The European Monetary Union and Imbalances

Is it an anticipation story?*

Daniele Siena†
Bocconi University

THIS DRAFT: August 15, 2012

Abstract

Within an estimated model of a small open economy inside the European Monetary Union, we study the contribution of anticipated and unanticipated shock on the dynamics of international macroeconomic variables. A business cycle model featuring incomplete international financial markets, price and investment adjustment costs, variable capital utilization, habit persistence, home bias and both tradable and non-tradable sector creates a comprehensive framework that allows us to distinguish between possible determinants of the accumulation of international imbalances inside the EMU. Focusing only on the ability of the model to replicate the path of the current account can be misleading. Real exchange rate reaction plays a crucial role in distinguishing between different possible sources of EMU imbalances. The model, estimated using a weighted average of Ireland, Portugal and Spain, shows that expected changes in productivity can’t be the major determinant of the current account deficit. Anticipated uncovered interest parity shock, interpreted as an expected decrease in the cost of borrowing, could instead be a major source of the imbalances experienced in EMU.

Keywords: current account, real exchange rate, EMU

JEL Classification: E32, E43, F32, F41

*I am indebted to my advisor Tommaso Monacelli for his excellent supervision. Thanks are due to the Organization for Cooperation and Development for their hospitality. Also, I would like to thank Sebastian Barnes, Artur Radziwill, Paolo Manasse, Luca Sala and the participants of the DEFAP-LASER TEAM summer school and Bocconi University Doctoral Workshops for valuable insights. This draft is preliminary and the usual disclaimer applies.

†Department of Economics, Università Bocconi, via Rontgen 1, 20136, Milan, Italy. Email: daniele.siena@phd.unibocconi.it
1 Introduction

...there is no simple structural relation, in our economy, between the trade balance and the terms of trade and suggests that one cannot characterize the relation between trade and prices without specifying the source of fluctuations.

Significant flows of capital and diverging current account balances have characterized the European Monetary Union (EMU) member countries since its formation. Starting in 1996, a group of countries, such as Ireland, Portugal, Spain and Greece, accumulated increasing current account deficits, while other countries, such as Germany, Netherlands and Austria, persistently increased the surpluses (figure 1(a)). The Great Recession revealed that the euro area countries that accumulated negative current account balances were those that suffered more from the crisis. As shown by figure 1(b), the positive correlation between current account deficit and harshness of the recent crisis seems to be specific to the European Monetary Union. Therefore, it is important to understand what have been the sources, and possibly the peculiarities, of the EMU current account imbalances. The main goal of the paper is to test theoretically the plausibility of different sources of current account imbalances among EMU countries before the beginning of the recent crisis.

The idea that capitals were flowing towards catching up countries with higher current or expected productivity growth, Blanchard & Giavazzi (2002), has partially lost empirical support. Zemanek, Belke & Schnabl (2009) and Berger & Nitsch (2010) suggest that in fact capitals were flowing towards countries not only with higher per capita GDP growth but also with higher domestic distortions. As in Giavazzi & Spaventa (2010) and Eichengreen (2010), we distinguish between types of current account imbalances depending on their trigger. There are ones driven by growth differentials that allow surplus countries to invest in future growth of the borrowing countries and others triggered by other factors that, instead

---

1 (Backus, Kehoe & Kydland 1994)
2 From now on we will not focus on Greece given some problem on the reliability of the data
3 Figure 1(b) shows that the positive correlation between current account balance of 2007 and the real GDP growth rate of the period 2007/11 it is lost when we consider other 13 developed countries that where having a similar pattern in the dispersion of the current account balances
4 For example relatively less flexible labor and product markets, higher level of employment protection and relatively less prudent fiscal policies. See also Schmitz & von Hagen (2009). Also Sodsriwiboon & Jaumotte (2010), Barnes, Lawson & Radziwill (2010), Barnes (2010) and Belke & Dreger (2011) focus on the dynamics and consequences of large current account deficits in the euro area but from a policy perspective.
of overcoming some economic constraint, might end up creating economic fragilities.

Therefore expectations play a crucial role in international flows of capitals and goods. We focus on understanding the role of anticipated shock as a source of the dynamics of these international variables. As in Hoffmann, Krause & Laubach (2011), henceforth HKL, we ask if changes in expectation can explain the path of current account imbalances. HKL study the role of imperfect expectation formation in the accumulation of global imbalances. We focus on the ability of anticipated shock, in a full information set-up, to explain what happened in small open economies inside a monetary union. Unlike from HKL, we believe that in order to understand the source of current account imbalances we need to look also at international relative prices to avoid possible interpretation mistakes. Relative prices, within and between countries, are crucial variables for the dynamics of international flows and by looking at the real exchange rate we can differentiate between similar reactions of the current account to different shocks.

The paper utilizes theoretical results to distinguish between different sources of current account imbalances. We want the model to reproduce the dynamics of a small set of key variables: current account, real exchange rate and output growth. Between 1996 and 2008, GDP growth above trend, current account deficit and real exchange rate appreciation have been common to some EMU countries such as Ireland, Portugal and Spain (henceforth IPS,
A two sector, tradable and non tradable, New Keynesian DSGE open economy model in a monetary union is constructed. Different features of open economy general equilibrium models are combined. Habit persistence, nominal and real rigidities, monopolistic competition, tradable and non tradable sector, home bias, variable capital utilization and incomplete financial markets are characteristics of the small open economy being analyzed. We then estimate the model on IPS data relying on Bayesian techniques that allow us to use carefully selected priors to reduce the short sample limitation. We accordingly use the estimated model to study the reactions to shocks. Different specifications and parameterizations of the

Figure 2: Ireland-Portugal-Spain

---

5Figure 2 shows the dynamics of the average current account, real exchange rate and GDP growth (detranded in a model consistent base) of a weighted average of Ireland Portugal and Spain. We used the annual HICP relative household consumption expenditure shares in the euro area totals as weights.

6Matheson (2010) shows that a small open economy model with a non tradable good sector fits the data better than the one-good model in a set of 3 small open economies. The model is in line with Smets & Wouters (2003), Galí & Monacelli (2005), Santacreu (2005), Galí & Monacelli (2008), Fuia & Monacelli (2008), Rabanal (2009), Merola (2010), Burriel, Fernandez-Villaverde & Rubio-Ramrez (2010).

7See An & Schorfheide (2007) for a discussion on Bayesian estimation in DSGE models
model are compared to test the robustness of the results. The results hold true for different combinations of estimated and calibrated parameters, for different utility specifications and for a reduced form of the model where we use just labor as a factor of production.

Anticipated as well as unanticipated productivity shock do not seem to be the source of current account deficits experienced inside the EMU before the 2008 crisis. On the other hand the anticipated shock in the uncovered interest rate parity is more likely an important source of those imbalances. Uncovered interest parity shock in a monetary union with a fixed nominal exchange rate can be interpreted, as shown later, as an anticipated risk premium shock. A decrease in the risk premium implies a lower cost of borrowing. It is plausible that agents, as soon as the monetary union became a credible agreement, anticipated a future drop in the borrowing cost. Interesting, if we allow in our model such an anticipated decrease in the risk premium, then the dynamics of the current account, real exchange rate and output mimic the IPS data.

The paper is organized as follows. Section 2 describes the economic environment in detail while section 3 illustrates the Bayesian estimation process. Section 4 analyzes which structural shock could explain the current account imbalance and presents the results of the different specification of the model analyzed. Section 5 concludes.

2 The Model

We construct a two sector New Keynesian Dynamic Stochastic General Equilibrium (DSGE) small open economy model. The domestic economy is part of a monetary union with the foreign economy which is, for analytical simplicity, the rest of the world (henceforth RoW).

The domestic representative household consumes, saves or borrows through domestic and foreign internationally traded bonds, supplies labor and decides the level of capital to use in production. Capital depreciates and investments are costly. The model features variable capital utilization and adjustment cost to investment to ensure that the model generates ag-

8Differently, for the US, Hoffmann et al. (2011) conclude that "global imbalances" can be largely explained with perceived change in trend growth rate. Their result is conditional on focusing just on the path of current account without trying to match a key variable as the real exchange rate
aggregate and sectoral co-movement in presence of anticipated shocks. The consumption good is a combination of non tradable and tradable good, which is itself a mix of home and foreign produced goods. There is no perfect substitutability between domestic and foreign tradable goods but their substitutability is higher than the one between tradable and non tradable goods. We introduce home bias for domestic tradable goods assuming that the purchasing power parity will not be necessarily satisfied.

Within each country there are two production sectors. Firms producing goods in both sectors are monopolistically competitive but can only adjust prices costly. They produce employing technology, labor and capital. Each sector is subject to both a common and a sector specific technology shock with unit root, allowing for permanent inflation differentials across countries and sectors.

There is a common monetary authority for the monetary union, the European Central Bank, that fixes the one period nominal interest rate targeting the euro area inflation, for which our small open economy contributes for only 10 percent. The assumption that the economy is small allows us to keep the model tractable and reflects the fact that the country’s economy is affected by the rest of the monetary union but cannot strongly influence it. The nominal exchange rate is fixed, given the participation to a monetary union, and becomes a key mechanism through which every price movement, in both sectors, will be reflected in movements of inflation, real interest rate and real exchange rate.

Finally we allow for perfect risk sharing within countries but incomplete international financial markets. There are only two international non contingent bonds, one foreign and one domestic, implying incomplete risk sharing in-between countries. Notice that through out the entire paper the * will characterize foreign variables, an overscript tilde indicates detrended variables, overscript hat defines deviation from steady state and lower case variables are the natural logarithm transformation.

---

2.1 Domestic Household

The domestic representative household maximizes the expected value of her lifetime utility

$$E_t \sum_{t=0}^{\infty} \chi_{t-1} \zeta_t U(C_t, L_t)$$

(1)

where $E_t$ denotes conditional expectation at date $t$, $U$ is the instantaneous utility function of the consumption index, $C$, and hours worked, $L$, and $\zeta_t$ is an intertemporal preference shock with law of motion:

$$\log \zeta_t - \rho \zeta_t \log \zeta_{t-1} + u_t^\zeta \quad \text{where} \quad u_t^\zeta \sim N(0, 1)$$

(2)

$\chi_t$ is the household’s endogenous discount factor. Following Uzawa (1968) and Schmitt-Grohe & Uribe (2003) we assume that agents become more impatient the higher is the average detrended consumption, $\tilde{C}_t^{10}$

$$\chi_t = \beta_t \chi_{t-1} \quad \text{where} \quad \beta_t \equiv \frac{1}{1 + \psi^\beta (\log \tilde{C}_t - \chi^\beta)}$$

(3)

The parameter $\psi^\beta$ determines the importance of detrended average consumption and, following Ferrero, Gertler & Svensson (2008), it is set sufficiently small to avoid strong interference with the dynamics of the model.

In the baseline model we allow for a utility function separable in consumption and hours worked which accounts for an $h$ degree of habit persistence in consumption $^{11}$

$$U(C_t, L_t) = \left\{ \log(C_t - hC_{t-1}) - \epsilon_t^L \psi^L \frac{L_t^{1+\nu}}{1+\nu} \right\}$$

(4)

where $\psi^L$ is a labor supply preference parameter, $\epsilon_t^L$ is a labor supply shock with law of motion:

$$\log \epsilon_t^L = \rho^L \log \epsilon_{t-1} + u_t^L \quad \text{where} \quad u_t^L \sim N(0, 1)$$

(5)

$^{10}$ This feature of the model ensures the presence of a stable non-stochastic steady state independent from initial conditions with incomplete financial markets. See Schmitt-Grohe & Uribe (2003) and Bodenstein (2011) for a detailed discussion on the topic. Notice that the detrended average consumption is treated as exogenous by the representative household.

$^{11}$ In section ?? we present the results of the model with a utility specification that allows for different intensities of the labor wealth effect. Jaimovich & Rebelo (2009) show that models with this type of utility in the presence of anticipated shock can better match the data.
\( C_t \) is a composite of consumption of non tradable goods, \( C_{N,t} \), and tradable goods, \( C_{T,t} \) which are also an compound of domestic-made and foreign-made tradable goods:

\[
C_t \equiv \left[ \gamma_{T,t} C_{T,t}^{\eta} + \eta_{N,t} C_{N,t}^{\eta} \right]^{\eta^{-1}} \quad \quad C_{T,t} \equiv \left[ \gamma_{h,t} C_{h,t}^{1-\eta} + \gamma_{f,t} C_{f,t}^{1-\eta}\right]^{\eta^{-1}}
\]

\( \eta > 0 \) is the elasticity of substitution between tradable and non tradable goods and \( \epsilon > 0 \) sets the substitutability between domestic and imported tradable goods. \( \gamma_{T,t} \), \( \gamma_{N,t} \), \( \gamma_{h,t} \) and \( \gamma_{f,t} \) are respectively the preference shares for tradable as a whole, non tradable, domestic tradable and foreign tradable goods.

Following Faia & Monacelli (2008) we also allow for a symmetric home bias, with respect to the RoW, in the share of home produced tradable goods.

Within each sector there are a continuum of different varieties of goods which are imperfectly substitutable:

\[
C_{f,t} \equiv \left[ \left( \frac{1}{1-n} \right) \frac{1}{n} \int_{n}^{1} C_{f,t}(i) \frac{di}{\phi} \right]^{\phi^{-1}} \quad \quad C_{h,t} \equiv \left[ \left( \frac{1}{1-n} \right) \frac{1}{n} \int_{0}^{n} C_{h,t}(i) \frac{di}{\phi} \right]^{\phi^{-1}}
\]

\[
C_{N,t} \equiv \left( \int_{0}^{1} C_{N,t}(i) \frac{di}{\phi} \right)^{\phi^{-1}}
\]

where \( \theta > 0 \) and \( \phi > 0 \) are the elasticity of substitution between varieties in the non tradable sector and in the tradable sector. There are two price indexes: a consumer price index (CPI), \( P_t \), and a domestic country price index for tradable good (TPI), \( P_{T,t} \):

\[
P_t = \left[ \gamma_{T,t} P_{T,t}^{1-\eta} + \gamma_{N,t} P_{N,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad \quad P_{T,t} = [\gamma_{h,t} P_{h,t}^{1-\epsilon} + \gamma_{f,t} P_{f,t}^{1-\epsilon}]^{\frac{1}{1-\epsilon}}
\]

In the appendix A we present the optimal allocation bundles in detail, including the foreign consumer allocation on domestically produced goods. The representative household maximizes her utility function subject to the following budget constraint:

\[
C_t + \frac{B_t}{P_t} - \frac{A_t}{P_t} I_t \leq W_t L_t + R_{t-1} B_t - R_{t-1} A_{t-1} + \left( r_t k_t u_t - \Psi(u_t) \right) k_{t-1}^p + \int_{0}^{1} \Gamma_{N,t}(i) + \int_{0}^{1} \Gamma_{h,t}(i) (6)
\]

where \( W_t \) is the real wage in terms of the CPI price index, \( \Gamma_{j,t}(i) \) are the real profits of monopolistic firm in both the tradable and non tradable sector\(^{12}\) \( j = \{ N, h \} \) is the set of

\(^{12}\)The shares can vary over time since they include deterministic preference shocks (see for example Rabanal (2009). Those are necessary to guarantee a balance growth path when the two sectors are allowed to grow at different rates as in our framework.

\(^{13}\)Shares of the monopolistic firm \( i \) are owned by domestic residents in equal proportions and are not traded internationally.
both non tradable and domestic tradable sector and will be used from now on to lighten the notation. Inside the domestic economy we allow for full insurance but the international financial market is incomplete. The domestic bond, \(B_t\), and the foreign bond, \((-A_t)\), are the only two internationally tradable assets. The one period return on the two types of bond is respectively \(R^B_t\) and \(R^A_t\).

The representative household owns physical capital \(k^p_t\) which accumulates according to

\[
k^p_t = (1 - \delta)k^p_{t-1} + \epsilon'_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t,
\]

where \(I_t\) is the investment in physical capital, \(\delta\) is the depreciation rate and \(S()\) is an adjustment cost function. We assume that \(S(z) = S'(z) = 0\) and \(S''(z) = \eta_k > 0\), where \(z\) is the economy’s steady state growth rate and \(\eta_k\) is the capital adjustment cost elasticity. \(\epsilon'_t\) is an investment specific technological shock that evolves according to log \(\epsilon'_t = \rho \epsilon'_{t-1} + u'_t\).

The household decides the capital utilization rate, \(u_t\), that determines the amount of physical capital to be transformed in effective capital that will be rented to firms at the rate \(r^k_t\):

\[
k_t = u_t k^p_{t-1}
\]

\(\Psi(u_t)\) in equation (6) is the cost of use of capital in units of consumption and following Christiano, Eichenbaum & Evans (2005) we assume that \(\Psi(u) = 0\) and \(\frac{\Psi'(u)}{\Psi''(u)} = \eta_u\), where \(u = 1\).

The representative household chooses processes \(\{C_t, L_t, B_t, A_t, u_t, k^p_t, I_t\}_{t=0}^{\infty}\) taking as given the set of processes \(\{P_t, W_t, r^k_t, R_t, R^B_t\}_{t=0}^{\infty}\) and the initial wealth \(B_0\) and \(A_0\), to maximize equation (1) subject to (4),(6),(7) and (3). The necessary first order conditions are listed in the appendix [A].

2.2 Firms

Production in both sectors is carried out by monopolistically competitive firms owned by the household. Different production technologies with a deterministic trend characterize the two sectors while we allow for a common trend for the overall economy. Production is carried out with the use of both capital, \(K_t\), and working hours \(L_t\):

\[
Y_{N,t} = A_{N,t}K_{N,t}^\alpha [X_t L_{N,t}]^{1-\alpha}
\]

\[
Y_{h,t} = A_{h,t}K_{h,t}^\alpha [X_t L_{h,t}]^{1-\alpha}
\]

where \(X_t\) is a common labor-augmenting technology process, \(A_{j,t}\) is the productivity variable for the tradable/non tradable sector. Productivity have a trend and a first order autoregres-
sive process:

\[ A_{N,t} = (1 + g^N)^t \tilde{A}_{N,t} \]  \hspace{1cm} (11)

\[ \log(\tilde{A}_{N,t}) = \rho_{AN} \log(\tilde{A}_{N,t-1}) + \upsilon_{AN}^t + \upsilon_{A}^t \]  \hspace{1cm} (12)

\[ A_{h,t} = (1 + g^T)^t \tilde{A}_{h,t} \]  \hspace{1cm} (13)

\[ \log(\tilde{A}_{h,t}) = \rho_{Ah} \log(\tilde{A}_{h,t-1}) + \upsilon_{Ah}^t + \upsilon_{A}^t \]  \hspace{1cm} (14)

The labor-augmenting technology follows:

\[ X_t = (1 + x)^t \tilde{X}_t \quad \text{where} \quad \tilde{X}_t = \upsilon_{X}^t \]  \hspace{1cm} (15)

The shocks are \textit{i.i.d.} normally distributed \( \upsilon_{AN}^t \sim N(0, \sigma_{AN}^2) \), \( \upsilon_{Ah}^t \sim N(0, \sigma_{Ah}^2) \), and \( \upsilon_{X}^t \sim N(0, \sigma_{X}^2) \). Notice that while the shocks to productivity, \( \upsilon_{AN}^t \) and \( \upsilon_{Ah}^t \), have a persistent but temporary consequence, the shock \( \upsilon_{X}^t \) leads to a permanent shift in the level of the common labor-augmenting technology without affecting the growth rate. The assumptions on the deterministic trends and the differences between sectors will be important later because they provide a model consistent based method to detrend the data when we proceed with the estimation.

Firms can change prices only costly and that implies a sluggish response of prices to shocks. We follow Rotemberg (1982) and we assume that the cost for price adjustment can be measured in terms of the output of the sector bearing the cost:

\[ \psi \left( \frac{P_{j,t}(i)}{\Pi P_{j,t-1}(i)} - 1 \right)^2 P_{j,t} Y_{j,t} \]  \hspace{1cm} (16)

where \( \psi \) measures the degree of price stickiness, \( \pi_j \) denotes the steady state inflation rate in the sector considered and \( Y_{j,t} \) is the aggregate demand for non tradable and home produced tradable goods.\footnote{\textsuperscript{14}}

Firms solve a two stage problem. In the first stage they minimize the real cost of production choosing in a perfectly competitive market the quantity of the two factors of production:

\[ \min_{k_{j,t},L_{j,t}} w_t L_{j,t} + \upsilon_t^k k_{j,t} \]

subject to (9) and (10).

The optimality condition gives us

\[ k_{j,t} = \frac{\alpha}{1 - \alpha \upsilon_t} L_{j,t} \]  \hspace{1cm} (17)

\[ \text{14} \text{The results are the same using Calvo style price stickiness. For simplicity we present here just the Rotemberg adjustment mechanism. The adjustment cost is a modified version of the general formulation of Ireland (2007) that allows us to avoid steady state effects of positive inflation rate.} \]
and the real marginal cost function

\[ MC_{j,t} = \left( \frac{1}{1 - \alpha} \right)^{1-\alpha} \left( \frac{\alpha}{\alpha} \right) w_{i}^{1-\alpha} r_{k}^{\alpha} \frac{A_{j,t} X_{1}^{1-\alpha}}{Y_{j,t}} \]  

(18)

In the second stage each firm \((i)\) chooses its price \(P_{j,t}(i)\) in order to maximize profits:

\[
E_{t} \left\{ \sum_{t=0}^{\infty} v_{0,t} \left[ \frac{P_{t,t}(i) Y_{j,t}(i)}{P_{t}} - MC_{j,t} Y_{j,t}(i) - \frac{\psi}{2} \left( \frac{P_{t,t}(i)}{P_{t,t-1}(i)} - 1 \right)^{2} \frac{P_{j,t} Y_{j,t}}{P_{t}} \right] \right\}
\]

subject to (9) and the total demand for the \(i\) non tradable/tradable good:

\[ Y_{j,t}(i) \leq \left( \frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\theta} Y_{j,t} \]  

(19)

2.3 Terms of trade, UIP, real exchange rate and current account

Understanding the reactions of international relative prices and current account to shocks plays a central role in the paper. Because of this we devote this section to introduce some important variable as the terms of trade, the real exchange rate, the relative price of traded and non traded goods and the current account. We then highlight the consequences of imperfect international financial market and explain the structure of the uncovered interest rate parity shock in a framework with fixed nominal exchange rate.

We start by defining the terms of trade as the price of imported over exported goods \(S_{t} \equiv \frac{P_{f,t}}{P_{h,t}}\). Following Faia & Monacelli (2008) the tradable price index over the price of the domestically produced tradable good can be written as a function of the terms of trade and parameters only:

\[
\frac{P_{T,t}}{P_{h,t}} = g(S_{t}) = [\gamma_{h,t} + \gamma_{f,t} S_{t}^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \quad \text{with} \quad \frac{\delta g(S_{t})}{\delta S_{t}} > 0
\]

(20)

\(J\) is the relative price of tradable over non tradable good \(J_{t} \equiv \frac{P_{T,t}}{P_{N,t}}\). The ratio of the CPI index over the price of non tradable goods then can be written as:

\[
\frac{P_{t}}{P_{N,t}} = m(J_{t}) = [\gamma_{T,t} J_{t}^{1-\eta} + \gamma_{N,t}]^{\frac{1}{1-\eta}} \quad \text{with} \quad \frac{\delta m(J_{t})}{\delta J_{t}} > 0
\]

(21)

The nominal exchange rate \(\epsilon\) is defined as the price of foreign currency in terms of home currency\(^{15}\). Assuming that the small open economy is part of a Monetary Union, the nominal exchange rate with the rest of the world is fixed. For simplicity we set \(\epsilon = 1\). We

\(^{15}\)An increase of \(\epsilon\) is a depreciation of home currency, while a decrease in the nominal exchange rate results in an appreciation of the home currency with respect to the foreign one.
assume that the law of one price holds \( P_{f,t}(i) = \varepsilon \cdot P_{f,t}^*(i) = P_{f,t}^*(i) \forall i \in [0,1] \). However notice that the Purchasing Power Parity (PPP) will not hold given our assumption of home bias.

The real exchange rate is defined as \( Q_t = \frac{P_t^*}{P_t} \). We can rewrite the real exchange rate as a function of \( S_t, J_t \) and the price variables under the control of the foreign economy, that in our framework are taken as exogenous:

\[
Q_t = \frac{S_t}{g(S_t)m(J_t)} P_t^* \frac{P_{f,t}}{P_{f,t}} \quad \text{with} \quad \frac{\delta Q_t}{\delta S_t} > 0, \quad \frac{\delta Q_t}{\delta J_t} > 0 \tag{22}
\]

Notice that in our case of imperfect financial market the link between the real exchange rate and the ratio of international marginal utilities of consumption is broken allowing the model to violate the risk sharing equation \( \hat{\lambda} \).\(^{16}\)

We now introduce an uncovered interest parity shock building from the work by Kollmann (2001), Kollmann (2005) and Bergin (2006). In our framework the uncovered interest parity condition, remembering that our small open economy is part of a monetary union, can be derived from the two euler conditions for the two assets:

\[
E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^B}{\pi_{t+1}} \right\} = E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{t+1}} \right\} \tag{23}
\]

Given our focus on the first order linear approximation around the steady state, equation (23) becomes \( \hat{R}_t^B = \hat{R}_t \) which omits the nonlinearities of marginal utilities and prices. Following a standard practice in new open economy macroeconomics models we insert an exogenous "risk premium term" to account for nonlinearities transforming the previous equation in our equilibrium condition:

\[
\hat{R}_t^B = \hat{R}_t + Risk_t \tag{24}
\]

where \( Risk_t = \rho_{RP} Risk_{t-1} + u_t^{Risk} \).

\(^{16}\)If we were in a model with perfect financial and insurance market with constant nominal exchange rate the risk sharing condition would be \( \zeta \frac{U_{C,t}^{U^*}}{U_{C,t}^U} = \frac{P_t^*}{P_t} \) where \( \zeta = Q_t \frac{U_{C,t}^{U^*}}{U_{C,t}^U} = 1 \) This equation states that a benevolent social planner would allocate consumption across countries in a way that the marginal benefit from an extra unit of consumption equals its marginal costs. With a time separable preferences and CRRA utility function we would have a positive correlation between the relative consumption and the real exchange rate. The data shows that this is not always the case and this goes under the name of Backus-Smith puzzle (Backus, Kehoe & Kydland 1993). Corsetti, Dedola & Leduc (2010) provide a complete overview of the literature, from the work of Cole & Obstfeld (1991) to most recent models.
From the budget constraint we can write the balance of payment condition, as share of steady state GDP, $Y$:

$$NX_t + \frac{R^B_{t-1}B_{t-1}}{YP_t} - \frac{R_{t-1}A_{t-1}}{YP_t} - \frac{B_t - A_t}{YP_t} = 0$$

(25)

where $nx_t$ denotes the real value of net exports as a ratio to $Y$ and it is equal to

$$NX_t = \frac{J_t}{g(S_t)m(J_t)} \frac{(Y_{h,t} - C_{h,t} - S_{f,t})}{Y}$$

(26)

We finally define the current account as the net change in real bond holding (scaled with $Y$):

$$CA_t = \frac{(B_t - A_t)}{PY} - \frac{(B_{t-1} - A_{t-1})}{PY}$$

(27)

### 2.4 Equilibrium in a Small Open Economy

In equilibrium we will have that firms meet the demand at selected prices clearing the market in both sectors. The following condition must hold:

$$Y_{N,t} = C_{N,t} + \psi (\frac{\pi_{N,t}}{\pi_{N}} - 1)^2 Y_{N,t}$$

(28)

$$Y_{h,t} = C_{h,t} + C_{h,t}^* + \psi (\frac{\pi_{h,t}}{\pi_{h}} - 1)^2 Y_{h,t}$$

(29)

Real GDP aggregates tradable and nontradable production weighted by the relative prices:

$$Y_t = \frac{P_{h,t}}{P_t} Y_{h,t} + \frac{P_{N,t}}{P_t} Y_{N,t} + I_t$$

(30)

Total labor supply must be equal to labor demand and the same holds for effective capital supply and demand which gives:

$$L_t = L_{N,t} + L_{h,t}$$

(31)

$$k_t = k_{N,t} + k_{h,t}$$

(32)

To complete the model we introduce a monetary policy rule. Our small open economy is part of a Monetary Union and we assume that the common Central Bank follows a simple Taylor rule with the union price index as a target:

$$R_t = R^p_{t-1} \left( \frac{\Pi^{EMU}_{t}}{\Pi^{EMU}_{t-1}} \right)^{(1 - \rho_r)\rho_s}$$

(33)

where $\Pi^{EMU}$ is the union inflation rate. $\Pi^{EMU} = \Pi_t^{\gamma_{ips}} (\Pi_t^{*})^{1 - \gamma_{ips}}$ where $\gamma_{ips}$ is the relative economic size of our domestic economy in the monetary union.
2.5 Detrending Equilibrium Conditions

The system of equilibrium conditions of the model is non-stationary. The reason is that the deterministic trends included in the sector productivities and the labor-augmenting technology make some variable grow as time elapses. Define $Z_t = \frac{X_t}{X_{t-1}}$ to be the growth rate of the real aggregate variable while $Z^j_t = \frac{X_tA_{j,t}}{X_{t-1}A_{j,t-1}}$ is the sector specific growth rate. To be able to use the standard solution techniques, we detrend the model.

In the appendix it is available the full list of detrended equilibrium conditions but, for instance, the production in the two sectors can be made stationary by defining:

$$\tilde{Y}_{N,t} = \frac{Y_{N,t}}{X_t(1 + g^N)_t} = \tilde{A}_{N,t}Z^{-\alpha}\tilde{K}^\alpha_{N,t}L^{1-\alpha}_{N,t}$$  \hspace{1cm} (34)

and

$$\tilde{Y}_{h,t} = \frac{Y_{h,t}}{X_t(1 + g^h)_t} = \tilde{A}_{h,t}Z^{-\alpha}\tilde{K}^\alpha_{h,t}L^{1-\alpha}_{h,t}$$  \hspace{1cm} (35)

where $\tilde{K}_{j,t} = \frac{K_{j,t}}{X_{t-1}}$.

3 Model Estimation

We rely on Bayesian techniques to estimate a subset of key parameters of the model for which there is both theoretical and empirical controversy. For the other set of parameters we use standard calibration methods. The reason for using calibration, which is the same as assigning infinitely tight priors, is that we are confident with the values used previously in the literature and we want to be able to compare our results with previous studies. The main advantage of using Bayesian estimation, with respect to classical methods, is that starting from a meaningful set of priors allows us to reduce the problem that we have to face due to the short sample data we are considering.

3.1 Data

Given our interest in the European Monetary Union we should be looking at data starting in January 1, 1999. Instead we consider the first quarter of 1996 as the first observation. Following Rabanal (2009) we assume that agents started to act as if they were in the EMU around 1996. We argue that this choice is even conservative, if something. On December 15-16, 1995, during the Madrid meeting, the European Council decided the name of the future common currency, the euro, and the timeline of the transition to a single currency.
Given our interest in estimating the importance of anticipation on the behavior of agents, we argue that start looking at data in the first quarter 1996 should be considered as a safe choice.

As the end of our sample we pick the last quarter of 2007, the beginning of the Great Recession. We claim that it’s important to focus on the pre-crisis period to understand why imbalances were actually accumulated without being influenced by the peculiarities of the crisis period. Linking the sources of the accumulating imbalances before the crisis and the reaction of the economy (and of current account) during the crisis is an interesting future question which will not be address in this paper. So we focus on the period between 1996:1 to 2007:4.

As shown by figure (1(a)) Ireland Portugal and Spain experienced a similar dynamics of the current account. Notice that following the construction of our model we should present the current account position only vis-a-vis the rest of the monetary union. Even if these data are not readily available we argue that for Spain and Portugal the analysis wouldn’t change. Using intra EU-12 trade balances as a proxy for intra euro area current account, as suggested by Ahearne, Schmitz & von Hagen (2008), we can show that the picture almost doesn’t change. Concerning Ireland we are aware of the importance of extra EMU trade (UK and USA) but we still include it in the analysis because more that 40 per cent of the inflows of capital (share of FDI by partner countries) are coming from the EMU partners. During the period under consideration IPS experienced similar dynamics of real exchange rate (with respect to EMU countries) and output growth. Accordingly through out the estimation we focus on a weighted average of these 3 countries using European Central Bank HCPI as weights.

We estimate the model using quarterly frequency for nine time series: growth rate of real GDP, growth rate of non tradable real output, change in the average weekly hours worked, growth rate of real consumption, growth rate of real investment, current account as a fraction of GDP, non tradable HICP inflation, the real exchange rate within EMU partners and 3-month money market interest rates for euro area countries. Following Beltran & Draper (2008) we also include in the estimation 3 time series coming from the exogenous foreign economy block, after having subtracted the IPS group, as an unrestricted autoregression: HICP EMU(-IPS) inflation, foreign tradable HICP EMU(-IPS) inflation and foreign growth rate of real consumption. We assume that foreign observables are generated by the following processes:

\[ \Pi_t^* = \rho_{\Pi} \Pi_{t-1}^* + u_{\Pi} \]
\[ \Pi_{T,t}^* = \rho_{\Pi_{T}} \Pi_{T,t-1}^* + u_{\Pi_{T}} \]
\[ c_t^* = \rho_{c} c_{t-1}^* + u_{c}^* \]

where \( u_{\Pi}, u_{\Pi_{T}} \) and \( u_{c}^* \) are independent and identically distributed random errors. Details
Table 1: Observable Variables and Model counterparts

<table>
<thead>
<tr>
<th>Observable variable</th>
<th>Model counterpart</th>
<th>Adjusting factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP growth IPS</td>
<td>$\Delta \tilde{y}_t$</td>
<td>$z$</td>
</tr>
<tr>
<td>Non tradable real GDP growth IPS</td>
<td>$\Delta \tilde{y}_{N,t}$</td>
<td>$z + g^{NT}$</td>
</tr>
<tr>
<td>Real consumption growth IPS</td>
<td>$\Delta \tilde{c}_t$</td>
<td>$z$</td>
</tr>
<tr>
<td>Real Investment growth IPS</td>
<td>$\Delta \tilde{I}$</td>
<td>$z$</td>
</tr>
<tr>
<td>Current account (%GDP)</td>
<td>$\Delta \tilde{c}_t$</td>
<td>$z$</td>
</tr>
<tr>
<td>3-month money market interest rate</td>
<td>$r_t$</td>
<td>$\log(r_{SS})$</td>
</tr>
<tr>
<td>NT HICP inflation IPS</td>
<td>$\tilde{\pi}_{N,t}$</td>
<td>$\pi_{IPS} - g^{NT}$</td>
</tr>
<tr>
<td>Δ Real exchange rate w/EMU partners</td>
<td>$\Delta \tilde{q}_t$</td>
<td>$\pi_{EMU} - \pi_{IPS} + g^{NT}$</td>
</tr>
<tr>
<td>Change avg weekly hours worked</td>
<td>$\Delta L_t$</td>
<td></td>
</tr>
</tbody>
</table>

| EMU(-IPS) T HICP inflation                | $\tilde{\pi}_{f,t}$   | $\pi_{EMU} - g^{T*}$ |
| EMU(-IPS) HICP inflation                  | $\tilde{\pi}_{t}^{*}$ | $\pi_{EMU}$    |
| EMU(-IPS) Real foreign consumption growth | $\tilde{c}_t^{*}$     | $z^{*}$         |

on the data are available in the appendix.

Table 1 lists the 12 observable variables and their model counterparts. Notice that the third column identifies the adjusting factor between the data and the model generated data that corresponds to the detrending factor. In fact $x$ and $g^d$ are the coefficient estimates on time when we fit a linear trend to the log of the trending variable. We can get for example $x$ by fitting a linear trend to $log(Y_t)$ using an ordinary least square regression:

$$\tilde{y}_t = log y_t^{data} - x_0 - xt = u_t^X$$

which implies that if we were in a deterministic world $x$ would be difference between the growth rate of the output in the real data and in the model:

$$\Delta y_t^{data} - \Delta \tilde{y}_t = x$$

3.2 Calibrated parameters

Table 2 summarizes the values and the sources of the calibrated parameters. We follow the estimation results of Smets & Wouters (2003) for different parameters. $v$, the inverse elasticity of work effort with respect to the real wage is set equal to 1. The depreciation rate, $\delta$, is 0.025 per quarter, implying a 10 per cent annual depreciation of capital. The monetary policy has a high degree of interest rate smoothing, $\rho_r = 0.94$, and strongly reacts to deviation of inflation from the target level, $\rho_{\pi}^{EMU} = 1.658$. 

16
The discount factor is endogenous and following Ferrero et al. (2008) we arbitrarily set $\chi^\beta = -1000$ and then calibrate $\psi = 1.0108 \cdot 10^{-5}$ to determine the steady state value of the discount factor equal to 0.99. In this way we ensure that the endogeneity of the discount factor does not significantly influence the medium term dynamics of the model.

As in Faia & Monacelli (2008) we set the elasticities of substitution between varieties in the same sector, $\theta$ and $\phi$, equal to 7.5 implying a steady state markup of 15 percent. Using the log linearized version of the pricing equations we get that the elasticities of inflation with respect to the real marginal cost are respectively $\frac{\theta - 1}{\psi}$ and $\frac{\phi - 1}{\psi}$. We can compare those with the slope of the Phillips curve we would get using a Calvo approach, $\frac{(1-p)(1-\beta p)}{p}$ where $p$ is the probability of not being able to reset prices. Assuming as in Angeloni, Aucremanne, Ehrmann, Gal, Levin & Smets (2006) that $p = 0.75$ it implies that the price stickiness parameter, $\psi$, is around 76.

For the share of tradable and non tradable goods in the consumption basket, $\gamma_{N,t}$ and $\gamma_{T,t}$, we use Eurostat HCPI item weights data. In the IPS the average share of tradable goods for the period 1996:2007 is 65.6 per cent. This number is slightly lower when instead we consider the entire euro area, $\gamma^*_{T,t} = 60.6$ per cent. Focusing on the tradable good sector we find that the share of imported goods is around 33.9 per cent for the IPS countries.

The last parameter we calibrate, $\gamma_{ips}$, is the weight of Ireland Portugal and Spain on the average EMU inflation. We calibrate it using Eurostat HCPI country weights and we set the parameter to 13.4 per cent.

3.3 Prior Distributions

Some structural parameter are central for shaping the reaction to shocks of those variables on which we are focusing the analysis. Empirical and theoretical studies provide a wide range of values for those parameters failing to give us a precise calibration on which we can rely. For this reason we perform a Bayesian estimation starting with previous studies as references for selecting priors. Two are the main focuses of the estimation procedure: finding the values of the elasticities and understanding the role of anticipated shocks in the EMU business cycle. Table 3 summarizes the prior of the parameters that we use in the estimation.

The elasticity of substitution between home and foreign produced tradable, the trade elasticity $\epsilon$, is a parameters for which the literature provides a large range of estimates. Most
Table 2: Calibrated Parameters

<table>
<thead>
<tr>
<th>Par</th>
<th>Value</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$</td>
<td>1</td>
<td>Inverse elasticity of labor supply</td>
<td>Smets &amp; Wouters (2003)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>-10000</td>
<td>Discount factor, arbitrary parameter</td>
<td>Ferrero et al. (2008)</td>
</tr>
<tr>
<td>$\psi^\beta$</td>
<td>$1.011 \cdot 10^{-5}$</td>
<td>Spillover effect of average detrended consumption on discount factor</td>
<td>Steady-state of $\beta = 0.99$</td>
</tr>
<tr>
<td>$\psi^L$</td>
<td>11.9783</td>
<td>Labor supply preference parameter</td>
<td>Ensures $L^{*a} = 0.236$, Eurostat 96/07</td>
</tr>
<tr>
<td>$\theta$</td>
<td>7.5</td>
<td>Elasticity between non tradables</td>
<td>Faia &amp; Monacelli (2008)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>7.5</td>
<td>Elasticity between tradables</td>
<td>Faia &amp; Monacelli (2008)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>75.73</td>
<td>Price stickiness</td>
<td>Faia &amp; Monacelli (2008)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Depreciation of capital</td>
<td>Smets &amp; Wouters (2003)</td>
</tr>
<tr>
<td>$\gamma_{T,t}$</td>
<td>0.656</td>
<td>Proportion of goods in IPS HICP</td>
<td>Eurostat 1996-2007</td>
</tr>
<tr>
<td>$\gamma_{T,t}^*$</td>
<td>0.606</td>
<td>Proportion of goods in EMU HICP</td>
<td>Eurostat 1996-2007</td>
</tr>
<tr>
<td>$\gamma_{f,t}$</td>
<td>0.3</td>
<td>Degree of openness</td>
<td>Eurostat 1996-2007</td>
</tr>
<tr>
<td>$\frac{IM}{Y}$</td>
<td>0.339</td>
<td>Average share of Imports on GDP</td>
<td>Eurostat 1996-2007</td>
</tr>
<tr>
<td>$\gamma_{ips}$</td>
<td>0.134</td>
<td>Average weight of IPS wrt EMU</td>
<td>Eurostat 1996-2007</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.94</td>
<td>AR interest rate</td>
<td>Smets &amp; Wouters (2003)</td>
</tr>
<tr>
<td>$\rho_{rEMU}^*$</td>
<td>1.658</td>
<td>Taylor rule inflation</td>
<td>Smets &amp; Wouters (2003)</td>
</tr>
</tbody>
</table>

Of the studies focus on the United States with respect to major trade partners. While the microeconomic and trade literature estimate large values, using disaggregated data, the international macroeconomic literature relying on aggregated data find lower values. In order to form a prior on possible values we look at studies in both fields. For example Broda & Weinstein (2006) find that the elasticity is decreasing over time and it ranges from values between 6.8 and 4 (when we consider three-digit goods). On the other hand Taylor (1999) estimates a long run elasticity of 0.39. Empirical literature was recently enriched with theoretical studies that showed that implied low trade elasticity could help explaining the Backus et al. (1993) puzzle ((Corsetti, Dedola & Leduc 2008) and (Benigno & Thoenissen 2008)) and the volatility of the real exchange rate((Thoenissen 2011)). We set a prior normal distribution with mean 1.5 (the most used value in calibrated exercise) and standard deviation of 1. We also set as boundaries the lower and upper extremes found by previous studies. Also the elasticity of substitution between tradable and non tradable goods, $\eta$, is an important parameter. Fortunately there is more consensus on the value of this parameter. Mendoza (1991), on a set of industrialized countries find a value of 0.74 while Stockman & Tesar (1995) estimate a lower elasticity of 0.44. Rabanal & Tuesta Retegui (2007) in a model made to understand the role of the non tradable goods in understanding the dynamics of the real

---

17 The list of cited studies is far from being complete but has the purpose of give a sense of the range of those estimates and some reference for further studies.
exchange rate estimate the parameter to be really low, 0.13. Combining this information we assume a gamma prior distribution with mean 0.5 and standard deviation 0.1 concentrating on values below 1.

We include in the estimation three other parameters from which previous studies do not give us a precise calibration. The habit in consumption choices, given that it can take values between zero and one, is assumed to have a beta distribution with mean 0.5 which is standard in the literature. Following Smets & Wouters (2003) we assume that the capital adjustment cost elasticity, $\eta_k$ is normally distributed with mean 4 and a wide standard deviation of 1.5. For the utilization rate elasticity to the rental rate of capital we define a variable $\eta_v$ such that $\eta_v = \frac{1-\bar{v}}{\eta_v}$ and estimate the new variable assuming a beta distribution with mean 0.5 and standard deviation 0.1.

For the set of priors on the autoregressive component and volatilities of shocks we follow a common practice in the estimated DSGE models. The prior distributions over the autoregressive coefficients of anticipated and unanticipated shock are assumed to be beta with prior mean of 0.7, while the volatilities have a positive support with the gamma distribution with mean 0.7 and standard deviation (0.3). Notice that we focus on unanticipated, four-quarters anticipated and twelve-quarters anticipated shocks. The choice of one year anticipation comes from the fact that we learned from Schmitt-Grohe & Uribe (2012) that those can account for more than 35% of the variance of output growth. We also introduce the three years anticipation because we want to understand the importance of agents anticipation in the first quarter of 1996 of the expected changes happening in the first quarter of 1999 (introduction of the euro).
Table 3: Prior and Posterior Distribution

<table>
<thead>
<tr>
<th>Estimated Parameters</th>
<th>Prior Distr.</th>
<th>Prior Mean</th>
<th>Prior St. Dev</th>
<th>Posterior Mean</th>
<th>Posterior Lower</th>
<th>Posterior Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$ T Vs NT</td>
<td>Gamma</td>
<td>0.500</td>
<td>0.1</td>
<td>0.9368</td>
<td>0.8135</td>
<td>1.0432</td>
</tr>
<tr>
<td>$\epsilon$ home VS foreign</td>
<td>Norm</td>
<td>1.500</td>
<td>0.5</td>
<td>1.8923</td>
<td>1.7907</td>
<td>1.9935</td>
</tr>
<tr>
<td>$h$ habit formation</td>
<td>Beta</td>
<td>0.500</td>
<td>0.1</td>
<td>0.9501</td>
<td>0.9463</td>
<td>0.9529</td>
</tr>
<tr>
<td>$\eta_v$ Utilization rate elast</td>
<td>Beta</td>
<td>0.500</td>
<td>0.1</td>
<td>0.3394</td>
<td>0.2313</td>
<td>0.4256</td>
</tr>
<tr>
<td>$\eta_h$ Capital adj cost elast</td>
<td>Norm</td>
<td>4.000</td>
<td>1.5</td>
<td>6.2320</td>
<td>4.9253</td>
<td>7.605</td>
</tr>
<tr>
<td>AR Coefficients</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{Ah}$ T Techn</td>
<td>Beta</td>
<td>0.7</td>
<td>0.1</td>
<td>0.4634</td>
<td>0.3417</td>
<td>0.6642</td>
</tr>
<tr>
<td>$\rho_{AN}$ NT Techn</td>
<td>Beta</td>
<td>0.7</td>
<td>0.1</td>
<td>0.7791</td>
<td>0.6872</td>
<td>0.8710</td>
</tr>
<tr>
<td>$\rho_{C}$ Preference</td>
<td>Beta</td>
<td>0.5</td>
<td>0.1</td>
<td>0.3895</td>
<td>0.2850</td>
<td>0.4815</td>
</tr>
<tr>
<td>$\rho_{\varepsilon,b}$ Risk Prem</td>
<td>Beta</td>
<td>0.7</td>
<td>0.1</td>
<td>0.5844</td>
<td>0.5088</td>
<td>0.6741</td>
</tr>
<tr>
<td>$\rho_{\epsilon_l}$ Invest</td>
<td>Beta</td>
<td>0.7</td>
<td>0.1</td>
<td>0.3821</td>
<td>0.2874</td>
<td>0.4783</td>
</tr>
<tr>
<td>$\rho_{\epsilon_L}$ Labor</td>
<td>Beta</td>
<td>0.7</td>
<td>0.1</td>
<td>0.6326</td>
<td>0.4896</td>
<td>0.7666</td>
</tr>
<tr>
<td>Standard Deviation (in percent)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\epsilon_A}$ Common Techn</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.0962</td>
<td>0.0424</td>
<td>0.1482</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{Ah}}$ T Techn</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.2432</td>
<td>0.1542</td>
<td>0.335</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{An}}$ NT Tech</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.2576</td>
<td>0.1786</td>
<td>0.3396</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_C}$ Preference</td>
<td>Gamma</td>
<td>1</td>
<td>0.5</td>
<td>0.1387</td>
<td>0.0492</td>
<td>0.2327</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{\varepsilon,b}}$ Risk Prem</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.4548</td>
<td>0.2050</td>
<td>0.7357</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_l}$ Invest</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.3919</td>
<td>0.1931</td>
<td>0.5517</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_L}$ Labor</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.3159</td>
<td>0.1016</td>
<td>0.5535</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_r}$ Int rate</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>1.0189</td>
<td>0.8985</td>
<td>1.1383</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_X}$ Trend shock</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.0114</td>
<td>0.0063</td>
<td>0.0158</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{4,A}}$ Ant Common Techn</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.1178</td>
<td>0.0530</td>
<td>0.1850</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{4,Ah}}$ Ant Ah</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.1848</td>
<td>0.0795</td>
<td>0.2859</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{4,An}}$ Ant An</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.3581</td>
<td>0.1719</td>
<td>0.5040</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{4,C}}$ Ant $\zeta$</td>
<td>Gamma</td>
<td>1</td>
<td>0.5</td>
<td>0.1776</td>
<td>0.0725</td>
<td>0.2831</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{4,I}}$ Ant I</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.8249</td>
<td>0.5185</td>
<td>1.0586</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{4,L}}$ Ant L</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.6161</td>
<td>0.3225</td>
<td>0.8836</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{4,\varepsilon,b}}$ Ant Risk Prem</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.2963</td>
<td>0.1493</td>
<td>0.4436</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{4,X}}$ Ant Trend shock</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.0103</td>
<td>0.052</td>
<td>0.0150</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{12,A}}$ Ant Common Techn</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.1369</td>
<td>0.0624</td>
<td>0.2091</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{12,Ah}}$ Ant Ah</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.1751</td>
<td>0.0772</td>
<td>0.2660</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{12,An}}$ Ant An</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.2017</td>
<td>0.0982</td>
<td>0.3054</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{12,C}}$ Ant $\zeta$</td>
<td>Gamma</td>
<td>1</td>
<td>0.5</td>
<td>0.1494</td>
<td>0.0556</td>
<td>0.2438</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{12,I}}$ Ant I</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.4820</td>
<td>0.2659</td>
<td>0.6679</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{12,L}}$ Ant L</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.5918</td>
<td>0.3609</td>
<td>0.8184</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{12,\varepsilon,b}}$ Ant Risk Prem</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.4449</td>
<td>0.2973</td>
<td>0.6008</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_{12,X}}$ Ant Trend shock</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.3</td>
<td>0.0112</td>
<td>0.0053</td>
<td>0.0161</td>
</tr>
</tbody>
</table>
3.4 Posterior Distribution

Table 3 presents the posterior distribution mean and standard deviation for the model’s parameter. We use 1 million draws from a random walk Metropolis Hastings chain algorithm of which we drop the first sixty per cent of the observations.

The estimated process for the productivity shock is more persistent in the non tradable than in the tradable sector and it is mostly driven by the unanticipated component. [to be completed following table 3]

4 What structural shocks can explain current account imbalances

Ireland Portugal and Spain, during the period under investigation, were accumulating current account deficit, experiencing real exchange rate appreciation and growing slightly above trend (figure 2). The goal of the paper is to investigate what structural shock, anticipated or unanticipated, can reproduce a similar pattern of those three key variables. We start by looking at the impulse response to different shocks and we claim that to be considered a major source of the imbalances the shocks need to induce a reaction of current account, real exchange rate and GPD growth similar to the one presented in figure 2.

4.1 Impulse Response

We study the dynamics of the model in response to a wide range of possible shocks for the estimated parametrization. For every shock we consider the unanticipated component but we also allow for the possibility that agents learn in advance about the shock that will come true only in the future. In this section we look at the baseline model with separable utility function and complete information in which agents perfectly forecast the future. Later we relax these two assumptions: in section 4.2 we show how the results change when we use a preference specification that allows us to control for the wealth elasticity of labor supply. In section 4.3 we assume imperfect information and slow learning by agents of the true technology process.

18 Notice that periods in which current account deficit increase coincide with periods in which output is growing above trend. In fact, during 2002 and 2003 GDP was growing below trend and IPS were experiencing a reduction in the deficit.
Sector specific technology, common technology, permanent trend, investment cost, risk premium, preferences and labor supply are the shocks we consider in our specification. For each shock we try to interpret what type of imbalances they generate.

As in Giavazzi & Spaventa (2010) and Eichengreen (2010), we distinguish between types of current account imbalances depending on their trigger. There are ones driven by growth differentials that allow surplus countries to invest in future growth of the borrowing countries and others triggered by other factors. We define good imbalances the ones resulting from a deeper financial integration that allows countries with better growth prospect to borrow from abroad to undertake profitable investment and to increase their consumption due to the anticipated wealth effect. In a way is like saying that good imbalances are the ones that comes from borrowing using as collateral future growth. If we believe in this definition it is not difficult to understand why anticipated productivity shock could be seen as the main source of good imbalances. In this section we want to test if the imbalances experienced by the IPS inside the EMU could be driven by unanticipated as well as anticipated productivity shocks. We first present the response of the model to an unanticipated shock in technology to understand in detail the forces behind the movements of the variables and then present, with figure 4, the response of the current account, the real exchange rate and output growth to all considered technology shocks.

Figure 3 shows a one standard deviation unanticipated shock in the tradable sector. Consumption and investment co-move resulting in output growing above trend. Tradable marginal costs of production decrease and due to the stickiness of prices and the imperfect substitutability of goods, tradable firms have excessive production. This pushes their prices and the demand for labor and capital down. Wages and cost of capital decrease generating a reduction in the marginal costs also in the non tradable sector. As a result prices go down in both sectors and the real exchange rate depreciates. Home produced tradable goods are now also internationally more competitive through a term of trade depreciation. The current account deteriorates due to the fact that the increase in wealth, resulting in increase in consumption and investment, exceed the increase in exports due to the gained comparative advantage. Although, notice that a sectoral Balassa-Samuelson effect is still active: non tradable goods become relative more expensive than tradable goods. Everything works in a very similar fashion when the shock is specific to the non tradable sector. The differences are that now the non tradable goods will become relatively cheaper with respect to the tradable goods and that, given the smaller weight of the non tradable sector in the economy, the increase in exports will exceed the increase in wealth, turning current account in surplus. Notice that the
terms of trade and the real exchange rate depreciate in response to both sector specific shocks.

Figure 3: Impulse response to an unanticipated tradable sector technology shock

Figure 4 summarizes the responses of current account, real exchange rate and GDP growth to all types of technology shock included in the model. The left column of the picture shows the responses to unexpected shocks while the right column focuses on anticipated shocks. Starting from the first line and moving vertically we look at sector specific shock in the tradable and non tradable sector, common shock in both sectors and a trend shock. What comes out of the picture is that none of the technology shock considered can jointly replicate figure 2.
The main problem comes from the inability of the model to generate a real exchange rate appreciation as a response to just a positive technology shock. In fact, for the estimated parametrization, we can’t rely on a strong wealth effect that pushes the increase in demand above the increase in supply of goods. As shown by Corsetti et al. (2008) the most important parameter governing the wealth effect is the elasticity of substitution between domestic and foreign tradable goods. We show in figure 5 that keeping fixed the calibration/estimation of the other parameters, the model is unable to generate a real exchange rate appreciation as a response to an unanticipated common technology shock for any value of the elasticity in the range considered as plausible by previous studies. An increase in the efficiency of production in our frameworks leads always to a decrease in the marginal cost (expected in the case of anticipated shock) that pushes prices down. Being fixed the nominal exchange rate all the
movements goes directly to the real exchange rate and the terms of trade. On the other hand notice from the same picture that the range of parameters for which the current account becomes negative in response to a positive technology shock (common to both sectors) is pretty wide.

Figure 5: Response of CA, GDP and RER to a common technology shock with varying $\epsilon$

Figure 6 summarizes the dynamics of the model in response to a drop in the risk premium, ($RP_t$), an increase in household preferences for consumption, ($\epsilon^\sigma$), a positive increase of investment technology, $\epsilon^I$ and finally a labor supply shock, $\epsilon^L_t$. Investment shock, as the productivity shock just considered, does not generate a real exchange rate appreciation. Labor supply moves current account and real exchange in the right direction but induce output growing below trend. Leaving aside the preference shock, due to the lack of clear interpretation, the most interesting results seems to come from the anticipated drop in the risk premium. Current account deteriorates, real exchange rate appreciates and output growth slightly above trend. The interesting pattern is that current account start to deteriorates and stays persistently in deficit position. The real exchange rate keep appreciating until the risk premium shock realizes.
Figure 6: Demand vs interest rate vs investment

An anticipated drop in risk premium can be plausibly interpreted as an expected decrease in the "risk free rate" that the IPS had to pay on domestic issued bond. Figure 7 shows that this is a reasonable assumption given the persistent and strong decrease in the 3 months Euro Inter Bank offered rate. In fact, if in 1996 agents in Spain Ireland and Portugal were expecting that the introduction of a common currency would decrease the cost of borrowing the path of current account balance, real exchange rate and output would have followed a similar path to the one experienced in reality.
Let’s say we don’t trust the estimation results. Here we show that the results are consistent for a broad range of elasticity of substitution between goods found in previous studies. For a meaningful range of parameters $0.5 \leq \eta \leq 1$ and $0.75 \leq \epsilon \leq 2$ the results are still consistent with the finding that positive TFP shock can hardly explain the current account deficit jointly with a persistence real exchange rate appreciation. Two interesting exercises are to test what happens to the model when we allow domestic and foreign tradable goods to be either strongly substitutable or complements. Current account reacts positively when goods are almost perfect substitutes and it reacts negatively if they are complements. These results are in line with previous studies and do not come as surprise. The ability of producing a almost fully substitutable tradable good with a marginal cost advantage increases exports and decreases imports by a bigger amount while, in the case of strong complementarity, the wealth effect increases consumption more than production generating current account deficit and real exchange rate depreciation.

4.2 Jaimovich and Rebelo preferences

An expected permanent shock in technology leads to a reduction in hours worked. Agents anticipate the increase in wealth and increase the demand of normal goods as leisure and goods. To understand the role played by the labor supply wealth effect for the results of our

---

19If the shock is anticipated and tradable goods are highly substitutable, current account starts with a deficit followed by a surplus as soon as the shock hits the economy

20For a further investigation of the mechanism described look at Corsetti et al. (2008)
model we substitute our utility function (equation 4) with the non separable specification introduced by Jaimovich & Rebelo (2009). To better compare it with our previous result we also introduce the modification proposed by Hoffmann et al. (2011) that allows for habit in consumption choices and trend in growth rate:

$$U(C_t, L_t) = \left\{ \left( C_t - hC_{t-1} \right) - \epsilon_t^L \psi^L L_t^{1+\nu} \Omega_t \right\}^{1-\sigma} - 1$$

where

$$\Omega_t = (C_t - hC_{t-1})^\mu \Omega_{t-1}^{1-\mu} (Z_t)^{1-\mu}$$

Utility depends on consumption at time $t$, $C_t$, a weighted component of average past consumption, $hC_{t-1}$, and hours worked $L_t$. Notice that past average consumption is perceived as external by the maximizing household which does not take into account the effects of his decision on the accumulation of average consumption. $\Omega_t$ controls the wealth effect on labor supply through the parameter $\mu \in [0, 1]$. By just changing $\mu$ we can account for two important classes of utility function used in the business cycle literature: King, Plosser & Rebelo (1988) types of preferences (KPR henceforth) when $\mu = 1$ and Greenwood, Hercowitz & Huffman (1988) when $\mu = 0$ (GHH henceforth). The inclusion of $Z_t^{1-\mu}$, where $Z_t = X_t / X_{t-1}$, allows the model to account for the possibility of having trend growth in labor augmenting productivity $X_t$.

In the appendix we present the full set of new necessary first order conditions for the representative household. As in the baseline model we proceed by calibrating some parameters and then estimate the model with Bayesian techniques. For the parameters overlapping with the baseline model we use the same calibration and the same set of priors in order to be able to compare better the results. The set of calibrated parameters overlaps with the baseline specification a part from $\sigma$, the intertemporal elasticity of substitution, that it’s set equal to 1. We also estimate two new parameters: $\mu$ and $\nu$. As explained before, $\mu$ plays an important role because it controls the degree of the wealth elasticity of the labor supply. As a prior we follow Schmitt-Grohe & Uribe (2012) setting a uniform distribution defined over the interval $(0, 1]$. $\nu$ determines the elasticity of labor supply when $\mu$ is equal to zero. Also for this parameter we take a wide prior imposing a uniform distribution over the interval $(1.1, 11)$. Table ??, available on request, summarizes the priors and posterior for all the estimated parameters.

The main advantage of using this specification for the utility function is that we are able to match better the data concerning the reaction of hours worked in the presence of anticipated

\(^{21}\text{In Jaimovich & Rebelo (2009) they impose } \mu > 0 \text{ in order to impose some weight on the KPR preferences which are growth consistent. Given our focus on low values of } \mu \text{ the introduction of } (Z_t)^{1-\mu} \text{ allows us to avoid problems coming from highly persistent deviations from the steady state growth path.} \)
shock. In fact now the model does not generate a strong reduction in hours worked when
the realization of the shock occurs. The main results still hold and are in line with what
presented previously. Unanticipated or anticipated technology shock can’t be the only major
source of the current account imbalance experienced inside the European Monetary Union.

4.3 Imperfect Information

Let’s assume now that there is imperfect information. Agents can now observe only the
level of technology but they can’t perfectly distinguish if the change is coming from the
trend or the cyclical component. Hoffmann et al. (2011) show that in such a framework
technology shock can explain the accumulation of current account deficit inside the US. We
are interested in understanding if that is the case also for Ireland Portugal and Spain. We
follow Boz, Daude & Durdu (2011) and Hoffmann et al. (2011) assuming that agents form
expectations about the shock through a filtering problem. Agents have available at time \( t \)
the entire history of TFP shocks and they know the underlying distribution of both trend
and cyclical shocks. At each point in time they use the new information available to form
the best possible estimation of the current level of trend growth and behave accordingly. Given
our focus on the linear solution the best estimate of the forecasting of agents is obtained by
the Kalman Filter according to the recursion used by Hoffmann et al. (2011)

\[
\tilde{A}_{j,t|t} = (1 - \kappa)\rho_{A_j} \tilde{A}_{j,t-1|t-1} + \kappa z_t \quad \text{where} \quad z_t = \ln(Z_t) = \ln\left(\frac{X_t}{X_{t-1}}\right)
\]  

(40)

\( \kappa \) represent the Kalman gain and it is given by the following equation

\[
\kappa = \frac{\sigma_1 - (1 - \rho_{A_j}) + \sigma_1 \sqrt{((1 - \rho_{A_j})/\sigma_1)^2 + 1 + 2(1 + \rho_{A_j})/\sigma_1}}{1 + \sigma_1 - (1 - \rho_{A_j}) + \sigma_1 \sqrt{((1 - \rho_{A_j})/\sigma_1)^2 + 1 + 2(1 + \rho_{A_j})/\sigma_1}}
\]  

(41)

where \( \sigma_1 \equiv \frac{\sigma_{A_j}}{\sigma_X} \) is the signal to noise ratio and measures the relative weights assigned to trend
growth relative to one time permanent change in the level of technology. Figure shows the
difference between the true dynamics of the shock (in the figure we present a one time level
shock) with respect to the perceived dynamics. As time goes by the agents learns that the
shock wasn’t permanent and converges to the right path.
The impulse response with imperfect information are less pronounced and more persistent but the main result of the paper doesn’t change. The hard task is not to replicate the dynamics of the current account, as it is shown in Hoffmann et al. (2011), but the joint dynamics of the current account and the real exchange rate. Using just imperfectly learned change in trend growth rates it is not sufficient, for the set of estimated and calibrated parameters presented above, to explain the current account deficit and the real exchange rate appreciation.

4.4 Robustness of the results

As a final robustness exercise we check what happens when we consider a model with labor as the only input of production. The reason why we do that is that some previous studies used a simplified model without capital in production to draw conclusions on international variables. Our results still hold when we use a simplified version but the model perform poorly when matching the moments of the data. As for the dynamics of the impulse response the introduction of capital and investment play an important role in the dynamics of the current account. In fact, if we exclude capital, the initial movement of the current account is much less pronounced: in the baseline model when a positive technology shock hits the economy, the marginal product of capital increases the incentives to invest in capital. Investment goes up with money borrowed from abroad. All this mechanism is absent if use labor as the only input of production. In our opinion, to be able to bring a model to the data with the goal of studying the international dynamics of the current account and relative prices, capital and investment sector can not be excluded.
5 Conclusions

Ireland, Portugal and Spain, small open economies member of the European Monetary Union, from the birth of the euro to the 2008 financial crisis, experienced large current account deficit, appreciated real exchange rate and output growth above trend. The goal of the paper is to construct an estimated model of a small open economy inside a monetary union that allows us to theoretically test among different reasonable determinants of those accumulated imbalances.

Capital’s flowing towards “catching-up” euro area countries with high actual or expected productivity growth seems to have lost empirical support. This paper shows that also theoretically, it is hard to replicate the joint path of current account, real exchange rate and output growth with unanticipated as well as anticipated productivity shocks. On the other hand the anticipated shock in the uncovered interest rate parity is more likely an important source of those imbalances. Uncovered interest parity shock in a monetary union with a fixed nominal exchange rate can be interpreted as an anticipated decrease in the cost of borrowing. It is plausible that agents, as soon as the monetary union became a credible agreement in 1996, anticipated a future drop in the borrowing cost. Interesting, if we allow in our model such an anticipated decrease in the risk premium, then the dynamics of the current account, real exchange rate and output mimic the IPS data.

A new distinction between types of imbalances is needed and this distinction is to be characterized by the sources of those imbalances. Future studies will go deeper on the understanding between the distinction between types of imbalances.

Appendices

A Equilibrium Conditions

We present here the full list of necessary and sufficient equilibrium condition.
A.1 Household

The representative household optimal demand within each sector and varieties of goods yields:

\[ C_{N,t} = \gamma_{N,t} (m(J_t))^\eta C_t \]  
\[ C_{h,t} = \gamma_{T,t} \gamma_{h,t} \left( \frac{m(J_t)}{J_t} \right)^\eta \frac{g(S_t)}{S_t} C_t \]  
\[ C_{f,t} = \gamma_{T,t} \gamma_{f,t} \left( \frac{m(J_t)}{J_t} \right)^\eta \left( \frac{g(S_t)}{S_t} \right)^\epsilon C_t \]

where \( P_t = \gamma_{T,t} P_{1_t}^1 + \gamma_{N,t} P_{1_t}^1 - \eta_{T,t} + \gamma_{h,t} P_{1_t}^f - \eta_{h,t} \) and \( P'_{T,t} = \gamma_{h,t} P_{1_t}^f + \gamma_{f,t} P_{1_t}^f - \epsilon \) are respectively the consumer price index and the tradable goods price index.

Similarly foreign demand for the home produced tradable can be written as:

\[ C^*_h = \gamma_{T,t} \gamma_{h,t} Q^\eta \left( \frac{m(J_t)}{J_t} \right)^\eta \frac{g(S_t)}{S_t} C^*_t \]

The first order necessary conditions of the maximization problem of the households are:

\[ \frac{1}{C_t} \frac{1}{C_{t-1}} - \frac{1}{h} \beta_t \frac{1}{C_{t+1}} - \frac{1}{h} C_t = \lambda \]
\[ \lambda_t = \beta_t E_t \left\{ \lambda_{t+1} \frac{R_{t}^B}{m_{t+1}} \right\} \text{ and } \lambda_t = \beta_t E_t \left\{ \lambda_{t+1} \frac{R_{t}}{m_{t+1}} \right\} \]
\[ r^k_t = \Psi'(u_t) \]
\[ 1 = q_t \epsilon_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \beta_t E_t \left\{ q_{t+1} \epsilon_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( r^k_{t+1} - \Psi(u_{t+1}) - q_t (1 - \delta) \right) \right] \right\} \]
\[ q_t = \beta_t E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( r^k_{t+1} u_{t+1} - \Psi(u_{t+1}) + q_t (1 - \delta) \right) \right] \]
\[ \zeta_t \epsilon_t L_t^r = \lambda_t W_t \]

where \( \lambda_t \) is the lagrangian multiplier associated with the budget constraint, \( Q_t \) the lagrangian multiplier associated with installed capital. We denote with \( q_t = \frac{Q_t}{\lambda_t} \) the Tobin’s Q, which is the value of installed capital in consumption units.

In equilibrium:

\[ K_t = u_t k^p_{t-1} \]
\[ k^p_t = (1 - \delta) k^p_{t-1} + I_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] \epsilon_t \]
\[ \beta_t = \frac{1}{1 + \psi^\delta \log \gamma_t - \psi^\delta \chi} \]
A.2 Firms

Firms produce in the two sectors with the following production functions:

\[ Y_{j,t} = A_{j,t} X_1^{1-\alpha} K_{j,t}^{\alpha} L_1^{1-\alpha} \]  \hspace{1cm} (A.14)

The optimality conditions of the cost minimization problem are:

\[ K_{j,t} = \frac{\alpha}{1-\alpha} \frac{W_t}{r_t^k} L_{j,t} \]  \hspace{1cm} (A.15)

and

\[ MC_{j,t} = \left( \frac{1}{1-\alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right)^{\alpha} \frac{W_t^{1-\alpha}}{A_{j,t} X_1^{1-\alpha}} \]  \hspace{1cm} (A.16)

When setting the prices, assuming a symmetric equilibrium in which each domestic producer sets optimally equal prices and produces the same level of output as other producers, the resulting equation are:

\[ \Pi_{N,t} = \frac{\Pi_{N,t} \Pi_{N} - 1}{\Pi_{N}^{N}} = E_t \left\{ \beta_t \frac{\lambda_{t+1}}{\lambda_t} \frac{m(J_t)}{m(J_{t+1})} Y_{N,t} Y_{N,t+1} \left( \frac{\Pi_{N,t+1} \Pi_{N,t}^{N} - 1}{\Pi_{N,t}^{N}} \right) + \frac{\theta}{\psi} MC_{N,t} m(J_t) - \frac{\theta - 1}{\psi} \right\} \]  \hspace{1cm} (A.17)

\[ \Pi_{h,t} = \frac{\Pi_{h,t} \Pi_{h} - 1}{\Pi_{h}^{N}} = E_t \left\{ \beta_t \frac{\lambda_{t+1}}{\lambda_t} \frac{m(J_t)}{m(J_{t+1})} Y_{h,t} Y_{h,t+1} \left( \frac{\Pi_{h,t+1} \Pi_{h,t}^{N} - 1}{\Pi_{h,t}^{N}} \right) + \phi MC_{h,t} m(J_t) g_t - \phi - 1 \right\} \]  \hspace{1cm} (A.18)

where we used \( \pi_{(N,h),t} = \frac{P_{(N,h),t}}{P_{(N,h),t-1}} \) and \( \frac{\partial \pi_{t+1}}{\partial \pi_{t}} = \beta_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{t+1}}{P_t} \).

A.3 Relative prices and International variables

The relevant relative prices are the terms of trade, the relative prices of TPI over the prices of home produced tradable goods, the relative price of tradable over non tradable goods and the CPI over the price of non tradable goods:

\[ S_t = \frac{P_{f,t}}{P_{h,t}} \]  \hspace{1cm} (A.19)

\[ g(S_t) = \frac{P_{T,t}}{P_{h,t}} = [\gamma_{h,t} + \gamma_{f,t} S_t^{1-\eta}]^{-1} \]  \hspace{1cm} with \( \frac{\delta g(S_t)}{\delta S_t} > 0 \)  \hspace{1cm} (A.20)

\[ J_t = \frac{P_{T,t}}{P_{N,t}} \]  \hspace{1cm} (A.21)

\[ m(J_t) = \frac{P_t}{P_{N,t}} = [\gamma_{T,t} J_t^{1-\eta} + \gamma_{N,t}]^{-1} \]  \hspace{1cm} with \( \frac{\delta m(J_t)}{\delta J_t} > 0 \)  \hspace{1cm} (A.22)

(A.23)
We define the real exchange rate in the monetary union as

\[ Q_t = \frac{P_t^*}{P_t} \]  \hspace{1cm} (A.24)

The balance of payment condition, as share of steady state GDP, \( Y \):

\[ NX_t + \frac{R_{t-1}B_t}{YP_t} - \frac{R_{t-1}A_t}{YP_t} - \frac{B_t - A_t}{YP_t} = 0 \]  \hspace{1cm} (A.25)

where \( nx_t \) denotes the real value of net exports as a ratio to \( Y \) and it is equal to

\[ NX_t = \frac{J_t}{g(S_t)m(J_t)} \left( \frac{Y_{h,t} - C_{h,t} - S_t C_{f,t}}{Y} \right) \]  \hspace{1cm} (A.26)

The current account as the net change in real bond holding, as share of steady state GDP, \( Y \):

\[ CA_t = \frac{(B_t - A_t)}{P_t Y} - \frac{(B_{t-1} - A_{t-1})}{P_t Y} \]  \hspace{1cm} (A.27)

A.4 Market Clearing and Monetary Policy Rule

Labor and capital can move freely across sectors and demand will equal labor when

\[ K_t = K_{N,t} + K_{h,t} \]  \hspace{1cm} (A.28)
\[ L_t = L_{N,t} + L_{h,t} \]  \hspace{1cm} (A.29)

In the non tradable and in the tradable sector market clears:

\[ Y_{N,t} = C_{N,t} + \frac{\psi_2}{2} \left( \frac{\pi_{N,t}}{\pi_N} - 1 \right)^2 Y_{N,t} \]  \hspace{1cm} (A.30)
\[ Y_{h,t} = C_{h,t} + C_{h,t}^* + \frac{\psi_2}{2} \left( \frac{\pi_{h,t}}{\pi_h} - 1 \right)^2 Y_{h,t} \]  \hspace{1cm} (A.31)

Real GDP aggregates tradable and nontradable production weighted by the relative prices:

\[ Y_t = \frac{J_t}{m(J_t)g(S_t)} Y_{h,t} + \frac{1}{m(J_t)} Y_{N,t} + I_t \]  \hspace{1cm} (A.32)

To complete the model we introduce a monetary policy rule, set by the European Central Bank:

\[ R_t = R_t^{\rho_E} \left( \frac{\Pi_t^{EMU}}{\Pi^{EMU}} \right)^{(1-\rho_e)\rho_e} \]  \hspace{1cm} (A.33)
B Detrended Equilibrium Conditions

In the model we allow for different grow rates in the two sectors and for a common trend in all aggregate variables induced by the technological process. To be able to solve the model with standard techniques we need first to make the model stationary by detrending each variable with its particular growth rate. We rewrite the equations using stationary transformation of each variable, indicated with a overwritten tilde.

B.1 Technologies and Prices

Before going into the details of each equation notice that given the technology process specified in equation (9) and (10), variables in the non tradable sector grow at rate \((1 + g^N)^t X_t\) while in the tradable sector the growing rate is \((1 + g^T)^t X_t\). The growth rate of aggregate variable \(\frac{X_t}{\Pi_t} = Z_t\) Assuming that CPI prices grow at \(\Pi^t\) then the two sectors specific prices grow at \(\frac{\Pi^t}{(1-g^j)^t}\).

B.2 Household

The demand functions for each type of good become:

\[
\tilde{C}_{N,t} = \tilde{\gamma}_{N,t} \left( \tilde{m}(\tilde{J}_t) \right)^\eta \tilde{C}_t \tag{B.1}
\]
\[
\tilde{C}_{h,t} = \tilde{\gamma}_{T,t}\tilde{\gamma}_{h,t} \left( \frac{\tilde{m}(J_t)}{J_t} \right)^\eta \tilde{g}(S_t)^\epsilon \tilde{C}_t \tag{B.2}
\]
\[
\tilde{C}_{f,t} = \tilde{\gamma}_{T,f,t} \left( \frac{\tilde{m}(J_t)}{J_t} \right)^\eta \left( \frac{\tilde{g}(S_t)}{S_t} \right)^\epsilon \tilde{C}_t \tag{B.3}
\]
\[
\tilde{C}^*_{h,t} = \tilde{\gamma}_{T,h,t}\tilde{\gamma}_{h,t} \tilde{Q}^\eta \left( \frac{\tilde{m}(J_t)}{J_t} \right)^\eta \tilde{g}(S_t)^\eta \tilde{S}_t^{\epsilon-\eta} \tilde{C}_t^* \tag{B.4}
\]

where
\[
\begin{align*}
\tilde{C}_{N,t} &= \frac{C_{N,t}}{(1 + g^N)^tX_t} & \tilde{C}_{h,t} &= \frac{C_{h,t}}{(1 + g^T)^tX_t} & \tilde{C}_{f,t} &= \frac{C_{f,t}}{(1 + g^{T*})^tX_t} \\
\tilde{C}^*_{h,t} &= \frac{C^*_{h,t}}{(1 + g^{T*})^tX^*_t} & \tilde{C}^*_{h,t} &= \frac{C^*_{h,t}}{X^*_t} & \tilde{C}^*_{t} &= \frac{C^*_{t}}{X^*_t} \\
\tilde{\gamma}_{N,t} &= \frac{\gamma_{N,t}}{(1 + g^N)^t(1-\eta)^tX_t} & \tilde{\gamma}_{T,t} &= \frac{\gamma_{T,t}}{(1 + g^T)^t(1-\eta)^tX_t} & \tilde{\gamma}_{h,t} = \gamma_{h,t} \\
\tilde{\gamma}_{f,t} &= \gamma_{f,t} \left(\frac{1 + g^T}{1 + g^{T*}}\right)^{(1-\epsilon)t} \left(\frac{\Pi}{\Pi^*}\right)^{(1-\epsilon)t} & \tilde{\gamma}^*_{T,t} = \gamma^*_{T,t} \left(\frac{1 + g^{T*}}{1 + g^T}\right)^{(1-\epsilon)t} \left(\frac{\Pi^*}{\Pi}\right)^{(1-\epsilon)t} \\
\tilde{\gamma}^*_{h,t} &= \gamma_{h,t} \left(\frac{1 + g^{T*}}{1 + g^T}\right)^{(1-\epsilon)t} \left(\frac{\Pi^*}{\Pi}\right)^{(1-\epsilon)t}
\end{align*}
\]

and

\[
\begin{align*}
\tilde{S}_t &= S_t \left(\frac{1 + g^{T*}}{1 + g^T}\right)^t \left(\frac{\Pi}{\Pi^*}\right)^t & \tilde{g}(S_t) = g(S_t) \\
\tilde{J}_t &= \left(\frac{1 + g^T}{1 + g^N}\right)^t & \tilde{m}(J_t) = \frac{m_t}{(1 + g^N)^t} \\
\tilde{Q}_t &= Q_t \left(\frac{\Pi}{\Pi^*}\right)^t
\end{align*}
\]

Notice that

\[
\begin{align*}
\tilde{\Pi}_t &= \frac{\Pi_t}{\Pi} \text{ because } \tilde{P}_t = \frac{P_t}{(\Pi)^t} \\
\tilde{\Pi}_{N,t} &= \Pi_{N,t} \frac{(1 + g^N)}{\Pi} \text{ because } \tilde{P}_{N,t} = \Pi_{N,t} \frac{(1 + g^N)^t}{\Pi} \\
\tilde{\Pi}_{h,t} &= \Pi_{h,t} \frac{(1 + g^T)}{\Pi} \text{ because } \tilde{P}_{h,t} = \Pi_{h,t} \frac{(1 + g^T)^t}{\Pi}
\end{align*}
\]

The first order conditions of the household:
\[
\begin{align*}
\zeta_t \frac{1}{\tilde{C}_t - \frac{h}{Z_t} \tilde{C}_{t-1}} - h \beta_t \zeta_{t+1} \frac{1}{\tilde{C}_{t+1} - h Z_{t+1} \tilde{C}_t} = \tilde{\lambda}_t \\
\tilde{\lambda}_t = \beta_t E_t \left\{ \tilde{\lambda}_{t+1} \frac{\Pi}{Z_{t+1}} \frac{R^B_t}{\Pi_{N,t+1} m_{t+1}} \right\} \\
R^B_t = R_t \\
r^k_t = \Psi'(u_t) \\
1 = q_t \epsilon_t \left[ 1 - S\left( \frac{I_t}{I_{t-1}} Z_t \right) - S'\left( \frac{I_t}{I_{t-1}} Z_t \right) \frac{\tilde{I}_t}{Z_t} \right] + \beta_t E_t \left\{ \frac{q_{t+1} \epsilon_{t+1}}{\tilde{\lambda}_{t+1}} \frac{1}{Z_{t+1}} S'\left( \frac{\tilde{I}_{t+1}}{I_t} Z_{t+1} \right) \frac{\tilde{I}_{t+1}^2}{I_t^2} Z_{t+1}^2 \right\} \\
q_t = \beta_t E_t \left[ \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{1}{Z_{t+1}} \left[ r^k_{t+1} u_{t+1} - \Psi(u_{t+1}) + q_{t+1}(1 - \delta) \right] \right] \\
\zeta_t \epsilon_t^L L^\nu_t = \tilde{\lambda}_t \tilde{W}_t \\
\end{align*}
\]

where

\[
\tilde{\lambda} = \lambda X_t \quad \tilde{I}_t = \frac{I_t}{X_t} \quad \text{and} \quad \tilde{w}_t = \frac{w_t}{X_t}
\]

The other equilibrium conditions will be:

\[
\begin{align*}
\tilde{K}_t &= u_t \tilde{k}^P_{t-1} \\
\tilde{k}^P_t &= (1 - \delta) \tilde{k}^P_{t-1} + \tilde{I}_t \left[ 1 - S\left( \frac{\tilde{I}_t}{I_{t-1}} \right) \right] \epsilon_t \\
\beta_t &= \frac{1}{1 + \psi^\delta \log C_t - \psi^\beta \chi} \\
\end{align*}
\]

where

\[
\tilde{K}_t = \frac{K_t}{X_{t-1}} \quad \text{and} \quad \tilde{k}^P_{t-1} = \frac{k^P_{t-1}}{X_{t-1}}
\]
B.3 Firms

Firms production function in the two sectors are:

\[
\tilde{Y}_{N,t} = \tilde{A}_{N,t} Z_t^{-\alpha} K_{N,t}^{\alpha} L_{N,t}^{1-\alpha} \\
\tilde{Y}_{h,t} = \tilde{A}_{h,t} Z_t^{-\alpha} K_{h,t}^{\alpha} L_{h,t}^{1-\alpha}
\]  
(B.15)  
(B.16)

where

\[
\tilde{Y}_{N,t} = \frac{Y_{N,t}}{(1 + g_N)^t X_t} \\
\tilde{A}_{N,t} = \frac{A_{N,t}}{(1 + g_N)^t} \\
\tilde{K}_{N,t} = \frac{K_{N,t}}{X_{t-1}} \\
\tilde{Y}_{h,t} = \frac{Y_{h,t}}{(1 + g_T)^t X_t} \\
\tilde{A}_{h,t} = \frac{A_{h,t}}{(1 + g_T)^t} \\
\tilde{K}_{h,t} = \frac{K_{h,t}}{X_{t-1}}
\]

Cost minimization and marginal cost:

\[
\tilde{K}_{N,t} = \frac{\alpha}{1 - \alpha} \frac{\tilde{W}_t}{r_t^{\alpha}} L_{N,t} \\
\tilde{K}_{h,t} = \frac{\alpha}{1 - \alpha} \frac{\tilde{W}_t}{r_t^{\alpha}} L_{h,t}
\]  
(B.17)  
(B.18)

\[
\tilde{MC}_{N,t} = \left( \frac{1}{1 - \alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right) \frac{\tilde{W}_t^{1-\alpha} r_t^{k \alpha}}{A_{N,t}}
\]  
(B.19)

\[
\tilde{MC}_{h,t} = \left( \frac{1}{1 - \alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right) \frac{\tilde{W}_t^{1-\alpha} r_t^{k \alpha}}{A_{h,t}}
\]  
(B.20)

where

\[
\tilde{MC}_{N,t} = MC_{N,t} (1 + g_N)^t \\
\tilde{MC}_{h,t} = MC_{h,t} (1 + g_T)^t
\]

Pricing equations become:

\[
\frac{\Pi_{N,t}}{\Pi_N} \left( \frac{\Pi_{N,t}}{\Pi_N} - 1 \right) = E_t \left\{ \beta_t \frac{\lambda_{t+1}}{\lambda_t} m(J_t) Y_{N,t+1} \left( \frac{\Pi_{N,t+1}}{\Pi_N} - 1 \right) \frac{\Pi_{N,t+1}}{\Pi_N} \right\} + \frac{\theta}{\psi} MC_{N,t} m(J_t) - \frac{\theta - 1}{\psi}
\]  
(B.21)

\[
\frac{\Pi_{h,t}}{\Pi_h} \left( \frac{\Pi_{h,t}}{\Pi_h} - 1 \right) = E_t \left\{ \beta_t \frac{\lambda_{t+1} Y_{h,t+1}}{\lambda_t Y_{h,t}} m(J_t) g(S_t) J_{t+1} \left( \frac{\Pi_{h,t+1}}{\Pi_h} - 1 \right) \frac{\Pi_{h,t+1}}{\Pi_h} \right\} + \frac{\phi}{\psi} MC_{h,t} m(J_t) g_t - \frac{\phi - 1}{\psi}
\]  
(B.22)
B.4 Relative prices and International variables

The balance of payment condition, as share of steady state GDP, \( Y \):

\[
\tilde{N}X_t + \frac{R_t^R \tilde{B}_{t-1}}{Z_t \tilde{Y}_t \tilde{P}_t} - \frac{R_{t-1} \tilde{A}_{t-1}}{Z_t \tilde{Y}_t \tilde{P}_t} - \frac{\tilde{B}_t - \tilde{A}_t}{\tilde{Y}_t \tilde{P}_t} = 0 \tag{B.23}
\]

The real value of net exports as a ratio to \( Y \) becomes

\[
\tilde{N}X_t = \frac{\tilde{J}_t}{g(S_t)\tilde{m}(J_t)} \frac{\left( \tilde{Y}_{h,t} - \tilde{C}_{h,t} - \tilde{S}_t \tilde{C}_{f,t} \right)}{\tilde{Y}_t} \tag{B.24}
\]

The current account, as share of steady state GDP, \( Y \) is:

\[
\tilde{C}A_t = \frac{\left( \tilde{B}_t - \tilde{A}_t \right)}{\tilde{Y}_t} - \frac{(\tilde{B}_{t-1} - \tilde{A}_{t-1})}{Z_t \tilde{P}_t \tilde{Y}} \tag{B.25}
\]

where

\[
\tilde{N}X_t = \frac{NX_t}{X_t} \quad \tilde{B}_t = \frac{B}{X_t} \quad \tilde{A}_t = \frac{A}{X_t} \quad \tilde{C}A_t = \frac{CA}{X_t}
\]

B.5 Market Clearing and Monetary Policy Rule

Labor supply and capital supply equalize labor demand and supply:

\[
\tilde{K}_t = \tilde{K}_{N,t} + \tilde{K}_{h,t} \tag{B.26}
\]
\[
L_t = L_{N,t} + L_{h,t} \tag{B.27}
\]

Same for demand and supply of non tradable and home tradable:

\[
\tilde{Y}_{N,t} = \tilde{C}_{N,t} + \frac{\psi}{2} \left( \frac{\pi_{N,t}}{\pi_N} - 1 \right)^2 \tilde{Y}_{N,t} \tag{B.28}
\]
\[
\tilde{Y}_{h,t} = \tilde{C}_{h,t} + \tilde{C}^*_t + \frac{\psi}{2} \left( \frac{\pi_{h,t}}{\pi_h} - 1 \right)^2 \tilde{Y}_{h,t} \tag{B.29}
\]
Real GDP:

\[
\tilde{Y}_t = \frac{\tilde{J}_t}{\tilde{m}(J_t)} \tilde{Y}_{h,t} + \frac{1}{\tilde{m}(J_t)} \tilde{Y}_{N,t} + \tilde{I}_t \tag{B.30}
\]

where

\[
\tilde{Y}_t = \frac{Y_t}{X_t}
\]

To complete the model we introduce a monetary policy rule, set by the European Central Bank:

\[
R_t = R_{t-1}^{p_r} \left( \frac{\bar{\Pi}_{t}^{EMU}}{\bar{\Pi}^{EMU}} \right)^{(1-\rho_r)p_r} \tag{B.31}
\]

C Log Linearized model

Here we present the entire set of equation log linearized around the steady state. Lower case letters denote log version of the capital letter and the upper hat indicates log deviation from steady state. The demand functions for each type of good become:

\[
\hat{c}_{N,t} = \eta \hat{m}(J_t) + \hat{c}_t \tag{C.1}
\]

\[
\hat{c}_{h,t} = \eta \hat{m}(j_t) - \eta \hat{j}_t + \epsilon \hat{g}(s_t) + \hat{c}_t \tag{C.2}
\]

\[
\hat{c}_{f,t} = \eta \hat{m}(j_t) - \eta \hat{j}_t + \epsilon \hat{g}(s_t) - \epsilon \hat{s}_t + \hat{c}_t \tag{C.3}
\]

\[
\hat{c}_{h,t}^* = \eta \hat{q} + \eta \hat{m}(j_t) - \eta \hat{j}_t + \eta \hat{g}(s_t) + (\epsilon - \eta) \hat{s}_t + \hat{c}_t \tag{C.4}
\]

The first order conditions of the household are:
\[(1 - h\beta z)\hat{\lambda}_t = \hat{\zeta}_t - h\beta zE_t[\hat{\beta} + \hat{\zeta}_{t+1}] + \frac{h}{z-h}\hat{c}_{t-1} + \left[\frac{1 + h^2\beta}{1 - \frac{h}{z}}\right]\hat{c}_t + \left[\frac{h\beta z}{1 - \frac{h}{z}}\right]E_t\hat{c}_{t+1} - \frac{h}{z-h}\hat{\zeta}_t\]  

\[\hat{\lambda}_t = \hat{\beta}_t + E_t\left\{\hat{\lambda}_{t+1} + \hat{\beta}B - \hat{\Pi}_{N,t+1} + \hat{m}_t - \hat{m}_{t+1}\right\}\]  

\[\hat{r}_t^k = \frac{\Psi''(1)}{\Psi'(1)}\hat{u}_t\]  

\[\hat{q}_t = (1 - \delta)\frac{z}{\beta}E_t\hat{q}_{t+1} + \left[1 - \beta(1 - \delta)\right]E_t\hat{q}_{t+1} + \hat{\beta}_t + E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t\]  

\[\hat{q} = \eta_K z^2\hat{z}_t - \hat{c}_t^I - \eta_K z^2\hat{c}_{t-1} + (1 - \beta)\eta_K z^2\hat{u}_t - \beta z^2\eta_K E_t\hat{u}_{t+1}\]  

\[\hat{\zeta}_t + \hat{\epsilon}_t + \nu\hat{\epsilon}_t = \hat{\lambda}_t + \hat{\omega}_t\]  

The other equilibrium conditions are:

\[\hat{k}_t = \hat{\omega}_t + \hat{k}_{t-1}^p\]  

\[\hat{k}_{t-1}^p = \frac{(1 - \delta)}{z}\left[\hat{k}_{t-1}^p - \hat{z}_t\right] + \frac{z - (1 - \delta)}{z}\left[\hat{\epsilon}_t + \hat{\epsilon}_t^I\right]\]  

\[\hat{\beta}_t = -\beta\hat{\psi}\hat{\beta}_t\]  

### C.1 Firms

Firms production function in the two sectors are:

\[\hat{y}_{N,t} = \hat{a}_{N,t} - \alpha\hat{Z}_t + \alpha\hat{k}_{N,t} + (1 - \alpha)\hat{l}_{N,t}\]  

\[\hat{y}_{h,t} = \hat{a}_{h,t} - \alpha\hat{Z}_t + \alpha\hat{k}_{h,t} + (1 - \alpha)\hat{l}_{h,t}\]  

Cost minimization and marginal cost:

\[\hat{k}_{N,t} - \hat{\lambda}_t = \hat{\omega}_t - \hat{r}_t^k\]  

\[\hat{k}_{h,t} - \hat{\lambda}_t = \hat{\omega}_t - \hat{r}_t^k\]  

\[\hat{m}_{C_{N,t}} = (1 - \alpha)\hat{w}_t + \alpha\hat{r}_t^K - \hat{a}_{N,t}\]  

\[\hat{m}_{C_{h,t}} = (1 - \alpha)\hat{w}_t + \alpha\hat{r}_t^K - \hat{a}_{h,t}\]
Pricing equation:
\[
\hat{\pi}_{h,t} = E_t \{\beta \hat{\pi}_{h,t+1}\} + \frac{\phi - 1}{\psi} [\hat{m}_{h,t} + \hat{\gamma}(s_t) - \hat{J}_t]
\] (C.20)
\[
\hat{\pi}_{N,t} = E_t \{\beta \hat{\pi}_{N,t+1}\} + \frac{\theta - 1}{\psi} [\hat{m}_{N,t} + \hat{m}_t]
\] (C.21)

C.2 Relative prices and International variables

Relative prices:
\[
\hat{s}_t = \hat{s}_{t-1} + \hat{\pi}_{f,t} - \hat{\pi}_{h,t}
\] (C.22)
\[
\hat{g}_t = \hat{g}_{t-1} + \hat{\pi}_{T,t} - \hat{\pi}_{h,t}
\] (C.23)
\[
\hat{j}_t = \hat{j}_{t-1} + \hat{g}_t - \hat{g}_{t-1} + \hat{\pi}_{h,t} - \hat{\pi}_{N,t}
\] (C.24)
\[
\hat{\pi}_{T,t} = \gamma_{h} \hat{\pi}_{h,t} + \gamma_{f} \hat{\pi}_{f,t}
\] (C.27)

where \(\gamma_T\), \(\gamma_h\) and \(\gamma_f\) are respectively the steady state share of tradable in CPI and share of home and foreign produced tradable goods in TPI.

International variables become:
\[
\hat{b}_t = \frac{1}{\beta z} \hat{b}_{t-1} + \hat{\pi}_x t
\] (C.28)
\[
\hat{\pi}_x t = \text{share}_n x [\hat{c}_{h,t} - \hat{c}_t - \hat{c}_{f,t}]
\] (C.29)
\[
\hat{c}_{a_t} = \hat{b}_t - \hat{b}_{t-1} \frac{1}{z}
\] (C.30)

C.3 Market Clearing and Monetary Policy Rule

Market clearing conditions
\[
\hat{K}_t = \gamma_N \hat{K}_{N,t} + \gamma_T \hat{K}_{h,t}
\] (C.31)
\[
\hat{L}_t = \gamma_N \hat{L}_{N,t} + \gamma_T \hat{L}_{h,t}
\] (C.32)
\[
\hat{Y}_{N,t} = \hat{C}_{N,t}
\] (C.33)
\[
\hat{Y}_{h,t} = \frac{c_h}{y_h} \hat{C}_{h,t} + \frac{c_h^*}{y_h} \hat{C}_{h,t}^*
\] (C.34)
Real GDP:

\[
\hat{y}_t = \frac{P_h y_h}{P} [j_t - g_t - m(j_t) + y_{h,t}] + \frac{P_N y_N}{P} [m(j_t) + y_{N,t}] + \frac{I}{y} \hat{I}_t
\]

(C.35)

Monetary policy rule (ECB):

\[
\hat{r}_t = \rho r_{t-1} + (1 - \rho) \rho_n \frac{I_{PS}}{I_{EMU}} [\hat{m}(j_t) - \hat{m}(j_{t-1}) + \hat{\Pi}_{N,t}]
\]

(C.36)

D Jaimovich and Rebelo Preferences Model

Here we list the first order conditions, the detrended version and the log linearization of the equilibrium equations for the household when we use utility function (38). We just specify those equations that are different with respect to the baseline model.

\[
\zeta [c_t - hC_{t-1} - \epsilon t^L \psi L_t^{1+v} \Omega_t]^{-\sigma} + \lambda_t^{JR} [c_t - h\bar{c}_{t-1}]^{1-\mu} \Omega_{t-1}^{1-\mu} Z_t^{1-\mu} = \lambda_t
\]  

(D.1)

\[
- \zeta [c_t - hC_{t-1} - \epsilon t^L \psi L_t^{1+v} \Omega_t]^{-\sigma} \epsilon t^L \psi L_t^{1+v} - \lambda_t^{R} L_t^\mu \Omega_t = \lambda_t W_t
\]  

(D.2)

\[
\zeta [c_t - hC_{t-1} - \epsilon t^L \psi L_t^{1+v} \Omega_t]^{-\sigma} \epsilon t^L \psi L_t^{1+v} - \lambda_t^{R} + \beta_t E_t \{ \lambda_{t+1}^{JR} (1 - \mu) [c_{t+1} - h\bar{c}_{t+1}]^{1-\mu} \Omega_{t+1}^{1-\mu} Z_{t+1}^{1-\mu} \} = 0
\]  

(D.3)

and finally the law of motion of \( \Omega_t \)

\[
\Omega_t = (C_t - h\bar{C}_{t-1})^\mu \Omega_{t-1}^{1-\mu} (Z_t)^{1-\mu}
\]  

(D.4)

where \( \lambda_t \) is the lagrangian multiplier associated with the budget constraint, and \( \lambda_t^{JR} \) is the multiplier attached to law of motion of \( \Omega_t \). The model allows for growth rate so we need to detrend it. Following the procedure explained in the previous appendix and changing variables in order to express the equations as functions of stationary variables we can rewrite the previous equation as follows:
The log linearized version of the first order conditions using JR preferences are the following:

\[
\zeta_t \left[ \tilde{C}_t - \frac{h}{Z_t} \tilde{C}_{t-1} - \epsilon_t^L \psi^L L_t^{1+v} \tilde{\Omega}_t \right]^{-\sigma} + \tilde{\lambda}^{JR}_t \mu \left[ \tilde{C}_t - \frac{h}{Z_t} \tilde{C}_{t-1} \right]^{\mu-1} \tilde{\Omega}_{t-1}^{-\mu} = \tilde{\lambda}_t \\
- \zeta_t \left[ \tilde{C}_t - \frac{h}{Z_t} \tilde{C}_{t-1} - \epsilon_t^L \psi^L L_t^{1+v} \tilde{\Omega}_t \right]^{-\sigma} \epsilon_t^L \psi^L (1 + v) L_t^2 \tilde{\Omega}_t = \tilde{\lambda}_t \tilde{W}_t \\
\zeta_t \left[ \tilde{C}_t - \frac{h}{Z_t} \tilde{C}_{t-1} - \epsilon_t^L \psi^L L_t^{1+v} \tilde{\Omega}_t \right]^{-\sigma} \epsilon_t^L \psi^L L_t^{1+v} - \tilde{\lambda}^{JR}_t + \beta_t E_t \left\{ \tilde{\lambda}^{JR}_{t+1} (1 - \mu) \left[ \tilde{C}_{t+1} - h \tilde{C}_t \right] \tilde{\Omega}_{t+1}^{-\mu} \right\} = 0 \\
\tilde{\Omega}_t = (\tilde{C}_t - h \tilde{C}_{t-1})^\mu \tilde{\Omega}_{t-1}^{-\mu}
\]

where

\[
\tilde{\lambda}^{JR}_t = \lambda^{JR}_t X^\sigma \\
\tilde{\lambda}_t = \lambda_t X^\sigma \\
\tilde{C}_t = \frac{C_t}{X_t} \quad \text{and} \quad \tilde{\Omega}_t = \frac{\Omega_t}{X_t}
\]

The log linearized version of the first order conditions using JR preferences are the following:

\[
\zeta M1^{-\sigma} \left[ \tilde{\xi}_t - \sigma \tilde{M}_t \right] + \lambda^{JR}_t \mu [\tilde{\lambda}^{JR}_t + (\mu - 1) \tilde{M}_2 + (1 - \gamma) \tilde{w}_{t-1} = \lambda \tilde{\lambda}_t \\
M1\tilde{M}_t = c \tilde{\xi}_t - \frac{hc}{\bar{z}} \tilde{\xi}_{t-1} \tilde{\xi}_t - \epsilon_t^L \psi^L L_t^{1+v} \omega [\tilde{c}_t + (1 + v) \tilde{L}_t + \tilde{w}_t] \\
M2\tilde{M}_2 = c \tilde{\xi}_t - \frac{hc}{\bar{z}} [\tilde{c}_t - \tilde{z}_t] \\
\tilde{\xi}_t - \sigma \tilde{M}_t + \tilde{c}_t^L + v \tilde{L}_t + \tilde{w}_t = \tilde{\lambda}_t + \tilde{w}_t \\
\zeta M1^{-\sigma} p s_t^L L_t^{1+v} \tilde{\xi}_t - \sigma \tilde{M}_t + \tilde{c}_t^L + (1 + v) \tilde{L}_t + \lambda^{JR}_t \tilde{\lambda}_t = \\
= E_t \left\{ \beta \lambda^{JR}_t z^{1-\sigma} (1 - \mu) \tilde{\beta}_t + \tilde{\lambda}^{JR}_{t+1} + \mu \tilde{M}_2 - \mu \tilde{w}_t \right\}
\]

E Data Sources

We list here the time series used in the estimation:

1. Gross Value Added, Eurostat, Millions of euro, Current prices, Seasonally adjusted and adjusted data by working days, without public administration and community services.

2. Gross Value Added, Eurostat, Millions of euro, chain-linked volumes, reference year 2000, Seasonally adjusted and adjusted data by working days, without public administration and community services.
3. Final consumption expenditure, Eurostat, Millions of euro, Current prices, Seasonally adjusted and adjusted data by working days.

4. Gross capital formation, Eurostat, Millions of euro, Current prices, Seasonally adjusted and adjusted data by working days.

5. Real Effective Exchange Rate, Eurostat, deflator: consumer price indices - 12 trading partners.

6. Real GVA in non tradable sector, Eurostat, Millions of euro, chain-linked volumes, reference year 2000, Seasonally adjusted and adjusted data by working days.

7. Harmonized Index Consumer Prices inflation [IPS, EURO AREA], Eurostat.

8. Harmonized Index Consumer Prices Services Inflation [IPS], Eurostat.

9. Average number of usual weekly hours of work in main job, by sex, professional status, full-time/part-time and occupation (hours), Eurostat.

10. Current Account per cent of GDP, Versus all countries of the world, Eurostat.

11. 3-month money market interest rates for euro area countries, Eurostat.

12. GDP Deflator = (1)/(2) [IPS, EURO AREA]

13. Real Consumption = (3)/(12) [IPS, EURO AREA]

14. Real Investment = (4)/(12) [IPS]

15. Harmonized Index Consumer Prices inflation EURO AREA, Eurostat

16. HICP - Country weights, Eurostat

References


