Asymmetric Monetary Policy Towards the Stock Market: A DSGE Approach*

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Abstract

In the aftermath of the financial crisis, it has been argued that a guideline for future policy should be to take the ‘a’ out of ‘asymmetry’ in the way monetary policy deals with asset price movements. Ravn (2011) provides empirical evidence for the US that during the period 1998-2008, a drop in stock prices increased the probability of a subsequent interest rate cut, while an increase in stock prices led to no policy reaction. In the present paper, I study the effects of such a policy in a DSGE model. The asymmetric policy rule introduces an important non-linearity into the model that cannot be ‘log-linearized away’. Instead, I solve the model using the method of Bodenstein et al. (2009). If the central bank reacts asymmetrically to asset prices, this asymmetry will be inherited by the business cycle: Booms in output and inflation will tend to be amplified, and recessions will be dampened. I further demonstrate how an asymmetric stock price reaction could be motivated by the desire of policymakers to correct for inherent asymmetries in the way stock price movements affect the economy. However, I also show that such a policy leads to asset price booms that are closely linked to the risk of creating moral hazard problems.

Keywords: Asymmetries, Monetary Policy, Asset Prices, DSGE Modelling.


*An extended appendix containing further details about the model and the solution method is available from the author’s webpage: https://sites.google.com/site/sorenhoveravn/. The author wishes to thank Henrik Jensen for competent and inspiring guidance, and Niels Blomquist, Martin Ellison, Antonio Mele, Tommaso Monacelli, Gert Peersman, Jesper Rangvid, and Emiliano Santoro for useful comments and discussions. Also thanks to seminar participants at the University of Oxford and Danmarks Nationalbank. All remaining errors are my own. The views expressed are those of the author, and do not necessarily correspond to those of Danmarks Nationalbank.

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1 Introduction

The recent financial and economic crisis has led to a certain degree of pondering and self-examination among macroeconomists. While the crisis surely does not invalidate everything we have learned about macroeconomics since 1936, as Barro (2009) eloquently puts it, it has led economists to reconsider some paradigms that once were common sense. As one example, the crisis has led to a revival of the debate about the possible role of asset prices in monetary policy. This debate goes back at least to Bernanke and Gertler (1999, 2001), who argue that asset prices should not enter the monetary policy rule, except insofar as these can be regarded as signals about future macroeconomic conditions. In contrast, Cecchetti et al. (2000) reach the opposite conclusion, as they find that the optimal monetary policy rule does include a reaction to the stock market.\footnote{The view of Bernanke and Gertler has been supported by, among others, Gilchrist and Leahy (2002) and Tetlow (2005), as well as in speeches by leading Federal Reserve officials (Kohn, 2006; Mishkin, 2008). The activist position of Cecchetti et al. has received support from Bordo and Jeanne (2002), Borio and White (2003) and, more recently, Pavasuthipaisit (2010).} Despite some enduring disagreement, it has been argued that a certain degree of consensus seemed to have been reached before the crisis. According to this consensus, central banks should not try to lean against perceived asset price bubbles; partly because these are extremely hard to identify in real time, and partly because of the difficulties in using monetary policy to ‘prick’ such bubbles. Instead, central banks should stand ready to cut interest rates aggressively after the bursting of bubbles in order to curb the effects on real economic activity and price stability.

In the aftermath of the crisis, however, this ‘pre-crisis consensus view’, as coined by Bini Smaghi (2009), has come under critique for involving an inherent asymmetry, in the sense that it calls for central banks to react only when asset prices go down. Bini Smaghi (2009), Issing (2009) and White (2009) all recognize this asymmetry, and question the validity of what used to be conventional wisdom. Ravn (2011) provides empirical evidence on this question. Building on the work of Rigobon and Sack (2003), he uses the method of identification through heteroskedasticity to test for an asymmetric reaction of the Federal Reserve to stock price changes. He finds that during the period 1998-2008, a 5 \% drop in the S&P 500 index increased the probability of a subsequent 25 basis point interest rate cut by 33 \%. On the other hand, he finds no significant policy reaction to stock price increases.

The present paper contributes to the recent debate by examining the effects of such an asymmetric policy in general equilibrium. Motivated by the empirical findings of Ravn (2011), I build a Dynamic Stochastic General Equilibrium (DSGE) model with an explicit role for asset prices. I then allow the central bank to follow a monetary policy rule with an asymmetric reaction to stock prices. This introduces an important non-linearity into the model that cannot be ‘log-linearized away’. As a result, it is not possible to solve the model using standard techniques. Instead, I apply the solution method of Bodenstein et al. (2009), which exploits the piecewise linearity of the model. Essentially, the model consists of two linear systems; one when stock prices are increasing (or constant), and another when...
they are decreasing. I then construct a shooting algorithm to detect the switching points between these systems in order to solve the model.

The theoretical analysis uncovers some interesting implications of the asymmetric policy. By reacting only to stock price drops, the central bank is able to obtain an outcome where booms in output and inflation are amplified, while recessions are dampened. In other words, the asymmetric policy translates into an asymmetric business cycle. In addition, the asymmetric policy gives rise to what I call an anticipation boom in asset prices. In the wake of an expansionary shock, the asset price jumps up. It turns out that this jump is larger in the model with an asymmetric policy rule than in the model with no reaction to stock price changes, despite the fact that in both cases, the policy reaction to stock prices is zero during the asset price boom. The anticipation boom, which measures the additional rise in asset prices when the asymmetric policy is introduced, can be attributed to forward-looking agents anticipating that whenever stock prices start falling, the central bank will cut the interest rate. This implicit, partial insurance against asset price drops amplifies the rise in asset prices immediately after the shock.

If the size of the policy reaction to stock price drops is as estimated by Ravn (2011), these effects are quantitatively quite small. In the literature, a remarkable divergence exists between the magnitude of the reaction to asset prices found in empirical studies, which is often quite small, and the values used in theoretical contributions, which are usually a lot larger. To bridge this gap, and to make my results of more general interest, I therefore also employ a value of the reaction parameter which is more in accordance with the values in other theoretical contributions. When this is done, the above effects are sizeable, especially when the economy is hit by monetary policy shocks.

Within the DSGE framework, I evaluate the possibility that the asymmetric policy could be a response to underlying asymmetries in the way stock prices influence the macroeconomy. Indeed, this possibility is suggested by Ravn (2011), who points to the financial accelerator of Bernanke et al. (1999) and to the stock wealth effect on consumption as potential sources of such an asymmetry. I show that if the financial accelerator is assumed to be stronger in times when net worth of firms is low, as has been suggested in the literature, the asymmetric policy is able to 'cancel out' this asymmetry. Even a modest degree of asymmetry in the financial accelerator is able to rationalize the result of Ravn (2011). I further discuss how asymmetric wealth effects on consumption due to loss averse private agents are another potential motivation for the asymmetric policy.

Although an asymmetric monetary policy can thus be a useful tool in order to neutralize other economic asymmetries, it also implies a salient risk of creating moral hazard problems by effectively insulating stock market investors from part of their downside risk. As a matter of fact, this has been a central element in the critique of the pre-crisis consensus. I briefly discuss this risk towards the end of the paper.

The remainder of the paper is structured as follows. Section 2 describes in detail the
DSGE workhorse model. Section 3 illustrates the dynamics of the model and the implications of introducing an asymmetric reaction to stock prices. In section 4, I discuss possible explanations for the asymmetric policy within the model framework. Section 5 concludes. The appendix contains details about the model and the solution method.

2 Model

The general equilibrium model is a version of the standard New-Keynesian sticky-price model with capital; augmented with the financial accelerator of Bernanke et al. (1999) in order to introduce a role for asset prices. An additional feature is that contracts are written in terms of the nominal interest rate as in Christensen & Dib (2008), introducing the debt-deflation channel of Fisher (1933). Christiano et al. (2010) find that this channel is empirically relevant. The model is in large part similar to that of Christensen and Dib (2008) or Gilchrist and Saito (2008). This has the advantage that the dynamics of this class of models is well described in the literature, allowing me to isolate the effects of the asymmetric monetary policy rule. Moreover, this allows me to calibrate the model using the parameter values estimated by Christensen and Dib for the US economy for most of the parameters. Finally, this class of models has typically been used in the literature to analyze whether central banks should react to asset prices. The stochastic part of the model is quite parsimonious, as only two shocks are included: a technology shock and a monetary policy shock. These two shocks, which can loosely be interpreted as a supply and a demand shock, are sufficient to highlight the effects of the asymmetric policy.

2.1 Entrepreneurs

In each period, entrepreneurs face a constant probability \((1 - \nu)\) of leaving the economy. As described by Bernanke et al. (1999), this assumption is made in order to ensure that entrepreneurs do not eventually accumulate enough capital to be able to finance their own activities entirely. Entrepreneurs produce the intermediate goods that the final goods producers take as input. Each entrepreneur hires labor, rents capital, and produces according to the following production technology:

\[
Y_t \leq (A_t H_t)^{1-\alpha} K_t^\alpha.
\]

The technology level \(A_t\) evolves according to

\[
\ln (A_t) \equiv (1 - \rho_a) A + \rho_a \ln (A_{t-1}) + \varepsilon_t^a,
\]

\[\text{See, among others, Bernanke and Gertler (1999), Gilchrist and Saito (2008), or Pivasuthipaisit (2010).}\]
where \( \varepsilon_t^u \) is a normally distributed shock to technology with mean zero. Bernanke et al. (1999) assume that entrepreneurs also work, so that the labor input into the production function is an aggregate of household and entrepreneurial labor; \( N_t = (N_t^E) \alpha (N_t^H)^{1-\alpha} \).

This assumption is made in order to ensure that new entrepreneurs start out with non-zero net worth. Instead, I follow Christensen and Dib (2008) and Gilchrist and Saito (2008) in setting \( \alpha = 0 \). I then further follow Christensen and Dib in allowing newly entering firms to inherit a portion of the net worth of those firms who exit the economy.\(^3\)

Entrepreneurs choose their inputs of capital and labor to maximize their profits, subject to the production technology. As there is perfect competition in the entrepreneurial sector, the price which they receive for their products will be equal to the marginal cost of producing the intermediate good. This gives rise to the following first-order conditions:

\[
mp_t = \frac{Y_t}{K_t} mc_t, \tag{3}
\]

\[
w_t = (1 - \alpha) \frac{Y_t}{H_t} mc_t, \tag{4}
\]

where \( mp_t \) denotes the real marginal productivity of capital, and \( mc_t \) is the real marginal production cost of entrepreneurs.

Each entrepreneur can obtain the capital needed for production in two ways: He can issue equity shares (internal financing), or he can borrow the money from a financial intermediary (external financing).\(^4\) Because internal financing is cheaper, as discussed below, entrepreneurs use all of their net worth, and borrow the remainder of their funding needs from the financial intermediary. The total funding needed by an entrepreneur is \( q_t K_{t+1} \), where \( q_t \) is the real price of capital as measured in units of consumption.\(^5\) If \( n_t \) denotes the net worth of the entrepreneur, the amount he needs to borrow is then \( q_t K_{t+1} - n_{t+1} \). Letting \( f_t \) denote the external financing cost of one extra unit of capital, the demand for external finance must satisfy the following condition in optimum:

\[
E_t [f_{t+1}] = E_t \left[ \frac{mp_{t+1} + (1 - \delta) q_{t+1}}{q_t} \right]. \tag{5}
\]

The numerator on the right-hand side is the marginal productivity of a unit of capital plus the value of this unit of capital (net of depreciation) in the next period. If this condition was not satisfied, the capital demand of entrepreneurs would be either zero or infinite.

As in Bernanke et al. (1999), the existence of an agency problem between borrower and lender renders external finance more costly than internal finance. While entrepreneurs observe the outcome of their investments costlessly, the financial intermediary must pay

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\(^3\)These differences are of little importance for the results.

\(^4\)Note that it is assumed that each entrepreneur has to refinance his entire capital stock each period. As pointed out by Bernanke et al. (1999), this assumption ensures that any financial constraint faced by the entrepreneur applies to the capital stock as such, and not just to the investment in any given period.

\(^5\)I use the terms price of capital, asset price and stock price interchangeably.
an auditing cost to observe this outcome. Entrepreneurs must decide - after observing the outcome - whether to report a success or a failure of the project, i.e. whether to repay or default on the loan. If they default, the financial intermediary pays the auditing cost, and then claims the returns to the investment. Bernanke et al. (1999) demonstrate that the optimal financial contract involves an external finance premium (the difference between the cost of internal and external finance) which depends on the entrepreneur’s net worth, and show that the marginal external financing cost is equal to the external finance premium times the opportunity cost of the investment; given by the risk-free real interest rate:

\[ E_t [f_{t+1}] = E_t \left[ \Psi \left( \frac{n_{t+1}}{q_t \bar{K}_{t+1}} \right) \frac{R_t}{\pi_{t+1}} \right], \]  

(6)

where the function \( \Psi (\cdot) \) describes how the external finance premium depends on the financial position of the firm. \( \frac{n_{t+1}}{q_t \bar{K}_{t+1}} \) denotes the ratio of the firm’s internal financing to its total financing, and is thus a measure of the leverage ratio. Bernanke et al. further demonstrate that \( \Psi (\cdot) < 0 \), implying that if firms’ net worth goes up (or, equivalently, their leverage ratio goes down), the external finance premium falls, and firms get cheaper access to credit. The reason is that as the entrepreneur puts more of his own money behind the project, thus lowering the leverage ratio, the agency problem between borrower and lender is alleviated. The entrepreneur’s incentive to undertake projects with a high probability of success increases, and as a result, the lender demands a lower return on the loans he makes.

It is useful to consider the log-linearized version of (13) in order to get an understanding of how the financial accelerator works:

\[ E \tilde{f}_{t+1} - \left( \tilde{R}_t - E_t \tilde{\pi}_{t+1} \right) = -\psi \left( \tilde{n}_{t+1} - \tilde{q}_t - \tilde{\bar{K}}_{t+1} \right), \psi > 0. \]  

(7)

As described in the appendix, \( \tilde{x}_t \) denotes the deviation of variable \( x_t \) from its steady state value \( x \). Equation (14), is the key to the financial accelerator mechanism. If the net worth of an entrepreneur goes up, the external finance premium which the firm has to pay decreases. This leads to an increase in the firm’s demand for external finance, which in turn leads to an increase in the firm’s stock of capital in the next period, and thus its production level. In this way, to the extent that movements in net worth are procyclical, the financial accelerator works to amplify business cycle movements. The parameter \( \psi \) plays a key role, as it measures the elasticity of the external finance premium to changes in firms’ leverage ratio. Thus, this parameter can be said to measure the ‘strength’ of the financial accelerator mechanism; as a higher elasticity implies a stronger effect on the business cycle of a given change in net worth.

The net worth of entrepreneurs consists of the financial wealth they have accumulated (i.e., profits earned in previous periods) plus the bequest \( \Upsilon_t \) they receive from entrepreneurs.

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6See Section 3 and Appendix A of Bernanke et al. (1999) for details about how to solve for the optimal contract and how to demonstrate that the external finance premium is a function of firms’ net worth.
leaving the economy:

\[ n_{t+1} = \nu \left[ f_t q_{t-1} K_t - E_{t-1} f_t (q_{t-1} K_t - n_t) \right] + (1 - \nu) \Upsilon_t. \]  

(8)

2.2 Households

A continuum (of unit length) of households derive utility from an index of the consumption goods produced by the retailers \( C_t \), leisure \( (1 - H_t) \), and real money holdings \( \frac{M_t}{P_t} \), and decide how much labor to supply to entrepreneurs producing intermediate goods. As all households are identical, they each solve the following utility maximization problem:

\[
\max_{C_t, H_t, D_t, \frac{M_t}{P_t}} U = E_0 \sum_{t=0}^{\infty} \beta^t u \left( C_t, H_t, \frac{M_t}{P_t} \right),
\]

(9)

with instantaneous utility function:

\[
u \left( C_t, H_t, \frac{M_t}{P_t} \right) = \frac{\gamma}{\gamma - 1} \ln \left[ C_t^\gamma + \left( \frac{M_t}{P_t} \right)^{\frac{\gamma-1}{\gamma}} \right] + \eta \ln (1 - H_t),\]

(10)

subject to the relevant budget constraint:

\[
C_t + \frac{M_t - M_{t-1}}{P_t} + \frac{D_t - R_t - D_{t-1}}{P_t} \leq \frac{W_t}{P_t} H_t + \Omega_t.
\]

(11)

\( D_t \) are deposits which are stored at a financial intermediary at the risk-free rate of interest \( R_t \). \( \Omega_t \) denotes dividend payments deriving from households’ ownership of retail firms. The first-order conditions of the household are presented in the appendix.

2.3 Capital Producers

The role of capital producers is to construct new capital \( K_{t+1} \) from final goods \( I_t \) and existing capital \( K_t \). As in Bernanke et al. (1999), it is implicitly assumed that capital producers rent existing capital from entrepreneurs within each period at a rental rate of zero. They face capital adjustment costs, implying a non-constant price of capital \( q_t \). I use the same quadratic functional form for the capital adjustment costs as Christensen and Dib (2008): \( \frac{\chi}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \). Profits of capital producers are then:

\[
\Pi_t^c = E_t \left[ q_t I_t - I_t - \frac{\chi}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \right].
\]

(12)

Choosing the level of investment that maximizes this expression results in the following equilibrium condition:
\[ E_t \left[ q_t - \chi \left( \frac{I_t}{K_t} - \delta \right) \right] = 1. \]  

Note that in the absence of adjustment costs, the parameter \( \chi \) equals zero, so the optimality condition collapses to \( E_t q_t = 1 \).\(^7\) This illustrates that capital adjustment costs are necessary to create a time-varying price of capital. Moreover, the condition is essentially a Tobin’s q relation, ensuring that the investment level is chosen so that the 'effective' price of capital (i.e., net of capital adjustment costs) is equal to 1.

### 2.4 Retailers

As mentioned at the beginning of this section, the retail sector is included in the model with the single purpose of creating price stickiness. Following Calvo (1983), price rigidity is introduced by assuming that in each period, only a fraction \( (1 - \xi) \) of firms in the retail sector are allowed to change their price. The price of firms who are not allowed to change their price is indexed with the steady state inflation rate \( \pi \). This problem gives rise to the following first-order condition for optimal price setting by firm \( i \):

\[
P^n_t (i) = \frac{\epsilon^p}{\epsilon^p - 1} \frac{E_t \left\{ \sum_{s=0}^{\infty} (\beta \xi)^s \lambda_{t+s} Y_{t+s} (i) P_{t+s} mc_{t+s} \right\}}{E_t \left\{ \sum_{s=0}^{\infty} (\beta \xi)^s \lambda_{t+s} Y_{t+s} (i) \pi^s \right\}}.
\]

The evolution of the aggregate price level is a weighted average of the price of those firms who are allowed to change their price in a given period, and of those who are not; whose prices are therefore indexed:

\[
P_t = \left[ (1 - \xi) (P^n_t)^{1-\epsilon^p} + \xi (P_{t-1} \pi)^{1-\epsilon^p} \right]^{1/(1-\epsilon^p)}.
\]

In the extended model appendix, I demonstrate how the log-linearized versions of (21) and (22) can be combined to yield a standard version of the New-Keynesian Phillips Curve.

### 2.5 Monetary Policy

While large parts of the model is similar to that in Christensen and Dib (2008), the specification of monetary policy differs substantially. They assume that the central bank follows a Taylor rule with a reaction to inflation, the output gap and money growth, and without interest rate smoothing. While I stick to the framework of the Taylor rule, I follow most of the literature in assuming no reaction to money growth. I further add interest rate smoothing, as this tends to improve the ability of Taylor rules to fit US data (Clarida et al., 1999; Christiano et al., 2010).

\(^7\)Recall that \( q_t \) is a real price measured in units of consumption. Hence, \( q_t = 1 \) will hold in the absence of adjustment costs, irrespective of the fact that the price level on consumption goods fluctuates due to the price stickiness faced by retailers.
To introduce the asymmetric policy discovered in section 3, I then augment the rule with a term that captures a (non-linear) reaction to stock price drops. Ravn (2011) attempts to control for the movements in the interest rate that are driven by macroeconomic variables such as output and inflation. Therefore, his result is interpretable as a reaction to stock prices on top of the reaction to those variables. Based on this line of argument, it therefore seems reasonable to interpret his result within the Taylor rule framework. This gives rise to the following monetary policy rule:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_r} \left[ \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{Y_t}{Y} \right)^{\phi_y} \left\{ \left( \frac{\Delta q_t}{q} \right)^{\phi_{q'}} \right\}^{1[\Delta q_t < 0]} \left\{ \left( \frac{\Delta q_t}{q} \right)^{\phi_{q''}} \right\}^{1[\Delta q_t \geq 0]} \right]^{(1-\rho_r)} \epsilon_t^\tau, \tag{16}
\]

where \( 1[X] \) is the indicator function; equal to 1 if \( X \) is true and zero otherwise. This captures that the central bank is reacting to the change in stock prices only when this change is negative. If it is positive, the stock price term cancels out. \( \epsilon_t^\tau \) is a normally distributed monetary policy shock with mean zero. The stated monetary policy rule allows for interest rate smoothing, as measured by the parameter \( \rho_r \). The parameters \( \phi_\pi \) and \( \phi_y \) measure the monetary policy reaction to deviations of inflation from its target level, and of output from its steady state level, respectively. Note that the steady state or natural level of output (\( Y \)) is below the efficient level of output (\( Y^* \)) due to the presence of monopolistic competition.

So far, the only motivation for including an asymmetric policy rule is the empirical relevance of such a rule demonstrated in section 3. In section 6, I speculate how such a policy rule could otherwise be motivated, for instance by assuming an asymmetric loss function of the central bank, or by the possible existence of underlying asymmetries in the economy. While this paper is the first to consider a Taylor rule with a reaction to stock price drops, Tetlow (2005) and Gilchrist and Saito (2008) suggest a Taylor rule augmented with a symmetric reaction to stock price changes, and apply this rule to models largely similar to the one outlined above.

### 2.6 Equilibrium and Model Solution

The 16 equilibrium conditions in 16 variables are summarized in the appendix. The equilibrium of the model consists of a vector of allocations \( \left( C_t, H_t, \frac{\lambda_t}{P_t}, Y_t, K_t, n_t, I_t \right) \) and prices \( \left( \pi_t, R_t, w_t, mc_t, mp_t, q_t, f_t, \lambda_t, \frac{\rho_t}{P_t} \right) \) such that those 16 equations are satisfied.

In the extended model appendix, I present the steady state of the model. The model is log-linearized around this steady state. However, the non-linear monetary policy rule

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8 This formulation of non-linearity is suggested by Wooldridge (2002), p. 537.
9 To be exact, in addition to these 16 variables one could include the equation describing the evolution of the technology level (2).
implies that even after log-linearization, an important non-linearity remains in the model. As a result, the model cannot be solved with standard techniques.

Instead, I solve the model using the approach of Bodenstein et al. (2009). This method exploits the piecewise linearity of the model, and was developed by Bodenstein et al. to deal with problems where the zero lower bound on interest rates is binding in a number of periods. As the only non-linearity in the present model is the monetary policy reaction to asset prices, the model in effect consists of two linear systems; one for when asset prices are decreasing, and one for when they are non-decreasing. Following Bodenstein et al. (2009), I first build a shooting algorithm in order to identify the 'turning points' in the evolution of the asset price following a shock; i.e. when the sign of $\Delta q_t$, and thus the monetary policy regime, shifts. For any initial guess of the turning points, the model is then solved using backward induction. If the initial guess turns out not to be consistent with the sign of $\Delta q_t$ shifting at that time, the guess is adjusted accordingly, and the process is repeated until the shifting criteria are satisfied. Details of the solution method are outlined in the appendix.\footnote{Another possible solution method is the endogenous regime-switching approach of Davig and Leeper (2006). They use numerical methods to solve a model where the monetary policy reaction to inflation depends on the lagged level of inflation. However, applying their solution method to a medium-scale DSGE model as the one outlined above involves considerable computational problems, as this approach suffers heavily from the curse of dimensionality. In future work, I plan to develop a solution of the model building on orthogonal collocation methods, Chebyshev or complete polynomials, and a sparse grid method along the lines of Smolyak (1963). This will provide important insights on the sensitivity of the results in the present paper with respect to the solution method employed.}

## 2.7 Calibration

As already mentioned, I obtain most of the parameter values from Christensen and Dib (2008), who estimate a model largely similar to the one outlined above using US data for the sample period 1979-2004. As a number of the parameters cannot be estimated, Christensen and Dib instead calibrate these parameters using values that are quite standard in the literature. The parameter values used in the calibration are presented in the appendix. The reader is referred to Christensen and Dib (2008) for a more detailed discussion of the parameter values.\footnote{Note that I set the steady state inflation rate to $\pi = 1$, while Christensen and Dib choose a value slightly above 1 to match historical data. I also change the value of $\nu$ from 0.9728 to 0.9853, as explained in the extended model appendix. These changes have little impact.} However, the parameter measuring the elasticity of the external finance premium with respect to changes in firms’ leverage position deserves special mention. I use the value $\psi = 0.042$ as estimated by Christensen and Dib. This value is somewhat smaller than the calibrated value used by Bernanke et al. (1999) and Gilchrist and Saito (2008) of $\psi = 0.05$. This implies that the financial accelerator mechanism is less strong in the present paper.

As for monetary policy, the policy rule in my model differs substantially from that of Christensen and Dib (2008), as already described. Therefore, I do not use their parameter estimates. Instead, I set $\phi_\pi = 1.5$ as suggested by Taylor (1993). Furthermore, I set...
$\phi_y = 0.2$, as recent empirical studies with US data seem to suggest that Taylor’s suggested value of 0.5 is probably too high (see e.g. Christiano et al. 2010). The interest smoothing parameter is set at 0.7, indicating an important role for interest rate smoothing as suggested by, among others, Clarida et al. (1999).

To correctly formulate the result of Ravn (2011) in terms of the Taylor rule, the point estimate of the reaction to stock price drops needs to be transformed. The interpretation offered by Ravn (2011) of the estimation result relies on the fact that the Federal Open Market Committee meets once every six weeks. The model of the present paper is formulated (and calibrated) in quarterly terms. This involves an implicit assumption that monetary policy can only be changed every 12 weeks; once per quarter. Thus, following the same line of argument as Ravn (2011), I arrive at a parameter value of $\phi_q = 0.0246$ whenever $\Delta \hat{q}_t < 0$.\textsuperscript{12} This value is quite low. As the study of Ravn (2011) is the first to identify a specific reaction to stock price drops, the literature does not offer much guidance on the magnitude of this parameter. However, some information can be obtained from Tetlow (2005) and Gilchrist and Saito (2008), who augment the Taylor rule with a symmetric reaction to the change in stock prices. Tetlow finds that if the reaction to stock prices is allowed to differ from zero, it should be quite large; always larger than 1. Gilchrist and Saito allow the parameter to take on values between 0.1 and 2.0. In other words, there seems to be a severe divergence between estimated and calibrated values of this parameter.\textsuperscript{13} To bridge this gap, and to broaden the scope of the paper, I therefore perform most of the simulations below for two different values of $\phi_q$; the one obtained from Ravn (2011) and a calibrated value of 0.5, which is more in line with the values used in the theoretical literature.

3 Dynamics of the Model

In this section, I investigate the dynamics of the model when the asymmetric monetary policy rule is in place. In linear models, the impulse response to a positive shock is by construction the mirror image of the response to a negative shock of the same type and size. In this model, instead, positive and negative shocks have different dynamic effects. As the central bank reacts only to falling asset prices, a shock that drives asset prices down will induce a stronger monetary policy reaction than a shock which leads to higher asset prices. Further, the adjustment back to the steady state will also differ, depending on whether asset prices are approaching their steady state value from above or below.

Before looking into the effects of the asymmetric policy, it is useful to study the effects of each shock in the linear, constant-parameter model. Figure 1 and 2 display the impulse

\textsuperscript{12}Ravn (2011) estimates the parameter measuring the reaction of the 3-month Treasury Bill rate to changes in the stock price at 0.0123. As the interest rate in the present model on average stays unchanged for half a quarter, this estimate must be multiplied by 2 when interpreted in the setup of the present model.

\textsuperscript{13}Indeed, if one were to use the result of Rigobon and Sack (2003) in the present setting, this would imply a (symmetric) value of $\phi_q = 0.0428$. 

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responses of some key endogenous variables to an orthogonalized shock to technology and monetary policy when the policy reaction to stock prices is always zero; $\phi_q = 0$.

Figure 1: Effects of a positive technology shock, constant-parameter model

Following a positive technology shock, Figure 1 illustrates that output rises, as does consumption and investment (not shown). The hump-shaped pattern of output is due to the assumption that the central bank prefers to smooth the interest rate. Inflation and the nominal interest rate both fall in response to this positive supply shock. The fall in the inflation rate is the source of the drop in net worth. Lower inflation implies a higher real cost of repaying outstanding debt, depressing the net worth of firms.\(^{14}\) This is the debt-deflation channel. As net worth goes down, the external finance premium increases due to more severe agency problems between borrower and lender, as described above. In turn, this dampens economic activity. Thus, the term financial accelerator is in fact misleading in the case of a technology shock when the debt-deflation channel is included, as in this case the fluctuations in output are actually attenuated. The presence of the debt-deflation channel is crucial for this result, as also demonstrated by Christiano et al. (2010). In a somewhat similar model, they find that the debt-deflation channel and the financial accelerator mechanism reinforce each other in the wake of shocks that drive output and inflation in the same direction, whereas they counteract each other after shocks that, like the technology shock, drive output and inflation in different directions.

\(^{14}\)Note that the consumer price index is the relevant price index for ‘deflating’ net worth, as the firms are eventually owned by households.
The technology shock leads to a boom-bust cycle in the asset price. The initial rise and fall in the price of capital is due to the investment boom following the technology shock. However, the price of capital 'undershoots' its steady state level for a number of periods. This undershooting is again due to the debt-deflation channel as the persistent drop in net worth leads to a persistent rise in the price of external funding, lowering the demand for capital (and thus, the asset price) even many periods after the shock.\textsuperscript{15} It may seem counterintuitive that net worth and the price of capital move in different directions. The explanation is that the initial (and numerically quite small) increase in the price of capital is the result of two opposing effects: While the positive technology shock increases investment and the price of capital; the resulting rise in the external finance premium has the exact opposite effect.

Figure 2: Effects of a contractionary monetary policy shock, constant-parameter model

Figure 2 illustrates the dynamics after a one-time positive innovation to monetary policy. As expected, the nominal interest rate jumps up, and then falls back gradually due to interest rate smoothing. In this case, the financial accelerator does work to amplify business cycle fluctuations. As output and inflation move in the same direction, this is in line with the predictions of Christiano \textit{et al.} (2010). The higher interest rate depresses economic activity and in particular investment, reducing the price of capital. This leads to a drop in the net

\textsuperscript{15}Indeed, this undershooting does not occur in the model of Gilchrist and Saito (2008), where the debt-deflation channel is not included.
worth of firms, which is further enhanced by the drop in inflation through the debt-deflation channel. Lower net worth increases the external finance premium, which further depresses investment and output. These dynamics explain why this mechanism is referred to as the financial accelerator.

3.1 Dynamics of the Asymmetric Model

Having discussed the effects of each shock in the constant-parameter model, I now turn to the study of how these effects are altered when the asymmetric monetary policy rule (16) is introduced. When computing impulse responses, I use the calibrated value of $\phi_q = 0.5$ in order to illustrate the effects of the asymmetric policy. For each shock, I compare the effects of positive and negative shocks on the dynamics of key endogenous variables. Consider first the effects of a technology shock.

![Figure 3: Effects of a technology shock, asymmetric model. Solid blue line: Positive shock. Dashed red line: Mirror image of negative shock.](image)

Figure 3 illustrates what happens after positive and negative technology shocks. The 'mirror image' of a negative shock is just the impulse responses of the negative shock multiplied by -1; facilitating comparison. As illustrated, the asymmetric policy has a dampening effect on contractions in output relative to expansions. A positive technology shock causes
output to increase by more than it decreases following a similar-sized negative shock. The explanation is that in the wake of a negative technology shock, asset prices are pushed down for a number of periods.\textsuperscript{16} Under the asymmetric policy, this drop in asset prices is met with an interest rate cut (although this cut is dominated by the increase in the interest rate as a reaction to the jump in inflation), spurring economic activity and thus dampening the initial economic slowdown. On the other hand, as asset prices rise following a positive technology shock, this induces no increase in the interest rate \textit{per se}. In other words, output contractions following technology shocks are mitigated by an interest rate reaction to asset prices, while output expansions are not. Also for inflation, increases will be larger than drops, as the interest rate reaction to asset prices exerts an upward pressure on inflation following a negative shock, but no corresponding downward pressure after a positive shock. While the asset price still displays a boom-bust cycle, the asymmetric policy implies that the decline following a negative shock is less severe than the boom following a positive shock. It thus seems that the policy reaction to asset price drops succeeds in mitigating these drops. The quantitative importance of the asymmetric policy is limited, however, as indicated by the small absolute distance between the impulse responses for the positive and (mirrored) negative shocks.

It is interesting to compare the effects on the asset price to the effects of a similar-sized shock in the constant-parameter model (Figure 1). As the negative shock induces a monetary policy reaction to the drop in stock prices, it is not surprising that the effects of a negative shock (Figure 3) is numerically smaller than the effects of a positive shock in the constant-parameter model. However, we also observe that the increase in the asset price following a positive shock is larger in the asymmetric model than in the constant-parameter model with no asset price reaction. As the asset price increases immediately after a positive technology shock, both models imply no reaction of monetary policy to this increase. In the asymmetric model, however, agents realize that whenever asset prices start to fall, this drop will be alleviated by a monetary policy reaction. This expectation drives up the asset price more than in the model where the reaction to asset prices is always zero, giving rise to an 'anticipation boom'. This anticipation boom measures the additional increase of the asset price in the asymmetric model, relative to its increase in the constant-parameter model, following a positive shock. Quantitatively, the anticipation boom is quite substantial under the calibration with $\phi_q = 0.5$; amounting to 26.4% when evaluated two periods after the shock; the last period before the asset price starts to fall and monetary policy actually starts reacting to asset price changes. If instead I use the estimated value of $\phi_q = 0.0246$, the number is reduced to 1.2%.

Consider finally the asymmetric effects on the two financial variables, net worth and the external finance premium. Recall that because of the debt-deflation channel, net worth is depressed after a positive technology shock, as the drop in inflation increases the real

\textsuperscript{16}Actually, as the figure illustrates, the asset price increases on impact following a negative shock, and only starts falling from period 2 onwards.
burden of firms’ debt repayments. However, it is apparent that the effect on net worth is much larger following a negative shock. After a positive shock, the drop in net worth is counteracted by the rise in the asset price. In the case of a negative shock, this effect is much weaker, as the drop in asset prices is much smaller. Indeed, after a negative shock, the asset price rises in the first period, which is exactly where most of the difference arises in the effects on net worth. As net worth is highly persistent, so is this difference. In turn, also the external finance premium is affected more by a negative shock, which is unsurprising given the movements in net worth.

Figure 4: Effects of a monetary policy shock, asymmetric model. Solid blue line: Contractionary shock. Dashed red line: Mirror image of expansionary shock.

Figure 4 illustrates the asymmetric effects of contractionary and expansionary monetary policy shocks. First of all, the differences between positive and negative shocks are substantially larger than for technology shocks for most variables. As in the constant-parameter model, output and inflation both drop following a contractionary monetary policy shock (an increase in the interest rate). An expansionary shock, however, induces an even larger increase in output and inflation. As was the case for technology shocks, then, the asymmetric policy implies that when output is driven by monetary policy shocks, booms become larger than recessions, once again creating an asymmetric business cycle. The explanation is linked to the movements in the asset price. Following a contractionary shock, the asset price goes down, inducing the central bank to cut the interest rate. This mitigates the initial
economic downturn caused by the shock, and also pushes inflation up. On the other hand, the rise in asset prices following an expansionary shock is not met with any monetary policy reaction, so the counteracting effect is not present. Note that while the nominal interest rate does not display a large, numerical difference, the real interest rate, which matters for consumption and investment decisions, is affected differently during expansionary and contractionary phases, as implied by the impulse responses for inflation.

Furthermore, adding to the asymmetric effects on output and inflation stemming from the monetary policy to reaction to asset prices, the drop in the external finance premium during expansions is much larger than the increase during contractions. In turn, this implies cheaper access to credit for firms, increasing the demand for capital, the investment level, and eventually output. In other words, when considering monetary policy shocks, the financial accelerator channel amplifies the asymmetric business cycle effects arising from the conduct of monetary policy. In section 4, we will see how this result is affected when the financial accelerator mechanism is itself assumed to be a source of asymmetry.

As in the case of technology shocks, an expansionary shock to monetary policy leads to an anticipation boom in asset prices. This is evident when comparing the effects of an expansionary shock in the asymmetric model (Figure 4) to the effects of monetary policy shocks in the constant parameter model (Figure 2). In the case of monetary policy shocks, the anticipation boom is evaluated one period after the shock; the last period before the asset price starts declining. The extra rise in asset prices is 28.0% when \( q \) is set to 0.5. Using the estimated value of \( q = 0.0246 \), this number drops to 1.1%.

The finding that under both types of shocks, the asymmetric policy amplifies booms relative to recessions contradicts most empirical evidence, which tends to find that recessions are sharper (and shorter) than booms (Neftci, 1984; Acemoglu and Scott, 1997). This suggests that an asymmetric policy of the type investigated above has not historically been driving the business cycle. For several reasons, this is not particularly surprising. First, the asymmetry discovered by Ravn (2011) is observed only during the period 1998-2008. Second, the asymmetry is of too little quantitative importance to be a dominant driver of the business cycle. On the other hand, the implications of the asymmetric policy are consistent with the results of Cukierman and Muscatelli (2008) and Wolters (2011), who find that the Federal Reserve has displayed a recession avoidance preference in the recent past. According to these studies, estimated reaction functions of the Federal Reserve indicate that US monetary policymakers tend to react more strongly to the output gap during recessions than during expansions. This creates outcomes that are in line with the impulse responses

17In fact, the period 1998-2008 was characterized by a large boom in the US economy, interrupted by a mild and short recession in 2001. For this period, the business cycle thus seems to be more in line with the predictions of the model. While the asymmetric reaction is unlikely to have been the driving force behind this, at least there seems to be no contradiction between model predictions and the data for this period.

18Note that the sample periods of these studies extend back to 1983 (Wolters, 2011) and 1987 (Cukierman and Muscatelli, 2008), respectively. On the other hand, Surico (2007) finds no evidence of a recession avoidance preference for the post-1980 period.
displayed above, suggesting that an asymmetric reaction to stock prices can be rationalized by recession avoidance preferences, as further discussed in the next section.

The emergence of the anticipation boom can be related to what Davig and Leeper (2006) call the *preemption dividend*. In their model, the central bank is assumed to react stronger to inflation if the lagged inflation level is above a certain threshold (the inflation target). Rational agents will embed this non-linearity in their inflation expectations. As a consequence, monetary policy will be more effective in bringing down inflation in the wake of an inflationary shock, compared to a situation with a linear reaction to inflation. As the central bank is able to successfully manage expectations, the actual increase in the interest rate does not have to be very large. In my setup, agents embed the monetary policy reaction to stock price drops in their expectations, leading to a larger increase in asset prices immediately after a positive shock. This happens despite the fact that when asset prices are increasing, as in the first period(s) after the shock, the actual monetary policy reaction to asset prices is zero in the asymmetric model as well as in the constant-parameter model. As the preemptive dividend of Davig and Leeper (2006), the anticipation boom arises solely due to the central bank’s ability to manage the expectations of private agents. In this way, the asymmetric monetary policy amplifies the boom-bust cycle in asset prices following a shock to the economy, thereby creating additional volatility in asset prices. Finally, it is worth mentioning that similarly to Davig and Leeper (2006), I find substantial differences between the impulse responses shown above, which take into account that agents anticipate the possibility of future regime switches, and the impulse responses (not shown) obtained when agents naively expect the present regime to be in place forever.

4 Potential Motivations and Implications

As demonstrated by the impulse responses in the previous section, reacting asymmetrically to asset prices can lead to a situation in which recessions are attenuated relative to expansions. This raises the question of whether one could think of the central bank as aiming to obtain exactly such an asymmetric outcome. This would then have to show up in the central bank’s underlying loss function. Usually, it is assumed that the central bank (implicitly or explicitly) minimizes a loss function where deviations of output and inflation from their target values are punished in a fully symmetric way (see, e.g., Woodford, 2003). Given a mapping from the parameter governing the central bank’s preference for output stability relative to inflation stability to the parameters of the Taylor rule, one can think of the Taylor rule as a tool used by the central bank to minimize a loss function of this type. It is, however, not given that the objective of the central bank should be perfectly symmetric, as also observed above. Among others, Blinder (1998), Ruge-Murcia (2004), and Surico (2007) suggest that the central bank could be seeking to minimize an asymmetric loss function. Ruge-Murcia assumes that the loss arising from inflation fluctuations is symmetric, but that
social loss is higher when unemployment (which he allows to enter the loss function in lieu of output) is above its natural level, compared to when it is below.

If the loss function of the central bank is of such an asymmetric type, this could serve as the motivation for a policy such as that discovered by Ravn (2011). Indeed, the central bank could adjust the parameters in its asymmetric Taylor rule (16) to obtain the outcome that minimizes the asymmetric loss function. In section 3, we saw how the asymmetric policy implied that booms not only in output, but also in inflation, tended to be stronger and longer than recessions. This would be consistent with a central bank that has a preference for booms and high inflation over recessions and low inflation.

Woodford (2003) shows that a symmetric loss function approximates the negative of the utility of the representative household in the basic New-Keynesian model, so that minimizing such a loss function is equivalent to maximizing the utility of the representative household. Accordingly, if the central bank minimizes an asymmetric loss function, this would also require a micro-foundation in order to be optimal. For example, Ruge-Murcia (2004) suggests that the motivation for the asymmetric loss function could be concerns about the costs of high unemployment. Another way to micro-found an asymmetric loss function is to assume that agents are loss averse with respect to changes in financial wealth. This possibility is discussed at the end of subsection 4.2. Surico (2007) discusses other possible sources of asymmetric welfare losses. In any case, the model outlined above does not include any features that could serve as a welfare-based motivation for an asymmetric loss function, and therefore is unable to explain why the central bank would adopt such a loss function.

If one is not willing to accept the notion of an asymmetric loss function, it is still possible to think of potential motivations for an asymmetric policy. One potential motivation for the central bank to obtain outcomes such as the ones illustrated in section 3 is the fact that natural or steady state output is lower than the efficient level of output. This gives the central bank an incentive to try to push output above its natural level, as in the well-known model of Barro and Gordon (1983). Other rationalizations derive from the fact that specifying a loss function of the central bank of the usual, symmetric form not necessarily implies that the tools of the central bank should also be symmetric. Indeed, if the central bank believes that certain asymmetries exist in the economy, for example that stock price drops and increases have asymmetric macroeconomic effects, an asymmetric policy might be seen as an attempt to correct for this inherent asymmetry, and in turn obtain a symmetric outcome. Ravn (2011) acknowledges this possibility, and points out two potential sources of asymmetric effects of stock price. In the following, I study each of them in more detail.

4.1 Asymmetric Effects of the Financial Accelerator

One channel which may give rise to asymmetric effects of stock price movements is the financial accelerator included in the model outlined above. The possibility of non-linear
balance sheet effects has received some attention in the literature, and was discussed by, among others, Bernanke and Gertler (1989), Gertler and Gilchrist (1994) and Bernanke et al. (1996). During a recession, when asset prices tend to be falling, more firms are likely to be liquidity constrained and in need of external financing. Moreover, small changes in the net worth of firms are likely to be more costly when the collateral value of firms is already low, and the agency costs of borrowing are already large. A final reason why the financial accelerator might be stronger when net worth is low is that ultimately, as firms’ net worth becomes ‘low enough’, a credit crunch might result. Peersman and Smets (2005) assess the empirical transmission effects of monetary policy in the euro area, and find that the financial accelerator effect does indeed seem to be stronger in recessions. Gertler and Gilchrist (1994) further provide empirical evidence that the performance of small firms are more sensitive to interest rate changes during economic downturns than in booms, suggesting that financial factors are more important in bad times. As discussed by Peersman and Smets (2005), such an asymmetry could potentially explain why monetary policy exerts a stronger effect on output during recessions than in booms.

In the model above, the importance of the financial accelerator is governed by the parameter $\psi$ in (7). As discussed in section 2, $\psi$ measures the elasticity of the external finance premium with respect to the net worth of firms. As the dependence of the external finance premium on net worth is the key to the financial accelerator mechanism, this elasticity measures the strength of the balance-sheet effect. The larger is $\psi$, the stronger is this effect. In other words, modelling an asymmetric financial accelerator amounts to allowing $\psi$ to take on different values. In light of the above discussion, I therefore allow $\psi$ to take on one value ($\psi_L$) for the case when net worth is above its steady state value, i.e., $\bar{b}_t > 0$, and a higher value ($\psi_H$) whenever $\bar{b}_t < 0$. This reflects that when net worth of firms is already low, the external finance premium becomes more sensitive to small changes in net worth, capturing the effects discussed above.

Under the assumption that the external finance premium depends non-linearly on net worth, the financial accelerator mechanism becomes itself a possible source of asymmetric business cycle fluctuations. In that case, one could view the asymmetric monetary policy reaction found by Ravn (2011) as an attempt to counter this inherent non-linearity and obtain symmetric outcomes. Consider first what happens under technology shocks. When the economy is hit by a positive technology shock, net worth drops below its steady state value, implying that the elasticity of the external finance premium becomes high. This exerts a downward pressure on investment and output, dampening the initial boom. In the case of a negative technology shock, this effect is much weaker, as the rise in net worth leads to a much smaller drop in the external finance premium. This implies that the dampening of the initial downturn is small, exposing the economy to a situation in which recessions are generally larger than booms. The central bank, trying to correct for this asymmetry, would

19Recall that $\bar{b}_t$ measures the deviation of net worth from its steady state value.
then be tempted to react to asset price drops, as this would provide exactly the 'missing link';

namely, a dampening of the initial bust following a negative technology shock. Note that the
debt-deflation channel is not critical for this conclusion. Without the debt-deflation channel,
net worth would be procyclical after technology shocks (this is also noted by Gilchrist and
Saito, 2008). An asymmetric financial accelerator would then amplify recessions more than
booms. Once again, this could be counteracted by the same type of asymmetric policy.

Similarly, after an expansionary monetary policy shock, net worth jumps up. In this
case, the balance-sheet effect is relatively weak, as the elasticity of the external finance
premium is low. The resulting amplification of the initial boom is limited. On the other
hand, the financial accelerator is much stronger following a contractionary monetary policy
shock due to the drop in net worth. Once again, the central bank will be tempted to make
up for this asymmetric outcome by reacting to stock price drops.

In sum, the asymmetric financial accelerator implies that under technology shocks,
booms are dampened more than recessions, while under monetary policy shocks, booms
are amplified less than recessions. In both cases, the result is that recessions are larger than
booms. As we saw in the previous section, the effects of the asymmetric policy were the
exact opposite. It therefore seems natural to ask: How severe should the asymmetry of the
financial accelerator be in order to 'rationalize' the asymmetric result of Ravn (2011); so that
the two asymmetries 'cancel out'? In order to quantify the necessary degree of asymmetry,
I fix $\psi_L = 0.042$. I then use impulse response matching of output and inflation responses
for positive and negative shocks in the asymmetric DSGE model to calibrate the value of
$\psi_H$ that would 'match' the asymmetric policy. This value can then be compared with $\psi_L$.

Table 1 shows the degree of asymmetry needed to optimally match the impulse responses of
output and inflation to technology shocks for different values of $q$, the reaction coefficient
of monetary policy to stock price changes.

<table>
<thead>
<tr>
<th>Value of $\phi_q$</th>
<th>Value of $\psi_L$</th>
<th>Calibrated value of $\psi_H$</th>
<th>Ratio $\frac{\psi_H}{\psi_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0246</td>
<td>0.042</td>
<td>0.043</td>
<td>1.02</td>
</tr>
<tr>
<td>0.50</td>
<td>0.042</td>
<td>0.059</td>
<td>1.40</td>
</tr>
</tbody>
</table>

As the table illustrates, the degree of asymmetry in the financial accelerator needed to
match impulse responses is quite sensitive to the choice of $\phi_q$. For the value found by Ravn
(2011), the balance-sheet channel needs to be only slightly asymmetric in order for the two
asymmetries to 'cancel each other out'. On the other hand, if $\phi_q$ is set at 0.50, the financial
accelerator needs to be 40 % stronger when net worth is low, compared to when it is high.

More specifically; for each of the two types of shocks, I focus on the impulse responses of output and
inflation. I then compute the sum of squared errors (SSE) between the impulse response to a positive shock
and the mirror image of the impulse response to a negative shock. For this, I use the values in the first 16
periods after the shock. Finally, I solve for the value of $\psi_H$ that minimizes the sum of the SSE’s.
To put these numbers in perspective, I look to the study by Peersman and Smets (2005), which is one of the few empirical investigations of possibly asymmetric balance-sheet effects.\footnote{The empirical results of Gertler and Gilchrist (1994) are very difficult to translate into the present context.} Peersman and Smets first show that a monetary policy shock that raises the interest rate by 1 %-point causes a drop in the growth rate of output of 0.22 %-points during a boom, but a much larger drop of 0.66 %-points during a recession. They then estimate how various measures of firms’ financial position contribute in explaining this asymmetry. They find that if firms’ leverage ratio increases by 5 % of its average value, the \textit{difference} between the effect on output growth of a monetary policy shock in booms and in recessions increases by 0.14 %-points; i.e. from the original 0.44 %-points to 0.58 %-points. In other words, the financial position of firms is able to account for substantial asymmetries over the business cycle, indicating that the financial accelerator effect is considerably stronger in recessions than in booms. In this light, a degree of asymmetry of only 2 % surely does not seem unlikely, and even a number such as 40 % under the calibration with $\phi_q = 0.5$ is not necessarily unrealistic.

Consider next the results when the economy is driven by monetary policy shocks:

<table>
<thead>
<tr>
<th>Value of $\phi_q$</th>
<th>Value of $\psi_L$</th>
<th>Calibrated value of $\psi_H$</th>
<th>Ratio $\frac{\psi_H}{\psi_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0246</td>
<td>0.042</td>
<td>0.045</td>
<td>1.07</td>
</tr>
<tr>
<td>0.50</td>
<td>0.042</td>
<td>0.097</td>
<td>2.31</td>
</tr>
</tbody>
</table>

If the monetary policy reaction parameter to stock price changes is set at $\phi_q = 0.50$, the balance-sheet effect has to be more than twice as strong during periods of low net worth in order to obtain symmetric outcomes of output and inflation under monetary policy shocks. The degree of asymmetry needed to explain a reaction of the size estimated by Ravn (2011) is again much more modest. While Peersman and Smets (2005) did identify an important asymmetry in the functioning of the financial accelerator over the business cycle, to answer the question of whether the effect is indeed more than twice as strong in recessions as in booms, more empirical work is needed.

### 4.2 Asymmetric Wealth Effects

Another possible source of asymmetric macroeconomic effects of stock price movements is the wealth effect on consumption. Shirvani and Wilbratte (2000) and Apergis and Miller (2006) provide empirical evidence that the wealth effect of stock prices is stronger when stock prices are declining than when they are increasing. One possible, theoretical explanation for this finding is provided by prospect theory (Kahneman and Tversky, 1979). Prospect theory introduces an inherent asymmetry in agents’ preferences, as the utility loss from...
bad outcomes is assumed to be larger than the utility gain from good outcomes. If agents display such loss aversion in consumption as suggested by, among others, Koszegi and Rabin (2009), this might give rise to non-linear effects on consumption from asset price movements. If asset prices decline, so does financial wealth and permanent income, and agents will have to cut their consumption level, even if this is painful. On the other hand, following a rise in asset prices, loss averse agents are likely not to increase their consumption level by as much, but instead engage in precautionary savings to cushion themselves against the risk of a drop in asset prices in subsequent periods. As a result, increases in asset prices have smaller effects on consumption, and hence on the macroeconomy, than asset price declines.

Gaffeo et al. (2010) show how loss aversion in consumption can be introduced into a Markov-switching DSGE model. They assume that agents evaluate their consumption level relative to a function of aggregate consumption in the last period, i.e. a version of external habit formation. However, extending the asymmetric model of the present paper to include wealth effects, reference-dependent preferences and regime-switching is not tractable. For this reason, I do not attempt to quantify the magnitude of the asymmetry of the wealth effect needed to explain the result of Ravn (2011). Instead, I provide a verbal discussion of how such asymmetric wealth effects would affect the present model, and whether these might serve as a motive for the asymmetric policy reaction. Following a monetary contraction, output declines, and so does the asset price, leading in turn to a substantial, negative wealth effect which further depresses output. For positive shocks, the wealth effect is smaller and the amplification mechanism less strong, implying smaller booms than recessions in output, all else equal. The asymmetric policy reaction has exactly the opposite implications, as discussed in section 3, and could therefore in principle be a result of the central bank seeking to ‘correct’ for asymmetric wealth effects.

When a positive technology shock hits, the initial rise in asset prices leads to a modest wealth effect. Because inflation declines following the shock, the real value of agents’ financial wealth increases, as agents are net lenders. This enhances the wealth effect, amplifying the increase in output. After some periods, however, output starts to decline, and is further depressed by the negative wealth effects arising from the concurrent drop in asset prices. Furthermore, as inflation is rising back towards its steady state level, this undermines the real value of agents’ financial wealth, enhancing the negative wealth effects further, and driving output rapidly back towards its steady state level. Following a negative shock, the drop in asset prices and the rise in inflation will lead to larger, negative wealth effects, and in turn a large initial recession. On the other hand, as inflation starts falling and asset prices go up, this leads only to small wealth effects, and thus a slower return of output to its steady state. Recessions are now likely to be both larger and longer than booms. A central bank reacting to asset price declines can dampen the initial recession, and further prolong booms by cutting the interest rate as output and asset prices fall back towards their steady

\footnote{For now, I ignore the small rise in the asset price in the first period following a negative technology shock.}
state levels. Thus, also in the case of technology shocks, the asymmetric policy reaction could in principle reflect the central bank’s desire to obtain more symmetric outcomes.

Gaffeo et al. (2010) demonstrate that in their model with reference-dependent utility and loss aversion, the optimal monetary policy is asymmetric over the business cycle. Importantly, they abstract from asset pricing issues, and therefore do not consider the possibility of asymmetric wealth effects. Instead, they show that loss aversion implies that both the intertemporal elasticity of substitution in consumption and the intratemporal marginal rate of substitution between consumption and leisure differ between contractions and expansions. As a result, the effect on output of monetary policy innovations is smaller during expansions, which is in accordance with empirical evidence. This implies that the optimal monetary policy should be reacting stronger to inflation during booms than in contractions, as the trade-off between inflation and output stabilization is more advantageous during booms. As their model does not include asset prices, their result obviously cannot be transferred directly to the question at hand here. Instead, it serves as an example of monetary policy operating asymmetrically in order to make up for asymmetries elsewhere in the economy; specifically, loss aversion in consumption.

According to the above explanation, asymmetric wealth effects arise through the effect of stock wealth on consumption. A related line of argument, also deriving from prospect theory, is that gains and losses in financial wealth might have direct, asymmetric effects on utility. Barberis et al. (2001) assume that agents derive (dis)utility from fluctuations in their financial wealth. Combining this with loss aversion, the loss in utility following from a drop in asset prices and financial wealth is larger than the utility gain from a similar-sized increase. As illustrated in section 3, the introduction of an asymmetric policy rule implies a dampening of the drops in asset prices and an amplification of the increases. If the central bank believes that agents have preferences of the type suggested by Barberis et al. (2001), the asymmetric policy could then be an attempt to cushion agents from the utility losses when asset prices decline. As agents are assumed to derive utility from changes in asset prices (as opposed to the level), this story would be consistent with the result that the central bank is reacting to changes in stock prices. Note the distinction that in this case, changes in asset prices would be entering the reaction function of the central bank not because of their effects on other variables of interest, such as output and inflation, but as a separate target variable entering the underlying loss function of the central bank. As briefly discussed above, loss aversion with respect to changes in financial wealth could therefore serve as a potential welfare-based motivation for an asymmetric loss function.

4.3 The Risk of Moral Hazard
A recurring topic in the post-crisis discussion of asset prices and monetary policy has been the risk of creating moral hazard problems. If an investor realizes that the central bank
reacts only to stock price drops, he will be tempted to hold more stock instead of bonds. If the stock price goes down, the central bank will cut the interest rate, lowering his alternative cost of holding stock. If instead the stock price increases, there is no corresponding rise in the interest rate. In this way, the asymmetric policy distorts the investor’s choice by covering part of his downside risk. The risk of moral hazard has been at the heart of the critique of the ‘pre-crisis consensus view’ of how monetary policy should deal with asset prices. Issing (2009) discusses the creation of such moral hazard problems, and advocates that monetary policy should be leaning against ‘headwind’ (asset price declines) as well as ‘tail wind’ (increases). Mishkin (2010) also acknowledges this risk, and White (2009) calls for monetary policy to be symmetric in the future. Farhi and Tirole (2010) discuss the moral hazard implications of what they call interest rate bailouts.

In the framework of the present paper, the emergence of an anticipation boom in asset prices in the wake of expansionary shocks is closely related to the issue of moral hazard. The anticipation boom arises exactly because investors realize that once asset prices start falling, the central bank will cut the interest rate, facilitating a ‘soft landing’ that insulates them from stock market losses. Essentially, stock market participants are prizing the future interest rate cut into the current stock price. The asymmetric policy leads to moral hazard problems, since for the single investor, it is optimal to behave (invest) more risky than what is socially optimal, as he is partly insured against bad outcomes (falling stock prices) by the central bank. The anticipation boom is testimony of this line of argument.

As shown in section 3, the magnitude of the anticipation boom is very sensitive to the size of the stock price reaction of monetary policy; $\phi_q$. Using the estimated result of Ravn (2011) implies a modest boom, while a very substantial boom results if the reaction parameter is chosen in accordance with calibrated values suggested in the literature. The latter is in line with the findings of Miller et al. (2001), who study the effects of the so-called Greenspan Put in a theoretical model.23 They show that if investors believe the Federal Reserve will act to prevent stock prices from falling by more than 25 % below their previous peak, this may push up stock prices by as much as 50 %, depending on the calibration.

A related issue is what will happen if the private sector begins to doubt the central bank’s commitment to the asymmetric policy. This approach has been used to study problems of loose commitment (or quasi-commitment) on behalf of the policymakers by Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010). Schaumburg and Tambalotti motivate loose commitment by assuming that with some probability, a new central bank governor is appointed in each period, who then reneges on the promises of his predecessor and commits to a new policy. As the asymmetric policy reaction to stock prices is closely related to the aforementioned Greenspan Put, this interpretation seems highly relevant also for my purposes: Given that the policy is directly named after the central bank governor, it seems

23The Greenspan Put is typically used to denote the perception that if stock prices fall suddenly and drastically, the Federal Reserve will react by cutting the interest rate aggressively. See Miller et al. (2001) for details.
reasonable that agents assign some non-zero probability to this policy not being continued when a new governor takes office.

In particular, imagine that agents start to fear that with some probability $\theta$, the central bank will stop reacting to asset price drops, and instead stick to a purely symmetric policy rule with no asset price reaction. The agents’ perceived policy rule will then be a weighted average of those two rules: $R_{t}^{\text{perceived}} = \theta R_t [\phi_q = 0] + (1 - \theta) R_t [\phi_q > 0]$. In the model above, if this happens outside the steady state (for instance, if the change in beliefs is triggered by a shock to the economy), the result will be a discrete drop in the asset price. Thus, not only does the asymmetric policy lead to a moral hazard-driven amplification of booms in asset prices, it might also lead to sudden asset price drops if the sustainability of the policy is questioned. Debortoli and Nunes (2010) demonstrate that even small deviations from the full commitment case (in this case, $\theta$ close to 0) have substantial effects.

The present paper follows most of the modern, macroeconomic literature by log-linearizing the equilibrium conditions around a steady state. Thus, by construction, the economy eventually returns to the original steady state following a shock. This inherent limitation prevents me from assessing the risk that as a result of the asymmetric policy reaction, the economy might end up in a different steady state than the original. For instance, one might suspect that the asymmetric policy would eventually drive the nominal interest rate to its zero lower bound. In relation to moral hazard problems, it would be interesting to analyze if the effects of the asymmetric policy on the behavior of investors could potentially drive the economy to a new steady state, and if so, how the new steady state would compare to the original in terms of social welfare. This type of analysis would require moving beyond the log-linear approach, and is left for future work.

In sum; while an asymmetric reaction to stock prices might serve to correct for other asymmetries, the risk of creating moral hazard problems must be taken properly into account. Given that the central bank wants to neutralize inherent asymmetries in the economy, it should instead attempt to do so by using other, less blunt tools than an asymmetric interest rate policy. As an example, if policymakers want to make up for an asymmetric financial accelerator mechanism, they should perhaps think of ways to mitigate the agency problem between borrower and lender which lies at the core of the problem. Thus, by putting more appropriate policy measures to use, the central bank would be able to obtain symmetric economic outcomes without giving rise to moral hazard problems. A comprehensive study of how asymmetric monetary policy can cause moral hazard problems by distorting the incentives of the individual investor would require an even richer microfoundation than that of the present paper, explicitly modelling the investor’s investment decision. While this is surely an interesting idea for future research, it is beyond the scope of the present paper.

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24 Recall that $q = 1$ in steady state.
25 To see this, note that taking a weighted average of the two policy rules simply amounts to changing the reaction parameter to stock price drops from $\phi_q$ to $(1 - \theta) \phi_q$. 

26
4.4 Implications for Equilibrium Determinacy

An interesting question is to what extent the asymmetric stock price reaction might lead to equilibrium indeterminacy. In the standard New-Keynesian model, the usual result is that to avoid indeterminacy, the nominal interest rate should be raised more than one-for-one in response to an increase in inflation, ensuring that the real interest rate is increased (see e.g. Clarida et al., 1999). This is the well-known Taylor Principle. However, Carlstrom and Fuerst (2005) show that this conclusion is overturned when capital and investment are added to the model. In particular, they find that if the central bank is reacting to expected (as opposed to current) inflation, equilibrium indeterminacy is by far the most likely outcome. Under their baseline calibration, determinacy is obtained only if the reaction parameter to expected inflation is within the extremely narrow band of \( 1 < \phi_\pi < 1.0027 \). On the other hand, Carlstrom and Fuerst show that if the central bank is reacting to current inflation, the Taylor principle is a necessary and sufficient condition for equilibrium determinacy.

Kurozumi and Van Zandweghe (2008) provide a qualification and an explanation of this result. The introduction of capital into the model introduces a 'cost channel' of monetary policy, whereby an increase in the interest rate increases the marginal production cost of firms, thus creating inflation. This involves a risk of self-fulfilling inflation expectations. However, Kurozumi and Van Zandweghe show that adding a reaction to current output, or adding interest rate smoothing, to a forward-looking Taylor rule solves the indeterminacy problem. The reason is that a rise in the interest rate in response to an increase in inflation expectations depresses current consumption and investment. With a policy reaction to current output, this 'demand channel' then in turn calls for a cut in the interest rate, dominating the cost channel and restoring equilibrium determinacy. Interest rate smoothing in a forward-looking rule means reacting to the lagged interest rate, which in turn implies reacting to current output and inflation, introducing the demand channel (provided a sufficient degree of smoothing). On the other hand, reacting to future output does not suffice.

In the model of the present paper, recall from (16) that monetary policy reacts to current inflation and current output, and further includes interest rate smoothing. According to the discussion above, then, equilibrium determinacy should be satisfied in the constant-parameter version with \( \phi_q = 0 \). Does the introduction of a non-linear stock price reaction overturn this conclusion? As argued below, the short answer to this question is 'no'.

Following Proposition 1 of Blanchard and Kahn (1980), evaluating equilibrium determinacy of rational expectations models essentially amounts to verifying that the number of eigenvalues of the matrix \( \Omega \) that lie outside the unit circle is equal to the number of non-predetermined (or non-state) variables of the model; where the matrix \( \Omega \) is the matrix governing the dynamics of the model (for details, see Blanchard and Kahn (1980) or the extended appendix to the present paper, available from the author's webpage). However, as

\[ \text{This is obtained with a Taylor rule with no reaction to output, although Carlstrom and Fuerst state that including an output reaction would have only minor effects on their conclusion.} \]
described in the appendix, due to the non-linearity of the policy rule, the dynamics of the present model does not boil down to one such matrix. This renders a traditional, formal investigation via the location of the eigenvalues impossible. However, it is still possible to characterize the way this non-linearity affects the question of equilibrium determinacy.

Consider first a linear stock price reaction. This does not change the cost channel of monetary policy described above. On the other hand, it enhances the demand channel by adding to it an extra dimension. A rise in the interest rate lowers the stock price, as future dividends are now discounted more heavily. Provided that the central bank is reacting to the ‘current stock price change’ (i.e., the change from the last period to the current), this calls for an interest rate cut; exactly as explained above for consumption and investment. In this sense, a linear reaction to stock price changes helps secure equilibrium determinacy.

Having established this, the question remains of whether the mere presence of asymmetric or regime-switching policy has implications for determinacy. Davig and Leeper (2007) show how the introduction of regime-switching in monetary policy alters the standard results for equilibrium determinacy through its effects on expectations formation; as rational agents will assign non-zero probability to the possibility of a regime change. In the present model, the asymmetric policy gives rise to an anticipation boom. Note, however, how this boom prevents the occurrence of self-confirming expectations and equilibrium indeterminacy: If expected inflation drops, the resulting interest rate cut gives rise to an anticipation boom in stock prices, but to no policy reaction to this boom. However, higher stock prices will in turn tend to push up output and inflation, counteracting the initial drop in inflation expectations. This can be called a ‘second-order demand channel effect’: Higher stock prices do not lead to an interest rate increase per se, but they push up other variables that will in turn induce the central bank to increase the interest rate.

In sum, the non-linear stock price reaction tends to strengthen the demand channel of monetary policy, which helps ensure equilibrium determinacy. This is true during expansions as well as contractions; either because of the direct implications of stock price drops for monetary policy, or because of the indirect effects of stock price increases on variables that in turn enter the monetary policy rule. Thus, the asymmetric policy seemingly does not lead to equilibrium indeterminacy.

5 Concluding Remarks

The present paper provides some theoretical inputs to the recent debate about the potential asymmetric reaction of monetary policy to stock prices. I demonstrate that an asymmetric policy of the type detected by Ravn (2011) for the US in the years 1998-2008 will translate into an asymmetric business cycle. Booms in output following expansionary shocks will tend to be amplified, while recessions will be dampened. A similar pattern emerges for inflation. This could be motivated by assuming that the desire of the policymaker is to
minimize an asymmetric loss function, or by the existence of other asymmetries in the economy. As an example, I compute the ‘degree of asymmetry’ of the financial accelerator mechanism of Bernanke et al. (1999) that would be needed to ‘rationalize’ an asymmetric policy. For an asymmetric reaction of the size estimated by Ravn (2011), the necessary degree of asymmetry is quite modest, and does not at all seem unrealistic.

However, I also find that the asymmetric policy gives rise to what I call an anticipation boom following an expansionary shock; i.e. an amplification of the boom in asset prices brought about by agents’ anticipation that once the boom in asset prices turns into a bust, the central bank will intervene by cutting the interest rate. This boom is related to the moral hazard problems that inevitably arise when investors realize that in effect, the central bank is covering part of their downside risk. As a result, while an asymmetric policy can help policymakers mitigate other asymmetries in the economy, more targeted measures than the interest rate are likely to be more appropriate to this end. Indeed, as witnessed by Issing (2009) and Mishkin (2010), the recent financial crisis seems to have drawn considerable attention to the need for taking the ‘a’ out of ‘asymmetry’ in monetary policy questions exactly for these reasons.

In an attempt to explain the intellectual foundation underlying the pre-crisis consensus that monetary policy should not lean against asset price movements, Bini Smaghi (2009) points to the fact that the New-Keynesian model framework, which has become the dominant theoretical workhorse for monetary policy analysis over the last ten years, has - at least until recently - failed to pay sufficient attention to financial markets. When financial frictions play only a small (or no) role in the model economy, the potential gains from reacting to asset prices are reduced markedly. In the recent past, however, and especially after the financial crisis, researchers have devoted a lot of attention to the introduction of a wide variety of financial factors into DSGE models (among many others, see Christiano et al., 2010 (and the references therein); Gertler and Kiyotaki, 2010; and Woodford, 2010). An interesting path for future research, bearing the considerations of Bini Smaghi (2009) in mind, is to investigate whether in such models with more detailed descriptions of the financial side of the economy, central banks could actually benefit from reacting to asset prices; i.e., whether the pre-crisis consensus will be overturned.

As a methodological issue, further research is needed to push forward the agenda of endogenous regime-switching initialized by Davìg and Leeper (2006). As already mentioned, extending their method in order to deal with the curse of dimensionality might facilitate a numerical solution of the model. Even more interesting would be to develop analytical tools for handling the problem, regardless of the dimensions of the model. These are directions I plan to follow in future work.

29
References


Bini Smaghi, Lorenzo. 2009. Monetary Policy and Asset Prices. Speech held at the University of Freiburg, Germany (October).


Kohn, Donald. 2006. Monetary Policy and Asset Prices. Speech held at the European Central Bank, Frankfurt, Germany (March).


Appendix: Model Details

This appendix contains details about the model and the solution method. A detailed model appendix is available from the author’s webpage; https://sites.google.com/site/sorenhoveravn/.

A1: Calibration

Table A1: Parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Capital share in production</td>
<td>0.3384</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
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</tr>
<tr>
<td>$\gamma$</td>
<td>Preference for consumption and real balances</td>
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</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
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</tr>
<tr>
<td>$\epsilon_p$</td>
<td>Elasticity of substitution between final goods</td>
<td>6</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Preference for leisure</td>
<td>1.315</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Entrepreneurs’ survival rate</td>
<td>0.9853</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Probability of not adjusting price</td>
<td>0.7418</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Persistence in technology process</td>
<td>0.7625</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Importance of capital adjustment cost</td>
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</tr>
<tr>
<td>$\psi$</td>
<td>Elasticity of ext. fin. premium wrt. leverage</td>
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</tr>
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<td>$\Psi$</td>
<td>Steady state external finance premium</td>
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</tr>
<tr>
<td>$\pi$</td>
<td>Steady state inflation rate</td>
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<td>$K/n$</td>
<td>Rate of capital to net worth in steady state</td>
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</tr>
<tr>
<td>$\rho_r$</td>
<td>Degree of interest rate smoothing</td>
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<tr>
<td>$\phi_\pi$</td>
<td>Monetary policy reaction to inflation</td>
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<td>$\phi_y$</td>
<td>Monetary policy reaction to output</td>
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<tr>
<td>$\phi_q$</td>
<td>Monetary policy reaction to stock price drops (estimated)</td>
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</tr>
<tr>
<td>$\phi_q$</td>
<td>Monetary policy reaction to stock price drops (calibrated)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

A2: Equilibrium Conditions

The 16 equilibrium conditions of the model are summarized below. The first four conditions are the first-order conditions of the household.\(^{27}\)

\[
\lambda_t C_t^{\frac{\pi - 1}{\epsilon}} + \lambda_t \left( \frac{M_t}{P_t} \right)^{\frac{\pi - 1}{\epsilon}} = C_t^{\frac{1}{\gamma}}, \tag{A1}
\]

\[
\frac{\eta}{1 - H_t} = \lambda_t w_t, \tag{A2}
\]

\(^{27}\) \(w_t = \frac{W_t}{\Pi_t}\) is the real wage.
\[
\frac{\lambda_t}{R_t} = \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \quad (A3)
\]

\[
\left( \frac{C_t}{N_t} \right)^{\frac{1}{\nu}} = \frac{R_t - 1}{R_t}, \quad (A4)
\]

\[
Y_t = (A_t H_t)^{1-\alpha} K_t^\alpha, \quad (A5)
\]

\[
m_p = \alpha \frac{Y_t}{K_t} m c_t, \quad (A6)
\]

\[
w_t = (1 - \alpha) \frac{Y_t}{H_t} m c_t, \quad (A7)
\]

\[
E_t[f_{t+1}] = E_t \left[ \frac{m_{t+1} + (1 - \delta) q_{t+1}}{q_t} \right], \quad (A8)
\]

\[
E_t[f_{t+1}] = E_t \left[ \Psi \left( \frac{n_{t+1}}{q_t K_{t+1}} \right) \frac{R_t}{\pi_{t+1}} \right], \quad (A9)
\]

\[
n_{t+1} = \nu \left[ f_t q_{t-1} K_t - E_{t-1} f_t (q_{t-1} K_t - n_t) \right] + (1 - \nu) Y_t, \quad (A10)
\]

\[
K_{t+1} = I_t + (1 - \delta) K_t, \quad (A11)
\]

\[
E_t \left[ q_t - \chi \left( \frac{I_t}{K_t} - \delta \right) \right] = 1, \quad (A12)
\]

\[
P_t^n = \frac{e^\nu}{e^\nu - 1} \frac{E_t \left\{ \sum_{s=0}^{\infty} (\beta \xi)^s \lambda_{t+s} Y_{t+s} m c_{t+s} P_{t+s} \right\}}{E_t \left\{ \sum_{s=0}^{\infty} (\beta \xi)^s \lambda_{t+s} Y_{t+s} \pi^s \right\}}, \quad (A13)
\]

\[
P_t = \left[ (1 - \xi) (P_t^n)^{1-e^\nu} + \xi (P_{t-1} \pi)^{1-e^\nu} \right]^{1/(1-e^\nu)}, \quad (A14)
\]

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_t} \left[ \frac{\lambda}{\pi} \phi_r \left( \frac{Y_t}{\pi} \right) \phi_r \left( \frac{\Delta q_t}{q} \phi_q \left\{ \begin{array}{l} 1 \left[ \Delta q_t < 0 \right] \\ 0 \left[ \Delta q_t \geq 0 \right] \end{array} \right) \frac{\Delta q_t}{q} \right)^{0} \right]^{(1-\rho_t)} e^{\xi_t}, \quad (A15)
\]

\[
Y_t = C_t + I_t. \quad (A16)
\]
A3: Log-linearized Equilibrium Conditions

The final step is to log-linearize the conditions describing the equilibrium; (A1)-(A16), around the steady state described above. For details, see the extended model appendix on the author’s webpage. In the following, \( \tilde{x}_t \) will denote the log-deviation of variable \( x_t \) from its value in the nonstochastic steady state; denoted \( x \).

\[
\gamma \tilde{x}_t + \lambda C (\gamma - 1) \left( \frac{R}{R - 1} \right)^{-1} \left( \frac{M_t}{P_t} \right) = (\lambda C (1 - \gamma) - 1) \tilde{C}_t, \tag{A30}
\]

\[
H \tilde{H}_t = (1 - H) \left( \tilde{x}_t + \tilde{w}_t \right), \tag{A31}
\]

\[
\tilde{\lambda}_t = E_t \tilde{\lambda}_{t+1} - E_t \tilde{\pi}_{t+1} + \tilde{R}_t, \tag{A32}
\]

\[
\frac{\tilde{R}_t}{R - 1} = \frac{1}{\gamma} \left( \tilde{C}_t - \frac{M_t}{P_t} \right), \tag{A33}
\]

\[
\tilde{Y}_t = (1 - \alpha) \tilde{A}_t + (1 - \alpha) \tilde{H}_t + \alpha \tilde{K}_t, \tag{A34}
\]

\[
\tilde{m} p_t = \tilde{Y}_t + \tilde{m} c_t - \tilde{K}_t, \tag{A35}
\]

\[
\tilde{w} = \tilde{Y}_t + \tilde{m} c_t - \tilde{H}_t, \tag{A36}
\]

\[
\tilde{f}_t = \frac{m p}{f} \tilde{m} p_t + \frac{1 - \delta}{f} \tilde{q}_t - \tilde{q}_{t-1}, \tag{A37}
\]

\[
E_t \tilde{f}_{t+1} - (\tilde{R}_t - E_t \tilde{\pi}_{t+1}) = -\psi \left( \tilde{n}_{t+1} - \tilde{q}_t - \tilde{K}_{t+1} \right), \tag{A38}
\]

\[
\tilde{n}_{t+1} = \frac{K}{n} \tilde{f}_t + \left( 1 - \frac{K}{n} \right) (\tilde{R}_{t-1} - \tilde{\pi}_t) + \left( 1 - \frac{K}{n} \right) (\tilde{q}_{t-1} + \tilde{K}_t) + \left[ 1 + \psi \left( \frac{K}{n} - 1 \right) \right] \tilde{n}_t, \tag{A39}
\]

\[
\tilde{K}_{t+1} = \frac{I}{K} \tilde{f}_t + (1 - \delta) \tilde{K}_t, \tag{A40}
\]

\[
\tilde{q}_t = \chi \left( \tilde{I}_t - \tilde{K}_t \right), \tag{A41}
\]
\[ \hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) \left[ \phi_p \hat{p}_t + \phi_q \hat{Y}_t + \phi_y [\Delta \hat{q}_t < 0] \Delta \hat{q}_t \right] + \varepsilon_t^r, \quad (A42) \]

\[ Y \hat{Y}_t = C \hat{C}_t + I \hat{I}_t, \quad (A43) \]

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \xi_1) (1 - \beta \xi)}{\xi} \hat{m}_c_t. \quad (A44) \]

**A4: The Solution Method**

As the solution method I use follows the work of Bodenstein *et al.* (2009), the rest of this appendix builds on their Appendix A. The log-linearized version of the model consists of equations (A30) – (A44). Note that the monetary policy condition (A42) is not linear, as the value of the parameter \( \phi_q \) depends on the sign of \( \Delta \hat{q}_t \). However, the model is piecewise linear, in the sense that given one of the two possible values of \( \phi_q \), all equations are linear. This is the key insight underlying the solution method. I can represent each of the two linear systems in the following way, stacking the 15 equations and 15 variables.\(^{28}\)

Let

\[ 0 = \bar{A} E_t s_{t+1} + \bar{B} s_t + \bar{C} s_{t-1} + \bar{D} \varepsilon_t \quad (A45) \]

describe the dynamics of the system when the asset price is non-decreasing, i.e. when the monetary policy reaction to the asset price is zero. Further, let

\[ 0 = A^* E_t s_{t+1} + B^* s_t + C^* s_{t-1} + D^* \varepsilon_t \quad (A46) \]

denote the equivalent system when the asset price is decreasing and monetary policy involves a non-zero reaction to the asset price. Here, the vector \( s \) contains all the relevant variables, as measured in log-deviations from their steady state values: \( s_t = \left[ R_t, \hat{n}_t, \hat{q}_t, \left( \frac{\hat{m}}{\hat{Y}} \right)_t, \hat{R}_t, \hat{C}_t, \hat{\lambda}_t, \hat{f}_t, \hat{Y}_t, \hat{I}_t, \hat{w}_t, \hat{\pi}_t, \hat{\pi}_c, \hat{m}_c, \hat{m}_c \right]^\prime \). The matrices \( \bar{A}, \bar{B}, \bar{C}, \) and \( A^*, B^*, C^* \) are \( N \times N \) coefficient matrices, where \( N = 15 \) is the number of variables. Finally, \( \varepsilon_t = [\varepsilon_t^a, \varepsilon_t^f]^\prime \) is the vector of shocks, and \( \bar{D} \) and \( D^* \) are \( N \times M \) coefficient matrices, with \( M = 2 \) representing the number of shocks. The elements of the coefficient matrices derive from the log-linear system of equations presented above. Note that the only difference between the two systems is the reaction of monetary policy to asset price changes; i.e., whether \( \phi_q = 0 \) in equation (A42) or not. This affects only the matrices multiplying \( s_t \) and \( s_{t-1} \).

In other words, \( \bar{A} = A^* \), and \( \bar{D} = D^* \). Further, the matrices \( \bar{B} \) and \( B^* \) differ in only one entry, and the same is true for \( \bar{C} \) and \( C^* \): If the monetary policy reaction function is listed

\(^{28}\)While the model originally consisted of 16 equations in 16 variables, the log-linearized model has only 15 equations in 15 variables, as equations (A13) and (A14) were combined to yield one log-linearized equation; (A44), making the variable \( \frac{\hat{m}_c}{\hat{Y}} \) redundant.
as the n’th equation in the system, and the price of capital appears as the m’th variable in the vector $s$, then these matrices differ only in the (n,m)’th entry.

As each of these two systems are linear, they can be solved separately using well-known methods such as the Toolkit method of Uhlig (1999), which I use, or the Gensys method of Sims (2002). The solutions can then also be written on matrix form, as the evolution of the endogenous variables are fully described by the lagged values of the state variables and the realizations of the shocks. Hence, the solutions to the above systems are, respectively:

$$s_t = P s_{t-1} + Q \varepsilon_t,$$

$$s_t = P^* s_{t-1} + Q^* \varepsilon_t.$$  \[A47\]

$$s_t = P s_{t-1} + Q \varepsilon_t,$$

$$s_t = P^* s_{t-1} + Q^* \varepsilon_t.$$  \[A48\]

Assume that a shock hits the economy in period 0. As the economy starts out in the regime with no reaction to stock price changes, the first regime change will occur the first time the change in the asset price ($\Delta q_t = q_t - q_{t-1}$) becomes negative. Depending on the shock, this may happen on impact or after a number of periods.\(^{29}\) Once the regime has shifted, it may shift back, or it may remain in the new regime.\(^{30}\) In principle, an arbitrary number of regime shifts might take place, depending on the evolution of the asset price.

In order to illustrate the idea behind the solution method, consider the evolution of the asset price following a positive technology shock; the lower left panel of figure 1 in the main text.\(^{31}\) Evidently, this impulse response involves two turning points; which I call $T_1$ and $T_2$, i.e. points where the sign of the change in the asset price switches. After the second turning point, the stock price is increasing, so the dynamics of the economy are described by the solution to the model with no reaction to asset prices (and no further shocks):

$$s_t = P s_{t-1}, \quad t > T_2.$$  \[A49\]

Consider now the dynamics for $T_1 < t \leq T_2$, for which the monetary policy reaction to asset prices is non-zero. I use backward induction to trace out the evolution of the endogenous variables in these periods. As no shocks are assumed to hit the economy outside period 0, it follows from (A47) that $s_{T_2+1} = P s_{T_2}$. This is useful in the last period before the shift ($t = T_2$), where the following is true:

$$0 = \overline{AE_t} s_{T_2+1} + B s_{T_2} + C s_{T_2-1}$$

\(^{29}\)Unless the asset price remains forever constant, however, it will happen sooner or later, as the asset price must return to its initial value.

\(^{30}\)Of course, the economy will eventually return to its steady state, where the regime is always that of a zero reaction to stock price changes.

\(^{31}\)The figure shows the impulse response of the asset price in the constant-parameter model. I first assume that the turning points under this model are equal to those when the asymmetric policy. I then later verify that this is in fact the case.
\[ s_{T_2} = \Gamma_1 s_{T_2-1}, \quad \Gamma_1 = -(\bar{A} \bar{P} + B^*)^{-1} C^*. \quad (A50) \]

In similar fashion, I can derive an expression for the second-last period before the shift \((t = T_2 - 1)\). Let \( A = -(B^*)^{-1} \bar{A} \), and \( C = -(B^*)^{-1} C^* \). Then:

\[
0 = \bar{A}E_1 s_{T_2} + B^* s_{T_2-1} + C^* s_{T_2-2} \quad \iff \\
\]

\[ s_{T_2-1} = (I - A \Gamma_1)^{-1} C s_{T_2-2}. \quad (A51) \]

Thus, by recursive substitutions, I can express the endogenous variables at any point in this interval as a function of their 1-period lagged values. In the general case, I get:

\[ s_t = \Gamma_{T_2-t+1} s_{t-1}, \quad T_1 < t \leq T_2, \quad (A52) \]

where, for each \( t \):

\[
\Gamma_{T_2-t+1} = (I - A \Gamma_{T_2-t})^{-1} C, 
\]

recalling the definition of \( \Gamma_1 \equiv (\bar{A} \bar{P} + B^*)^{-1} C^* \). In fact, the recursivity of the problem allows me to write \( s_t \) for each period in this interval as a function of \( s_{T_1+1} \); the first period in this interval:

\[ s_t = \left( \prod_{i=1}^{t-1} \Gamma_{T_2-i} \right) s_{T_1+1}. \quad (A53) \]

In period \( T_1+1 \), the values of the endogenous variables are 'inherited' from the dynamics in the previous interval. For \( t \leq T_1 \), when the policy reaction to asset prices is again zero, I can similarly compute the value of \( s_t \) in each period recursively as a function of \( s_{T_1+1} \). From \((A52)\), I get the following expression, which is needed to describe the last period before this first shift:

\[ s_{T_1+1} = \Gamma_{T_2-T_1} s_{T_1}. \quad (A54) \]

Performing recursive operations in a similar fashion to above provides me with the following expression for \( s_t \):

\[ s_t = \Theta_{T_1-t+1} s_{t-1}, \quad 2 \leq t \leq T_1, \quad (A55) \]

where, for each \( t \):

\[
\Theta_{T_1-t+1} = (I - \hat{A} \Theta_{T_1-t})^{-1} \hat{C},
\]
and where \( \hat{A} = -(\tilde{B})^{-1} \tilde{A} \); \( \hat{C} = -(\tilde{B})^{-1} \tilde{C} \); and:

\[
\Theta_1 \equiv -(\tilde{A} \Gamma_{T_2-T_1} + \tilde{B})^{-1} \tilde{C}.
\]

Finally, the special case where \( t = 1 \) is the only time at which the shocks take on non-zero values. I use (A45) and (A55) as well as the assumption that the economy starts out in steady state in period 0, implying that \( s_0 = 0 \). I then obtain an expression for \( s_1 \) as a function of the time 1-innovations:

\[
0 = \tilde{A}s_2 + \tilde{B}s_1 + \tilde{C}s_0 + \tilde{D} \xi_1
\]

\[\iff\]

\[
s_1 = \left( I - \hat{A} \Theta_{T_1-1} \right)^{-1} \hat{D} \xi_1.
\] (A56)

- where \( \hat{D} = -(\tilde{B})^{-1} \tilde{D} \). Finally, I obtain:

\[
s_t = \left( \prod_{i=1}^{t-1} \Theta_{T_1-i} \right) \left( I - \hat{A} \Theta_{T_1-1} \right)^{-1} \hat{D} \xi_1, \ 2 \leq t \leq T_1.
\] (A57)

As mentioned in the main text, the model is solved in practice by making use of a shooting algorithm to find the turning points. An initial guess for each of the turning points is needed. Given the initial guess, I then solve for \( s_t \), \( \forall t \). It is then easy to verify whether this initial guess was correct or not by simply checking whether the sign of \( \Delta q_t \) actually does shift for \( t = T_{initial\ guess} \). If this is the case, I keep the solution. If not, I adjust my initial guess, and I 'shoot' again, until the condition is satisfied.