Profit-Sharing as the Optimal Wage Contract*†

Kenjiro Hori Faculty of Economics and Politics University of Cambridge

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Abstract

The aim of the paper is to challenge the current conjecture in corporate strategy that it is optimal for a firm to adopt a hire-and-fire employment policy. A general model of firms facing demand uncertainty is built to show that, for both risk-neutral and risk-averse workers, the optimal strategy for a profit-maximising firm is to offer a profit-sharing wage contract. This provides more stable employment for workers, compared with an economy where firms offer fixed-wage contracts. Moreover for risk-neutral workers, an explicit expression for the optimal profit-share ratio is found, which is shown to occur at a point where the fixed-component of the wage is driven down to the out-of-work income. One implication of this is that full employment would be achieved if unemployment benefit can be set at zero.

1 Introduction

Conjectures in economic theory change with experience. This is certainly true in the field of the theory of the firm. In the 1980s when the world feared the domination of the Japanese corporations, many claimed that Japan's structure of shared economy, with its life-time employment, seniority-based income structure, and rotation-based in-house training, was the secret behind the phenomenal recovery of post-war Japanese economy. In the 1990s it was the turn

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[†]This is a shorter version of the original paper Hori (2003a). The original extends the work by applying the model to Weitzman's (1985) macroeconomic model of competitive monopolistic firms and CES demand functions. This allows us to demonstrate specifically the results attained here. In particular, we are able to provide an explicit closed-form expression for the optimal profit-share parameter for risk-neutral workers.

of the US corporations and their flexible labour market to claim victory. Many advocates preached the benefits of the hire-and-fire corporation culture: that not only it would lead to efficient production within the firms, but there would also be an efficient allocation of resources in the economy as a whole. The aim of this paper is to challenge this hypothesis, particularly in the face of demand uncertainty. For workers facing uncertain future, it is not too difficult to make the case for a certain degree of job security, or at least a remuneration increase to compensate for the imposed labour flexibility. The main question of this paper is whether it is also in the interest of the firm facing uncertainty to adopt a non-hire-and-fire employment policy. If it turns out to be so then this will lead to a Pareto superior employment contract.

This paper shows that one could indeed incentivise firms to behave as a labour-hoarding firm without requiring any employment commitment by the firms, by using a linear profit-sharing wage contract which is a sum of a fixed component of wage and an aggregate-performance related bonus which is the profit-sharing component. The model is a very general one with the only other requirement being that the revenue function be concave. The basic argument is as follows. Given such a wage contract and a realised market demand, a profitmaximising firm would adjust its employment level by equating its marginal revenue of an extra employee with the fixed-wage component level. An increase in this fixed-wage component then, while giving workers higher total remuneration, would increase the probability of redundancy in times of negative shocks in the market demand. The net welfare effect of this for the workers thus seems ambiguous. The paper however shows that employment would always occur where this net effect is positive. This means that workers will be willing to trade some of their fixed component of wage for a higher level of profit-sharing, as long as their participation constraint is satisfied. Thus in going from a fixedwage contract to a profit-share contract, the workers benefit from higher bonus payouts and greater stability in employment, while losing out from a decreased fixed-wage component. The overall effect of these on the utility level is zero when their participation constraints bind. The firm on the other hand loses out from being required to share a larger portion of the aggregate profit, but it also gains from the lower fixed-wage payouts, while the increased level of employment has no marginal effect on its profit. Thus there is a net social gain of increased employment stability, which in a monopolistic goods market the firm claims, at least until the fixed-wage component is driven down to the out-of-work income (such as unemployment benefit). The corresponding profit-sharing level is the optimal profit-share ratio. One implication of this is that if the unemployment benefit can be set at zero, then we would have a pure profit-sharing full employment economy. The paper then extends this argument to cover risk-averse workers, where it is shown that profit-sharing is still optimal, despite the effects of the changes in the variance of the wage income.

The resulting effects for employment of applying profit-sharing wage contracts is identical to those of labour-hoarding, where the employment level is kept constant for a negative shock in demand, at least up to a threshold point. There is much evidence of these effects in the literature. An often cited work is Fay and Medoff (1985), which in its survey of the U.S. manufacturing sector conclude that between 4 and 8% of the labour paid can be classified as hoarded. This result is supported by subsequent studies by Fair (1985), Burnside, Eichenbaum and Ronelo (1993) and Sbordone (1996). Others such as Hart and Malley (1996) and Vecchi (2000) undertake international comparisons and show that the Japanese firms do reduce their workforce size much less in economic downturns than in countries such as the U.S., U.K. or Germany. This paper addresses the question of why such behaviour is observed. Most, including Okun (1962), Le Roy Miller (1971), Fay and Medoff (1985) and Sbordone (1996) argue that labour hoarding is optimal behaviour for a firm, given contractual commitments, hiring and training expenses, lay-off costs and firm-specific human capital investment costs. The fact that the last of these is significantly higher in Japan than in the west (as suggested by Hashimoto and Raisian (1985), Mincer and Higuchi (1988), Koike (1988)) is then often cited as the reason why labour-hoarding is observed to be more prevalent in Japan.

The argument in this paper is that even without these costs, firms will optimally keep the employment level stable for a negative shock smaller than a threshold level. The main point is that a wage contract can be devised such that firms choose to offer greater employment security. This is in line with the argument proposed by Weitzman (1984). In his book, Weitzman claims that in comparison to the fixed-wage economy, firms in a profit-share economy would provide higher employment stability due to its lower marginal cost of an extra worker. Weitzman (1995) provides some literature survey on the empirical studies testing this, the major one being Kruse (1993). A formal macroeconomic model of his profit-share economy, using competitively monopolistic firms and CES demand functions, is outlined in Weitzman (1985). Here a more general model is developed with conditions on production and demand functions relaxed. As already stated an introduction of participation constraints for the workers allows a trade-off between profit-sharing and the fixed-wage component levels, and the resulting Pareto-optimal outcome turns out to be that of a positive profit-share. Furthermore, where Weitzman (1985) states that he does not have "a formal theory that would explain ... why a society chooses a particular [profit-sharing] configuration...", here we provide an explicit expression for the optimal profit-share parameter for risk-neutral workers.

The paper is organised as follows. Section 2 develops the general model of profit-share economy with risk-neutral workers. Section 3 extends the model to risk-averse workers. Section 4 then concludes.

2 Model with Risk-Neutral Workers

2.1 The Problem

We start with homogeneous firms with concave revenue functions R = py, where $p \equiv p(y)$ is the market price, $y \equiv y(N)$ is the firms' production function, and N is the number of employees. Note that in a monopolistic setting, such as the one assumed by Weitzman (1985), y(N) itself need not be concave as long as p is downward sloping. These firms face uncertainty in the demand market which is represented by a multiplicative parameter ϕ , such that the stochastic market price for the output goods equals ϕp . ϕ is thus assumed to be stochastically distributed, with $\phi \in [0, \infty]$, $E[\phi] = 1$, and its probability density function $f(\phi)$ is known by both the firm and the workers. We assume no production uncertainties, which would exclude any problems associated with moral hazard.

The workers, who are also homogeneous, are assumed to receive w_{out} when out-of-work. This can be thought of as the unemployment benefit provided by the state. They are also assumed to hold a reservation utility below which they will not supply labour. To begin with workers are assumed to be risk-neutral, and therefore the reservation utility is a reservation expected wage level w_{res} . Clearly $w_{res} \geq w_{out}$.

We also follow Weitzman (1985) and set the wage contract offered by the firm to take the following form,

$$w = F + \lambda \left(\frac{\phi R - FN}{N}\right) \tag{1}$$

where F is the fixed-wage component and R is the non-stochastic part of the revenue function given by R = py. Then $\phi R - FN$ is the aggregate profit for the particular state of nature ϕ . λ of this, where $0 \le \lambda \le 1$, is shared equally among the N workers as aggregate performance-related bonus payment. Clearly when $\lambda = 0$ then the wage contract becomes that of fixed-wage.

Now the wage and employment determination process is modelled as follows. First the profit-sharing ratio λ is determined by the firm, and the form of wage contract (1) is made public. Next we assume there to be a market clearing process in w_{res} , given λ , in the labour market such that the full employment equilibrium is reached. In other words the fixed-wage component F is driven down to the point where each firm employs N_{full} workers, where nN_{full} is the total workforce population, with the corresponding expected wage income being the full equilibrium reservation wage w_{res} . In this case w_{out} will indeed be the unemployment benefit. Now we let the demand uncertainty ϕ be revealed.

¹Weitzman (1985) assumes the firms to be *competitive monopolists*, where a large number of firms operate in individual differentiated goods markets. The labour market on the other hand is assumed perfectly competitive.

Seeing the actual demand level, the firm will then adjust its employment level to $N \leq N_{full}$ to maximise its profit. Here we rule out re-employment of redundant workers by other firms with positive demand shocks $\phi > 1$ (in which case N will be greater than N_{full}). This is justified by either assuming ϕ to be a uniform shock to the economy, in which case the restriction is by the total size of the labour market, or that while ϕ is unique to the individual firms, n is large and there is sufficient friction in the inter-industry labour mobility that re-employment is too costly for that period. This ensures that the redundant workers spend at least one period out-of-work.

This problem is solved backwardly:

Stage 3 Given (F, λ) and the realised ϕ , the firm maximises π with respect to N.

$$\max_{N(\phi)} \quad \pi(F, \lambda, \phi) = \phi R(N) - wN \tag{2}$$

subject to a full employment labour market constraint,

$$N(\phi) \le N_{full} \tag{3}$$

By substituting for w this profit function can also be written as,

$$\pi(F, \lambda, \phi) = (1 - \lambda) \left\{ \phi R(N) - FN \right\} \tag{4}$$

Stage 2 Given λ , the risk-neutral workers can work out for themselves the resulting employment level $N(\phi)$ for each realised ϕ and the particular values of F. Their participation constraint is then,

$$Ew(F,\lambda) = \int_0^\infty \overline{w}(\phi) f(\phi) d\phi \ge w_{res}$$
 (PC1)

where the average income $\overline{w}(\phi)$ for the state of nature ϕ is given by,

$$\overline{w}(\phi) = \left(\frac{N(\phi)}{N_{full}}\right) w + \left(1 - \frac{N(\phi)}{N_{full}}\right) w_{out}$$
 (5)

Here again w_{out} is the income outside work (e.g. unemployment benefit) and w_{res} is the reservation expected wage income level for the full employment labour market equilibrium. For each particular λ , the workers will then accept the value of F that will satisfy this.

Stage 1 The firm, knowing the workers' behaviour in Stage 2, will maximise the expected profit level $E\pi$ with respect to λ ,

$$\max_{\lambda} E\pi(F,\lambda) = \int_{0}^{\infty} (1-\lambda) \left\{ \phi R(N) - FN \right\} f(\phi) d\phi \qquad (6)$$

subject to (PC1) and (2).

In the analysis below we suppress the argument ϕ in the optimal employment level $N(\phi)$ and the average income $\overline{w}(\phi)$ for notational simplicity.

2.2 Solving the Problem

Let us first investigate N. Given the assumption for the shape of the revenue function $R \equiv py$, whether the firms operate as competitive monopolists or in perfect competition, the profit-function $\pi = (1 - \lambda)(\phi py - FN)$ for given F, λ , and ϕ is a single-peaked concave function in N. This shifts upwards and to the right with an increase in ϕ . Then N are the labour input levels where the peaks are attained, unless it is constrained by N_{full} ,

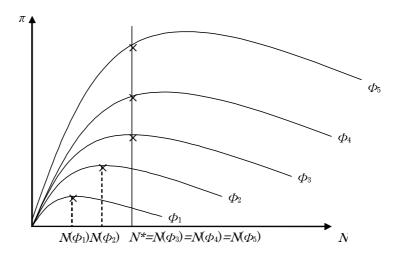


Figure 1: π vs. N for varying ϕ , given (ω, λ)

For values of ϕ for which $N < N_{full}$, N is then determined by (2), or using (4),

$$R'(N) = \frac{F}{\phi} \tag{7}$$

Note that this is independent of λ . N thus only depends on λ insofar as it affects the equilibrium fixed-wage component F determined in the participation constraint (PC1). For values of ϕ for which $N=N_{full}$, the marginal revenue $\phi R'$ will be greater than the fixed-wage component F. We define $\overline{\phi}$ for the particular value of ϕ such that the peak occurs exactly at N_{full} , i.e.,

$$\overline{\phi} = \frac{F}{R'(N_{full})} \tag{8}$$

In Figure 1, $\overline{\phi}$ equals ϕ_3 . In this paper we assume a normalisation such that at the level of F where $\lambda = 0$, $\overline{\phi}$ would equal 1. This is simply stating that when the wage contract is a fixed-wage, then some levels of redundancies will

take place with probability $\frac{1}{2}$. As later seen a positive λ would lead to a reduced probability of redundancy, which would imply that $\overline{\phi} \leq 1$.

Note that in Figure 1, as F increases the peaks of the π curves would shift to the left. Hence,

$$\frac{\partial N}{\partial F} \begin{cases} < 0 & \text{for } \phi \le \overline{\phi} \\ = 0 & \text{for } \phi > \overline{\phi} \end{cases} \tag{9}$$

and,

$$\frac{\partial \overline{\phi}}{\partial F} > 0 \tag{10}$$

Now we are ready to solve our problem. First consider the maximisation (6). Differentiating $E\pi$ with respect to λ subject to (2) gives,

$$\frac{dE\pi}{d\lambda} = -\frac{E\pi}{1-\lambda} - (1-\lambda)\overline{N}\frac{dF}{d\lambda} \tag{11}$$

where \overline{N} is the average number of employees,

$$\overline{N} = \int_0^\infty Nf(\phi)d\phi \tag{12}$$

Here we have used the fact that although N is also affected by changes in λ via F, there is no marginal effect of an increase in N on the aggregate profit level $\phi R - FN$ as this is continuously being maximised. Next consider the participation constraint (PC1). Here F adjusts such that $\frac{dEw}{d\lambda} = 0$. Now Ew can be rewritten as,

$$Ew = \int_0^\infty \left[\frac{N}{N_{full}} \left(w - w_{out} \right) \right] f(\phi) d\phi + w_{out}$$
 (13)

Then, by noting that from (1) $\frac{\partial w}{\partial \lambda} = \frac{\phi R - FN}{N}$, $\frac{\partial w}{\partial F} = 1 - \lambda$ and $\frac{\partial w}{\partial N} = -\lambda \left(\frac{\phi R - FN}{N^2}\right)$ (the last of which again takes into account of (2)),

$$\frac{dEw}{d\lambda} = \frac{\partial Ew}{\partial \lambda} + \frac{dF}{d\lambda} \frac{dEw}{dF}
= \int_{0}^{\infty} \frac{N}{N_{full}} \left(\frac{\phi R - FN}{N}\right) f(\phi) d\phi + \frac{dF}{d\lambda} \left\{ \int_{0}^{\infty} \frac{N}{N_{full}} (1 - \lambda) f(\phi) d\phi \right.
+ \int_{0}^{\infty} \frac{1}{N_{full}} \frac{\partial N}{\partial F} (w - w_{out}) f(\phi) d\phi - \int_{0}^{\infty} \frac{N}{N_{full}} \frac{\partial N}{\partial F} \lambda \left(\frac{\phi R - FN}{N^{2}}\right) f(\phi) d\phi \right\}
= \frac{1}{N_{full}} \frac{E\pi}{1 - \lambda} + \frac{1}{N_{full}} \frac{dF}{d\lambda} \left\{ (1 - \lambda) \overline{N} + (F - w_{out}) \overline{N}' \right\}$$
(14)

where \overline{N}' is the average change in employment level,

$$\overline{N'} = \int_0^{\overline{\phi}} \frac{\partial N}{\partial F} f(\phi) d\phi \tag{15}$$

which we already know to be negative. Here we have used the fact that $\frac{\partial N}{\partial F} = 0$ for $\phi > \overline{\phi}$. Hence we have for $\frac{dEw}{d\lambda} = 0$,

$$\frac{E\pi}{1-\lambda} + \frac{dF}{d\lambda} \left\{ (1-\lambda)\overline{N} + (F - w_{out})\overline{N'} \right\} = 0 \tag{16}$$

Combining (16) with (11) gives the following result,²

$$\frac{dE\pi}{d\lambda} = \Delta \overline{N'} \frac{dF}{d\lambda} \tag{17}$$

where

$$\Delta = F - w_{out}$$

i.e. the extra income from the fixed-wage component when in work. The interpretation of this result is as follows. In (11) when the value of λ is increased, the firm loses from being required to share a larger proportion of the aggregate profit, but if the sign of $\frac{dF}{d\lambda}$ is negative it gains from the fall in the fixed-wage component level. The fall in F would also affect the number of employees, but as already mentioned the marginal effect of this on the profit level is zero due to the maximisation in (2). The mirror image of this is for the workers the first two terms of eqn (14): an increase in λ increases their bonus payout, but decreases their income from fixed-wage component if again $\frac{dF}{d\lambda}$ is negative. These are simply welfare transfers between the firm and the workers. However in addition the workers also benefit from the increased employment level (the third term in (14)). Thus there is a net social welfare gain for the firm and the workers of the workers' benefit from increased employment. The firm having all the bargaining power restricts the workers' utility at the reservation level (i.e. the participation constraint binds, and hence (16)), and thus it claims the extra rent, as shown in (17).

To investigate (17), first consider (PC1) at $\lambda = 0$, i.e. the fixed-wage contract w = F. Then using (13), (PC1) becomes,

$$\Delta \frac{\overline{N}}{N_{full}} = w_{res} - w_{out} \tag{18}$$

$$E\pi = \int_{0}^{\infty} (\phi R - w_{out} N) f(\phi) d\phi - (w_{res} - w_{out}) N_{full}$$

Totally differentiating this with respect to λ and using (7) again yields (17).

²Another way of getting to this result is to note that, in satisfying the participation constraint of the workers, the firm's expected profit is also equivalent to,

Thus if the risk-neutral workers' reservation utility level w_{res} was equal to (greater than) the out-of-work income w_{out} , then the fixed-wage component F at $\lambda = 0$ will also have to be equal to (greater than) w_{out} . We exclude the case $w_{res} < w_{out}$ as a condition for positive labour supply by the workers. Thus at $w_{res} = w_{out}$, (17) implies that the firm will be indifferent offering the fixed-wage contract. For $w_{res} > w_{out}$,³ we have a more interesting result:

Proposition 1 For $w_{res} > w_{out}$ the firm will optimally choose to offer a profit-sharing wage contract.

Proof. This will be true if (17) is positive at $\lambda = 0$ for $w_{res} > w_{out}$. It is easier to see this if we rewrite (17) as,

$$\frac{dE\pi}{d\lambda} = \frac{-\Delta \overline{N'} \frac{\partial Ew}{\partial \lambda}}{\frac{dEw}{dE}} \tag{19}$$

This simply uses the fact that (PC1) implies that $\frac{dEw}{d\lambda} = \frac{\partial Ew}{\partial \lambda} + \frac{dF}{d\lambda} \frac{dEw}{dF} = 0$. Then as we already know that $\overline{N'}$ is negative, and that $F > w_{out}$ for $w_{res} > w_{out}$, it suffices to show that $\frac{\partial Ew}{\partial \lambda}$ and $\frac{dEw}{dF}$ are both positive at $\lambda = 0$. We know these from (14):

$$\frac{\partial Ew}{\partial \lambda} = \frac{1}{N_{full}} \frac{E\pi}{1-\lambda} \tag{20}$$

$$\frac{dEw}{dF} = \frac{1}{N_{full}} \left\{ (1 - \lambda) \, \overline{N} + \Delta \overline{N'} \right\} \tag{21}$$

Thus $\frac{\partial Ew}{\partial \lambda}$ is unambiguously positive. To see the sign of $\frac{dEw}{dF}$ requires a little more thought. At $F=w_{out}$ this is also unambiguously positive. An increase in F reduces the first term (as N decreases), while making the second term negative. Thus at some point the sign of $\frac{dEw}{dF}$ turns negative. In the limit as F becomes very large, the first-order condition (7) implies that $N\to 0$ for all ϕ . As this implies that $\overline{N'}$ also become zero, $\frac{dEw}{dF}$ will thus approach zero asymptotically. So it seems that the sign of $\frac{dEw}{dF}$ is indeterminate. Fortunately it turns out that we can state unambiguously, that for feasible values of F, $\frac{dEw}{dF}$ is always positive. To see this, first rewrite Ew in (13) as

$$Ew(F) = \Delta \frac{\overline{N}}{N_{full}} + \frac{\lambda}{N_{full}} \int_{0}^{\infty} (\phi R - FN) f(\phi) d\phi + w_{out}$$

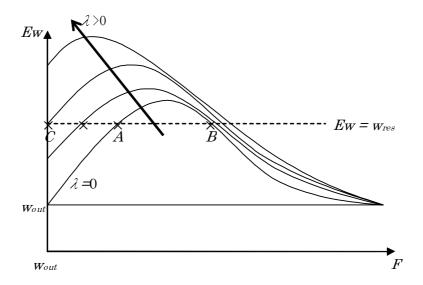
We denote Ew with the argument F to remind ourselves that we are considering the Ew curve with respect to F, for a given value of λ . At $F = w_{out}$, $\Delta = 0$

³This is possible from our earlier assumption of search friction which ensures that the workers spend at least one period out-of-work after being made redundant.

and this equals,

$$Ew(w_{out}) = \frac{\lambda}{N_{full}} \int_0^\infty (\phi R - w_{out} N) f(\phi) d\phi + w_{out}$$
 (22)

This is increasing in λ , and is equal to w_{out} at $\lambda = 0$. At the other extreme, as again eqn (7) implies that for F very large N decreases to zero (and thus $R \to 0$ also), $Ew(\infty)$ will asymptotically approach w_{out} . This is true for all values of λ . So Ew(F) for different values of λ look like this:



Note that as we know that $\frac{\partial Ew}{\partial \lambda} > 0$ for any values of F (eqn (20)), the curve shifts upwards with an increase in λ . Now the solutions for the participation constraint $Ew = w_{res}$ occur where the horizontal line w_{res} crosses the Ew curves. At $\lambda = 0$, for $w_{res} > w_{out}$ there are clearly two solutions.⁴ The question is at which values of F (A or B in Figure 2) would the firm operate. It turns out that the firm will always choose the lower value of F, as at a given λ , differentiating (6) by F subject to (2) gives,

$$\frac{dE\pi}{dF} = -(1-\lambda)\,\overline{N}\tag{23}$$

which is negative. Hence we have proved that for $w_{res} > w_{out}$, at $\lambda = 0$ the firm will always operate at a value of F where $\frac{dEw}{dF} > 0$. Together with the positive $\frac{\partial Ew}{\partial \lambda}$ in (20), this implies that $\frac{dE\pi}{d\lambda} > 0$ in eqn (19). Thus the firm will optimally choose to offer a profit-sharing wage contract $\lambda > 0$.

⁴We disregard the cases where w_{res} is higher than the maximum possible Ew (i.e. at the peak), as then there will be no firms operating in the fixed-wage economy. We can assume that the labour market competition will ensure this.

Intuitively what we are stating here can be summarised as follows. We know that the expected wage Ew for the workers increases with the share-parameter λ for a fixed F. In contrast an increase in the fixed-wage component F affects Ew in two different ways: (i) positively by increasing the wage level, and (ii) negatively by increasing the probability of being made redundant. What the proof of above proposition has shown is that the net effect of these is always positive at $\lambda = 0$. Then risk-neutral workers will allow a decrease in F (as long as $F > w_{out}$) for an increase in λ in keeping their Ew constant, i.e. $\frac{dF}{d\lambda}$ is indeed negative unambiguously. Then as we have already seen there is an overall welfare gain from becoming a profit-sharing firm, which is the expected profit gain for the firm.

Now it follows that,

Proposition 2 The optimal profit-share occurs at the point where the fixed-wage component F equals the out-of-work income w_{out} .

Proof. We have seen in the above proof that as λ increases, F at which Ew equals w_{res} decreases (i.e. the participation constraint solution shifts from point A towards point C in Figure 2). This continues until the Ew(F) curve has risen enough that this solution does equal w_{out} (i.e. point C). Let us call the value of λ for which this occurs λ^* . For λ greater than λ^* there will only be one solution for the participation constraint $Ew = w_{res}$, for which $\frac{dEw}{dF}$ is negative. Thus for these values of λ , $\frac{dE\pi}{d\lambda} < 0$ in eqn (19) and the firm will have no more incentive to increase profit-sharing. Thus the optimal profit-share occurs at $\lambda = \lambda^*$, or at the value of λ such that $F = w_{out}$ in (PC1).

One can also see this by investigating (19). We have already stated that from (20), $\frac{\partial Ew}{\partial \lambda}$ is unambiguously positive. (21) also shows that $\frac{dEw}{dF}$ would never take the value $+\infty$. Thus the only time that $\frac{dE\pi}{d\lambda}$ in (19) would equal 0 is when $\Delta=0$, i.e. $F=w_{out}$. In fact we can find an explicit expression for the optimal profit-share parameter λ^* . As we know what Ew is at $F=w_{out}$ from eqn (22), λ^* is the value of λ at which this equals w_{res} , i.e.,

$$\lambda^* = \frac{(w_{res} - w_{out}) N_{full}}{\int_0^\infty (\phi R - w_{out} N) f(\phi) d\phi}$$
(24)

One of the implications of Proposition 2 is the following,

Proposition 3 Full employment is achieved at $w_{out} = 0$.

Proof. Proposition 2 implies that with $w_{out} = 0$, λ will be driven up to a point where F is driven down to 0 (i.e. a pure profit-share contract). At this point the first-order condition (7) for the profit function implies that for

all values of ϕ , the firm will wish to employ the maximum possible employment level, i.e. N_{full} . Thus full employment is achieved regardless of the fluctuations in the market demand.

The full employment profit-share ratio is then (24) when $w_{out} = 0$, i.e.,

$$\lambda^* = \frac{w_{res} N_{full}}{R\left(N_{full}\right) \int_0^\infty \phi f(\phi) d\phi} = \frac{w_{res} N_{full}}{R\left(N_{full}\right)}$$
(25)

We now proceed to see if these results continue to hold for risk-averse workers.

3 Extension to Risk-Averse Workers

Instead of demanding a constant expected wage level, risk-averse workers will demand a fixed level of certainty equivalent. This can be approximated by (see for example Milgrom and Roberts (1992) Ch7),

$$CE = Ew - \frac{r}{2}Var[w] \tag{26}$$

where r is the constant risk-aversion parameter, and the variance term is given by,

$$Var[w] = \int_0^\infty \overline{w}^2 f(\phi) d\phi - [Ew]^2$$
 (27)

where \overline{w} is defined in (5). Substituting this we can expand (27) to,

$$Var[w] = \int_{0}^{\infty} \left\{ \left(\frac{N}{N_{full}} \right) (w - w_{out}) + w_{out} \right\}^{2} f(\phi) d\phi - [Ew]^{2}$$

$$= \int_{0}^{\infty} \left\{ \left(\frac{N}{N_{full}} \right) (w - w_{out}) \right\}^{2} f(\phi) d\phi + 2w_{out} (Ew - w_{out}) + w_{out}^{2} - [Ew]^{2}$$

$$= \int_{0}^{\infty} \left\{ \left(\frac{N}{N_{full}} \right) (w - w_{out}) \right\}^{2} f(\phi) d\phi + (Ew - w_{out})^{2}$$
(28)

The participation constraint for the risk-averse workers is then,

$$CE(F,\lambda) = Ew - \frac{r}{2}Var[w] = CE_{res}$$
 (PC2)

As before, as the firm alters the share-parameter λ the workers will adjust F to keep CE at CE_{res} . Thus,

$$\frac{dCE}{d\lambda} = \frac{dEw}{d\lambda} - \frac{r}{2} \frac{dVar[w]}{d\lambda} = 0$$
 (29)

We already know $\frac{dEw}{d\lambda}$ from (14). Thus (29) becomes,

$$\frac{1}{N_{full}} \frac{E\pi}{1-\lambda} + \frac{\overline{N}}{N_{full}} \frac{dF}{d\lambda} \left\{ (1-\lambda)\overline{N} + \Delta \overline{N'} \right\} - \frac{r}{2} \frac{dVar[w]}{d\lambda} = 0$$

Combining with (11) gives the following result,⁵

$$\frac{dE\pi}{d\lambda} = \Delta \overline{N'} \frac{dF}{d\lambda} - \frac{r}{2} N_{full} \frac{dVar[w]}{d\lambda}$$
 (30)

As in the risk-neutral workers case, an increase in λ reduces the firm's profit by a decrease in its share of the aggregate profit, but the firm also benefits from a fall in the fixed-wage component level F if the sign of $\frac{dF}{d\lambda}$ is negative. The mirror image of this is that the workers benefit from a larger bonus payout but suffer from a reduction in their fixed-wage component income. In addition the workers benefit from the increased employment level caused by the drop in F, as already seen before. However risk-averse workers are also affected by the effects that the increase in λ has on the variance of w. As the participation constraint (PC2) ensures that the net effect on the workers' certainty equivalent is nil, the firm claims any net welfare gain, which is eqn (30).

Unfortunately the sign of $\frac{dVar[w]}{d\lambda}$ is indeterminate. Clearly as λ increases, Var[w] increases as the size of the variable part of the wage contract increases. On the other hand a decrease in F (again if $\frac{dF}{d\lambda}$ is negative) reduces the variance, as the difference between the in-work wage level w and out-of-work income w_{out} decreases. The effect of the resulting increase in employment level N is also ambiguous, as while a smaller pie of the aggregate profit means a reduced variance, an increased probability of being paid the higher in-work wage increases the variance.

We are, however, still able to state the following,

Proposition 4 For $CE_{res} > w_{out}$ the firm will optimally choose to offer a profit-sharing wage contract.

Proof. Firstly (PC2) implies that $\frac{dCE}{d\lambda} = \frac{\partial CE}{\partial \lambda} - \frac{dF}{d\lambda} \frac{dCE}{dF} = 0$, and thus,

$$\frac{dF}{d\lambda} = \frac{-\frac{\partial CE}{\partial \lambda}}{\frac{dCE}{dF}} \tag{31}$$

$$E\pi = \int_{0}^{\infty} \left(\phi R - w_{out} N\right) f(\phi) d\phi - \left\{ C E_{res} - \left(w_{out} - \frac{r}{2} Var[w]\right) \right\} N_{full}$$

Totally differentiating this with respect to λ and using (7) again yields (30).

⁵Again one could have derived at the same result by realising that, in satisfying (PC2) the firm's expected profit is equivalent to,

Next we expand (30) to get,

$$\frac{dE\pi}{d\lambda} = \Delta \overline{N'} \frac{dF}{d\lambda} - \frac{r}{2} N_{full} \left\{ \frac{\partial Var[w]}{\partial \lambda} + \frac{dF}{d\lambda} \frac{dVar[w]}{dF} \right\}
= -\frac{r}{2} N_{full} \frac{\partial Var[w]}{\partial \lambda} + \frac{dF}{d\lambda} \left\{ \Delta \overline{N'} - \frac{r}{2} N_{full} \frac{dVar[w]}{dF} \right\}$$

Then by substituting (31),

$$\frac{dE\pi}{d\lambda} = \frac{-\frac{r}{2}N_{full}\frac{\partial Var[w]}{\partial \lambda}\frac{dCE}{dF} - \frac{\partial CE}{\partial \lambda}\left\{\Delta\overline{N'} - \frac{r}{2}N_{full}\frac{dVar[w]}{dF}\right\}}{\frac{dCE}{dF}}$$
(32)

We want this to be positive at $\lambda = 0$. Now Appendix A shows that the numerator of (32) at a given λ equals the following,

$$Numerator = \frac{\Delta}{N_{full}} \frac{E\pi}{1-\lambda} \left\{ -\overline{N'} + rCov\left[\overline{w}, \frac{\partial N}{\partial F}\right] \right\} + \frac{r}{N_{full}} \left\{ E\pi Cov\left[\overline{w}, N\right] - \overline{N}Cov\left[\overline{w}, \pi\right] \right\}$$
(33)

which is also shown to be positive at $\lambda=0$. Thus for (32) to be positive at $\lambda=0$, it suffices to show that the denominator $\frac{dCE}{dF}$ is positive at this point. To do so first expand CE in (26) at $\lambda=0$ (i.e. w=F) using (13) and (28),

$$CE = \left\{ \Delta \frac{\overline{N}}{N_{full}} + w_{out} \right\} - \frac{r}{2} \left\{ \Delta^2 \left[\int_0^\infty \left(\frac{N}{N_{full}} \right)^2 f(\phi) d\phi - \left(\frac{\overline{N}}{N_{full}} \right)^2 \right] \right\}$$
(34)

Then at $F = w_{out}$, $\Delta = 0$ and hence $CE = w_{out}$. At the other extreme, as again eqn (7) implies that for $F \to \infty$, $N \to 0$, CE would asymptotically approach w_{out} again. However in contrast to the risk-neutral case (Figure 2), for large r this approach will take place from below the $CE = w_{out}$ line. To see what happens in the middle we can differentiate CE with respect to F for given λ using eqns (38) and (40) in Appendix A, which at $\lambda = 0$ is,

$$\frac{dCE}{dF} = \frac{dEw}{dF} - \frac{r}{2} \frac{dVar[w]}{dF} \\
= \frac{1}{N_{full}} \left(\overline{N} + \Delta \overline{N'} \right) - \frac{r}{N_{full}} \left(Cov \left[\overline{w}, N \right] + \Delta Cov \left[\overline{w}, \frac{\partial N}{\partial F} \right] \right)$$

This is unambiguously positive at the point $F = w_{out}$, as then $\Delta = 0$ and $\overline{w} = w_{out}$, the latter making all covariance terms zero,

$$\frac{dCE}{dF} = \frac{\overline{N}}{N_{full}} > 0 \tag{35}$$

Hence the CE(F) curve for $\lambda = 0$ will look something like this,

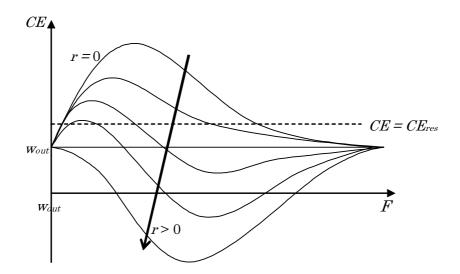


Figure 3: CE vs. F at $\lambda = 0$ for different values of r

The CE curve shifts down with an increase in r as for given F and λ , $\frac{\partial CE}{\partial r} = -\frac{1}{2}Var[w] < 0$. Strictly speaking as N also depends on F from the first-order condition (7), eqn (34) is a polynomial in F of higher order than two, and thus the CE curve may have multiple maxima / minima than as depicted in Figure 3. However the important point is that the lowest participation constraint solution to $CE = CE_{res}$, the solution the firm will prefer as again $\frac{dE\pi}{dF} < 0$ from (23), occurs at a value of $F > w_{out}$ for which $\frac{dCE}{dF}$ is positive when $CE_{res} > w_{out}$. Thus now with both the numerator and the denominator unambiguously positive at $\lambda = 0$, (32) is positive for all values of r at $\lambda = 0$, and hence the firm will optimally choose to offer a profit-sharing contract $\lambda > 0$. Finally note from Figure 3 that when $CE_{res} = w_{out}$, the participation constraint solution is at $F = w_{out}$. From this point the firm is unable to decrease F further to offset any possible increase in λ , and hence in this case the wage contract will remain that of the fixed-wage.

One interesting consideration is to see what happens when the workers are highly risk-averse. In such cases CE would approximately equal $-\frac{r}{2}Var[w]$, the maximum value of which is zero attained when $F = w_{out}$ and $\lambda = 0$, i.e. a fixed-wage paying the out-of-work income. In Figure 3 this is the case where the CE curve is at all times below the line $CE = w_{out}$ (the lowest curve shown). Then the only possible solution to (PC2) is $CE_{res} = w_{out}$, and hence the optimal wage contract in this extreme case is that of a fixed wage.

Now in contrast to the risk-neutral case where the optimal profit-share was shown to occur at the point where F is driven down to w_{out} , here with risk-averse workers it is not possible to make such a statement. This is because in the

risk-neutral case, as seen in eqn (17), the firm is able to unambiguously increase its expected profit (the welfare gain of extra job security that is claimed by the firm) by increasing λ and decreasing F, at least until F is driven down to w_{out} . However in the risk-averse case the increase in expected profit, shown in eqn (30), is net of the change in the wage variance. In this case $\frac{dE\pi}{d\lambda}$ in (30) may well take the value zero before F reaches w_{out} . One may deduce from this argument that the higher the value of F, the more likely that the optimal profit-sharing is attained at a higher value of F. An example of this can be constructed using Weitzman's model (1985) of profit-share economy (see Hori (2003a)). It also follows from this that the policy implication for risk-neutral workers, of full employment when w_{out} equals zero, cannot be stated for risk-averse workers.

4 Conclusion

In concluding this paper, I refer to my personal experience of working for a large Japanese bank. My salary consisted of two components, a fixed salary and biannual bonus payments. The bonus payments, at least for us younger cohorts, were independent of individual performances, but varied according to the firm's aggregate performance, albeit with stickiness. This variable component could be as much as a third of my annual income. The wage contract was thus very much a profit-sharing contract as modelled here.

In 1990, when I joined the bank at the height of the asset price boom, it was unthinkable that any of us would work for anyone other than the firm we had just joined (or at least its subsidiaries) for the rest of our working lives. By mid-1990s when it became clear that the Japanese banks were emburdened with huge non-performing debts, many workers in the firm began to receive calls from headhunters offering jobs at foreign banks. These invariably offered a much higher salary. This could partially be because they were targeting higher-ability workers, while in large Japanese firms workers in their first 10 to 15 years are paid similar wages depending mostly on their age. However the decision these workers had to make was whether they would take a job that pays a higher salary but with a possibility of being out of work after a year, or stay with the firm paying a lower salary but with job security. The result of this paper indicates that if the workers are invariant between the two wage contracts, then the firm offering the profit-sharing wage contract would have a higher expected profit. The reason for this was given in two steps. First it was shown that, with the only assumption being that the revenue function be concave, employment would always occur at a point where the workers are willing to sacrifice some of their fixed component of the wage for a larger profit-share. It follows then that with increasing profit-sharing, there will be a net welfare gain for the firm as a whole. This is because while for the firm the effects are a decrease in their share of aggregate profit but also a decrease in fixed-wage component payouts, for riskneutral workers the effects are an increase in their bonus payouts, a decrease in their fixed-wage component income, and an increase in employment stability. The first two terms are simply monetary transfers and thus they cancel out, leaving the last term as the positive social welfare effect which the firm claims. The firm would thus continue to increase its profit-sharing ratio until it is no longer possible to decrease the fixed component of the wage further, i.e. when the fixed-wage component is driven down to the out-of-work income, such as the unemployment benefit. This in turn implies that if the unemployment benefit was set at zero, we would have a pure-profit-sharing full employment economy. In practice our fixed-wage components in the bank were never as low as the unemployment benefit. However this can be explained by extending our model to risk-averse workers. An increase in profit-sharing would have an effect on the variance of the wage income, which if positive (i.e. negative in utility terms) would offset the gain from increased job security. Then the optimal profit-share would occur at a higher fixed-wage component than the out-of-work income level.

The question remains then as to why we still seem to be heading towards the corporation strategy of hire-and-fire. One major factor would be that of moral hazard. Paying an equal share of the aggregate profit in a non-cooperative setting would lead to what is often termed the $\frac{1}{n}$ -problem, if the workers' individual outputs were affected by a random factor and their effort levels were not observable by the firm. Wage contracts offered to us by foreign firms typically included a possibility of an own-performance related bonus payouts, which can be designed to alleviate this asymmetric information problem (see for example Prendergast (1999)). It can indeed be shown that in this setting, firms are better off offering a relative-performance wage contract rather than a profit-sharing one, which is essentially a wage contract with positive own-performance related bonus parameter and negative profit-share parameters. This is elaborated in Hori (2003b).⁶ Another possible explanation is that the negative shocks to the Japanese economy in the early 1990s was possibly much larger than the threshold level (denoted by ϕ in the text), beyond which redundancy is necessary even with profit-sharing wage contracts. After more than 13 years of negligible growth Japanese firms still only tinker their employment levels by restricting new recruitment or seconding workers to subsidiaries. This paper suggests that a more rigorous restructuring is necessary, however not to the levels suggested by the advocates of hire-and-fire corporation strategy.

⁶It can of course be argued that the Japanese firms tackled this problem in ways other than design of wage contracts, such as small-team based performance evaluation and centrally controlled promotion schemes. The former would strengthen monitoring within the team, while the latter would discourage shirking much in the way modelled in the *Tournament Theory* of Lazaer and Rosen (1981).

A Derivation of eqn (33)

Expand the numerator of (32),

$$Numerator = -\frac{r}{2}N_{full}\frac{\partial Var[w]}{\partial \lambda}\frac{dCE}{dF} - \frac{\partial CE}{\partial \lambda}\left\{\Delta\overline{N'} - \frac{r}{2}N_{full}\frac{dVar[w]}{dF}\right\}$$

$$= -\frac{r}{2}N_{full}\frac{\partial Var[w]}{\partial \lambda}\left\{\frac{dEw}{dF} - \frac{r}{2}\frac{dVar[w]}{dF}\right\}$$

$$-\left\{\frac{\partial Ew}{\partial \lambda} - \frac{r}{2}\frac{\partial Var[w]}{\partial \lambda}\right\}\left\{\Delta\overline{N'} - \frac{r}{2}N_{full}\frac{dVar[w]}{dF}\right\}$$

$$= -\frac{r}{2}N_{full}\frac{dEw}{dF}\frac{\partial Var[w]}{\partial \lambda} - \Delta\overline{N'}\frac{\partial Ew}{\partial \lambda}$$

$$+\Delta\frac{r}{2}\overline{N'}\frac{\partial Var[w]}{\partial \lambda} + \frac{r}{2}N_{full}\frac{\partial Ew}{\partial \lambda}\frac{dVar[w]}{\partial \lambda}$$

$$(36)$$

But we know from (14) that,

$$\frac{\partial Ew}{\partial \lambda} = \frac{1}{N_{full}} \frac{E\pi}{1-\lambda} \tag{37}$$

$$\frac{dEw}{dF} = \frac{1}{N_{tull}} \left\{ (1 - \lambda)\overline{N} + \Delta \overline{N'} \right\}$$
 (38)

Substituting these yields,

$$Numerator = -\frac{r}{2} \left\{ (1 - \lambda) \overline{N} + \Delta \overline{N'} \right\} \frac{\partial Var[w]}{\partial \lambda} - \Delta \frac{\overline{N'}}{N_{full}} \frac{E\pi}{1 - \lambda}$$

$$+ \Delta \frac{r}{2} \overline{N'} \frac{\partial Var[w]}{\partial \lambda} + \frac{r}{2} \frac{E\pi}{1 - \lambda} \frac{dVar[w]}{dF}$$

$$= -\frac{r}{2} (1 - \lambda) \overline{N} \frac{\partial Var[w]}{\partial \lambda} - \Delta \frac{\overline{N'}}{N_{full}} \frac{E\pi}{1 - \lambda}$$

$$+ \frac{r}{2} \frac{E\pi}{1 - \lambda} \frac{dVar[w]}{dF}$$

$$(39)$$

We can also substitute for the derivatives of Var[w]. To do this, first evaluate the following derivatives of $\overline{w} = \left(\frac{N}{N_{full}}\right)(w - w_{out}) + w_{out}$ using again $\frac{\partial w}{\partial \lambda} = \frac{\phi R - FN}{N}$, $\frac{\partial w}{\partial F} = 1 - \lambda$ and $\frac{\partial w}{\partial N} = -\lambda \left(\frac{\phi R - FN}{N^2}\right)$,

$$\begin{split} \frac{\partial \overline{w}}{\partial \lambda} &= \left(\frac{1}{N_{full}}\right) \frac{\pi}{1 - \lambda} \\ \frac{\partial \overline{w}}{\partial F} &= \frac{1}{N_{full}} \left\{ (1 - \lambda) N + \frac{\partial N}{\partial F} \left[(w - w_{out}) - N\lambda \left(\frac{\phi R - FN}{N^2} \right) \right] \right\} \\ &= \frac{1}{N_{full}} \left\{ (1 - \lambda) N + \Delta \frac{\partial N}{\partial F} \right\} \end{split}$$

where once again $\Delta = F - w_{out}$. Then differentiating (27),

$$\frac{\partial Var[w]}{\partial \lambda} = 2 \left\{ \int_{0}^{\infty} \overline{w} \frac{\partial \overline{w}}{\partial \lambda} f(\phi) d\phi - Ew \frac{\partial Ew}{\partial \lambda} \right\}
= 2Cov \left[\overline{w}, \frac{\partial \overline{w}}{\partial \lambda} \right]
= \frac{1}{1 - \lambda} \frac{2}{N_{full}} Cov \left[\overline{w}, \pi \right]
\frac{dVar[w]}{dF} = 2Cov \left[\overline{w}, \frac{\partial \overline{w}}{\partial F} \right]
= \frac{2}{N_{full}} \left\{ (1 - \lambda) Cov \left[\overline{w}, N \right] + \Delta Cov \left[\overline{w}, \frac{\partial N}{\partial F} \right] \right\}$$
(40)

Substituting these into (39) gives,

$$\begin{aligned} Numerator &= -r\frac{\overline{N}}{N_{full}}Cov\left[\overline{w},\pi\right] - \Delta\frac{\overline{N'}}{N_{full}}\frac{E\pi}{1-\lambda} \\ &+ r\frac{1}{N_{full}}E\pi Cov\left[\overline{w},N\right] + r\frac{E\pi}{1-\lambda}\frac{\Delta}{N_{full}}Cov\left[\overline{w},\frac{\partial N}{\partial F}\right] \\ &= \frac{\Delta}{N_{full}}\frac{E\pi}{1-\lambda}\left\{-\overline{N'} + rCov\left[\overline{w},\frac{\partial N}{\partial F}\right]\right\} \\ &+ \frac{r}{N_{full}}\left\{E\pi Cov\left[\overline{w},N\right] - \overline{N}Cov\left[\overline{w},\pi\right]\right\} \end{aligned} \tag{41}$$

which is eqn (33). Now as higher ϕ implies higher \overline{w} , π , N and $\frac{\partial N}{\partial F}$ (c.f. (9)), all covariance terms are positive. Thus the only problem term is the fourth one involving $Cov[\overline{w}, \pi]$. However we can approximate the variance of a function of a stochastic variable by (c.f. Mood, Graybill and Boes (1974)),

$$Var[f(X)] = \{f'(\overline{X})\}^2 Var[X] + \text{ Higher Orders}$$

Here we have,

$$Var[\pi] \approx \left\{ \frac{d\pi}{dN} \left(\overline{N} \right) \right\}^2 Var[N] = 0$$

and thus

$$Cov\left[\overline{w},\pi\right] = Corr\left[\overline{w},\pi\right] \sqrt{Var[\overline{w}]} \sqrt{Var[\pi]} \approx 0$$

Therefore the numerator of (32) is positive at $\lambda = 0.\blacksquare$

References

- [1] Burnside, C., Eichenbaum, M. and Robelo, S. (1993), "Labor Hoarding and the Business Cycle", *Journal of Political Economy* **101**, 245-273
- [2] Fair, R.C. (1985), "Excess Labor and the Business Cycle", American Economic Review 75, 239-245
- [3] Fay, J.A. and Medoff, J.L. (1985), "Labor and Output over the Business Cycle: Some Direct Evidence", American Economic Review, 75, 638-55
- [4] Hart, R.A. and Malley, J. (1996), "Excess Labour and the Business Cycle: A Comparative Study of Japan, Germany, the United Kingdom and the United States", *Economica*, **63**, 325-42
- [5] Hashimoto, M. and Raisian, J. (1985), "Employment Tenure and Earning Profiles in Japan and the United States", American Economic Review, 75, 721-35
- [6] Hori, K. (2003a), "Profit-Sharing as the Optimal Wage Contract", PhD Thesis, University of Cambridge
- [7] Hori, K. (2003b), "Why is Profit-Sharing Not More Common?", PhD Thesis, University of Cambridge
- [8] Koike, K. (1988), Understanding Industrial Relations in Modern Japan, New York: St Martin's Press
- [9] Kruse, D.L. (1993), Profit Sharing: Does it Make a Difference? Kalama-zoo, Mich.
- [10] Lazear, E. P. and S. Rosen (1981), "Rank-order Tournaments as Optimal Labor Contracts", *Journal of Political Economy*, **89**, 841-864
- [11] Le Roy Miller, R. (1971), "The Reserve Labour Hypothesis: Some Tests of its Implications", *The Economic Journal* 81, 17-35
- [12] Milgrom, P. and J. Roberts (1992), Economics, Organization and Management, Prentice-Hall International
- [13] Mincer, J. and Higuchi, Y. (1988), "Wage Structure and Labour Turnover in the United States and Japan", Journal of Japanese and International Economies, 2, 97-133
- [14] Mood, A. M., F. A. Graybill and D. C. Boes (1974), *Introduction to the Theory of Statistics*, McGraw-Hill Book Company, 3rd ed.

- [15] Okun, A. (1962), "Potential GNP: Its Measurement and Significance", Proceedings of the Business and Economic Statistics Section
- [16] Prendergast, C. (1999), "The Provision of Incentives in Firms", Journal of Economic Literature, 37, 7-63
- [17] Sbordone, A.M. (1996), "Cyclical Productivity in a Model of Labor Hoarding", Journal of Monetary Economics 38, 331-61
- [18] Vecchi, M. (2000), "Increasing Returns, Labour Utilization and Externalities: Procyclical Productivity in the United States and Japan", *Economica* **67**, 229-244
- [19] Weitzman, M.L. (1984), The Share Economy, Harvard University Press
- [20] Weitzman, M.L. (1985), "The Simple Macroeconomics of Profit Sharing", The American Economics Review, 75(5), pp937-53
- [21] Weitzman, M.L. (1995), "Incentive Effects of Profit Sharing", in Horst Siebert (ed.), Trends in Business Organization: Do Participation and Cooperation Increase Competitiveness? International Workshop, Tubingen: Mohr, pp51-78