditions 43-46.*

Under these circumstances, the dimensions of policy available to the union-wide planner are altered in that the local taxation versus borrowing dimension is no longer free. Instead all cross-sectional policy is conducted via transfers, both in the current time period and the future commitment to transfer behaviour. The influence of future transfer behaviour on current policy implies that the commitment to operate optimal policy from a *timeless perspective* has more tangible implications than in the cooperative case. The union-wide policy maker has an incentive to promise future transfers to incentivise local governments to behave appropriately. In the absence of commitment there is nothing to enforce adherence to said promises.

4. Calibration

Bayesian estimation of an optimal policy model from a *timeless perspective* is fraught with difficulty[24] instead I solve the model numerically via calibration. As the fiscal union with the richest data sources, particularly due to detailed state fiscal data, I use observations on the US to calibrate the parameter values. Each state is to be considered an approximately atomistic locale. The time period is considered to be one year in order to match frequency of disaggregated state and local government fiscal data; accordingly, I set $\beta = 0.96$.

The Frisch elasticity of labor supply is assumed to take a value of one-third by setting $\varphi = 3$; see Domeij and Floden (2006) for recent evidence. This implies that agents spend on average one-third of their unit time endowment working. I calibrate the steady state debt to GDP ratio, $\Omega = 0.6$, to be inline with the average US general government debt to GDP of ratio for the period of the great moderation (1990-2007). I assume it is apportioned between the different levels of government inline the expenditure share, α_G; the results are not sensitive to this. I set the steady state ratio of general government purchases of goods and services to output as $\chi = 0.22$ [25], similarly in line with the recent US experience.

The share of local government expenditure in total government spending is carefully constructed from the Census Bureau's *State and Local Government Finances* database and *Consolidated Federal Funds Report* database rather than using the numbers from the national accounts. In keeping with the modeling assumptions, state and local expenditure is adjusted such that intergovernmental revenue from the federal government that was used to finance expenditure on goods and services is netted out and treated as a federal spending. This gives a figure of $\alpha_G = 0.55$.

To calibrate α, I use the Commerce Department's *Commodity Flow Survey* (CFS) on trade in goods between states. I disaggregate the data by industry and consider the value of total US interstate flows. To

[24] Julliard (2007) offers a discussion of the difficulties associate with Bayesian estimation of optimal policy regimes. The greatest difficulty lies with how to incorporate any past policy commitments when initialising the estimation. The Julliard (2007) solution proposes is to assume policy follows some a simple rule for an earlier period in the sample before switching to an optimal policy regime at fixed date. This may be reasonable in the context of optimal monetary policy, whereby the introduction of, say, an inflation targeting regime could be used as the break point in the sample. However, in the case of a fiscal union an equivalent break point may be difficult to find in sample.

[25] All expenditure data used is net of interest and transfer payments to private entities. Intergovernmental expenditure is netted out to prevent double counting

Table 1: Calibrated Values

Parameter	Description	Value	Parameter	Description	Value
	Model			*Exogenous Processes*	
α	Share of spending on local goods	0.3	ρ_a^U	Persistence of union wide TFP shocks	0.82
α_G	Local share of government spending	0.55	ρ_a^U	Persistence of idiosyncratic TFP shocks	0.48
β	Discount rate	0.96	ρ_g^U	Persistence of union wide G shocks	0.91
φ	Inverse Frisch elasticity of labour substitution	3	ρ_g^U	Persistence of idiosyncratic G shocks	0.84
ϵ	Curvature of the transfer function	1.5	$s_{a,U}^2$	Variance of union wide TFP shocks	0.00025
σ	Elasticity of substitution in the traded basket	3	$s_{a,l}^2$	Variance of idiosyncratic TFP shocks	0.00049
	Steady State		$s_{g,U}^2$	Variance of union wide G shocks	0.00011
Ω	Steady state gen. govt. debt to GDP ratio	0.6	$s_{g,l}^2$	Variance of idiosyncratic G shocks	0.00037
χ	Government spending share of GDP	0.22			

Notes: The model parameters are as defined in section 2, steady state parameters are defined in AppendixB and exogenous process are defined in this section. As described in the text, the calibration is based upon my and other authors observations of the US data.

limit double-counting flows that are caused by re-exports, i.e. a good moving from warehouse to warehouse in different states, I limit my attention only to primary and secondary industries and exclude wholesalers. This data is problematic in the sense includes flows of intermediate goods and goods that are imported or exported abroad that also move between two US destinations. To deal with the former I scale the data by the gross output net of international exports. For the latter I net off a fixed share of the value of the relevant industry imports and exports from abroad from the value of the total interstate flow such that the share netted off is equal to the ratio of the remaining flow to gross output. This is equivalent to saying the same share of foreign trade is shipped between US states as domestic trade. This process gives a figure for α of 0.61; approximately sixty percent of state output is shipped to other states. However, the CFS is only covers shipments of physical goods which are more likely to move across state lines. Services account for 67% of US GDP but there are no data sources on interstate service trade. In this regard, a figure of 0.61 is likely to be an overestimate and so it is best to think of this as an upper bound for α. A lower bound would be to assume that services are traded no more across state-lines than they are across US national borders. About 6% of US services output is exported, which gives a lower bound for α at 0.21. I calibrate to the mid-point of these two bounds and set $\alpha = 0.4$.

There is still uncertainty in the international trade literature over the appropriate value for calibrating σ, the elasticity of substitution between tradeable goods.[26] The issue is more complicated in this context as the the relevant trade flows are not between nations but between states. I therefore rely on the recent work of Yilmazkuday (2012) which uses CFS data to estimate the trade elasticity of substitution for US interstate trade and suggests a figure of $\sigma = 3$. This is lower than the figure often estimated for international trade flows but still within the (wide) range of estimates in the literature.

I assume that the exogenous processes for TFP and public consumption are, in logarithmic terms, the sum of two first order auto-regressive processes representing a union wide and idiosyncratic locale specific cyclical processes respectively:

$$
\begin{aligned}
log(A_t^i) &= log(A) + a_t^U + a_t^i \\
a_t^U &= \rho_a^U a_t^U + \epsilon_t^U,\ \epsilon_t^U \sim NID(0, s_{a,U}^2) \\
a_t^i &= \rho_a^l a_t^i + \epsilon_t^i,\ \epsilon_t^i \sim NID(0, s_{a,l}^2)
\end{aligned}
$$

[26] Corsetti et al (2008) offer a discussion.

$$
\begin{aligned}
log(\bar{G}^i_t) &= log(\bar{G}) + g^U_t + g^i_t \\
g^U_t &= \rho^U_g g^U_t + \varepsilon^U_t,\ \varepsilon^U_t \sim NID(0, s^2_{g,U}) \\
g^i_t &= \rho^l_g a^i_t + \varepsilon^i_t,\ \varepsilon^i_t \sim NID(0, s^2_{g,l})
\end{aligned}
$$

Where the variables without time subscript are the steady state values, I set $A = 1$ for convenience, and the random innovations are assumed to be uncorrelated. I calibrate these processes using Kalman filtered log annual real per capita US data. For the purposes of this exercise it is necessary to construct estimates of total government spending by state, $\bar{G}^i_t$, and the break down in spending between different levels governments. As above, I do this using the Census Bureau's *State and Local Government Finances* database and *Consolidated Federal Funds Report* database. I convert these numbers into real terms using the appropriate government spending deflators from the US national accounts. As above, the state and local spending is adjusted to exclude intergovernmental spending by the federal government and interest payments and transfers to private individuals are not included. This gives a series for real government purchases by state and the split between the share financed by state and local governments and the share financed by the federal government (which can be thought of as federal transfers). State level real GSP is data is obtained from the BEA's *regional accounts* database.

Constructing a 50 state filter is numerically challenging due to the size of state space. Instead, I setup the Kalman filter the series on a state-by-state pairwise basis with the aggregate federal series. All series are filtered assuming a smoothed trend plus a cyclical term and irregular. The cyclical component of the federal series are first order auto-regressive; local series cyclical components are decomposed as the sum the corresponding federal cyclical component and an idiosyncratic cycle. The estimation is iterated to normalise the federal process across states: I first filter every state on an unrestricted basis and take the average of the federal parameters. I then re-estimate the model for all states but restrict the behaviour of the federal series to correspond to estimated average. The critical features of the data are laid out in figure 2.

The institutional structure of fiscal policy in the US is similar to the case with local balanced budget rules and federal borrowing so I calibrate the remaining parameters using this version of the model. Recall that the presence of local borrowing rules prohibits strategic behaviour on the part of local governments. In order to solve the model numerically, I approximate the continuum of locales by discretising the cross-section into 50 evenly sized units (reflecting a calibration on US data). One can view this equivalently either as 50 clusters of locales whereby all shocks are perfectly correlated or as 50 locales that are sufficiently small such that the approximation of atomistic behaviour is not unreasonable[27]. The approximation by discretising the space of locales with this level of granularity does not appear to have meaningful implications for the welfare results or the calibrated parameters.

I simulate the economy removing the non-stationary components of the simulated series via a Beveridge-Nelson decomposition. I then match the key features of the data with the moments of the stationary component of the simulation. The public consumption processes are calibrated to match the behaviour of the cyclical idiosyncratic and aggregate public consumption processes for the average state. The parameters for aggregate and idiosyncratic productivity process are chosen to match the features of the output processes.

What remains is to calibrate the size of the transfer friction. Under the theoretical setup considered, the co-movement between the local public consumption requirement and federal transfers is perhaps not best metric to consider, as a proportion $1-\alpha_G$ of any public consumption shock can be met by transfers regardless

[27]Given the institutional structure of the US the latter is a more appealing interpretation; even the largest US state, California, only makes up just 8% of aggregate GDP.

Table 2: Features of aggregate and state-level US data; (1981-2009).

Data Feature	Value
Persistence of cyclical component of US aggregate output	0.80110
Persistence of cyclical component of US aggregate general government spending	0.91222
Average persistence of cyclical component of state output	0.73832
Average persistence of cyclical component of state general government spending	0.85096
Variance of cyclical component of US aggregate output	0.00120
Variance of cyclical component of US aggregate general government spending	0.00168
Average variance of innovations to idiosyncratic cyclical component of state output	0.00161
Average variance of cyclical component of state general government spending	0.00246
Correlation between cyclical component of transfers and output	-0.27044

Notes: Features of the cyclical component of Kalman filtered US state data. The average refers to the mean of the parameter estimates across the 50 states.

of the friction. Instead, as a measure of how flexibility of transfers in response to a idiosyncratic shock it is better consider the co-movement between transfers and output. In this regard, the transfer friction is chosen to match the correlation between the stationary component of transfers and the stationary component of idiosyncratic output seen in the data.[28]

5. Dynamics

In order to gain an understanding for the dynamic response of relevant endogenous variables to shocks I consider the impulse responses when a proportion, half for the sake of symmetry, of locales are hit by a 1% shock to their respective idiosyncratic exogenous productivity and government consumption processes. The persistence of these shocks are line with the calibration in the previous section. Given the modeling assumptions, the locales form into two groups - the shocked and unshocked respectively. By symmetry optimal policy is identical for any given locale contained within a group and, hence, the model effectively collapses into a two cross-section case which is straightforward to solve numerically. I restrict my attention to asymmetric, idiosyncratic shocks rather than to aggregate movements to investigate interesting dynamics that stem from the dual-government feature of this model . Indeed, under cooperation, the model with an aggregate shock collapses to a single economy case that has been studied extensively elsewhere in the literature[29].

For the sake of brevity, I primarily present results for variables of interest for the union aggregate and the locales that have been shocked. Figures 2 and 3 contain comparable impulse responses for the cooperative and strategic cases respectively. The equivalent impulse responses for unshocked locales are generally symmetric under cooperation. However, in the strategic case asymmetries between the shocked and unshocked locales play a key role in understanding the dynamics of the model; therefore I present additional impulse responses for some strategic cases in figure 4.

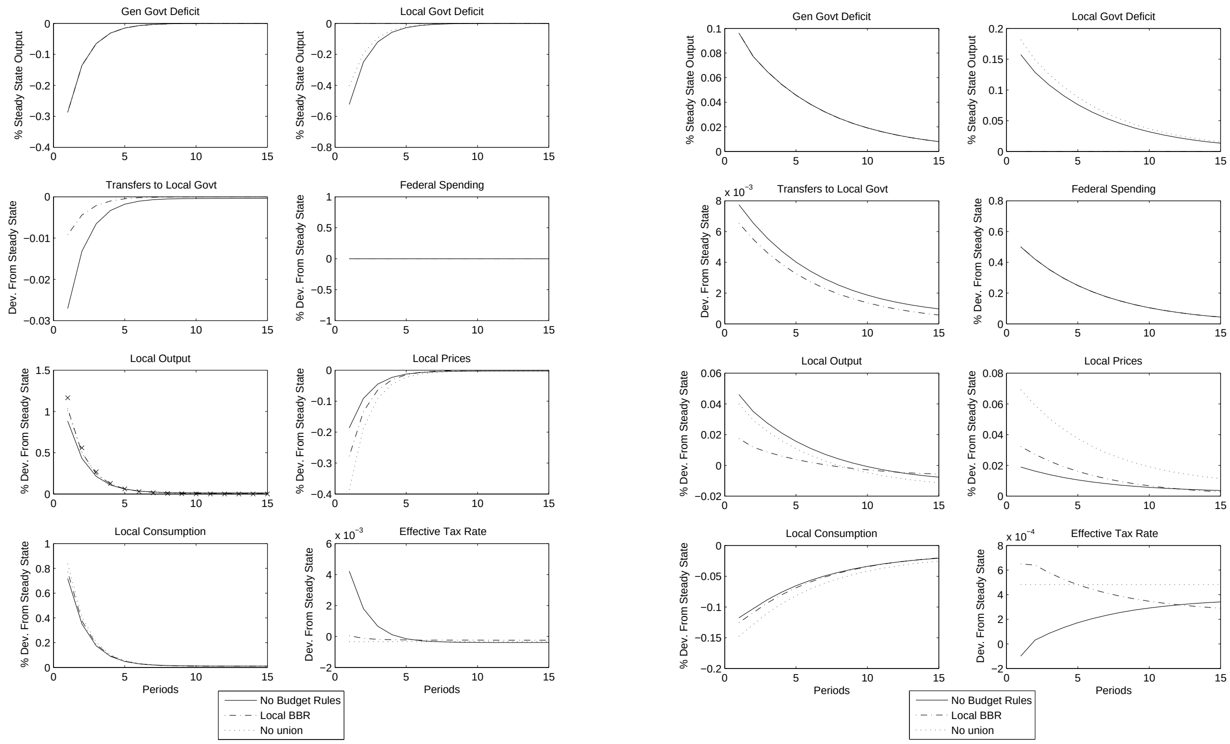
This exercise provides some key intuition behind the stabilising mechanisms available to the union-wide policymaker as well as the policy trade-offs faced under different institutional borrowing regimes. It is also illustrative of the distinction between cooperative and strategic local policy. I present impulse responses

[28]Note, this correlation represents an accurate description of the cost of transferring resources only to the extent that the federal policymaker is operating in an optimal fashion. However, given that the transfer friction was largely motivated as a simplistic attempt to capture the political economy inefficiencies surrounding intergovernmental transfers; that the federal policy choices observed in the data maybe suboptimal is capturing the political constraints the friction is attempting to encompass in the first place.

[29]For cases with unrestricted borrowing much of the intuition can be found in Lucas and Stokey (1983); Stockman (2001) has a discussion on the impact of the balanced budget rules in this context.

Figure 2: Impulse responses under cooperative policy

Response to a 1% productivity shock Response to 1% public consumption shock



Notes: The impulses responses correspond to the model defined in section 2, exogenous variables follow the stochastic processes defined in section 4. The steady state is symmetric and as defined in AppendixB. By local, I refer to the response of the endogenous variables in an arbitrary shocked locale. Periods are considered to correspond to a year. Unrestricted borrowing is assumed to involve a Federal BBR.

for the following cases under cooperative policy: (i) a federal BBR is present (equivalent to unrestrained borrowing); (ii) local BBRs are present and federal borrowing is permitted; (iii) no fiscal union ($\alpha_G = 0$, $\epsilon \to \infty$) with local borrowing. Under strategic policy, the cases are: (i) no restrictions on borrowing; (ii) a federal BBR is present; (iv) no union with local borrowing.

5.1. Cooperative policy

Under cooperative policy the objectives of the union-wide planner are in line with the bi-dimensional stabilising motives discussed previously. These objectives are threefold: first, policy instruments are directed towards limiting the fluctuations in distortionary taxes both inter-temporally and cross-sectionally. Second, the policymaker has the incentive to limit variation in the relative price of local goods, both in order to reduce the price of union-wide consumption and because labour supply is partially determined by local consumer price. Third, the policymaker attempts to limit fluctuations in output in excess of those warranted by productivity changes.

The policy problem is simplest when there are local balanced budget rules: on a cross-sectional basis in response to idiosyncratic shocks the union-wide planner tradeoffs using federal transfers to stabilise tax rates and prices across locales against the inefficiency of deviating from the optimal ratio between local and federal spending in each locale. While on an inter-temporal basis federal borrowing is used to smooth over the aggregate component of the shocks. If local borrowing is permitted, the aggregate inter-temporal aspect is altered only in the sense that the the aggregate local debt stock is used to smooth aggregate fluctuations as opposed to federal borrowing, which is the intuition presented in proposition 4. However, on a cross-sectional basis, in response to idiosyncratic shocks tax rates can now either be smoothed by increasing the local debt stock or via a transfer. This implies that the tradeoff remains between smoothing local taxes and the inefficiency of transfers; however, in the case of temporary shocks local debt can be used as a buffer to defer some the transfer and thus limit the inefficiency. The intuition above explains the behaviour of the impulse responses of the two fiscal union cases displayed in figure 2.

For example, in the case of the public consumption shock, resources are transferred to locales requiring a greater level of government spending in order to harmonise prices and hours worked across the union both contemporaneously and in future periods. When borrowing is permitted, the local policymaker also borrows to smooth out a portion of the shock. This stabilising effect is lost when a local BBR is present, hence the transfers are more generous to compensated for the loss of borrowed financing. However, the larger the transfer the greater the efficiency loss from the transfer friction which causes the union-wide planner to only imperfectly supplement the lack of local borrowing with an increased transfer.

The converse is true in the case of a productivity shock; a local BBR makes transfers less aggressive.[30] The reasoning here is that local borrowing is actively used to supplement transfers; debt is used as a counter-cyclical tool and low productivity locales used deficit financed taxes cuts in order to boost production an equalise the supply of goods, prices and hours worked. This tax subsidising behaviour increases union-wide output increasing resources available to pay for transfers. This implies that transfers are larger which can in turn can finance more subsidising tax cuts to low productivity locales boosting overall output again and creating a feedback loop. On the other hand the group of locales that receive a positive to productivity boost save more resources by running a local surplus which is then slowly transferred to the low productivity locales to pay down the deficit they acquired.

Furthermore, although it is hard to observe in figure 2, this use of local debt as a stabilising tool implies transfers have a permanent component. The intuition here here is along the same line of the classic inter-temporal tax smoothing motive of Barro (1979) which implies unit roots in fiscal variables. However, in this case the motive is not inter-temporal but intra-union. As discussed, changes in relative local government debt are used to optimally smooth over idiosyncratic shocks on an inter-temporal basis; in future periods the union-wide planner will then transfer resources to indebted locales in order to limit the tax distortion arising from interest payments. Transfers will be used erode the debt stock until the point where the marginal gain from a decrease in the level of debt is equal to the marginal cost of the transfer necessary to achieve it. However, once this point is reached transfers have still not returned to the steady state level as it is still optimal to provide resources to pay a portion of the interest bill on the remaining debt stock. In essence what is happening is that the economy has converged to a new steady state where an idiosyncratic shock has left one group of locales relatively more indebted than the other and there are permanent transfer payments to the indebted group to help pay interest on part of the debt in order to tax rates smooth across the locales. This leads to a temporary shock having a permanent effect on transfer policy.

This result is driven entirely by differences in relative debt stocks. The general government deficit behaves in a different fashion as it adjusts to offset fluctuations in the aggregate tax distortion. This explains why the response to a shock is identical regardless of whether local BBRs or a federal BBR is present, there is an almost identical inter-temporal tax-smoothing motive at the aggregate level. However, it is also a feature of the first order approximation failing to include the cost of transfers in the approximate deficit (due its quadratic nature); the case with local BBRs has larger transfers which would lead to a marginal increase in

[30]It is worth noting that case with a local BBRs is nested in the case equivalent to unrestrained borrowing. The planner could choose to implement it but it is not optimal. This laid is out explicitly in the impulse responses: the terms of trade are more volatile in response to idiosyncratic shocks when there is a local BBR than without.

aggregate borrowing.

The behaviour of the effective tax rates themselves reflect the analysis above; incomplete markets leads to debt stocks adjusting to equalise the marginal efficiency of government revenues leading to a permanent component in the effective tax rates (as in, for example, Aiyagari et al (2002)). A standard result that follows from a tax-smoothing motive. Tax rates do not follow a perfect random walk due to the presence of transfers which lead to two additional stationary processes being added to the tax rate. First, the inefficiency of providing the temporary component of asymmetric transfers needs to be paid for by an increase in taxes. Second, the presence of transfers allows for subsidising of locales via changes in tax rates. The intuition behind the latter in the case of a productivity shock is described above. However, it is also apparent in the face of a public consumption shock whereby demand for a particular locale's good goes up. This can be partly offset by increased contemporaneous transfers but also by a tax cut financed borrowing partly paid for by future transfers allows for output today to be subsidised.

This leads to the somewhat quizzical result that in a fiscal union the local multiplier, i.e. the response of local output, from a spending shock is reduced by imposing local BBRs in an optimal policy arrangement both relative to the unrestricted borrowing and no union case. The lack of accompanying expansionary tax cuts explains this result against the former but against the no union case the difference in tax rates is much smaller. The mechanism instead comes from labour supply: in the no union case the public consumption shock raises the price of locale consumption and causes workers to increase labour supply to compensate which in turn increases output; which is the well known mechanism for a positive fiscal multiplier in an RBC model (see Woodford (2011)). This mechanism still exists in the fiscal union, however, the union-wide policymaker tries to limit the fluctuation in relative prices (and hence consumer prices) by increases in transfers which in turn reduces the labour supply response and hence the fiscal multiplier.

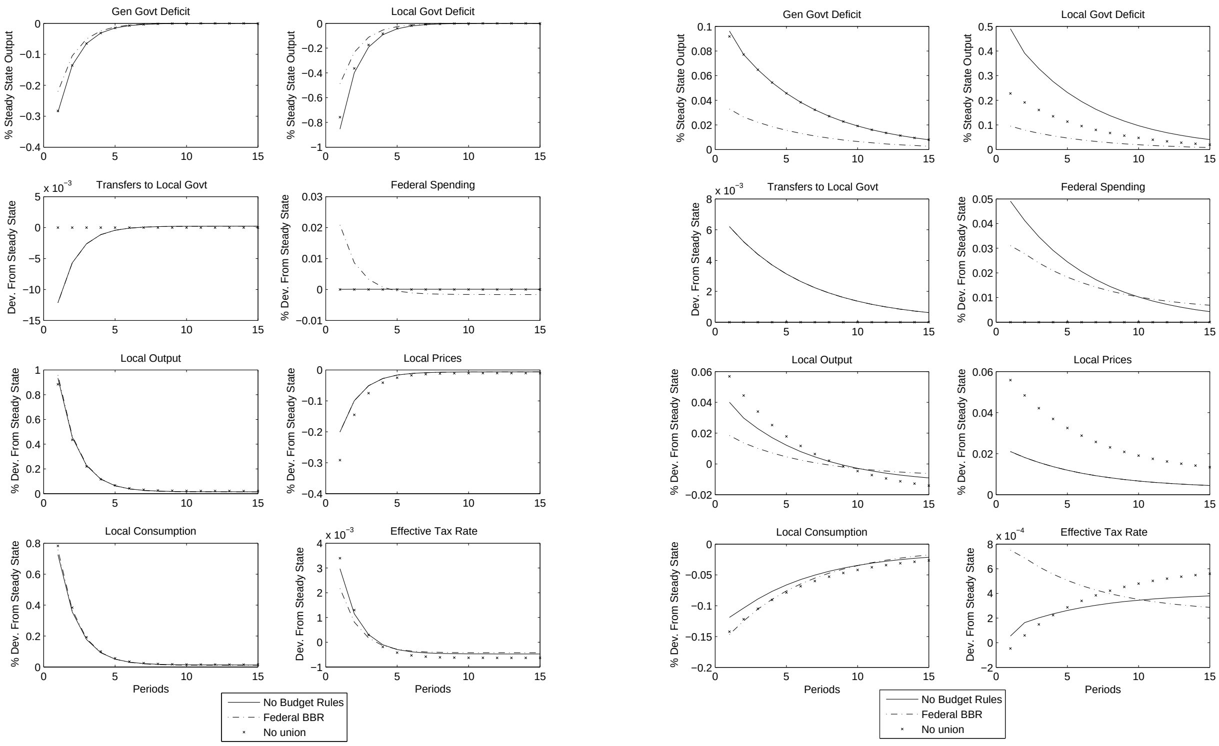
It is worthwhile elaborating a little further on the case where there is no union. Ferrero (2009) considers this case in greater detail and the results here match that intuition; to summarise, the lack of fiscal union implies there is no means to transfer resources across the locales, instead intra-union stabilisation must be carried out by altering marginal tax rates at the expense of changes in the debt stock. As there are no further temporary interactions between stabilisation policy and output, the tax-rates follow a perfect random walk. This still allows for the impact of shocks to be distributed across the union. For example, a positive productivity shock increases the relative price of the output of unshocked locales which reduces the real burden of outstanding debt. However, in the no-union set up there is no instrument which allows temporary for cross-sectional stabilisation; tax rates are changed only once and permanently in response to temporary shock. This lack of cross-sectional smoothing also manifests itself as an increase in the volatility of the terms of trade in response to a shock in comparison to the case where a fiscal union exists. In the union model, transfers perform the role of moving demand around by changing the share of public consumption that needs to come form local output; this demand shifting allows for the price response to idiosyncratic shocks to be ameliorated, stabilising terms of trade across the union. The welfare implications of the no union case and the extent to which the ability to transfer resources offsets the ability to be able to borrow is elaborated on in further detail in section 6.

5.2. The strategic case

Much of the analysis of the economy carries from the cooperative policy environment into the strategic case. The key difference is that the union-wide policy maker loses direct control of local tax and borrowing dimension. This intensifies the policy tradeoffs with respect to transfers. In the cooperative case transfer are used in conjunction with variations in local tax rates for the purposes of stabilising output and prices on a intra-union basis. In the strategic case, the union-wide policy maker uses transfers not only for the purposes of stabilising asymmetric shocks but also to offset the inefficient strategic behaviour of local governments.

Figure 3: Impulse responses under strategic policy

Response to a 1% productivity shock Response to 1% public consumption shock



Notes: The impulses responses correspond to the model defined in section 2, exogenous variables follow the stochastic processes defined in section 4. The steady state is symmetric and as defined in AppendixB. By local, I refer to the response of the endogenous variables in an arbitrary shocked locale. Periods are considered to correspond to a year.

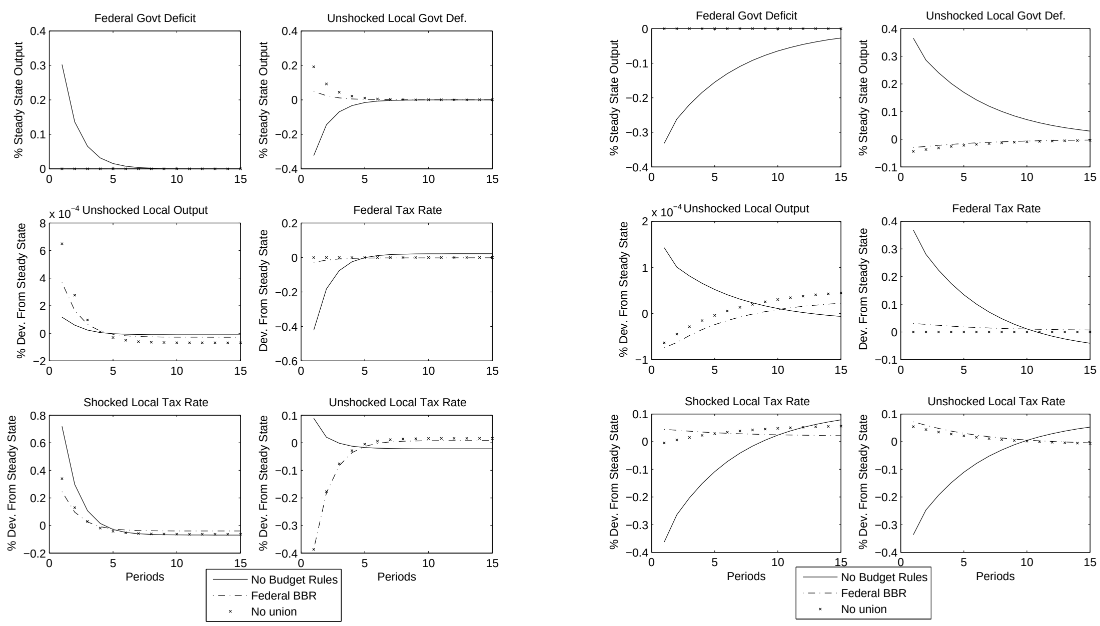
As as a result, the contrast between strategic and cooperative policy is starkest when there is no union, as in this case transfers cannot be used as an instrument to move towards cooperative policy. Under cooperative policy there is no desire on the parts of local governments to exploit asymmetric shocks to their advantage; as discussed, this leaves only an inter-temporal tax-smoothing motive and tax rates follow a perfect random walk. The temporary excess movements in taxes in seen in figure 3 is purely a response to strategic considerations.

Under the calibration used here the net effect of the externalities discussed in section 3 is to encourage the governments in the shocked locales to opt for more aggressive policy ignoring the impact this has on other locale's price levels. In other words, externality 2 acts as the dominant force. In the event of a positive productivity shock the local policymaker reduces debt levels and in order enjoy the lower prices that result from a reduced tax burden in future periods. Inspecting figure 3, this effect manifests itself as an increase in the fiscal surplus of locales that receive a positive productivity shock relative to the cooperative case. The converse is true for the the unshocked locales, it is more costly for policymakers is these locals to to target a new steady state with a permanently lower price level. Instead externality 1 dominates, tax cuts and deficits are used in the short term to stabilise the relative price of production but also to accumulate debt in order to have a lower future disutility of labour.

What is interesting is that, in the case of a positive productivity shock, this strategic behaviour causes the locales to over-stabilise on an intra-union basis in the short-term. That is to say that output each locale and the relative price level is closer to symmetry than the cooperative case; although it comes at the cost of converging to a new steady state values that has increased asymmetry. This implies that in the short-run there is less aggressive transfer payments compared to the cooperative case with local borrowing but in the

Figure 4: Additional impulse responses under strategic policy

Response to a 1% productivity shock Response to 1% public consumption shock



Notes: See note to figure 3. Additionally, the responses of locales that did not receive a shock are denoted “unshocked” and correspondingly the responses of locales that did receive a shock are denoted “shocked”.

long-run the permanent components of transfers are larger than they otherwise would be on order to offset the asymmetry in debt levels.

The local government does not fail to internalise the fact that its borrowing behaviour will influence future federal government transfers. Since the federal government is assumed to be unable to commit to credible punishment strategies for individual locales; in equilibrium it has an incentive to increase transfers to more indebted locales in order to minimise variations in distortionary taxes. As a result, not only does the local government fail to fully internalise the cost of its tax behaviour on union-wide welfare, but does so fully aware that excess borrowing today will be met by future transfers.

In the short-term this implies that transfers have a dual effect relative to the no union case. First, they are used to increase the cost of strategic behaviour; for example, by denying resources from locales that choose to set strategically low taxes. Second, they reduce the impact of differences in debt levels once the economy has re-converged to the steady state. This encourages locales to manipulate their debt stock more aggressively; those locales that are attempting to reduce their debt stock will try to run larger surpluses in order to pay for future transfers. This is evident if one compares the response of a local deficit between the no-union and unrestricted fiscal union models in the event of a positive productivity shock. On a net basis, however, the former must dominate and transfers work to offset strategic behaviour and improve welfare relative to the no union case, since the federal government can always choose not to transfer resources.

The federal government also uses its balance sheet to offset strategic behaviour. When borrowing is unrestricted the federal government conducts deficit financed tax cuts which make strategic local tax cuts on the part of unshocked locales more costly - due to resultant increases in labour supply - and coupled with a transfer this pushes unshocked locales into a surplus relative to the deficit they run in the no-union case. Figure 4 includes additional graphs to illustrate this. The overall response of the general government deficit is similar to the no union case; however, strategic excess borrowing is effectively mutualised and shifted away from unshocked locales to the federal government. This is welfare enhancing as the burden of paying federal debt falls upon the entire union as such the asymmetric tax distortions that arise from differing intra-union local debt levels are reduced.

When the federal government is unable to borrow this policy tool unavailable; the federal government mutualises spending instead and increases its overall share of government purchases. This serves to reduce the local deficit in unshocked locales but the efficiency cost of altering the relative size of the federal government implies that this approach is less effective as an increase in federal borrowing. It is also requires resources to pay the cost of the transfer friction which implies that tax levels in the economy as a whole cannot rise as much as they otherwise would, as output needs to be higher. This reduces the general government surplus.

The story with a public consumption shock is similar if inverted. Unshocked locales see an opportunity to target an environment where they have a permanently lower cost of consumption. The shocked locales instead run deficits and run up debt to work less in future. The federal government does much the same thing in terms of transfers and when it can borrow it uses its balance sheet in a similar fashion to shift the debt burden towards the unshocked locales by reducing its borrowing. When borrowing is prohibited it instead uses federal spending cuts to force through a similar adjustment.

6. Welfare

This section considers the welfare implications of the different policy regimes. I continue to define the exact union-wide expected welfare as V_0^U; however, given that the policy functions are approximated, one can only attempt an approximation of the welfare function. A second-order approximation is required - to the first-order expected welfare losses are necessarily zero in expectation when optimal policy is applied. However, the approximate quadratic welfare should be consistent with the linearised policy functions; in the sense that the policy functions do indeed maximise the objective. As Benigno and Woodford (2012) show, a "naive" second order approximation of V_0^U about the non-stochastic steady state does not, in general, achieve this and nor does it guarantee that the welfare ranking of policies will be correct. Instead, the appropriate objective is a second order approximation of the Lagrangian corresponding to the union wide planner's problem.

For an arbitrary optimal policy plan p, representing one of the classes of optimal policy considered in section 3, denote the quadratic welfare loss as $W^p(X_0)$ as a second order approximation of the corresponding Lagrangian, as defined in AppendixA, less the present value of the objective evaluated at the steady state. Recall that the steady state is invariant to p. The term X_0 denotes the initial values of the state variables, extended to included past Lagrange multipliers corresponding to the commitment to conduct policy from *timeless perspective.* As all optimal policy plans have a identical commitment requirement it is not necessary to impose any penalties on policy plans that would erstwhile not be time consistent. The initial conditions still need to be determined however. One could initialise at the steady state but this is still arbitrary and may favour certain policy plans; given that the economy should fluctuate about the steady state values this approach would penalise plans which have a low curvature of W^p . Instead, I follow the solution in Benigno and Woodford (2012) and calculate expected welfare by integrating over a distribution of initial state vectors. The non-stationary nature of the model implies that the actual set state vector contains unit root processes. To deal with this I take a Beveridge-Nelson decomposition of the state vector to divide it between a trend component and a stationary (cycle) component. The latter is then used to define a distribution of initial state vectors and solve for expected welfare[31]; denote this expectation $\bar{W}^p$.

Given these definitions one can express the cost of a policy plan, p, relative to an a benchmark optimal policy, b, in terms of a share of steady state consumption, λ^p:

[31] This is done numerically, the model is simulated 1000 times for 500 periods initialising at the steady state. The Beveridge-Nelson is applied to the final period value to isolate the stationary component and welfare is evaluated using this value as an initial condition. The expected welfare is then calculated by taking the mean of the simulations.

Table 3: Welfare losses under different policy regimes

	Cooperative Cases			Strategic Cases		
	Unrestricted	Local BBRs	No Union	Unrestricted	Federal BBR	No Union
$\bar{W}_t^p$	-0.2589	-0.2603	-0.3147	-0.2995	-0.3307	-0.4090
λ^p	0.0000%	0.0057%	0.2230%	0.1623%	0.2868%	0.5986%
from aggregate shocks	-0.1335	-0.1336	-0.1336	-0.1585	-0.1651	-0.1707
from idiosyncratic shocks	-0.1253	-0.1266	-0.1811	-0.1410	-0.1656	-0.2383

Notes: The optimal policy regimes are as defined in section 3. Unrestricted refers to the case when local governments can borrow in the cooperative case and where both levels of government borrow in the strategic case. The quadratic welfare cost is calculated as a second order approximation of the optimal policy Lagrangian with dynamics determined by approximated linear policy functions. The welfare loss in units of steady state consumption is calculated as in equation 47. The loss from idiosyncratic shocks is calculated by setting the variance of aggregate shocks to zero and re-evaluating welfare; the method is inverted for the loss from aggregate shocks. The total loss may not be the sum of the parts due to differences in rounding.

$$ln((1-\lambda^p)C_F) - ln((1-\lambda^p)\alpha) - \frac{1}{1+\varphi}[\frac{Y}{A}]^{1+\varphi} + \bar{W}^b = ln(C_F) - ln(\alpha) - \frac{1}{1+\varphi}[\frac{Y}{A}]^{1+\varphi} + \bar{W}^p \tag{47}$$

where variables without time subscripts denote steady values. The case where policy is cooperative and local borrowing is permitted is the most flexible institutional setup in the sense that the alternative optimal policy plans considered are all implementable in this environment. Correspondingly, this also must imply that the quadratic loss is minimised under this plan, hence it serves as reasonable benchmark case. In order to calculate welfare numerically, I retain the technique of approximating the continuum of locales by a discretised cross-section of 50 evenly sized units

Table 3 presents the results for the quadratic welfare loss for the 6 policy plans under consideration, as well as the relative loss as a proportion of steady state consumption and the proportion of the loss accruing to aggregate and idiosyncratic shocks. By and large these results reflect the discussion in section 5; particularly regarding the welfare ordering. The key result is that under cooperative policy if transfers are present the welfare cost of introducing balanced budget rules at the local level is almost negligible when compared to the cost of strategic interaction or a lack of transfers.

The differences between welfare in the cooperative cases are determined in how well the policy plan can adjust to idiosyncratic shocks. Aggregate shocks are dealt with in an identical fashion in all policy plans by using the general government deficit to tax smooth over time. The only cooperative policy under which aggregate shocks would have different welfare implications is if no level of government could borrow. With regards to idiosyncratic shocks, comparing the case with local balanced budget rules versus no-union policy, under the calibration used here, the ability to transfer resources is a much more effective tool at intra-union stabilisation than the flexibility offered by local debt. In general, this is not too surprising, as local borrowing in isolation is an imperfect cross-sectional stabilisation tool with respect to idiosyncratic shocks as in the long-run the debt has to paid back using distortionary taxation and cross-sectional variations in distortionary taxes are inefficient. However, local borrowing is still welfare enhancing and serves to compliment transfer policy, as is evident if one compares the welfare under unrestricted borrowing case to local BBRs. The reason for this is that transfers can be used to offset the long-run costs of variation in the debt stock; while borrowing allows for temporary tax subsidising behaviour to offset the impact of transfers.

The strategic case has similar key result: even if a fully fledged fiscal union with transfers is in place the efficiency cost of strategic behaviour on the part of local policymakers suggest that it is optimal to prohibit local borrowing as a policy tool to enforce cooperation rather than allow local governments control over their own borrowing. Strategic behaviour generates inefficiencies both in response to aggregate and idiosyncratic shocks due to the externalities discussed in section 5. In this regard, restricting the federal governments ability to borrow is additionally costly as the federal governments balance sheet can be used to mutualise excessive deviations in idiosyncratic debt levels or offset strategically motivated changes in the aggregate debt level on the part of local governments. Even so, with borrowing prohibited the federal government is still able to use is control over the union wide tax rate, as well as transfer policy to offset some of the cost of strategic behaviour.

The losses relative to the benchmark as proportion of steady state consumption are small, but not dissimilar to results from representative agent models elsewhere in the literature. To give context, compare these results to Stockman's (2001) finding that introducing a BBR in a similar model featuring a single fiscal authority was equivalent to around 1% of consumption. The order of magnitudes are smaller here, but then no case considered completely rules out all forms of borrowing, so that is to be expected. What this does suggest is that the 0.0057% consumption cost of imposing local BBRs when fiscal transfers are present is also negligible when compared to the welfare cost of preventing borrowing at all levels of government.

Of course, from a welfare perspective, the efficiency of transfers in overriding the inflexibility of local balanced budget rules depends on the size ν, the curvature of the transfer friction which determines the cost of transfer policy. It is worth noting that even if ν was infinite the unrestricted fiscal union under cooperation still is more efficient than the no union case since a proportion $1 - \alpha_G$ of all idiosyncratic public consumption shocks are paid for by the federal government which is acts as an additional form of cross-sectional stabilisation. The question is, at what point are transfers sufficiently costly such that the union-wide policymaker is unable to use them to offset the cost of a local budget rule? Solving for this value gives a figure of $\nu = 22.6$. It seems unreasonable have a strong prior over the value of ν but a natural way of assessing its validity to see what it implies for the behaviour transfers relative to the data. When calibrating the model, the correlation of the simulated cyclical component of transfers with that of the simulated cyclical component of output in the model with local BBRs was matched to that seen in the average US state. Repeating this exercise using a figure of $\nu = 22.6$ gives a correlation of just -0.0859, less than a third of that seen in the data. This does suggest that a value of ν of that magnitude is too high.

However, there are two caveats to this. First, taking the estimation results at face value, it is important to note that the US has a well developed federal system with institutions designed to limit the political economy considerations that surround transfer policy. There is no guarantee fiscal unions elsewhere can achieve a similar level of efficiency. Second, the estimated correlation from the data is also subject to uncertainty; it is worthwhile noting that while the average correlation is substantially larger than -0.0859, in the pairwise estimates for individual states correlations of this magnitude were observed. Therefore a value of $\epsilon = 22.6$ may not be completely unreasonable. As a result, the evidence presented here is not conclusive proof that one should not worry too much about imposing local borrowing rules in a fiscal union but it questions whether local BBRs are a strictly terrible idea.

7. Conclusions

The purpose of this paper is to investigate optimal policy in a RBC model of a fiscal union with two levels of government. Fiscal instruments are restricted to only allow for distortionary taxes and one period debt in order to allow for tax-smoothing to become a key driver of government policy. I consider how borrowing restrictions and the presence of transfers, with a quadratic cost to deviating from an optimal ration of spending by each level of government, affects the dynamics of the economy and the welfare criterion. I consider both the case when policy is set cooperatively and strategically where local governments set local policy in order to maximise local welfare.

The main findings are: *(1)* when policy is conducted on a cooperative basis welfare is virtually insensitive to introduction of local balanced budget rules. Transfers are effective in offsetting the loss of flexibility offered by variations in local debt levels. This result is dependent on the calibrated value on curvature of the transfer friction, but the size of the friction required to overturn this result seems large in comparison to the features of US data. *(2)* When policy is conducted on a strategic basis, federal borrowing is a useful tool to correct for strategic behaviour by local governments. Federal debt can be used to mutualise deficits and hence prevent inefficient variation in local debt levels. *(3)* The intra-union tax smoothing motive of the union wide planner implies that transfers has have a unit root component when local governments can borrow, even if exogenous fluctuations in the economy are temporary. Local variations in debt levels are partly met by transfers in order to limit costly fluctuations in intra-union tax levels that result from differing interest bills.

These findings have a two implications that should be highlighted. First, regarding the well developed literature on assessing the impact of fiscal rules empirically. That these studies have often focused on budget restrictions imposed at a sub-sovereign level suggests their results need to be treated carefully. The model suggests balanced budget rules have rather limited implications for economic activity when they are imposed on a sub-national authority which has access to (imperfect) transfer payments. Thus scaling up the empirical results from sub-national rules and assuming that the economic impact will be equivalent at the national level is maybe inappropriate. The results presented here also provide a potential explanation as to why the empirical literature has struggled to find a significant impact of balanced budget rules on economic volatility.

Second, from the more general perspective of policymakers, is that if one wishes to forge a fiscal union imposing balanced budget restrictions on fiscal entities at lower levels is not necessarily a bad policy choice. Particularly, if there are concerns over whether policymakers at different levels of governments can cooperate. Local budget rules serve as means to reduce the inefficiency arising from strategic interactions subject to transfer being sufficient to offset them. However, the optimal policies presented rely on general government borrowing on some level and transfers of resources between fiscal entities. Indeed, a reasonably well function transfer system is critical to the results the presented, as discussed, if the efficiency loss from transfers is sufficiently high then it may not be optimal to restrict borrowing at any level. A further point is that there are potential political ramifications in terms of the unit root present in optimal transfers policy. While the ex-ante initial insurance of a fiscal union may be appealing it may not be politically possible for a new system to be set up whereby certain regions can receive net transfers indefinitely in response to a temporary shock.

When considering these points it is also worth bearing in mind that the model presented here is highly stylised and there is plenty of scope for additional features to be added in future research to capture additional drivers cyclical fluctuations; sluggish price and wage adjustment, real rigidities such as habit formation and capital accumulation to name but a few. Adding these features would allow for a richer tradeoff of transfers versus idiosyncratic debt accumulation and may have implications for the relative welfare ordering. Although, while such modeling features are still possible within the same basic framework, they also would imply that local policy is no longer time consistent and one would need to make assumptions about the degree, and form, of commitment available to local policymakers. Another issue to consider how optimal policy changes when local debt carries default risk. This may significantly alter the potential desirability of transfers as a policy tool.

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AppendixA. The Lagrangian for a generic the optimal policy problem

Consider cooperative optimal policy from a timeless perspective as laid out in definition 3. The union-wide planners problem is to choose a sequence of endogenous choice variables:

$$X = \{\{Y_t^i, S_t^i, \tau_t^i, G_t^i, h(i), b_t^i\}_i, \tau_t^U, G_t^U, C_{F,t}, b_t^U\}_{t=0}^{\infty}$$

In order to maximise union wide welfare, given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ln(C_{F,t}) + (1-\alpha)\int_0^1 ln(S_t^i)di - \frac{1}{1+\varphi}\int_0^1 [\frac{Y_t^i}{A_t^i}]^{1+\varphi} di\} \tag{A.1}$$

Subject to the following constraints from the behaviour of private agents and the definition of government debt ($\forall i \in [0,1]$):

$$Y_t^i = G_t^i + (S_t^i)^{\sigma}(G_t^U + C_{F,t}) + \frac{(1-\alpha)}{\alpha}(S_t^i)C_{F,t} \tag{A.2}$$

$$(Y_t^i)^{\varphi} = \frac{\alpha (A_t^i)^{\varphi+1}(1-\tau_t^i-\tau_t^U)}{C_{F,t}S_t^i} \tag{A.3}$$

$$([\frac{(G_t^i - \tau_t^i Y_t^i)}{C_{F,t}S_t^i}] + \frac{b_{t-1}^i}{C_{F,t}})) = \beta E\left[\frac{1}{C_{F,t+1}}\right] b_t^i \tag{A.4}$$

$$\bar{G}_t^i = G_t^i + h_t(i)G_t^U - \nu((1-\alpha_G)G_t^i - (\alpha_G)h_t^i G_t^U)^2 \tag{A.5}$$

$$1 = \int_0^1 (S_t^i)^{1-\sigma} di \tag{A.6}$$

$$\int_0^1 h_t^i di = 1 \tag{A.7}$$

$$\frac{G_t^U - \tau_t^U \int_0^1 \frac{Y_t^i}{S_t^i} di}{C_{F,t}} - \frac{b_{t-1}^U}{C_{F,t}} = \beta E\left[\frac{1}{C_{F,t+1}}\right] b_t^U \tag{A.8}$$

A BBR at the local level be incorporate by modifying constraint A.4 into $([\frac{(G_t^i - \tau_t^i Y_t^i)}{C_{F,t}S_t^i}] + \frac{b_0^i}{C_{F,t}})) = \beta E\left[\frac{1}{C_{F,t+1}}\right] b_0^i$, and dropping the sequence $\{\{b_t^i\}_i\}_t$ from the set of choice variables. The same adjustment but to equation A.8 and to the sequence $\{b_t^U\}_t$can be applied to incorporate a federal BBR. Generically we can rewrite this as a dynamic control problem. Let $\xi_t = \{A_t^i, \bar{G}_t^i\}_i = \{\xi_t^i\}_i$ be the vector of exogenous of shocks and X_t be the vector of endogenous choice variables at time t. We can rewrite the policy problem as:

$$V_0^U(y_0; \xi_0) = max\left[E_0 \sum_{t=0}^{\infty} \beta^t U(X_{t+1}, X_t, X_{t-1}; \xi_t)\right]$$

subject to the set of constraints (2)-(8) which we rewrite as as:

$$E_t f(X_{t+1}, X_t, X_{t-1}; \xi_{t+1}, \xi_t, \xi_{t-1}) = 0 \tag{A.9}$$

Hence, the Lagrangian can be written as:

$$\mathcal{L} = max\left[E\sum_{t=0}^{\infty}\beta^t(U(X_{t+1}, X_t, X_{t-1}; \xi_t) + \phi_t f(X_{t+1}, X_t, X_{t-1}; \xi_{t+1}, \xi_t, \xi_{t-1}))\right]$$

Where $\beta^t\phi_t$ denote the vector of Lagrange multipliers corresponding to the constraints above. This implies a set of first order conditions in the form:

- For $t > 0$

$$\begin{aligned} 0 &= E_t[U_1(X_{t+1}, X_t, X_{t-1}; \xi_t)] + E_{t-1}[\phi_{t-1} f_1(y_t, y_{t-1}, y_{t-2}; \xi_t, \xi_{t-1}, \xi_{t-1})] \\ &\quad + E_t[\phi_t f_2(X_{t+1}, X_t, X_{t-1}; \xi_{t+1}, \xi_t, \xi_{t-1})] + E_t[\phi_{t+1} f_3(X_{t+2}, X_{t+1}, X_t; \xi_{t+2}, \xi_{t+1}, \xi_t)] \end{aligned}$$

- For $t = 0$

$$\begin{aligned} 0 &= E_t[U_1(X_{t+1}, X_t, X_{t-1}; \xi_t)] \\ &\quad + E_t[\phi_t f_2(X_{t+1}, X_t, X_{t-1}; \xi_{t+1}, \xi_t, \xi_{t-1})] + E_t[\phi_{t+1} f_3(X_{t+2}, X_{t+1}, X_t; \xi_{t+2}, \xi_{t+1}, \xi_t)] \end{aligned}$$

The difference between the first order conditions between periods $t = 0$ and all future periods is the root of the timer inconsistency in a generic optimal policy problem. The union wide planner would choose a different policy where he or she to reoptimise at at future date than the one he or she would prefer to commit to today. Note that if there are no lagged Lagrange multipliers in the first order conditions, which is equivalent to forward looking variables in the constraints being independent of policy, there is no time inconsistency issue. Which explains why the local governments problem in the strategic case is time consistent. As laid out in the text, the approach to deal with this issue is to frame policy in a timeless perspective. This is equivalent to extending the state vector to include past Lagrange multipliers from previous period when optimising. Rather than assume ϕ_{-1} is a vector of a zeros when first optimising. This equivalent to imposing an additional constraint on policy such that the private sector's expectations of the endogenous choice variables in period -1 is rational. This guarantees that the first order optimisation conditions are invariant over time.

To give context to the time inconsistency problem relative to the model here, consider the first order conditions to cooperative problem (a federal BBR is imposed to prevent indeterminacy). Let the variables $\beta^t\phi^i_{1t}$ to $\beta^t\phi^i_{4t}$ be the Lagrange multipliers on constraints A.2 to A.5 and $\beta^t\phi_{5t}, \beta^t\phi_{6t}, \beta^t\phi_{7t}$ correspond to constraints A.7 and A.8. The steady state must satisfy the following first order conditions ($\forall i \in [0, 1]$):

wrt Y^i_t :

$$\phi^i_{1t} \quad + \varphi\phi^i_{2t}(Y^i_t)^{\varphi-1} = \phi^i_{3t} \quad \frac{\tau^i_t}{C_{F,t}S^i_t} + \phi_{7t}\frac{\tau^U_t}{S^i_t} + \frac{1}{Y^i_t}[\frac{Y^i_t}{A^i_t}]^{\varphi+1}$$

wrt S^i_t:

$$\begin{aligned} \phi^i_{2t}\frac{\alpha(A^i_t)^{\varphi+1}(1-\tau^i_t-\tau^U_t)}{C_{F,t}(S^i_t)^2} - \phi^i_{3t}[\frac{(G^i_t-\tau^i_tY^i_t)}{C_{F,t}(S^i_t)^2}] &= (1-\sigma)\phi_{5t}(S^i_t)^{-\sigma} + \\ & \phi^i_{1t}[(\sigma)(S^i_t)^{\sigma-1}G^U_t + C_{F,t}(\frac{(1-\alpha)}{\alpha} + (\sigma)(S^i_t)^{\sigma-1})] - \frac{\phi_{7t}\tau^U_tY^i_t}{(S^i_t)^2} - (1-\alpha)(S^i_t)^{-1} \end{aligned} \tag{A.10}$$

wrt τ^i_t:

$$\frac{\phi^i_{2t}\alpha(A^i_t)^{\varphi+1}}{S^i_tC_{F,t}} = \phi^i_{3t}\frac{Y^i_t}{S^i_tC_{F,t}} \tag{A.11}$$

wrt G^i_t:

$$\frac{\phi^i_{3t}}{C_{F,t}S^i_t} = \phi^i_{1t} + \phi^i_{4t}(1-2\nu(1-\alpha_G)[(1-\alpha_G)(G^i_t) - (\alpha_G)(h^i_tG^U_t)] \tag{A.12}$$

wrt h_t^i :

$$\phi_{6t} = \phi_{4t}^i(G_t^U + 2\nu(\alpha_G)(G_t^U)[(1-\alpha_G)(G_t^i) - (\alpha_G)(h_t^i G_t^U)] \tag{A.13}$$

wrt b_t^i:

$$E(\frac{\phi_{3t+1}^i}{C_{F,t+1}}) = E(\frac{\phi_{3t}^i}{C_{F,t+1}}) \tag{A.14}$$

wrt τ_t^U:

$$\int_0^1 \frac{\phi_{2t}^i \alpha(A_t^i)^{\varphi+1}}{S_t^i C_{F,t}} di = \phi_{7t} \int_0^1 \frac{Y_t^i}{S_t^i} di \tag{A.15}$$

wrt G_t^U:

$$\phi_{7t} = \int_0^1 \phi_{1t}^i (S_t^i)^\sigma di + \int_0^1 \phi_{4t}^i (h(i) + 2\nu(1-\alpha_G)(h_t^i)[(1-\alpha_G)(G_t^i) - (\alpha_G)(h_t^i G_t^U)] di \tag{A.16}$$

wrt $C_{F,t}$

$$\begin{aligned} \frac{1}{C_{F,t}} + \int_0^1 \phi_{3t-1}^i E_{t-1}\left[\frac{b_{t-1}^i}{C_{F,t}^2}\right] di \quad = \quad & \int_0^1 (\phi_{1t}^i (S_t^i)^\sigma + \frac{(1-\alpha)}{\alpha}(S_t^i)\phi_{1t}^i - \\ & \phi_{2t}^i \frac{\alpha(A_t^i)^{\varphi+1}(1-\tau_t^i-\tau_t^U)}{(C_{F.t})^2 S_t^i} + \phi_{3t}^i \frac{S_t^i(G_t^i - \tau^i Y_t^i - \varsigma_t^i) + b_{t-1}^i}{C_{F,t}^2}) di \end{aligned} \tag{A.17}$$

The only lagged Lagrange multipliers present in this problem correspond to the first order condition on consumption. This is because the pricing kernel for the yield on government debt depends on future expectations of aggregate consumption via the Euler condition. The time inconsistency, that would otherwise emerge, is fairly subtle in the sense that that the union-wide planner has incentive to use consumption surprises to manipulate the interest rate on government borrowing.

However, the issue is more relevant when the strategic case is considered. Allowing for strategic interactions increases the number of constraints the union-wide planner faces with the addition of four extra constraints that correspond to optimal, strategic, behaviour of local governments given their objectives:

wrt Y_t^i :

$$\theta_{1t}^i + \varphi(Y_t^i)^{\varphi-1}\theta_{2t}^i = \frac{\tau_t^i}{S_t^i C_{F,t}}\theta_{3t}^i + \frac{1}{Y_t^i}[\frac{Y_t^i}{A_t^i}]^{1+\varphi} \tag{A.18}$$

wrt S_t^i :

$$\frac{(Y_t^i)^\varphi \theta_{2t}^i}{(S_t^i)^2} + \frac{(1-\alpha)}{S_t^i} = \frac{(G_t^i - \tau_t^i Y_t^i)\theta_{3t}^i}{C_{F,t}(S_t^i)^2} + \sigma\theta_{1t}^i (S_t^i)^{\sigma-1}(G_t^U + C_{F,t}) + \frac{(1-\alpha)}{\alpha} C_{F,t}\theta_{1t}^i \tag{A.19}$$

wrt τ_t^i :

$$\frac{\alpha\theta_{2t}^i (A_t^i)^{\varphi+1}}{C_{F,t} S_t^i} = \frac{\theta_{3t}^i Y_t^i}{C_{F,t} S_t^i} \tag{A.20}$$

wrt b_t^i

$$\frac{\theta_{3t}^i}{E(C_{F,t+1})} = E\left[\frac{\theta_{3t+1}^i}{C_{F,t+1}}\right] \tag{A.21}$$

Where $\{\theta_{1t}^i, \theta_{2t}^i, \theta_{3t}^i\}$ correspond the Lagrange multipliers on the local governments problem. The optimal policy Lagrangian is straightforward to modify in order to take this additional constraints by simply treating the sequence of local Lagrange multipliers as an additional sequence of choice variables $\Theta = \{\{\theta^i_{1t}, \theta^i_{2t}, \theta^i_{3t}\}_i\}_{t=0}^{\infty} = \{\Theta_t\}_{t=0}^{\infty}$. The behaviour of local governments can then be incorporated as a set of additional constraints:

$$E_t\tilde{f}(X_{t+1}, X_t, X_{t-1}, \Theta_{t+1}, \Theta_t, \Theta_{t-1}; \xi_{t+1}, \xi_t, \xi_{t-1}) = 0 \tag{A.22}$$

$$\mathcal{L} = max\left[E\sum_{t=0}^{\infty}\beta^t(U(X_{t+1}, X_t, X_{t-1}; \xi_t) + \tilde{\phi}_t\tilde{f}(X_{t+1}, X_t, X_{t-1}, \Theta_{t+1}, \Theta_t, \Theta_{t-1}; \xi_{t+1}, \xi_t, \xi_{t-1}))\right]$$

Which can be solved using exactly the same techniques as the cooperative case. Time inconsistency is a greater issue here in that forward looking variables appear not just in determining the interest rate but also in the local governments optimality condition with respect to borrowing, equation A.21. This equation states that local governments will borrow up to the point where the current marginal efficiency of government debt is equal to is expect discounted future value. The time inconsistency emerges because the union-wide policy maker has an incentive to promise future transfers in order to alter θ^i_{3t} in order to incentivise local governments to behave properly. Considering optimal policy from a *timeless perspective* is equivalent to commitment to adhere to such promises not just made at time $t = 0$ and in future but any promises made periods prior to optimisation.

AppendixB. The existence of a symmetric, deterministic steady state

Let variables without time subscripts denote the steady state values. In the deterministic steady state the vector of exogenous disturbances are equal across locales and over time ($A^i = 1,\ ,\ \bar{G}^i = \chi Y^i > 0$). Where χ is the share of steady state output devoted to general government consumption. General government debt is a fixed fraction of output $B = \Omega Y^i$, I assume this is apportioned equally between the federal and local governments in line with their optimal shares in public consumption; i.e. $b^i = \alpha_G \Omega Y^i$. Symmetry can be used to eliminate the cross-section indices $Y^i = Y$, $\tau^i = \tau^L$, & $G^i = G^L\ \forall i \in [0, 1]$. It also implies no relative price differentials $S^i = 1$. It should also be obvious that:

$$h^i = 1,\ \forall i \in [0, 1]$$

The optimal levels of government consumption are $G^i = \alpha_G\chi Y^i$ and $G^U = (1 - \alpha_G)\chi Y$. Using the demand condition:

$$Y = G^L + G^U + \frac{1}{\alpha}C_F \Leftrightarrow \alpha(1-\chi)Y = C_F \tag{B.1}$$

Let the steady state local tax rate be equal to $\tau^i = \tau^L$ with τ^U the steady state federal tax rate, one can also define the aggregate tax rate $\bar{\tau} = \tau^L + \tau^U$. Using the local budget constraint A.4 and the stable debt in steady state we have:

$$\tau^L Y - G^L = \alpha_G(1-\beta)\Omega Y$$

Substituting in optimal government consumption behaviour and the debt level constraint we have:

$$\tau^L Y - \alpha_G\chi Y = \alpha_G(1-\beta)\Omega Y$$

Which implies the steady state local tax rate can be expressed as:

$$\tau^L = \alpha_G(1-\beta)\Omega + \alpha_G\chi$$

By applying equation A.8 and repeating the procedure becomes apparent that $\tau^U = (1-\alpha_G)(\chi+(1-\beta)\Omega)$ and the the aggregate tax rate as $\bar{\tau} = (1-\beta)\Omega + \chi$. Under these conditions it is possible to completely determine output as a function of χ and Ω using constraint A.3:

$$Y = \left[\frac{(1-\chi-(1-\beta)\Omega)}{(1-\chi)}\right]^{1/(\varphi+1)} \tag{B.2}$$

Equation B.2 also allows us to define what is meant by a feasible steady state:

Definition 7. *A steady state indexed by the government debt to output ratio, Ω, and the public consumption to output ratio, χ, is considered feasible if the level of of steady state output that satisfies equation B.2 is a positive, real number.*

This basically states that the fiscal demands on the economy are sufficiently small that the private sector can produce the resources to satisfy it.

What remains is to show that these conditions satisfy the f.o.c. defined above. In general, for a set of steady choice variables that satisfy the constraints on the economy, there exists a set of Lagrange multipliers such that this steady state implies satisfies the optimality conditions for the union wide planner. To see an example of this consider the first order conditions on the cooperative policy problem defined in AppendixA, utilising the fact that in the steady state all Lagrange multipliers will be constant and invariant across i.

wrt Y_t^i :

$$\phi_1 \quad +\varphi\phi_2(Y)^{\varphi-1} = \phi_3 \quad \frac{(1-\beta)\Omega+\alpha_G\chi}{\alpha(1-\chi)Y} + \phi_7(1-\alpha_G)\chi + [Y]^{\varphi}$$

wrt S_t^i:

$$\begin{aligned} \phi_2\frac{(1-(1-\beta)\Omega-\chi)}{(1-\chi)Y} - \phi_3[\frac{(1-\beta)\Omega Y}{\alpha(1-\chi)Y}] \quad &= \quad (1-\sigma)\phi_5 + \phi_1[(\sigma)(1-\alpha_G)\chi Y + \\ &\alpha(1-\chi)Y(\frac{(1-\alpha)}{\alpha} + (\sigma)] - \phi_7(1-\alpha_G)\chi Y + (1-\alpha) \end{aligned}$$

wrt τ_t^i:

$$\alpha\phi_2 = \phi_3 Y \tag{B.3}$$

wrt G_t^i:

$$\frac{\phi_3}{\alpha(1-\chi)Y} = \phi_1 + \phi_4 \tag{B.4}$$

wrt h_t^i

$$\phi_6 = \phi_4(1-\alpha_G)\chi Y \tag{B.5}$$

wrt b_t^i:

$$\phi_3 = \phi_3 \tag{B.6}$$

wrt τ_t^U:

$$\frac{\phi_2}{(1-\chi)Y} = \phi_7 Y \tag{B.7}$$

wrt G_t^U:

$$\phi_7 = \phi_1 + \phi_4 \tag{B.8}$$

wrt $C_{F,t}$

$$1 = \phi_1(1-\chi)Y - \phi_2\frac{(1-(1-\beta)\Omega-\chi)}{(1-\chi)Y} - \phi_3\frac{(1-\beta)\Omega}{\alpha(1-\chi)} \tag{B.9}$$

These represent a linear system of 9 equations with 7 unknowns; however, the two first order conditions on local debt are redundant leading to 8 equations with 7 unknowns. This reduces to a unique solution after some tedious algebra. To see this note that it is possible to expressϕ_2 and ϕ_3 in terms of ϕ_7:

$$\phi_1 \quad +\varphi\phi_2(Y)^{\varphi-1} = \phi_3 \quad \frac{(1-\beta)\Omega+\alpha_G\chi}{\alpha(1-\chi)Y} + \phi_7(1-\alpha_G)\chi + [Y]^\varphi$$

$$\phi_1 \quad = \quad \phi_7((1-\beta)\Omega+\chi-\varphi(1-\chi)(Y)^{\varphi+1}) + [Y]^\varphi$$

$$\phi_3 = \phi_7\alpha(1-\chi)Y$$

$$\phi_2 = \phi_7(1-\chi)Y^2 \tag{B.10}$$

$$-(1-(1-\beta)\Omega-\chi)(1-\chi)Y\phi_7 = \phi_2$$

Using the f.o.c wrt Y_t^i, C_t and the above conditions one can obtain two simultaneous equations in terms ofϕ_1 and ϕ_7:

$$\phi_1 \quad = \quad \phi_7((1-\beta)\Omega+\chi-\varphi(1-\chi)(Y)^{\varphi+1}) + [Y]^\varphi$$

$$\phi_1 = \frac{1}{(1-\chi)Y} + \phi_7$$

we can therefore solve for ϕ_7:

$$1 = [\phi_7((1-\beta)\Omega+\chi-\varphi(1-\chi)(Y)^{\varphi+1}) + (Y)^\varphi](1-\chi)Y - (1-\chi)Y\phi_7$$

$$\frac{1}{(1-\chi)Y} - (Y)^\varphi = [\phi_7((1-\beta)\Omega+\chi-\varphi(1-\chi)(Y)^{\varphi+1})] - \phi_7$$

In the steady state we also know that:

$$(Y)^{\varphi+1} = \frac{(1-(1-\beta)\Omega-\chi)}{(1-\chi)}$$

This implies:

$$\phi_7 = \frac{1}{(1+\varphi)(1-((1-\beta)\Omega+\chi))}((Y)^\varphi - \frac{1}{(1-\chi)Y})$$

And correspondingly:

$$\phi_1 = \frac{1}{(1+\varphi)(1-((1-\beta)\Omega+\chi))}((Y)^{\varphi} - \frac{1}{(1-\chi)Y}) + \frac{1}{(1-\chi)Y}$$

$$\phi_2 = \frac{(1-\chi)Y^2}{(1+\varphi)(1-((1-\beta)\Omega+\chi))}((Y)^{\varphi} - \frac{1}{(1-\chi)Y})$$

$$\phi_3 = \frac{\alpha(1-\chi)Y}{(1+\varphi)(1-((1-\beta)\Omega+\chi))}((Y)^{\varphi} - \frac{1}{(1-\chi)Y})$$

Using the f.o.c. wrt G_t^U we obtain:

$$\phi_4 = \phi_7 - \phi_1 = -\frac{1}{(1-\chi)Y}$$

and hence:

$$\phi_6 = \frac{-(1-\alpha_G)\chi}{(1-\chi)}$$

Lastly, note that one can express ϕ_5 as a function of the remaining multipliers using the f.o.c. wrt S_t^i:

$$\begin{aligned}\phi_2\frac{(1-(1-\beta)\Omega-\chi)}{(1-\chi)Y} - \phi_3[\frac{(1-\beta)\Omega Y}{\alpha(1-\chi)Y}] &= (1-\sigma)\phi_5 + \phi_1[(\sigma-1)(1-\alpha_G)\chi Y + \\ &\quad \alpha(1-\chi)Y(\frac{(1-\alpha)}{\alpha} + (\sigma-1)] -\end{aligned}$$

$$\begin{aligned}\phi_5 &= \phi_2\frac{(1-(1-\beta)\Omega-\chi)}{(1-\sigma)(1-\chi)Y} - \phi_3[\frac{(1-\beta)\Omega Y}{(1-\sigma)\alpha(1-\chi)Y}] + \frac{\phi_7(1-\alpha_G)\chi Y}{(1-\sigma)} - \frac{(1-\alpha)}{(1-\sigma)} \\ &\quad +\phi_1[(1-\alpha_G)\chi Y + (1-\chi)Y(\frac{(1-\alpha)}{(\sigma-1)} + 1]\end{aligned}$$

This combination of Lagrange multipliers and choice variables defines the steady state for the cooperative optimal policy problem where a federal BBR is present. A similar exercise can be conducted for the other cooperative policy problems. For the strategic case, I calculate the corresponding values of the local government's Lagrange multipliers in AppendixC.2.

AppendixC. Proofs of propositions

AppendixC.1. Proof of Proposition 4

Proposition 4: *For a cooperative optimal policy: (i) the relative shares of general government debt held by the federal (b_t^U) and local governments ($\int_0^1 b_t^i di$) are indeterminate. (ii) a federal BBR when coupled with unconstrained local borrowing can achieve an identical welfare level and private sector dynamics to unconstrained borrowing at all levels of government .*

Consider a version of the economy where borrowing occurs at both levels of government, local and federal budget constraints of the form:

$$G_t^U - \tau_t^U \int_0^1 \frac{Y_t^i}{S_t^i} di + b_{t-1}^U = R_t^{-1} b_t^U$$

$$\frac{G_t^i}{S_t^i} - \tau_t^i \frac{Y_t^i}{S_t^i} + b_{t-1}^i = R_t^{-1} b_t^i$$

Where R_t is interest on one-period bonds. Assume there exists a feasible stochastic sequence X that solves the optimal cooperative policy problem which includes a debt plan $\{\{b_t^i\}_i, b_t^U\}_{t=0}^{\infty}$. Now imagine an alternative sequence $\tilde{X}$ with an altered debt plan in the form $\{\{\tilde{b}_t^i\}_i, \tilde{b}_t^U\}_t = \{\{b_t^i + \delta_t^i\}_i, b_t^U - \delta_t\}_t$ where $\{\{\delta_t^i\}_i, \delta_t\}_t$ is an arbitrary sequence constrained such that:

- the debt levels remain appropriately bounded.
- The level of general government debt is unchanged: $\int_0^1 \delta_t^i di = \delta_t$
- The change in the federal debt is redistributed among locales in accordance with their share in nominal output: $\frac{\delta_t^i}{\delta_t} = \frac{Y_t^i/S_t^i}{\int_0^1 Y_t^j/S_t^j dj}$.

In addition, under $\tilde{X}$ taxes are altered such that the sequence satisfy the government budget constraint; namely, the new sequence of tax rates can be defined as: $\{\{\tilde{\tau}_t^i\}_i, \tilde{\tau}_t^U\}_{t=0}^{\infty} = \{\{\tau_t^i + \omega_t\}_i, \tau_t^U - \omega_t\}_{t=0}^{\infty}$ where $\omega_t = \frac{\delta_{t-1} - R_t\delta_t}{\int_0^1 Y_t^i/S_t^i di}$. The remaining elements of $\tilde{X}$ are identical to X.

To prove (i) one needs to check whether $\tilde{X}$ is feasible and leads to an identical welfare outcome as X. Since only the tax and debt elements of differ from X, it must be the case that constraints A.2,A.5, A.6 and A.7 all hold since X is feasible and these equations are identical under $\tilde{X}$. By construction government budget constraints A.4 and A.8 also hold and since the overall tax burden in each locale is unchanged $(\tilde{\tau}_t^i + \tilde{\tau}_t^U = \tau_t^i + \tau_t^U)$ the supply condition A.3 is satisfied; hence, $\tilde{X}$ is feasible.

Define the sequence of Lagrange multipliers corresponding to X as $\Phi = \{\{\phi_{1t}^i, \phi_{2t}^i, \phi_{3t}^i, \phi_{4t}^i\}_i, \phi_{5t}, \phi_{6t}, \phi_{7t}\}_{t=0}^{\infty}$ where ϕ_{1t}^i to ϕ_{4t}^i are the Lagrange multipliers on A.2- A.5 and ϕ_{5t} to ϕ_{7t} are the multipliers on A.6 - A.8. The first order condition on τ_t^i and τ_t^U can be rewritten as:

$$\frac{\phi_{2t}^i \alpha (A_t^i)^{\varphi+1}}{S_t^i C_{F,t}} = \phi_{3t}^i \frac{Y_t^i}{S_t^i C_{F,t}}$$

$$\int_0^1 \frac{\phi_{2t}^i \alpha (A_t^i)^{\varphi+1}}{S_t^i C_{F,t}} di = \frac{\phi_{7t}}{C_{F,t}} \int_0^1 \frac{Y_t^i}{S_t^i} di$$

these conditions can be combined to give:

$$\int_0^1 \phi_{3t}^i \frac{Y_t^i}{S_t^i} di = \phi_{7t} \int_0^1 \frac{Y_t^i}{S_t^i} di \tag{C.1}$$

To check that welfare is unchanged consider the absolute differential of the Lagrangian with respect to δ_t. By the envelope theorem:

$$\frac{d\mathcal{L}}{d\delta_t} = [\phi_{7t} - \int \phi_{3t}^i \frac{d\delta_i}{d\delta_t}] - \beta[\phi_{7t+1} - \int \phi_{3t+1}^i \frac{d\delta_i}{d\delta_t}]$$

Noting that $\frac{d\delta_i}{d\delta_t} = \frac{Y_t^i/S_t^i}{\int_0^1 Y_t^j/S_t^j dj}$, C.1 implies that $\frac{d\mathcal{L}}{d\delta_t} = 0$. Therefore, for any optimal sequence X any marginal change in debt in the form δ_t results in an allocation that remains optimal. Since a marginal change from any optimal sequence returns another optimal sequence and the Lagrangian is continuous, step changes in δ_t similarly result in an optimal sequence. An alternative way to see this is to guess that the sequence $\tilde{X}$ results in a modified sequence of Lagrange multipliers $\tilde{\Phi} = \{\{\phi_{1t}^i, \phi_{2t}^i, \frac{\phi_{3t}^i \tau_t^i}{(\tau_t^i + \omega_t)}, \phi_{4t}^i\}_i, \phi_{5t}, \phi_{6t}, , \frac{\phi_{7t}\tau_t^U}{(\tau_t^U - \omega_t)}\}_{t=0}^{\infty}$ and verify that the first order conditions continue to hold - which is a matter of tedious algebra. That changes in δ_t are irrelevant for welfare implies that there is indeterminacy in the relative debt shares.

To prove (ii) note that the above implies that for any optimal sequence it is possible to construct an arbitrary sequence $\{\delta_t\}_t$ which leaves the total amount of general government debt constant and shifts relative share between government levels with no welfare cost and identical private sector outcomes unchanged. A federal BBR requires federal debt to be time invariant, such that $b_t^U = \bar{b}$. From any given optimal sequence with unconstrained borrowing, we could choose $\delta_t = b_t^U - \bar{b}$ to achieve the same optimal allocation with a federal BBR.

AppendixC.2. Proof of Proposition 5

Proposition 5. *Consider the set of feasible symmetric steady states indexed by the debt-to-GDP ratio, Ω. The following is true: (i) there exists a Ω such that the marginal efficiency of additional government debt from the perspective of the local policymaker, θ_3^i, is zero. Denote the corresponding steady state effective tax rate $(\tau^l + \tau^U)$ asτ^*. This tax rate is contained in the unit interval: $\tau^* \in (0,1)$. (ii) In the steady state where the effective tax burden, $\tau^l + \tau^U$, is zero (corresponding to $\Omega < 0$) $\theta_3^i > 0$.*

Consider the same symmetric steady state as defined in AppendixB; we can rewrite the local foc as:

wrt Y_t^i :

$$\theta_1 + \varphi(Y)^{\varphi-1}\theta_2 = \frac{\tau^l}{\alpha(1-\chi)Y}\theta_3 + [Y]^\varphi$$

wrt S_t^i :

$$(Y)^\varphi\theta_2 + (1-\alpha) = \frac{(\alpha_G\chi Y - \tau^l Y)\theta_3}{\alpha(1-\chi)Y} + \sigma\theta_1(G + C_{F,t}) + \frac{(1-\alpha)}{\alpha}C_{F,t}\theta_{1t}^i$$

wrt τ_t^i :

$$\alpha\theta_2 = \theta_3 Y$$

wrt b_t^i

$$\theta_3 = \theta_3$$

Using the first order condition eliminate θ_2:

$$\theta_1 = \left[\frac{\tau^l}{(1-\chi)Y} - \varphi(Y)^\varphi\right]\frac{\theta_3}{\alpha} + [Y]^\varphi$$

$$\frac{(1-\alpha)}{Y} - \frac{(\alpha_G\chi - \tau^l)\theta_3}{\alpha(1-\chi)Y} + \frac{(Y)^\varphi\theta_3}{\alpha} = \theta_1[\sigma((1-\alpha_G)\chi + \alpha(1-\chi)) + (1-\alpha)(1-\chi)]$$

Let $\Upsilon = ((1-\alpha)(1-\chi) + \sigma((1-\alpha_G)\chi + \alpha(1-\chi))) > 0$ and substitute in for θ_1:

$$\frac{(1-\alpha)}{Y} - \frac{(\alpha_G\chi - \tau^l)\theta_3}{\alpha(1-\chi)Y} + \frac{(Y)^\varphi\theta_3}{\alpha} = \Upsilon\left[\frac{\tau^l}{(1-\chi)Y} - \varphi(Y)^\varphi\right]\frac{\theta_3}{\alpha} + \Theta[Y]^\varphi$$

Assuming we operate in an environment whereby the federal government does not borrow:$\tau^l = \alpha_G\chi + (1-\beta)\Omega$

$$\frac{\alpha(1-\alpha)}{Y} - \alpha\Upsilon[Y]^\varphi = \left(\Upsilon\left[\frac{\alpha_G\chi + (1-\beta)\Omega}{(1-\chi)Y} - \varphi(Y)^\varphi\right] - \frac{(1-\beta)\Omega}{(1-\chi)Y} - (Y)^\varphi\right)\theta_3$$

We can now solve for τ^*:

$$\theta_3 = 0 \Rightarrow \frac{\alpha(1-\alpha)}{Y} = \alpha\Theta[Y]^\varphi$$

Using the definition of steady state output $(Y)^{\varphi} = \frac{(1-\tau^*)}{(1-\chi)Y}$. This implies:

$$(1-\alpha) = \Upsilon \frac{(1-\tau^*)}{(1-\chi)}$$

Hence,

$$\tau^* = 1 - \frac{(1-\alpha)(1-\chi)}{\Upsilon}$$

Since, $\Upsilon > (1-\alpha)(1-\chi)$ and Υ, $(1-\alpha)(1-\chi) > 0$ it follows that $\tau^* \in (0,1)$. To prove *(ii)* note the expression for θ_3 can be expanded further by using the relationship: $(Y)^{\varphi} = \frac{(1-\chi-(1-\beta)\Omega)}{(1-\chi)Y}$:

$$\theta_3 = \frac{\alpha(1-\alpha)(1-\chi) - \alpha\Upsilon(1-\chi-(1-\beta)\Omega)}{(\Upsilon\left[\alpha_G\chi + (1-\beta)\Omega - \varphi(1-\chi-(1-\beta)\Omega)\right] - (1-\chi))}$$

If the the steady state tax rate is zero then $\chi + (1-\beta)\Omega = 0$, which implies:

$$\theta_3 = \frac{\alpha\Upsilon - \alpha(1-\alpha)(1-\chi)}{(\Upsilon\left[(1-\alpha_G)\chi + \varphi\right] + (1-\chi))}$$

Both the denominator and the numerator of this expression are strictly positive, hence $\theta_3^i > 0$.