Coordination Failure and the Financial Accelerator*

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Abstract

The paper considers the effect of short-term uncoordinated creditors in a real business cycle model with leveraged borrowers. Creditors (banks) receive imperfect signals regarding the profitability of borrowers (goods producing firms) and, based on this imperfect signal and their beliefs of other creditors actions, choose between rolling over and foreclosing on their loan. The global games methodology is employed to determine uniquely the behaviour of creditors in equilibrium. The paper finds that illiquidity risk for a borrower is higher when i) creditors are able to make better use of the collateral in the event of foreclosure, ii) the cost to the firm of uninstalling capital is higher, and iii) when the ratio of short-term to long-term creditors is higher. Due to the uncoordinated actions of creditors, there exists an inefficient incidence of premature foreclosure in the model. This inefficiency generates an illiquidity risk premium on external finance for firms. As firms become more leveraged, the size of the inefficiency increases as does the illiquidity risk. The interaction between the illiquidity risk premium and the evolution of firms’ net worth amplifies and propagates productivity shocks in the economy, similar in nature to the financial accelerator. The model also introduces an illiquidity shock, which originates from the credit market. Simulated method of moments is used to estimate the properties of the illiquidity shock, while impulse response analysis shows that the illiquidity shock generates highly persistent and hump-shaped output dynamics.

Keywords: Financial accelerator, Business cycles, Global games, Coordination failure

JEL Classification: D82, E32, E44, G12

1 Introduction

Financial institutions, governments and firms which borrow in credit markets face illiquidity risk (or rollover risk) - the risk of a panic induced run by creditors. Illiquidity risk is the result of two features: First, many institutions have shorter maturities on their liabilities than on their

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assets. By extension, this means that many institutions have assets which are less liquid than their liabilities. If creditors want to withdraw their credit before the assets yield their full return, the liquidated value of the assets will be insufficient to meet all creditors demands. The second feature is that large institutions often obtain external funds in relatively small amounts from very many creditors. This generates a coordination problem among the creditors. The interplay between these two features generates the incidence of solvent borrowers forced into failure by panicked creditors - a so called coordination failure.

Coordination failure is an extremely important characteristic of credit markets, and illiquidity risk is a significant risk faced by banks, governments and firms in need of external finance. Coordination failure and illiquidity risk have undoubtedly been a critical feature of the recent financial crisis. Given the importance of this problem therefore, it seems inconsistent that it has received such little attention in benchmark models used by macroeconomists.

The novel contribution of this paper is to consider, for the first time, the effect of short-term uncoordinated creditors in a real business cycle (RBC) framework. The model demonstrates that endogenous coordination failure in credit markets can be incorporated into this standard macroeconomic model in a rigorous and micro-founded manner. The result of allowing for coordination failure in credit markets is that productivity shocks are propagated and amplified through the economy, like a financial accelerator, while shocks originating directly from the credit market - illiquidity shocks - lead to highly persistent and hump-shaped output dynamics.

More specifically, the credit markets in this model are assumed to have the two features described in the opening paragraph: First, the debt contracts which firms issue (in order to purchase capital stock) give creditors the option to reconsider their investment, and pull out before the capital has been put to productive use. Second, it is assumed that firms borrow from a large number of anonymous creditors (banks) in the market. The firm then chooses the debt contract to maximize its expected profits subject to meeting the creditors participation constraint. The global games methodology is employed to determine uniquely under what conditions creditors rollover and under what conditions they foreclose. The paper finds that illiquidity risk for a borrower is higher when i) creditors are better able to make use of the collateral in the event of foreclosure, ii) the cost to the firm of uninstalling capital is higher (i.e. the tangibility of capital is lower), and iii) when the ratio of short-term to long-term creditors is higher. Due to the uncoordinated actions of creditors, there exists an inefficient incidence of premature foreclosure in equilibrium. This inefficient cost generates an illiquidity risk premium on external funds for firms. As firms become more leveraged, the size of the inefficiency increases as does the premium that external creditors demand to compensate for the increased illiquidity risk. The interaction between the illiquidity risk premium and the evolution of firms net worth drives the propagation and amplification of business cycles in this model. The model is able to address two further policy relevant questions: First, what is the effect of an illiquidity shock, originating in the credit markets, on the wider economy? Second, what are the welfare implications of regulating the ratio of short-term to long-term funding?

At this stage it is important to be explicit about what this model does not do. The paper does
not explain endogenously the shift towards shorter-term funding that has occurred over the past two decades in developed credit markets - the choice of funding maturity by borrowers is exogenously determined in the model. This would certainly be a useful area of future research. The paper also abstracts from many of the important features of modern macroeconomics models - nominal price and wage rigidities, consumption habits etc. - choosing instead the canonical RBC framework. This allows the mechanism by which coordination failure interacts in the model to be studied and understood with clarity. de Groot (2010) extends this paper in order to study the role of monetary policy in the presence of coordination failure in credit markets. Finally, it should be made clear that the credit market in the DSGE structure is made up of a sequence of static global games. The approach taken here should not be confused with the literature on dynamic global games.

The remainder of the paper is organized as follows: Section 2 reviews the related literature and Section 3 gives a model overview. Section 4 sets out the microfoundations of the credit market, solves for the ex ante probability of coordination failure and derives the debt contract. Section 5 aggregates the structure outlined in Section 4, introduces households and capital producers and presents the general equilibrium model. Section 6 conducts impulse response analysis of technology and illiquidity shocks, and uses simulated method of moments to estimate the stochastic illiquidity shock process. Section 7 concludes.

2 Related Literature

This paper draws heavily on two disparate areas of theoretical literature: The business cycle literature on the role of credit market conditions in the propagation of cyclical fluctuations and the global games literature on the role of coordination failure in credit markets.

Fisher (1933), writing during the great depression, was one of the first economists to give a central role to credit-market conditions in the propagation of cyclical fluctuations. However, it was not until Bernanke and Gertler (1989), that a theoretical model of the propagation mechanism was introduced. There are three seminal papers on which the majority of the recent literature on has built. These are Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999) (from now on, BGG).

In Bernanke and Gertler (1989) borrowers/firms with external finance requirements face the costly state verification problem of Townsend (1979). This motivates an inverse relationship between the borrower’s wealth and the expected agency cost of the borrower/lender relationship. In their model, agency costs are equated with monitoring/bankruptcy costs. In particular, a potential borrower with low net worth needs to rely relatively heavily on external finance; he thus faces a large risk of bankruptcy and a high premium on external finance. A number of other papers show that this basic analysis can be extended without affecting the qualitative results: For example, Gertler (1992) is the most relevant for this paper as it considers the case of multiperiod financial contracts.

The key novelty of my paper is that it replaces the costly state verification of lending assumption
with the coordination failure model of credit markets. In other respects the model in this paper utilizes several features of the BGG model, notably the central role for the endogenous evolution of borrower’s net worth in macroeconomic dynamics. In these models, the net worth of borrowers responds to changes in the valuation of real and financial assets that they hold. This element was added to the theoretical literature by Kiyotaki and Moore (1997). Kiyotaki and Moore analyze a stylized example in which land serves both as a factor of production and as a source of collateral for loans to producers. In their economy, a shock (to productivity, for example) lowers the value of land and hence producer’s collateral. This leads in turn to tighter borrowing constraints. In my model, it is capital that serves both as a factor of production and as a source of collateral.

Next, we turn attention to the coordination failure literature on which this paper builds. Coordination failure is an important feature of credit markets, which to date, has been neglected by dynamic general equilibrium macroeconomic models. The theoretical literature instead employs partial equilibrium analysis. Coordination failure is the source of bank runs, and was first formalized in a theoretical framework by Diamond and Dybvig (1983). This framework has been extended to other sources of coordination failure, including credit runs and the pricing of debt for non-financial corporations. The drawback of the Diamond and Dybvig model was that it predicted multiple equilibria. This result is problematic for the purposes of this paper since indeterminacy in the equilibrium prevents the ability to calculate the probability of a run and therefore the correct price of credit ex ante. Carlsson and van Damme (1993) showed that introducing noise into private signals in coordination games can lead to a unique equilibrium. Building on the insights of Carrlson and van Damm’s global games, Morris and Shin (2000, 2004, 2006, 2010), Goldstein and Pauzner (2005) and Rochet and Vives (2004) have all confronted the issue of solvent but illiquidity-vulnerable borrowers, in different settings.

The drawback of the global games models of coordination failure are that they are, in general, very stylized, finite period and partial equilibrium models. In contrast, my paper brings the insights on coordination failure, studied extensively in the global games literature, into a canonical RBC model. This has previously been used to understand credit cycles and financial accelerators but has, to date, not dealt with the issues of coordination failure in credit markets.

Finally, there is a growing empirical literature estimating the importance of illiquidity in firms’ credit spreads. See particularly Longstaff, Mithal and Neiss (2005), Collin-Dufresne, Goldstein and Martin (2001) and Ericsson and Renault (2006).

2.1 Global games: a brief description

This subsection provides a brief, non-technical, overview of the theory of global games, and the importance of the global games methodology for the model presented in this paper. The definitive guide to global games and its numerous applications is provided by Morris and Shin (2006). The economic phenomena of credit runs and illiquidity crisis come about because of creditors’ uncertainty about other creditors’ actions. These phenomena are therefore naturally modelled as games of incomplete information, where a player’s payoff depends on his own action, the actions of others,
and economic fundamentals. Harsanyi (1967) introduced the idea that rational behaviour in such environments depends not only on economic agents’ beliefs about economic fundamentals, but depends also on higher order beliefs - players’ beliefs about other players’ beliefs, players’ beliefs about other players’ beliefs about other players’ beliefs, and so on.

Global games, first studied by Carlsson and van Damme (1993), represent an environment in which these higher order beliefs can be tractably analyzed. Uncertain economic environments are summarized by a state and each player observes a different signal of the state with a small amount of noise. Assuming that the noise signal is common knowledge among the players, each player’s signal generates beliefs about fundamentals, beliefs about other players’ beliefs about fundamentals, and so on.

The benchmark result of the global games methodology is as follows: In a binary action continuum player game with strategic complementarities where each player has the same payoff function, there is a unique equilibrium where each player chooses the action that is a best response to a uniform belief over the proportion of his opponents choosing each action. Thus, when faced with some information concerning the underlying state, the prescription for each player is to hypothesize that the proportion of other players who will opt for a particular action is a random variable that is uniformly distributed over the unit interval and choose the best action under these circumstances. These beliefs and the actions they elicit are dubbed *Laplacian*.

The importance of the global games analysis for the purposes of this paper is that it is able to resolve the issue of self-fulfilling beliefs and multiple equilibria. If one set of beliefs brings about one outcome, while another set of self-fulfilling beliefs brings about another outcome, then there is apparent indeterminacy in the theory. While the beliefs may be fully rational, there is no guidance on which outcome will transpire without an account of how the initial beliefs are determined.

The indeterminacy of beliefs in many models (for example, Diamond and Dybvig mentioned above) with multiple equilibria are the consequence of two common modelling assumptions. First, the economic fundamentals are assumed to be common knowledge, and second, economic agents are assumed to be certain about others’ behaviour in equilibrium. These assumptions allow agents actions and beliefs to be perfectly coordinated and invites the multiplicity of equilibria. The global games methodology escapes this problem, by pinning down which set of self-fulfilling beliefs will prevail in equilibrium.

### 3 Model overview and basic assumptions

The model in this paper is a variant of the canonical Real Business Cycle (RBC) model, modified to allow for endogenous coordination failure in credit markets. In the limit, the cyclical properties of the model revert to those of the textbook RBC model. In de Groot (2010), the model is extended to included features of the New Keynesian framework, and specifically, implications for monetary policy are considered.

Figure 1 is a stylized diagram of the flows in the economy. The basic structure of the model is
as follows: There are four types of agents, called households, entrepreneurs, capital producers and financial intermediaries. The representative household lives forever, works, consumes and saves (in the form of risk-free interest-bearing deposits held at financial intermediaries). The financial intermediaries use households deposits to finance lending to entrepreneurs. In fact, financial intermediaries in this model have the technology for goods production. However, it is assumed that, ex ante the productivity of financial intermediaries is below the expected productivity of firms. It is therefore optimal ex ante for the financial intermediaries to lend to firms rather than produce themselves.

It is the relationship between entrepreneurs (borrowers) and financial intermediaries (creditors) that is central feature of this model. Entrepreneurs manage firms. At the start of each period, entrepreneurs purchase capital using their own net worth and borrowing from financial intermediaries. Capital purchased is used in combination with hired labour to produce output at the end of the period, by means of a constant-returns to scale (CRS) technology.

To endogenously motivate the existence of a risk premium on external funds, the model adopts a simple coordination problem faced by financial intermediaries. There are four important ingredients: First, that many uncoordinated financial intermediaries provide credit to each firm. Second, that creditors have the option to pull out (or foreclose) on the investment prior to production by the firm, but following the receipt of an (imperfect) signal regarding the firm’s profitability. Third, the firm’s capital stock, which acts as collateral for the external finance, is relatively intangible once installed but prior to production. This means that if a sufficient number of creditors foreclose, there is insufficient capital to continue producing, and creditors which rolled over receive nothing. Fourth, a foreclosing creditor is able to seize the capital (collateral) of the firm, and produce output using its own production technology. Importantly, this generates strategic complementarities in the decisions of creditors whether to rollover or foreclose: for a creditor, the incentive to foreclose is highest when the number of other creditors foreclosing reaches the level at which the firm can no longer operate. This generates the possibility of inefficient panic-induced credit runs.

Firms (and financial intermediaries) that get the opportunity to begin production, produce a homogenous output good that can be used as either a consumption or investment good. To close the model, capital producers purchase investment goods and old capital (from firms and financial intermediaries) to create new capital, via a production technology that exhibits capital adjustment costs. Each period, a fixed proportion of entrepreneurs exit and consume their net worth. The following period, new entrepreneurs enter.
4 Demand for capital, coordination failure and net worth

Having described the general set up of the model, this section provides a detailed analysis of the debt contracting problem between a single entrepreneur and a continuum of financial intermediaries. I show that, under the assumption of coordination failure among creditors, the debt contract provides a monotonically increasing relationship between the firm’s capital to net worth ratio and the illiquidity risk premium it pays on external funds.

Timing in the credit market can be broken down into three stages. At stage 1, entrepreneurs borrow from financial intermediaries to purchase capital. At stage 2, the firm experiences an idiosyncratic productivity shock (observed imperfectly by creditors), and the creditors decide whether to rollover or foreclose. At stage 3, firms that survive stage 2 and financial intermediaries that have seized a firm's productive assets produce. When we insert this into the DSGE framework, stages 1 to 3 all happen in the space of one "macro" period (or quarter). The timing can be thought of as $t$, $\tau$ and $t + 1$ with $t < \tau < t + 1$. However, for notational clarity, the remainder of this section will suppress all time subscripts, and I will refer to the start, interim and end of period dates.

Finally, it is worth noting that in this section, I derive the debt contracting problem assuming no aggregate uncertainty (i.e. $R^K$ is known). This is done for clarity. In the full general equilibrium model, aggregate uncertainty is introduced as it will be a key driver of the dynamic model. However, excluding aggregate uncertainty at this stage does not alter the properties of the debt contract derived below.
4.1 Payoffs for creditors

A entrepreneur uses her own net worth, \( N \) and borrowing, \( B = QK - N \) from financial intermediaries to purchase capital, \( K \) at price \( Q \). The ex post gross return on capital is \( \omega R^k \), where \( \omega \in [0, \infty) \) is an idiosyncratic shock with \( E(\omega) = 1 \) and \( R^k \) is the economy-wide return. Let \( F(\omega) \) and \( f(\omega) \) be the cdf and pdf of \( \omega \) respectively. External finance is provided by a continuum of uncoordinated financial intermediaries (creditors) of mass 1. There are two types of financial contract, \( D \in \{S, L\} \). A proportion, \( 0 < z < 1 \) are short-term (or active) contracts (type \( S \)) and \( 1 - z \) are long term (or passive) contracts (type \( L \)). Ex ante, the financial intermediaries do not know which contract they hold, although they do know the proportion of each type of contract, \( z \) in the market.\(^1\)

Following the purchase of capital, the firm observes its idiosyncratic productivity, \( \omega R^k \). Each creditor, \( i \) observes the idiosyncratic productivity shock with an imperfect signal, \( \omega^i = \omega + \varepsilon^i \), where \( \varepsilon^i \sim U[-\varepsilon, \varepsilon] \). Each creditor also observes whether he is an active or passive creditor. An active creditor has the choice, conditional on its signal, \( \omega^i \) and its beliefs regarding the actions of other creditors, of rolling over its loan or foreclosing on the firm. A passive creditor must automatically rollover its loan.

The financial contract between an entrepreneur and a financial intermediary is as follows: Suppose all creditors rollover their loans. The equilibrium debt contract may be characterized by a gross non-default loan rate, \( Z^r \), and a threshold value of the idiosyncratic shock \( \omega \), call it \( \overline{\omega} \), such that for values of the idiosyncratic shock greater than or equal to the value \( \overline{\omega} \), the firm is able to repay the loan at the contractual rate, \( Z^r \). That is, \( \overline{\omega} \) is defined by:

\[
\overline{\omega} R^k QK = Z^r B
\]  \hspace{1cm} (1)

When \( \omega \geq \overline{\omega} \), the firm repays the creditor the promised amount \( Z^r B \) and keeps the difference.

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\(^1\)Another way of motivating the two types of financial contracts - \( S \) and \( L \) - is as follows: Suppose the financial intermediary does not know its own idiosyncratic productivity ex ante, and observes it only at the interim stage. Ex ante, the financial intermediary knows that it has a return to capital of \( \gamma R^k \) with probability \( z \) and 0 with probability \( 1 - z \). A financial intermediary that observes 0 for its own productivity will, for all signals of the firm’s productivity, be weakly better off by rolling over its loan, and will therefore act like a passive (long-term) creditor.
equal to $\omega R^k Q K - Z^r B$. If $\omega < \overline{R}$, the firm cannot pay the contractual return and thus declares insolvency. In this situation, the creditors receive $\omega R^k Q K$ and the firm receives nothing.

However, a proportion, $z$ of the creditors have the choice of rolling over or foreclosing. The financial contract specifies that a foreclosing creditor is able to seize assets of the firm up to the value $\overline{R} I K$. The financial intermediary is able to put the seized capital to productive use - it has an ex post gross return per unit of capital from goods production of $\gamma R^k$, where $0 \leq \gamma < 1$. This means that a foreclosing creditor receives a gross return $\gamma \overline{R} I K Q K$ (or $\gamma Z^r$).

Once the firm has purchased and installed its capital (prior to the realization of the idiosyncratic shock), it is costly to remove. This is captured by $0 \leq \lambda \leq 1$. When $\lambda = 1$ there is no cost of removing capital and when $\lambda = 0$ the capital has become worthless outside of the firm for the remainder of the period. $\lambda$ therefore measures the degree of tangibility of the firm’s assets (or the firm specificity of capital). The result is that, at the interim stage, the firm’s asset value is $\lambda Q K < Q K$.

Suppose a proportion $0 < p < 1$ of the active creditors foreclose. The firm is left with capital equal to $\min\{0, \left(1 - \frac{\overline{R} I K}{\lambda}\right)\} K$ for use in production and has debt obligations of $\overline{R} I K Q K (1 - zp)$. If the firm cannot meet its obligations to provide collateral of value $\overline{R} I K$ to a foreclosing creditor or pay the non-default loan rate, $\overline{R} I K Q K$ to a rolled over or passive creditor, then the remaining value of the firm is divided equally among the creditors.

The complete set of payoffs is presented in Table 1, normalizing by $R^k Q K$.

<table>
<thead>
<tr>
<th>Payoffs from Rollover</th>
<th>Payoffs from Foreclosure</th>
<th>Range of $p$</th>
<th>Range of $\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{R}$</td>
<td>$\gamma \overline{R}$</td>
<td>when $0 \leq p \leq \frac{\lambda}{\overline{R} I K}$ and $\omega \geq \frac{\overline{R} I K (1 - zp) \lambda}{\lambda - zp \overline{R} I K}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{\omega}{(1 - zp)} \left(1 - \frac{\overline{R} I K}{\lambda}\right)$</td>
<td>$\gamma \overline{R}$</td>
<td>when $0 \leq p \leq \frac{\lambda}{\overline{R} I K}$ and $\omega &lt; \frac{\overline{R} I K (1 - zp) \lambda}{\lambda - zp \overline{R} I K}$</td>
<td></td>
</tr>
<tr>
<td>$0$</td>
<td>$\frac{\gamma \lambda}{zp}$</td>
<td>when $\frac{\lambda}{\overline{R} I K} &lt; p \leq 1$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Payoffs to the creditors

Note that if $\lambda > z \overline{R}$, the firm is not vulnerable to a credit run. This means that even if all the creditors foreclosed, the firm would be able to meet its obligations to return the collateral and still have a positive level of capital stock. In this scenario there is no coordination problem, since a creditor no longer has to consider the actions of the other creditors in its decision to rollover or foreclose.

It should now be clear why the uncoordinated nature of creditors creates a tension in the model. If a sufficient number of creditors foreclose (i.e. the firm gets liquidated), those that chose to rollover receive nothing. An example of the payoff structure (when the firm remains liquid) is shown in Figure 2.

In order to implement the global games methodology and solve for the unique switching equi-

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2It is assumed that $\gamma < E(\omega) = 1$, otherwise, the financial intermediary would be made better off by purchasing capital for itself directly at the start of the period, and ignoring its financial intermediation role.
librium (between complete foreclosure and all creditors rolling over), the model has to meet six conditions set out in Morris and Shin (2006). As I progress with the description of the model, I will highlight the relevant conditions the model meets. Table 1 and Figure 2 highlight the first such condition: the payoffs of both actions are monotonic and nondecreasing in $\omega$. Since the payoff to rolling over is nondecreasing in the state variable, $\omega$, and the payoff to foreclosure is constant in $\omega$, the net incentive to rollover is increasing in $\omega$, given opponents’ actions, $p$.

**Condition 1** State monotonicity (Assumption A2 in Morris and Shin, 2006, p65).

Figure 3 plots the net payoff (payoff from rolling over less payoff from foreclosure) for creditor $i$ for a given $\omega$ over the range of actions of other creditors, $p$. It is clear that the net incentive to switch strategies from rolling over to foreclosing is highest when the firm is on the verge of collapsing at the interim period (when $p = \frac{\lambda \omega}{z}$). It is clear therefore that this model is not "action" monotonic, and thus violates condition A1 of Morris and Shin (2005, p65). However, the model does satisfy the weaker action single crossing condition. This has been referred to by Goldstein and Pauzner (2005), with an application to bank runs, as an example of one-sided strategic complementarities.


Finally, it is worth noting at this stage that the firm will make positive net profits:

$$
\left(1 - \frac{zp\omega}{\lambda}\right) \omega R^k Q K - \sigma R^k Q K (1 - zp)
$$

when $\lambda > \sigma$ and when $\lambda > \sigma$, $0 \leq p \leq \frac{\lambda}{z\sigma}$ and $\omega \geq \frac{\sigma(1-zp)\lambda}{\lambda-zp\sigma}$, and 0 in every other state.

The goal from this point is to write a debt contract ($\omega, K$) that maximizes the firm’s expected profits, conditional on the actions of creditors. $\sigma$ is therefore an endogenous variable. The three key exogenous parameters are $\gamma$ (financial intermediaries’ own productive capabilities), $\lambda$ the tangibility of the firm’s capital stock at the interim period, and $z$ the proportion of short-term contracts in the market.

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3The method used in solving my model borrows from the Goldstein and Pauzner's (2005) paper.
4.2 Informational assumptions

4.2.1 Perfect information

It is worth considering, briefly, the model under the assumption of perfect information. If creditors observe $\omega$ perfectly at the interim stage, then if $\omega < \gamma \overline{\omega}$, it is optimal to foreclose, irrespective of the actions of other creditors (i.e. it is a dominant strategy to foreclose). Let $\omega_L = \gamma \overline{\omega}$ and let the interval $[0, \omega_L)$ denote the lower dominance region. The model does not strictly have an upper dominance region - a value of $\omega$ above which it is a dominant strategy to rollover, irrespective of the actions of other creditors. Even as $\omega \to \infty$, if all other creditors foreclose ($p = 1$), it is optimal for creditor $i$ to foreclose as well. In reality this appears a very unlikely outcome, and it seems reasonable to assume a value of $\omega$ above which any creditor would be irrational to contemplate foreclosing. Rochet and Vives (2004) and Goldstein and Pauzner (2005) faced a similar problem. Following them, I make a further assumption that when the firm is solvent, there exists a credible potential lender (e.g. a government institution) which can cover the liquidity needs of the firm. While in equilibrium, this potential lender is never used, the model has an upper dominance region, $(\omega_H, \infty]$ with $\omega_H = \overline{\omega}$. The model then meets the next condition:

**Condition 3** Uniform Limit Dominance (Assumption A4* in Morris and Shin, 2006, p67)

Technically, upper and lower dominance regions are required to guarantee a unique switching equilibrium. Having identified reasonable upper and lower dominance regions for $\omega$, it is clear that when $\omega \in [\omega_L, \omega_H]$ there is a coordination problem among creditors, and with perfect information, there are multiple equilibria. This is problematic since it means it is not possible to determine the ex ante probability of a credit run on the firm by creditors. The ex ante probability of a run on the firm is needed to price the debt. As mentioned, this is overcome by imposing a small amount
of noise into the signals received by creditors. It is the assumption of perfect information that
does creditors actions and beliefs to be perfectly coordinated, inviting multiplicity of equilibria.
Imperfect information escapes this problem and pins down the unique equilibrium that will prevail.

4.2.2 Imperfect information

As stated previously, let creditor \( i \) observe \( \omega^i = \omega + \varepsilon^i \), where \( \varepsilon_i \sim U [-\varepsilon, \varepsilon] \).

**Condition 4** Monotone Likelihood Property (Assumption A7 in Morris and Shin, 2006, p69).

The uniform distribution meets the requirements on the noise distribution. The model also
meets the following requirement:

**Condition 5** Continuity (Assumption A5 in Morris and Shin, 2006, p65).

Finally, I will show below that the model also meets:

**Condition 6** Strict Laplacian State Monotonicity (Assumption A3 in Morris and Shin, 2006, p65).

This says that there exists a unique \( \omega^* \) that solves \( \int_{p=0}^{1} \pi (p, \omega^*) dp = 0 \), where \( \pi (., .) \) is the net
incentive to rollover (payoff from rolling over minus the payoff from foreclosure). Given that the
model meets conditions 1 – 6 (or A1*, A2, A3, A4*, A5, A7 as set out by Morris and Shin ,2006), it
is possible to invoke the following theorem:

**Theorem 7** The model that meets conditions 1 – 6 has a unique "switching" equilibrium in which
creditors foreclose if they observe a signal below threshold \( \omega^* (\psi) \) and do not foreclose above, where
\( \psi = \{\omega, \gamma, \lambda, z \} \). Given \( \psi \), the proportion of creditors that foreclose depends only on the fundamen-
tals. It is given by:

\[
p (\omega, \omega^* (\psi)) = \begin{cases} 
\begin{array}{ll}
\frac{1}{2} + \frac{\omega^*(\psi) - \omega}{2\varepsilon} & \text{if } \omega^*(\psi) - \varepsilon \leq \omega \leq \omega^*(\psi) + \varepsilon \\
0 & \text{if } \omega > \omega^*(\psi) + \varepsilon
\end{array}
\end{cases}
\]

4.3 Computing the roll-over threshold, \( \omega^* (\psi) \)

To compute the threshold value, \( \omega^* (\psi) \), consider that creditor \( i \) with signal \( \omega^i = \omega^* (\psi) \) must be
indifferent, in expectation, between rolling over and foreclosing his investment. This creditor’s
posterior distribution of \( \omega \) is:

\[
\omega \mid \omega_i \sim U [\omega^* (\psi) - \varepsilon, \omega^* (\psi) + \varepsilon]
\]

Moreover, creditor \( i \) believes that the proportion of agents that foreclose, as a function of \( \omega \), is
\( p (\omega, \omega^* (\psi)) \). Also, it must hold that creditor \( i \)’s posterior distribution of \( p \) is \( p \sim U [0, 1] \). It
is therefore possible to set up the indifference condition (condition 6), \( \int_{p=0}^{1} \pi(p, \omega) \, dp = 0 \) that is uniquely solved by the threshold, \( \omega^* \):

\[
\int_{p=\frac{\omega}{\omega + z}}^{1} \frac{-\gamma \lambda}{zp} \, dp + \int_{p=0}^{\frac{\omega}{\omega + z}} \left( \min \left\{ \frac{\omega}{\left(1 - zp\right)} - \frac{zp\omega}{\lambda} \right\} \right) \, dp = 0
\]

If we assume that the indeterminate region is bounded by the upper and lower dominance regions, such that \( \gamma \overline{\omega} \leq \omega^*(\overline{\omega}) \leq \overline{\omega} \), then (2) is solved by:

\[
\omega^* + \frac{\omega^*(\overline{\omega} - \lambda)}{\lambda} \ln \left(1 - \frac{\lambda}{\overline{\omega}}\right) - \gamma \lambda \left[1 - \ln \left(\frac{\lambda}{\overline{\omega}}\right)\right] = 0
\]

or,

\[
\omega^* = \frac{\gamma \lambda^2 \left[1 - \ln \left(\frac{\lambda}{\overline{\omega}}\right)\right]}{\lambda + (\overline{\omega} - \lambda) \ln \left(1 - \frac{\lambda}{\overline{\omega}}\right)}
\]

when \( \varepsilon \) is arbitrarily close to 0. Equation (4) is the first key result of the model, and I devote some space to understanding its properties. The proposition below provides the properties of the threshold, \( \omega^*(\psi) \) (proofs are provided in Appendix A):

**Proposition 8**

i) \( \omega^*(\psi) \) is increasing in \( \overline{\omega} \), and \( \frac{\partial \omega^*(\psi)}{\partial \overline{\omega}} > \gamma \).

ii) \( \omega^*(\psi) \) is increasing in \( \gamma \).

iii) \( \omega^*(\psi) \) is decreasing in \( \lambda \).

iv) \( \omega^*(\psi) \) is increasing in \( z \).

The foreclosure threshold, \( \omega^*(\psi) \) is determined by the endogenous \( \overline{\omega} \), and the three exogenous parameters of the model, \( \gamma, \lambda \) and \( z \). The relationship between \( \omega^* \) and the endogenously determined \( \overline{\omega} \) is important for generating the positive relationship between the illiquidity risk premium and the firm’s leverage, which will be made clearer below. Importantly, \( \omega^* \) increases in \( \overline{\omega} \) at a rate faster than \( \gamma \). Remember that \( \gamma \overline{\omega} \) is the (efficient) foreclosure threshold (i.e. the foreclosure threshold in the presence of perfect coordination). Thus the size of range of idiosyncratic productivity shocks, \( \omega \) that result in inefficient foreclosure, \( [\omega^*(\overline{\omega}), \gamma \overline{\omega}] \) increases as \( \overline{\omega} \) increases. Parts ii) iii) and iv) of the proposition have an intuitive explanation. The parameter \( \gamma \) measures the goods producing productivity of the financial intermediaries. A higher \( \gamma \) therefore leads to a higher incidence of foreclosure, since the creditors are more productive than a larger proportion of the firms in the economy. Consider the extreme scenarios. If \( \gamma = 0 \), there is no incentive to foreclose for the creditor since they cannot generate any value from the capital - a creditor with \( \gamma = 0 \) acts like a passive creditor. At the other extreme, when \( \gamma \rightarrow 1 \), the financial intermediary has the same level of productivity as the average firm ex ante, making firm vs. financial intermediary goods production close substitutes.

The parameter \( \lambda \) measures the tangibility of the capital stock at the interim period. Increasing \( \lambda \) (increasing the interim tangibility of the capital stock) therefore decreases the incidence of foreclosure. This is because, for a higher \( \lambda \), a larger proportion of creditors’ must foreclose in order
to tip the firm into failure - the potential coordination failure in the market is therefore lower. In Figure 3, a rise in \( \lambda \) would be illustrated by a rightward shift of the minimum point. Similarly with \( z \) in part \( iv \), as the proportion of short-term creditors in the market decreases, the potential for a panic driven run on the firm is reduced. In the limit, there exists a sufficient proportion of passive (long-term) creditors, \( 1 - \xi \), such that an interim period run on the firm is not achievable, even if all \( \xi \) short-term creditors foreclose.

### 4.4 Debt contract

In the previous section, an ex ante threshold switching value of \( \omega, \omega^* \) for creditors’ actions was derived, for a given \( \overline{\omega} \). In this section we endogenize the non-default loan rate, \( Z^r \). From rearranging equation (1), and employing the definition, \( B = QK - N \), it is clear that \( Z^r \) is a function of two endogenous variables, \( K \) and \( \overline{\omega} \):

\[
Z^r = \frac{\overline{\omega} R^k QK}{QK - N}
\]

This section shows how the firm chooses \( K \) and \( \overline{\omega} \) to maximize its expected profits, subject to a relevant market participation constraint for creditors. This section derives the key relationship between the illiquidity risk premium and the leverage of the firm.

First, it is necessary to calculate the ex ante (expected) payoff for creditor \( i \) of holding the firm’s debt. When doing this, we set the noise term, \( \varepsilon \) arbitrarily close to 0. This remains consistent with the existence of a unique switching equilibrium. With \( \varepsilon \to 0 \), the ex ante probability that creditor \( i \) will rollover at the interim stage is \( \Pr (\omega_i > \omega^*) = 1 - F(\omega^* (\overline{\omega})) \), where \( F(.) \) is the cdf of \( \omega \). We begin by assuming that \( \gamma \overline{\omega} < \omega^* (\overline{\omega}) < \overline{\omega} \), and verify later this is the case. Since financial intermediaries are perfectly competitive by assumption, it must hold that in equilibrium, the creditor earns an expected return equal to the risk-free rate, \( R \) which it offers to depositors. This implies that:

\[
R^k QK \times \left[ \overline{\omega} \Pr (\omega \geq \overline{\omega}) + E (\omega \mid \overline{\omega} > \omega \geq \omega^* (\overline{\omega})) (F (\overline{\omega}) - F (\omega^* (\overline{\omega}))) + \gamma \lambda F (\omega^* (\overline{\omega})) \right] \tag{5}
\]

The first term in square brackets is the outcome where the firm is solvent, creditors rollover and receive the non-default loan return, \( \overline{\omega} R^k QK \). The second term is the outcome where creditors have rolled over but the firm is unable defaults as it is unable to repay the full non-default rate. The third term in square brackets is the outcome where the creditors foreclose and produce themselves. The cutoff, \( \overline{\omega} \) determines the division of expected non-coordination-failure profits, \( R^k QK \) between the firm and the creditor. Let the expected non-coordination-failure share of profits going to the creditor be:

\[
\Gamma (\overline{\omega}) = \int_0^{\overline{\omega}} \omega f(\omega) d\omega + \int_{\overline{\omega}}^{\infty} f(\omega) d\omega \tag{6}
\]
where \( 0 < \Gamma (\omega) < 1 \). Clearly, \( \Gamma' (\omega) = 1 - F (\omega) > 0 \). Next, let the expected costs of coordination failure to be:

\[
G (\omega^* (\omega)) = \int_0^{\omega^* (\omega)} (\omega - \gamma \lambda) f (\omega) d\omega
\]

and note that:

\[
G' (\omega^* (\omega)) = (\omega^* - \gamma \lambda) f (\omega^*) \frac{d\omega^*}{d\omega}
\]

The net share of profits going to the creditor are \( \Gamma (\omega) - G (\omega^* (\omega)) \). Analogously, the expected share of profits going to the firm is \( 1 - \Gamma (\omega) \). The debt contracting problem can then be written as:

\[
\max_{K, \omega} (1 - \Gamma (\omega)) R^k Q K \quad \text{subject to} \quad [\Gamma (\omega) - G (\omega^* (\omega))] R^k Q K = R (Q K - N)
\]

where the constraint is equation (5). It is possible to rewrite this problem by first explicitly defining the premium on external funds, \( s = R^k / R \) and then normalizing by net worth, \( k = Q K / N \):

\[
\max_{k, \omega} \mathcal{L} = (1 - \Gamma (\omega)) sk + \xi [(\Gamma (\omega) - G (\omega^* (\omega))) sk - (k - 1)]
\]

where \( \xi \) is the Lagrangian multiplier on the constraint that lenders earn their required rate of return in expectation.\(^6\) The first order conditions (FOCs) are:

\[
\begin{align*}
\omega & : \quad \Gamma' (\omega) - \xi [\Gamma' (\omega) - G' (\omega^* (\omega))] = 0 \quad (11) \\
K & : \quad [(1 - \Gamma (\omega)) + \xi (\Gamma (\omega) - G (\omega^* (\omega)))] s - \xi = 0 \quad (12) \\
\xi & : \quad [\Gamma (\omega) - G (\omega^* (\omega))] sk - (k - 1) = 0 \quad (13)
\end{align*}
\]

Assuming an interior solution, the FOC with respect to \( \omega \) implies it is possible to write the Lagrange multiplier, \( \xi \) as a function of \( \omega \):

\[
\xi (\omega) = \frac{\Gamma' (\omega)}{\Gamma' (\omega) - G' (\omega^* (\omega))}
\]

Differentiating \( \Gamma (\omega) - G (\omega) \), there exists an \( \overline{\omega} \) such that:

\[
\Gamma' (\omega) - G' (\omega^* (\omega)) \leq 0 \quad \text{for} \quad \overline{\omega} \leq \omega \leq \overline{\omega}
\]

implying that the net payoff to the creditor reaches a global maximum at \( \overline{\omega} \). Since \( \Gamma (\omega) - G (\omega^* (\omega)) \) is decreasing on \((\overline{\omega}, \infty)\), the lender would never choose \( \omega > \overline{\omega} \). Thus, it holds that \( \xi (\omega) > 0 \) in

\(^6\)Equation (10) is very similar in structure to the profit maximization problem faced by the firm with costly state verification in BGG (see Appendix A, pp1381-1382). The key difference is the functional form of the function, \( G (\omega) \). In my model, \( G (\cdot) \) captures the inefficiency cost of coordination failure and in the BGG model, it is the bankruptcy/monitoring cost.
equilibrium. Next, using the FOC with respect to \( k \), the risk premium, \( s \) is written as a function of \( \omega \):

\[
s(\omega) = \frac{\xi(\omega)}{1 - \Gamma(\omega) + \xi(\omega) (\Gamma(\omega) - G(\omega^*(\omega)))}
\]

The derivative is:

\[
s'(\omega) = s(\omega) \frac{\xi'(\omega)}{\xi(\omega)} \left( \frac{1 - \Gamma(\omega)}{1 - \Gamma(\omega) + \xi(\omega) (\Gamma(\omega) - G(\omega^*(\omega)))} \right)
\]

Finally, using the FOC with respect to \( \xi \), and \( s(\omega) \), the capital to net worth ratio, \( k \) is written as a function of \( \omega \):

\[
k(\omega) = 1 + \frac{\xi(\omega) (\Gamma(\omega) - G(\omega^*(\omega)))}{1 - \Gamma(\omega)}
\]

Again, computing the derivative:

\[
k'(\omega) = \frac{\xi'(\omega)}{\xi(\omega)} (k(\omega) - 1) + \frac{\Gamma'(\omega)}{1 - \Gamma(\omega)} k(\omega)
\]

**Proposition 9** \( s'(\omega) > 0 \) and \( k'(\omega) > 0 \). It follows that \( s = \chi(k) \), where \( \chi'(k) > 0 \).

Proposition 9 is the second important result of this paper. It says that the illiquidity risk premium increases with the capital to net worth ratio (or leverage ratio) of the firm. A firm which wishes to increase its capital purchases for a given level of net worth (i.e. increase its leverage), must also offer creditors a higher proportion of its capital as collateral for the loan. This in turn reduces the firms ability to meet the obligations of foreclosing creditors. Creditors, knowing ex ante that the risk of a run on the firm is higher will therefore demand a higher premium to compensate for the illiquidity risk that is inherent in the loan. At this point, it is possible to get a sense of how shocks are propagated in this model. Suppose for example, there is a negative, economy-wide productivity shock. This reduces firms’ net worth, which drives up the need for external finance by firms. However, this increases the illiquidity risk in the credit market, and drives up the spread between the return on capital and the risk-free rate. Firms cut back on production, which in turn lowers firms’ net worth further in the next period, and increasing further illiquidity risk in the credit markets. A complete discussion of this propagation mechanism will be discussed in more detail in Section 6.

**Proposition 9** generates two further points of interest. First, that \( \xi \) can be interpreted as the shadow price of net worth, and second, that it is possible to back out the non-default loan rate, \( Z^r \).

First, denote the illiquidity risk premium:

\[
\frac{R^k}{R} = s(\omega)
\]

where:

\[^7\text{See Appendix A for relevant proofs.}\]
\[ s(\omega) = \frac{\xi}{1 - \Gamma + \xi (\Gamma - G)} \]
\[ s^{-1}(\omega) = \frac{(1 - \Gamma)}{\xi} + (G - G) \]

By multiplying both sides by \( QK \), so that \( R^k QK = s(\omega) RQK \), it is evident that there exists a wedge between total profits, \( R^k QK \) and total finance required, \( QK \), above the risk-free rate, \( R \).

Second, denote the capital to net worth ratio as:
\[
\frac{QK}{N} = k(\omega)
\]
where:
\[
k(\omega) = 1 - \frac{\xi (\Gamma - G)}{1 - \Gamma}
\]  \hspace{1cm} (18)

Combining equations (17) and (18), it is possible to derive the following expression:
\[
(1 - \Gamma) R^k QK = \xi RN
\]  \hspace{1cm} (19)

The relationship says that the maximized expected profit for the firm, \( (1 - \Gamma) R^k K \) is given by the product of \( \xi \) and net worth. This shows that the Lagrange multiplier, \( \xi \) can be interpreted as the shadow price (internal value) of net worth (internal funds). We can also derive an expression for the credit spread, \( Z^r / R \). Using equation (1):
\[
\frac{Z^r}{R} = \frac{\pi R^k QK}{QK - N R} \frac{1}{QK - N R}
\]

it is possible to substitute in (17) and (18) to give:
\[
\frac{Z^r}{R} = \frac{\omega}{(\Gamma (\omega) - G (\omega))}
\]

Up to this point in the paper, the model has been kept as general as possible. However, to implement the model, it is necessary to specify the distribution from which firms draw their idiosyncratic productivity shocks. For this I use a lognormal distribution, which will be useful once we consider the credit market structure developed here in the context of the log-linearized RBC framework.

### 4.4.1 The log-normal distribution for \( \omega \), an example and comparative statics

Suppose \( \omega \) is distributed log-normally. Under the assumption that \( \ln(\omega) \sim N \left(-\frac{1}{2}\sigma^2, \sigma^2\right) \), it follows that \( E(\omega) = 1 \), and:
\[ E(\omega \mid \omega > x) = \frac{1 - \Phi \left( \frac{1}{\sigma} \left( \ln x - \frac{\sigma^2}{2} \right) \right)}{1 - \Phi \left( \frac{1}{\sigma} \left( \ln x + \frac{\sigma^2}{2} \right) \right)} \]  

(20)

where \( \Phi(.) \) is the c.d.f. of the standard normal. Using this, it is possible to obtain:

\[
\begin{align*}
\Gamma(\bar{\omega}) &= \bar{\omega}[1 - \Phi(\bar{\tau})] + \Phi(\bar{\tau} - \sigma) \\
G(\omega^*(\bar{\omega})) &= \Phi(z^* - \sigma) - \gamma \lambda \Phi(z^*)
\end{align*}
\]

(21)

(22)

where \( \bar{\tau} \) and \( z^* \) are related to \( \bar{\omega} \) through \( \bar{\tau} \equiv \left( \ln \bar{\omega} + \sigma^2 / 2 \right) / \sigma \) and \( z^* \equiv \left( \ln \omega^*(\bar{\omega}) + \sigma^2 / 2 \right) / \sigma \) respectively. Differentiating with respect to \( \bar{\omega} \) gives:

\[
\begin{align*}
\Gamma'(\bar{\omega}) &= [1 - \Phi(\bar{\tau})] - \bar{\omega} \phi(\bar{\tau}) \bar{\omega}' + \phi(\bar{\tau} - \sigma) \bar{\omega}' \\
G'(\omega^*(\bar{\omega})) &= \phi(z^* - \sigma) z'^* \omega'^* - \gamma \lambda \phi(z^*) z'^* \omega'^*
\end{align*}
\]

(23)

(24)

where \( \bar{\omega}' = 1 / (\sigma \bar{\omega}) \) and \( z'^* = 1 / (\sigma \omega^*) \)

The function, \( s = \chi(k) \), represents the supply curve for loanable funds. As demonstrated in Proposition 9, the supply curve is upward sloping - the higher the price of loanable funds, \( R^k / R \) (the return on capital relative to the risk free rate), the larger the quantity of loanable funds financial intermediaries are willing to supply the market. The comparative statics of the supply curve, \( \chi(k) \) relies on numerical methods. Despite the need for numerical methods, much of the intuition derives from the behaviour of \( \omega^* \) which was discussed in detail in the previous section. Figure 4 provides the graphical representation of these comparative statics.
Figure 4: Comparative Statics for the Supply of Capital

In each of the quadrants, one of the (upward sloping) supply curves is the benchmark parameterization: $\sigma^2 = 0.28, \gamma = 0.2, \lambda = 0.3, z = 1$.\(^8\) The quadrants show the effects on the supply curve for shifts in $\gamma, \lambda, N$ and $z$ respectively. The thinnest lines show the curve for the lowest value of the target parameter, and the thickest line for the highest value of the target parameter.

The demand for loanable funds (demand for capital) is represented by a horizontal line, parameterized to be 2% above the risk-free rate. The demand curve is horizontal, owing to the CRS nature of the production technology. The equilibrium of the loanable funds market in the benchmark parameterization implies an approximate capital to net worth ratio of 2 for the firm (or equivalently, a leverage ratio of 0.5). It also corresponds to $\bar{\omega} \approx 0.5$ and $\omega^* = 0.245$. Coordination failure in the credit market leads to inefficient firm failures since $0.245 = \omega^* > \lambda \omega \approx 0.1$.

The top left quadrant shows the effect of a shift in $\gamma$, the goods-producing productivity of the financial intermediaries. A rise in $\gamma$ leads to a steepening of the cost of funds curve. This is because the net payoff from foreclosing for a creditor (and therefore the risk of complete foreclosure by all creditors) has increased. Ex ante, creditors are therefore willing to supply less funding for a given price. The top-right quadrant shows the effects of a shift in $\lambda$, the tangibility of the firm’s capital at the interim period. When the assets remain more liquid between capital installation and production, there is less risk that a firm is unable to meet the foreclosure obligations of creditors.

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\(^8\)These are simply illustrative parameterizations. In Section 6, the deep parameters will be chosen to match steady steady moments of the DSGE model with long-run averages in US data.
This reduces the risk of a run on the firm and lowers the cost of borrowing external funds. In equilibrium, this results in higher external borrowing, and therefore higher leverage. We can think of a negative shock to $\lambda$ as capturing a market freeze in a financial crisis. The resaleability of assets drops sharply in a crisis, and thus drives up the cost of funds, forcing borrowers to lower investment and deleverage. The bottom-left quadrant is the effect of a change in the firm’s net worth, $N$. For example, an exogenous (or non-fundamental) rise in asset prices, $Q$ would increase firms’ net worth, lower their borrowing costs and expand their capital purchases. Finally, the bottom-right quadrant analyzes the effect of a change in the proportion of short-term debt contracts, $z$ that are issued. Less short term debt eases the coordination problem, since more creditors are passive (i.e. they passively rollover their credit). This again lowers borrowing costs for the firm.

It is important to note there is a distinct difference between the scenario in which all creditors are passive (or long-term, $z = 0$), and the scenario in which creditors are fully coordinated. Passive creditors have no outside option following the realization of the idiosyncratic firm shock and therefore are also better off to remain invested. Fully coordinated creditors however, have the possibility of seizing the capital and producing themselves when the idiosyncratic shock is sufficiently low, i.e. when $\omega < \gamma\omega$. A market with fully coordinated short-term creditors is therefore a preferable market structure to one in which all debt is long-term. The trade-off between the costs of coordination and the benefits of foreclosing on a unprofitable firm early, from a welfare perspective, warrants further investigation.

4.4.2 Costly state verification vs. coordination failure

At several points throughout this paper, I have mentioned the similarities and differences between BGG and my model. Here, I take the opportunity to formalize some of these differences. The standard friction assumed in the models by Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997) and BGG is an agency cost in the borrower lender relationship, sometimes rationalized as a monitoring cost or bankruptcy cost. More specifically, BGG use the costly state verification model of Townsend (1979). The choice of this friction is two-fold: first, there is empirical evidence for monitoring costs and second, it allows for the inclusion of credit market frictions in a business cycle model in a tractable manner.

However, it is clearly a stylized mechanism which ignores other imperfections that exist in many credit markets. In contrast, my paper derives a model with endogenous coordination failure among creditors - an important feature of modern credit markets. Coordination failure has been used as a substitute for the agency cost model, although there is no reason that the two frictions could not coexist in a model. My paper makes two contributions in this regard. First, it shows an alternative mechanism by which shocks are propagated via the financial sector. Second, and more importantly, my model generates three parameters, $\gamma$, $\lambda$ and $z$ which are unique to the model presented in this paper. These parameters proxy important characteristics of credit markets, and therefore allow us to study the endogenous effects of credit markets on the real economy in more depth.

As well as a descriptive comparison of the two methods, it is also possible to compare the
mechanisms at the technical level. The differences in the credit markets reduces to the function, \( G(\cdot) \), which in both models measures the inefficient cost of the market friction:  

\[
\text{Costly state verification:} \quad G^{CSV}(\omega) = \left( \mu \int_0^\omega \omega f(\omega) d\omega \right) R^h QK
\]
\[
\text{Coordination failure:} \quad G^{CF}(\omega^*(\omega)) = \left( \int_0^{\omega^*(\omega)} (\omega - \gamma \lambda) f(\omega) d\omega \right) R^h QK
\]

where the monitoring cost is equal to a proportion, \( \mu \) of the realized gross profits of the firm. Figure 5 and 6 plot the payoff functions of the CSV debt contract and the coordination failure (CF) debt contracts, over the range of \( \omega \), in comparison to a standard debt contract. The standard debt contract is represented by a solid line. The payoffs of the augmented debt contracts are shown with dashed lines. In the left panel, the agency cost, \( G^{CSV}(\omega) \) is captured by the shaded area, while in the right panel, the cost associated with coordination failure, \( G^{CF}(\omega^*(\omega)) \) is measured as the area of the larger shaded triangle minus the area of the smaller shaded triangle. Importantly, \( dG(\omega^*(\omega))/d\omega > 0 \) holds for both forms of credit market frictions. It is this property that drives the positive relationship between the external finance premium (or illiquidity risk premium in the coordination failure model) and the leverage of the firm.

\[ \text{Figure 5: CSV contract} \quad \text{Figure 6: CF contract} \]

5 Aggregation and the general equilibrium environment

In this section, the structure of the aggregate economy is described, considering the problems of households, entrepreneurs, capital producers and financial intermediaries in turn. The full set of equilibrium conditions, non-stochastic steady state conditions and log-linearized equilibrium conditions are set out in Appendix B.

5.1 Households

The household sector is reasonably conventional. There is a continuum of households of mass 1. Each household, \( h \) works, consumes, and saves using a financial intermediary that pays a riskless

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9Coordination failure also affects the other equilibrium relationships of the general equilibrium model in a different way to the agency cost mechanism. The technical analysis presented in this section pertains only to the static and partial equilibrium analysis. A discussion of the difference in other aggregate equilibrium relationships is reserved for Appendix B.
return. $C_h^t$ is household consumption, $H_h^t$ is household labour supply, $W_t$ is the wage for household labour, and $D_t^h$ is deposits held at financial intermediaries, $R_t$ is the riskless rate of return on deposits held during period $t$, and $\Pi_t$ is the transfer from exiting entrepreneurs. The household’s objective function is:

$$\max E_t \sum_{k=0}^{\infty} \beta^k \left\{ \ln \left( \frac{C_h^t}{1+\varepsilon} \right) + \frac{(H_h^{S,t+k})}{1+\varepsilon} \right\}$$

The household budget constraint is given by:

$$C_i^t = W_t H_h^{S,t} + R_t D_t^h - D_{t+1}^h + \Pi_t$$

The household chooses $\{C_h^t, D_t^h, L_t^h\}$ to maximize (25) subject to (26). Solving the household’s problem yields the FOCs for consumption/savings and labour supply respectively:\textsuperscript{10}

$$\frac{1}{C_t^h} = \beta E_t \left\{ \frac{1}{C_{t+1}^h} \right\} R_{t+1}$$

$$W_t = C_t^h \theta \left( H_t^{hS} \right)^\varepsilon$$

Aggregation across households is straight forward (households are identical) and the aggregate equilibrium conditions are:

$$\frac{1}{C_t} = \beta E_t \left\{ \frac{1}{C_{t+1}} \right\} R_{t+1}$$

$$W_t = C_t \theta \left( H_t^{hS} \right)^\varepsilon$$

where $H_t^S = \int_0^1 H_t^{hS} dh$, $C_t = \int_0^1 C_t^h dh$ and $D_t = \int_0^1 D_t^h dh$ are aggregate variables.

### 5.2 Entrepreneurs

There are a continuum of entrepreneurs of mass 1. Entrepreneurs manage the production of goods. The production of goods uses capital constructed by capital producers and labour supplied by both households and entrepreneurs. Entrepreneurs purchase capital from capital goods producers and finance the expenditures on capital with both entrepreneurial net worth and debt.

Entrepreneurs are risk neutral. To ensure that entrepreneurs do not accumulate enough funds to finance their expenditures on capital entirely from net worth, it is assumed that each entrepreneur survives until the next period with probability $v$. New entrepreneurs enter to replace those who exit. Since the debt contracting problem is ill-defined when net worth is zero, entrepreneurs are

\textsuperscript{10}To facilitate aggregation in the model, it is assumed that households sign a contract, $(W_t, L_t)$ with a firm at the start of each period (i.e. before the realization of the firm’s idiosyncratic productivity shock). It is assumed that the final producer (whether firm or financial intermediary), honours the contract.
endowed with $H_t^e$ units of labour that are supplied inelastically as a managerial input to the good production at entrepreneurial wage $W_t^e$.

An entrepreneur, $e$ starts period $t$ with capital, $K_t$ purchased from capital producers at the end of period $t-1$ and produces goods, $Y_t$ with labour and capital. Labour $L_t$ is a composite of household labour, $H_t$ and entrepreneurial labour, $H_t^e$:

$$L_t^e = (H_t^h)^{1-\Omega} (H_t^e)^{\Omega}$$

The entrepreneur’s project is subject to an idiosyncratic shock, $\omega_t$ which affects both the production of goods and the effective quantity of capital held by the entrepreneur. It is assumed that $\omega_t^e$ is i.i.d. across time and entrepreneurs and $E(\omega_t^e) = 1$. The entrepreneurs production function for goods is given by:

$$Y_t^e = \omega_t^e A_t (K_t^e)^{\alpha} (L_t^e)^{1-\alpha}$$

where $A_t$ is exogenous technology common to all entrepreneurs. An entrepreneur’s revenue (if creditors rollover) is:

$$\omega_t^e \left[ A_t (K_t^e)^{\alpha} (L_t^e)^{1-\alpha} + Q_t (1-\delta) K_t^e \right]$$

In period $t$, the entrepreneur chooses the demand for both household and entrepreneurial labour to maximize expected profits given capital, $K_t^e$ acquired in the previous period.

$$W_t = \phi_t(.) (1-\alpha) \frac{F(K_t^e, L_t^e)}{L_t^e}$$

where $\phi_t(.) = 1 - G(\omega^* (\bar{w}_t^e))$. At the end of period $t$, the entrepreneur purchases capital $K_{t+1}^e$ from capital producers at price $Q_t$. The capital is used as an input to the production of goods in period $t+1$. The entrepreneur purchases capital, $Q_t K_{t+1}^e$ partly with net worth, $N_{t+1}^e$ and partly by issuing debt, $B_{t+1}^e$:

$$Q_t K_{t+1}^e = N_{t+1}^e + B_{t+1}^e$$

The entrepreneur’s capital purchase decision depends on the expected rate of return on capital and the expected marginal cost of finance. The rate of return on capital between period $t$ and $t+1$, $R_{t+1}^k$ depends on the marginal profit from the production of goods and the capital gain. The expected rate of return on capital is given by:

$$E_t R_{t+1}^K = E_t \left( \alpha \frac{F(K_{t+1}^e, L_{t+1}^e)}{K_{t+1}^e} + (1-\delta) \frac{Q_{t+1}}{Q_t} \right)$$

In the presence of short-term debt and coordination problems among creditors, the marginal cost of external funds (which includes an illiquidity risk premium) depends on the entrepreneur’s balance
sheet condition. The details of the marginal cost of external funds were given in Section 4. In equilibrium, the cost of external funds between period $t$ and $t+1$ is equal to the expected rate of return on capital. The illiquidity risk premium on external funds, $s_t$ is then defined as the ratio of the entrepreneur’s cost of external funds to the cost of internal funds, where the latter is equal to the cost of funds in the absence of coordination problems, $R_{t+1}$:

$$s_t^e \equiv \frac{E_t \left( R^K_{t+1} \right)}{R_{t+1}}$$

In the absence of coordination problems, $s_t = 1$. Section 4 shows that the cost of external funds depends on the entrepreneur’s balance sheet condition. In particular, the illiquidity risk premium increases when a smaller fraction of capital expenditures is financed by the entrepreneur’s net worth:

$$s_t^e = s \left( \frac{Q_t K^e_t}{N^e_t} \right)$$

where $S(.)$ is an increasing function for $N^e_t < Q_t K^e_{t+1}$.

It is important to show that it is possible to aggregate across entrepreneurs’ equilibrium conditions. BGG were a little inaccurate when they state on p1350 that the proportionality assumption between monitoring costs and the realized gross payoff to capital is important for aggregation. In fact, the requirement is slightly weaker. Instead, it is only important that the participation constraint of creditors can be written in the form $[X(\overline{c})] * R^K Q K^e = R (Q K^e - N^e)$, (see equation (5) or the first equation on p1381 in BGG). To understand why, consider the two equilibrium conditions, $R^K R = s(\overline{c})$ and $\frac{K^e}{N^e} = s(\overline{c})$ given in Section 4. Since both $R^K$ and $R$ are economy-wide variables, it must hold that $\overline{c} = \overline{c}$ where $\overline{c} = \int_0^1 \overline{c}^e de$. Remember that, ex ante, all firms are identical except for their net worth, $N^e$. Since $\overline{c}^e = \overline{c}$, it must therefore hold that $\frac{K^e}{N^e} = s(\overline{c})$, or $\frac{K^e}{N^e} = \frac{K}{N}$. In other words, at the equilibrium, all firms, despite differing net worth positions, must choose debt contracts which ensure symmetry of the non-default loan rate, $Z^e$ and of the capital to net worth (leverage) ratio. Since this holds, it must hold that:

$$[X(\overline{c})] * R^K Q K^e = R (Q K^e - N^e)$$

where $K$ and $N$ are aggregate capital stock and aggregate net worth respectively. Having proved that aggregation, it is possible to derive the aggregate net worth of entrepreneurs at the end of period $t$. This is the sum of equity held entrepreneurs who survive from period $t - 1$ and the
aggregate entrepreneurial wage: 11

\[ N_{t+1} = uV_{t+1} \]
\[ N_{t+1} = u \left[ \phi_t(.) R_t^K Q_{t-1} K_t - R_t (Q_{t-1} K_t - N_t) \right] \]

In Section 4, expected profits of the firm, \( V_t = (1 - \Gamma_t(.)) R_t^K Q_{t-1} K_t \). Above, I have simply substituted for the financial intermediaries’ participation constraint.

### 5.3 Capital producers

A representative capital producer purchases investment goods and old capital, and uses a constant-returns-to-scale capital production function, \( \eta(I_t/\phi_t(.) K_t) \phi_t(.) K_t \), to produce new capital. 12 The function \( \eta(.) \) is increasing and concave. When there are no adjustment costs, the price of capital, \( Q_t = 1 \) for all \( t \). Aggregate capital accumulation is then:

\[ K_{t+1} = (1 - \delta) \phi_t(.) K_t + \eta \left( \frac{I_t}{\phi_t(.) K_t} \right) \phi_t(.) K_t \]

Taking the price of capital, \( Q_t \) as given, capital producers choose inputs \( I_t \) and \( K_t \) to maximize profits from the formation of new capital:

\[ \max_{K_t,I_t} \Pi_t^K = Q_t \eta \left( \frac{I_t}{\phi_t(.) K_t} \right) \phi_t(.) K_t - I_t \]

The FOC with respect to \( I_t \) gives \( Q_t \):

\[ Q_t = \frac{1}{\eta' \left( \frac{I_t}{\phi_t K_t} \right)} \]

and the FOC with respect to \( K_t \) gives:

\[ I_t = Q_t \eta \left( \frac{I_t}{\phi_t K_t} \right) \phi_t K_t \]

### 5.4 Financial intermediaries

There are a continuum of financial intermediaries of mass 1. Each period, financial intermediary \( i \) receives deposits, \( D^h \) from household \( h \) and uses these funds to provide loans to firms. The financial intermediary fully diversifies its lending portfolio by lending an equal fraction to each firm \( h \) in \( H \). Financial intermediaries are also assumed to be perfectly competitive which ensures that they make

---

11 For the parameterization, I assume that \( \Omega \) (the share wage income accruing to the entrepreneur) is arbitrarily close to 0, since the entrepreneurial wage is only introduced no entrepreneur has zero net worth, which would be problematic for the debt contracting problem.

12 Notice that coordination failure causes physical retardation in the quality of capital each period, above the natural rate of depreciation, which is captured by the term \( \phi_t(.) < 1 \).
zero profits in expectation on each loan contract, and exactly zero profits on their aggregate lending portfolio.

What I have neglected to clarify so far is how aggregate uncertainty is dealt with (i.e. uncertainty about $R_{t+1}^K$) in the model. Initially, we expressed the non-default loan rate as:

$$Z_t^* B_{t+1} = \bar{\omega} R_{t+1}^K Q_t K_{t+1}$$

From this, it is clear that it is the ex post realization of $R_{t+1}^K$ that determines $\bar{\omega}$, not $E_t R_{t+1}^K$. It is therefore the entrepreneur/firm that bears all the aggregate risk. This means that the loan rate $Z_t^*$ adjusts countercyclically to macroeconomic conditions - a realization of $R_{t+1}^K$ that is lower than expected raises $Z_t^*$ to compensate for the increased default probability. This means that in effect the debt contract is state-contingent, with a participation constraint (5) for each realization of $R^K$. In effect this says that after the realization of $R_{t+1}^K$ (and $\omega_{t+1}^e$) when the creditor decides to rollover or foreclose, the loan rate for the second half of the project is higher than the first half, if $R_{t+1}^K < E_t R_{t+1}^K$.

Financial intermediaries are identical, implying that $B_t = \int_0^1 \int_0^1 B_t^{fj} df dj$. In equilibrium, household deposits at financial intermediaries equal total loanable funds supplied to firms:

$$D_t = B_t$$

### 5.5 Shock processes

The model allows for two sources of variation: shocks to economy wide technology, $A_t$ and shocks to the rollover threshold, $\omega_t^*$ (which I call an illiquidity shock). The exogenous shock process for technology is given by:

$$A_t = A_t^{\rho_A} e^{\varepsilon_t^A}$$

where:

$$\varepsilon_t^A \sim i.i.d. N(0, \sigma_A^2)$$

In Section 4 an endogenous rollover threshold, $\omega^*$ was derived. There are two potential sources of a shock to $\omega_t^*$, either as an exogenous shock to the parameters $\gamma$ or $\lambda$:

$$\omega^* \left( \bar{\omega}_t, N_t^\gamma \right) = \frac{\gamma N_t^\gamma \lambda^2 \left[ 1 - \ln \left( \frac{\lambda}{\bar{\omega}_t} \right) \right]}{\lambda + (\bar{\omega}_t - \lambda) \ln \left( 1 - \frac{\lambda}{\bar{\omega}_t} \right)}$$

$$\omega^* \left( \bar{\omega}_t, N_t^\lambda \right) = \frac{\gamma (\lambda N_t^\lambda)^2 \left[ 1 - \ln \left( \frac{\lambda N_t^\lambda}{\bar{\omega}_t} \right) \right]}{\lambda N_t^\lambda + (\bar{\omega}_t - \lambda N_t^\lambda) \ln \left( 1 - \frac{\lambda N_t^\lambda}{\bar{\omega}_t} \right)}$$
where $E_t(N_i) = 1$ for $i = \gamma, \lambda$ and $N_i = \frac{\partial N_i}{\partial t} \exp(\varepsilon_i)$, where $\varepsilon_i \sim N(0, \sigma_i^2)$. A temporary increase in the productivity of the financial sector (an increase in $\gamma$) would increase the risk of coordination failure in credit markets, as previously explained in Section 4. Similarly, a temporary fall in the tangibility of installed capital (a fall in $\lambda$) would also make lending in credit markets more risky (see also Section 4). This second shock can be thought of more generally as a scenario in which the (financial) assets on an institution's balance sheet become more illiquid due to a sudden fall in confidence in the market.

5.6 Additional aggregation

Since firms identically choose the same ratio of capital to labour in production, independent of net worth, it holds that: $H_t^D = \int_0^1 H_t^D dj$ and $K_t = \int_0^1 K_t^d de$. In equilibrium, the labour market clears, implying that $H_t^S = H_t^D$.

Ex post goods production is not identical across producers. Firms with $\omega^j \geq \omega^*$ produce output:

$$Y^j = \omega^j A_t \left( K_t^j \right)^{\alpha} \left( L_t^j \right)^{1-\alpha}$$

When $\omega^j < \omega^*$, the financial intermediaries takeover production and produce output:

$$Y^j = \gamma A_t \left( \lambda K_t^j \right)^{\alpha} \left( \lambda L_t^j \right)^{1-\alpha}$$

Aggregating across $j$ gives economy wide output:

$$Y_t = \int_0^1 Y_t^j dj$$

$$= \{ E(\omega | \omega > \omega^*) Pr(\omega > \omega^*) + \gamma \lambda Pr(\omega \leq \omega^*) \} A_t K_t^a L_t^{1-\alpha}$$

$$= \phi_t(.) F(K_t, L_t)$$

$$= (1 - G_t(.)) F(K_t, L_t)$$

Finally, $E_t R_{t+1}^k$ and the equilibrium wage rate in the economy are:

$$E_t R_{t+1}^k = E_t \left( \frac{\alpha F(K_{t+1}, L_{t+1}) + (1 - \delta) Q_{t+1}}{Q_t} \right)$$

$$W_t = \phi_t(1 - \alpha) \frac{F(K_t, L_t)}{L_t}$$

respectively. It is important to note that $R_t^k$ is not the aggregate expected return on capital. Instead, the aggregate expected return on capital is $\phi_t(.) R_t^k$ Instead, $R_t^k$ is interpretable as the frictionless (non-coordination failure) return on capital.
5.6.1 Aggregate resource constraint

In order to ensure that the model is internally consistent, it is possible to derive the aggregate resource constraint from the representative household budget constraint:

\[ C_t = W_t L_t + R_t D_t - D_{t+1} \]

Substituting for \( D_t, D_{t+1} \), and \( \Pi_t = (1 - \nu) V_t \) gives:

\[ C_t = W_t L_t + R_t (Q_{t-1} K_t - N_t) - (Q_t K_{t+1} - N_{t+1}) + (1 - \nu) V_t \]

Since \( N_{t+1} = \nu V_t, N_{t+1} + \Pi_t = V_t \):

\[ C_t = W_t L_t + R_t (Q_{t-1} K_t - N_t) - Q_t K_{t+1} + \phi_t(\cdot) R_t^t Q_{t-1} K_t - R_t (Q_{t-1} K_t - N_t) \]

Since the production function exhibits CRS, it is possible to invoke Euler’s Theorem, that \( F(K, L) = F_K K + F_L L \), and using definitions for \( W_t \) and \( R_t^t \), gives:

\[
C_t = \phi_t(\cdot) F_L (K_t, L_t) L_t - Q_t K_{t+1} + \phi_t(\cdot) \left( \frac{F_K (K_t, L_t) + (1 - \delta) Q_t}{Q_{t-1}} \right) Q_{t-1} K_t \\
= \phi_t(\cdot) (F_L (K_t, L_t) L_t + F_K (K_t, L_t) K_t) - Q_t K_{t+1} + \phi_t(\cdot) (1 - \delta) Q_t K_t
\]

Finally, using the capital accumulation equation:

\[ C_t = \phi_t(\cdot) F (K_t, L_t) - Q_t \eta \left( \frac{I_t}{\phi_t K_t} \right) \phi_t K_t \]

or:

\[ Y_t = C_t + I_t \]

5.7 Parameterization

Many of the underlying parameters of this model are standard features of any DSGE model, and as such, I employ values commonly used in the DSGE literature. For the capital share of income, \( \alpha = 0.35 \). Since one time period in the model is equal to one quarter the subjective discount factor is \( \beta = 0.99 \), the depreciation rate of capital is \( \delta = 0.025 \) and the persistence parameter of the technology shock is \( \rho_a = 0.95 \). There is less consensus in the literature regarding the parameters of the household utility function and the elasticity of the price of capital with respect to the investment-capital ratio. Taking from Gali (2008), \( \theta = 1 \), the elasticity of labour supply is \( 1/\varepsilon = 1/3 \), and taking from BGG, \( \eta = 0.25 \).13

13While I have chosen parameters values commonly used in the literature, its should be noted that the amplifying effect of coordination failure on output depends crucially on the elasticity of labour supply. When the labour supply is relatively elastic (\( \varepsilon \) is small) then the amplification effect is much larger.
The non-standard parameters of the model, $\gamma$, $\lambda$, $\nu$, $\sigma^2$ capture characteristics of the credit market.\textsuperscript{14} These parameter values are not directly observable. Instead, these values are calculated indirectly in order to match four steady state moments of the model with long-run averages in US data. Three of these state state moments are taken from BGG. The fourth is chosen to match evidence from Berger et al. (1996):

1. Risk spread, $R^k - R$ equal to 2%,

2. Annualized business failure rate, $F(\bar{z}) = 3\%$,

3. Capital to net worth ratio, $K/N = 2$,

4. Average recovery ratio of liquidated firms, $\int_0^{\omega^*} \frac{\gamma \lambda}{\omega} f(\omega) d\omega = 0.5$.

The first approximately matches the historical spread between the prime lending rate and the six-month treasury bill rate. The second is approximately the historical corporate bankruptcy rate. The third approximately matches the leverage ratio $(1 - N/K)$ of 0.5. The fourth moment approximates estimates produced by Berger et al. (1996). Berger et al. estimates what percentage of the book value each category of asset on the balance sheet produces when a firms’ activities are discontinued. In terms of the model, this equates to calculating the ratio of $\omega R^k QK$ (the value of the asset before exit) to $\gamma \lambda R^k QK$ (the value of the asset after exit), averaged across all the firms that failed at the interim stage, $\omega \in (0, \omega^*)$.

Matching these four moments implies that the following parameter values should be used: $\gamma = 0.44$, $\lambda = 0.38$, $\nu = 0.954$ and $\sigma^2 = 0.12$. Two of these variables, $\nu$ and $\sigma^2$ also appear in the Bernanke et al. (1999) model. The parameterization used here have lower values for $\nu$ and $\sigma^2$ with 0.945 vs. 0.973 and 0.12 vs. 0.28 respectively. The full list of parameters used in the benchmark impulse response analysis for the model are given in Table 2.

\textsuperscript{14}In the benchmark parameterization, it is assumed that all creditors are short-term (active) creditors, $z = 1$.
Table 2: Baseline parameterization of the model

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ Capital share of income</td>
<td>0.35</td>
<td>Standard</td>
</tr>
<tr>
<td>$\beta$ Discount rate</td>
<td>0.99</td>
<td>Standard</td>
</tr>
<tr>
<td>$\delta$ Depreciation of capital</td>
<td>0.025</td>
<td>Standard</td>
</tr>
<tr>
<td>$\theta$ Weighting of labour in the utility function</td>
<td>1</td>
<td>Standard</td>
</tr>
<tr>
<td>$1/\varepsilon$ Labour supply elasticity</td>
<td>1/3</td>
<td>Gali (2008)</td>
</tr>
<tr>
<td>$v$ Transfer from firms to households</td>
<td>0.954</td>
<td>Indirectly determined</td>
</tr>
<tr>
<td>$\sigma^2$ Variance of idiosyncratic shock</td>
<td>0.12</td>
<td>Indirectly determined</td>
</tr>
<tr>
<td>$\gamma$ Productivity of financial intermediaries</td>
<td>0.44</td>
<td>Indirectly determined</td>
</tr>
<tr>
<td>$\lambda$ Interim period tangibility of capital</td>
<td>0.38</td>
<td>Indirectly determined</td>
</tr>
<tr>
<td>$z$ Proportion of short-term creditors</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_a$ AR(1) parameter of technology shock</td>
<td>0.95</td>
<td>Standard</td>
</tr>
<tr>
<td>$\rho_R$ AR(1) parameter of rollover shock</td>
<td>0.95</td>
<td>-</td>
</tr>
</tbody>
</table>

This parameterization also implies that 0.4\% ($= F(\omega^*)$) of firms are liquidated early every period. Also, since the product of $\gamma$ and $\lambda$ is quite small, it implies that the proportion of firms that are "efficiently" liquidated is very close to zero. Thus, almost all of the 0.4\% of firms that are liquidated early are the result of inefficient coordination failure.

It is also possible to compare the elasticity of the external finance premium with respect to leverage, $\chi^{CSV}$ in BGG with the elasticity of the illiquidity risk premium with respect to leverage this model, $\chi^{CF}$:

$$E_t r^k_{t+1} - r_{t+1} = \chi^X (q_t + k_{t+1} - n_{t+1})$$

where $X = CSV$ or $CF$. In BGG, $\chi = 0.05$, while in this model it is 0.29. The rollover risk premium is therefore more sensitive in this model to changes in leverage than the external finance premium is in BGG. However, it should be noted that $\chi$ is sensitive to the inclusion of nominal frictions in the model. Excluding nominal rigidities from BGG alters the appropriate values for the exogenous parameters, $\mu$, $\sigma^2$ and $v$. To ensure the steady state moments $R^k - R$, $K/N$ and $F(\overline{z})$ still match long-run averages in the US data, $\mu$ rises from 0.12 to 0.17, $\sigma^2$ falls from 0.28 to 0.12 and $v$ falls from 0.973 to 0.956. This raises the $\chi^{CSV}$ from $\chi = 0.05$ to $XXX$.

6 Impulse response analysis

This section analyzes impulse response functions to negative technology shocks and illiquidity shocks to explore the role of coordination failure in credit markets and its effects on macroeconomic variables. Each panel shows a grid of nine subpanels, which show the responses of output, consumption,
investment, the expected return on capital \( (E(R_k)) \) and the safe return \((R)\), the illiquidity risk premium, price of capital, capital stock, net worth and the capital to net worth ratio \((QK - N)\) in terms of percentage deviations from the steady state of a 1% deviation of the exogenous shock variable from its steady state, respectively. Time is measured in quarters, and the y-axis is measured in percentage deviation from the steady state.

6.1 Exogenous technology shock

Figure 6 shows the result of a 1% negative technology shock. The dotted line shows the response of the RBC model, while the solid line shows the response of the full model with coordination failure. In the standard RBC model, the transitory shock to technology causes an immediate fall in output and asset prices. Along the transition path, output and asset prices return to their initial steady-state levels. With no short-term creditors and no coordination problems in credit markets, the illiquidity risk premium is constant at zero. When the credit market contains short-term creditors and suffers from the possibility of coordination failure, the response of output to a transitory technology shock is amplified. This is because the negative shock to technology causes a fall in asset prices (net worth) and leads to a rise in the illiquidity risk premium. This amplified response represents distortions in the resource allocation induced by coordination failure in the credit markets. Asset prices and investment - variables that are closely linked to the behaviour of creditors - deviate from their efficient levels by a larger amount in the presence of coordination failure. It is important to note that the magnitude of the response of output is sensitive to the elasticity of labour supply - a more elastic labour supply would generate an exaggerated response in output.
Figure 6: Response to a transitory shock to technology (full model and RBC model)

Figure 7 again shows the result of a 1% negative technology shock. In this figure, the dotted line shows the response of the BGG model.\textsuperscript{15} The paths of the real variables are very similar. The largest difference is the effect of the risk premium, which has a larger response in the model with coordination failure. However, there is less persistence in the coordination failure model in terms of investment, asset prices, net worth and leverage.

\textsuperscript{15}The BGG model has been modified to provide an accurate comparison with the model of coordination failure. First, nominal rigidities have been stripped from the model. Second, BGG argue that several terms, including $\xi_t$, can be ignored from the impulse response analysis as they are only of second order importance. Here I report impulse responses with a fully linearized model (i.e. including $\xi_t$ as an endogenous variable).
6.2 Exogenous illiquidity shock

Figure 8 shows the response of the economy to a positive 1% transitory shock to $\gamma$ (the goods producing productivity of the financial sector). For illustrative purposes, I have used a persistence parameter of $\rho_{\mu\gamma} = \rho_{\mu\lambda} = 0.95$. In the subsequent section, simulated method of moments will be employed to estimate the parameters of the illiquidity shock process from the US.

A higher $\gamma$ can represent a financial sector that is more fragile (or sensitive) to macroeconomic events. It can also be interpreted as a fall in confidence by creditors (firms’ productivity is suddenly lower, relative to the financial sector expectations), leading to a rise in the rollover threshold and a rise in the number of firms that are subject to panic induced failure due to foreclosing creditors. As a result of the initial shock, the illiquidity risk premium on lending rises (the expected return on capital rises by more than the risk-free rate). Output also falls. Household consumption rises as a result of the rise in the risk-free rate, which means that the fall in output, is the result of a sharp collapse in investment spending. Importantly, output has a hump-shaped response to an illiquidity shock and its deviation from the steady state is highly persistent. de Groot (2010) shows that if
the incidence of illiquidity shocks in the economy are high, then it will have significant implications for monetary policy responses.

Figure 8: Response to a transitory rollover shock (γ)
(full model)

Figure 9 presents the response of the model to a shock to λ, the tangibility of capital. A sudden drop in λ can be interpreted as a sudden fall in liquidity in the credit market, as creditors can no longer easily access their collateral. This again, generates heightened tensions in the credit market, and an increased incidence of panic based premature failure. Qualitatively, the response is similar to that seen in Figure 8. Output displays a highly persistent, hump-shaped trajectory, while consumption rises as a result of the shock. This is because it acts in the same way on the model, by directly shifting the rollover threshold, ω*. However, a 1% shock to the tangibility of capital has a much larger initial impact on both the illiquidity risk premium and the response of output.
In the next section, some of these impulse responses shall be subject to robustness analysis, and how sensitive they are to the parameterization of the model.

6.3 Estimation of the illiquidity shock process

In the previous section we showed the impulse responses of a negative illiquidity shock. In this section, I estimate the unknown parameters, $\rho_N$ and $\sigma_N$, using simulated method of moments following Lee and Ingram (1991). The deep parameters, as well as the parameters for the technology shock process are held fixed at the values presented in Table 2.
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation of output</td>
<td>1.66</td>
</tr>
<tr>
<td>Cross correlation, $\text{corr}(y_t, y_{t-1})$</td>
<td>0.91</td>
</tr>
<tr>
<td>Standard deviation of risk premium</td>
<td>0.32</td>
</tr>
<tr>
<td>Cross correlation, $\text{corr}(rp_t, y_t)$</td>
<td>0.06</td>
</tr>
<tr>
<td>Cross correlation, $\text{corr}(rp_t, y_{t+1})$</td>
<td>-0.20</td>
</tr>
<tr>
<td>Cross correlation, $\text{corr}(rp_t, y_{t+2})$</td>
<td>-0.40</td>
</tr>
<tr>
<td>Cross correlation, $\text{corr}(rp_t, y_{t+3})$</td>
<td>-0.53</td>
</tr>
<tr>
<td>Cross correlation, $\text{corr}(rp_t, y_{t+4})$</td>
<td>-0.54</td>
</tr>
</tbody>
</table>

Table 3: US data moments

Source, Stock and Watson (1999), US data for the period 1953Q1 to 1996Q4, where output is real GDP and the risk premium is the commercial paper/treasury bill spread.

An exactly identified estimation of the illiquidity shock process (with shocks to $\gamma$) gives $\rho_N = 0.94$ and $\sigma_N = 1.23$.

[XXX Results of overidentified simulation estimations are forthcoming XXX]

7 Results and conclusions

The partial equilibrium model of the credit market presented in this paper provides several key insights into the nature of the coordination problem among creditors providing finance for leveraged firms. When short-term creditors observe a noisy signal, there exists a unique switching equilibrium between rollover and foreclosure. This switching threshold was shown to be a function of three exogenous parameters and one endogenous variable. The incidence of coordination failure was found to be higher i) when the creditors themselves had better productive potential, ii) when the capital stock at the interim stage was more intangible, and iii) when the ratio of short-term to long-term creditors in the market was higher.

The unique switching equilibrium made it possible to calculate the ex ante probability of a credit run. Given this information, it was possible for the firm to offer financial intermediaries a loan contract which specified a loan rate (and collateral value) for a given amount of external finance demanded, relative to the firm’s own net worth. As a result of endogenous coordination failure, creditors demanded an illiquidity risk premium on their funds. This premium was found to increase as the leverage of firms increased.

Importantly, the partial equilibrium model of endogenous coordination failure in credit markets was constructed with the specific aim of being usable in a DSGE framework. This paper achieves this aim by modelling the credit market as a sequence of static global games, aggregating across many
borrowers (firms) and adding households and capital producers to complete the general equilibrium structure.

The model then allows for the analysis of impulse responses of technology and illiquidity shocks. The model showed that coordination problems in credit markets induce a greater amplification of the technology shock than seen in a standard RBC model. The response was qualitatively similar (although not quantitatively similar) to the response of a technology shock in the BGG financial accelerator. The coordination failure model is therefore an alternative way of rationalizing a financial accelerator type mechanism. This result implies that further research would be desirable to identify the relative importance of the costly state verification model and the coordination failure model in driving these macroeconomic dynamics.

The model also allowed for the study of illiquidity shocks originating in the credit market. This impulse response analysis showed that a shock emanating from the credit market leads to hump-shaped and highly persistent output dynamics. This result highlights the importance of the financial sector in macroeconomic dynamics, and the dangers of instability and fragility in the financial sector of an economy. Simulated method of moments were used to estimate the structure of the illiquidity shock process.

There are three important extension which are worth pursuing with this line of research. First is to extend the general equilibrium framework to include more realistic features (nominal friction etc.) and allow for the analysis of monetary policy responses. This extension is forthcoming in de Groot (2010). Second, is to study the welfare implications of short-term creditors in the model, and possibly endogenize the maturity choice for firms. Third, is to include coordination failure not just in the credit market for firms, but also in the funding markets for banks and other financial institutions.

References


8 Appendix A

8.1 Behaviour of $\omega^*$ ($\psi$)

This section provides the proofs of Proposition 8.

In equilibrium, creditors switch behaviour between rolling over and foreclosing on their loan at the threshold, $\omega^*$:

$$\omega^* = \frac{\gamma \lambda^2 \left[ 1 - z \ln \left( \frac{\lambda}{\omega} \right) \right]}{\lambda + (\overline{w} - \lambda) \ln (1 - \frac{\lambda}{\omega})}$$

Proposition 8 i) states that $\omega^*$ is increasing in $\overline{w}$:

$$\frac{\partial \omega^*}{\partial \overline{w}} = \frac{\gamma \lambda^2 \left[ \lambda \ln \left( \frac{\lambda}{\omega} \right) + z \ln \left( 1 - \frac{\lambda}{\omega} \right) \right]}{\overline{w} \left( \lambda + (\overline{w} - \lambda) \ln (1 - \frac{\lambda}{\omega}) \right)^2} > 0$$

**Proof.** $\omega^*$ is increasing in $\overline{w}$.

It is clear to see that $\frac{\partial \omega^*}{\partial \overline{w}} > 0$ iff:

$$\lambda \ln \left( \frac{\lambda}{\omega} \right) + z \ln \left( 1 - \frac{\lambda}{\omega} \right) \left( \overline{w} \ln \left( \frac{\lambda}{\omega} \right) - \lambda \right) > 0$$

We can rearrange the expression in the following form: Simplifying gives:

$$\ln \left( \frac{\lambda}{\overline{w} - \lambda} \right) > - \ln \left( \frac{1 - \frac{\lambda}{\omega}}{\frac{\lambda}{\omega}} \right) \ln \left( \frac{\lambda}{\overline{w}} \right)$$

It is a standard result that $\ln (1 + h) < h$ for $h > -1$. Setting $h = -\frac{\lambda}{\overline{w}} > -1$, it is clear that $- \ln \left( \frac{1 - \frac{\lambda}{\omega}}{\frac{\lambda}{\omega}} \right) > +1$. This means that for $\omega^*$ to be increasing in $\overline{w}$, we require that:

$$\ln \left( \frac{\lambda}{\overline{w} - \lambda} \right) > \ln \left( \frac{\lambda}{\overline{w}} \right)$$

This gives $\lambda^2 > 0$ which holds by assumption. ■

A stricter condition also holds which states that $\frac{\partial \omega^*}{\partial \overline{w}} > \gamma$. This is important because it states that $\omega^*$ is increasing at a faster rate than the efficient threshold, $\gamma \overline{w}$. For the creditor, this generates a trade-off in the debt contract she is willing to hold. A higher $\overline{w}$ increases the non-foreclosure share of gross profits flowing to the creditor, but also increases the probability of the firm failing prematurely.

Proposition 8 ii), which states that $\omega^*$ is increasing in $\gamma$, is self evident. Proposition 8 iii) states that $\omega^*$ is decreasing in $\lambda$:

$$\frac{\partial \omega^*}{\partial \lambda} = \frac{\gamma \lambda^2 \left[ (1 - 2 \ln \left( \frac{\lambda}{\omega} \right)) + z \ln \left( 1 - \frac{\lambda}{\omega} \right) \left( \overline{w} - \lambda + 2 \ln \left( \frac{\lambda}{\omega} \right) \right) \ln \left( \frac{\lambda}{\omega} \right) \right]}{\left( \lambda + (\overline{w} - \lambda) \ln (1 - \frac{\lambda}{\omega}) \right)^2}$$
\textbf{Proof.} $\omega^*$ is decreasing in $\lambda$.

It is clear to see that $\frac{\partial \omega^*}{\partial \lambda} < 0$ over the range $\lambda \in (0, 1)$ iff:

$$
\left(1 - 2 \ln \left(\frac{\lambda}{\overline{\omega}}\right)\right) + \ln \left(1 - \frac{\lambda}{\overline{\omega}}\right) \left(\frac{\overline{\omega}}{\lambda} + \left(1 - 2\frac{\overline{\omega}}{\lambda}\right) \ln \left(\frac{\lambda}{\overline{\omega}}\right)\right) < 0
$$

[A proof of this proposition has not been completed, although this result has been verified numerically]. As $\lambda$ falls, the ability of the firm to meet its obligations to foreclosing creditors falls. This means that the risk to a creditor that chooses to rollover rises, since less creditors need to foreclose for a rolled over creditor’s investment to become worthless. \qed

Finally, Proposition 8 \textit{iv}) states that $\omega^*$ is increasing in $z$:

$$
\frac{\partial \omega^*}{\partial z} = \frac{\gamma \lambda^2}{z (\lambda + (\overline{\omega} - \lambda) \ln (1 - \frac{1}{\overline{\omega}}))} > 0
$$

\textbf{Proof.} $\omega^*$ is increasing in $z$.

It is clear to see that $\frac{\partial \omega^*}{\partial z} > 0$ iff:

$$
\lambda + (\overline{\omega} - \lambda) \ln \left(1 - \frac{\lambda}{\overline{\omega}}\right) > 0
$$

Rearranging, we get:

$$
\left(1 - \frac{\lambda}{\overline{\omega}}\right) < -\frac{\lambda}{\ln (1 - \frac{1}{\overline{\omega}})}
$$

Again, using the fact that $\ln (1 + h) < h$ for $h > -1$, the RHS of the above inequality must be greater than one. Since $0 < \frac{\lambda}{\overline{\omega}} < 1$ by assumption, it must be that the inequality holds. \qed

\section{Appendix B}

\subsection{Equilibrium conditions}

This section lists the equilibrium conditions for the model (using the calibration that $\Omega$ is arbitrarily close to 0):

\textbf{Consumption savings:}

$$
\frac{1}{C_t} = \beta E_t \left\{ \frac{1}{C_{t+1}} \right\} R_{t+1}
$$

\textbf{Labour market equilibrium condition:}

$$
(1 - \alpha) \frac{Y_t}{L_t} = \theta C_t L_t^\xi
$$

\textbf{Expected rate of return on capital:}

$$
\ldots
$$
\[ E_t R_{t+1}^K = E_t \left( \alpha \frac{Y_{t+1}}{\phi_t(.)K_{t+1}} + (1 - \delta) Q_{t+1} \right) \] (29)

where:

\[ \phi_t (\omega^* (\varphi_t), N_t) \equiv \{ E (\omega | \omega > \omega^* (\varphi_t, N_t)) \Pr (\omega > \omega^* (\varphi_t, N_t)) + \gamma \lambda \Pr (\omega \leq \omega^* (\varphi_t, N_t)) \} \]

\[ \equiv 1 - \Phi (z^* (\varphi_t, N_t) - \sigma) + \gamma \lambda \Phi (z^* (\varphi_t, N_t)) \]

where:

\[ z^* (\varphi_t, N_t) \equiv (\ln \omega^* (\varphi_t, N_t) + \sigma^2 / 2) / \sigma \]

Aggregate resource constraint:

\[ Y_t = C_t + I_t \] (30)

Production function:

\[ Y_t = \phi_t(.) A_t K_t^\alpha L_t^{1-\alpha} \] (31)

Capital accumulation:

\[ K_{t+1} = (1 - \delta) \phi_t(.) K_t + \eta \left( \frac{I_t}{\phi_t(.) K_t} \right) \phi_t(.) K_t \] (32)

External finance premium:

\[ E_t \left( \frac{R_{t+1}^K}{R_{t+1}} \right) = E_t (s (\varphi_t+1, N_{t+1})) \] (33)

\[ \frac{K_{t+1}}{N_{t+1}} = E_t (k (\varphi_t+1, N_{t+1})) \] (34)

Net worth:

\[ N_{t+1} = v (\phi_t(.) R_{t}^K Q_{t-1}K_t - R_t (Q_{t-1}K_t - N_t)) \] (35)

Investment-Q:

\[ Q_t = \frac{1}{\eta'} \left( \frac{I_t}{\phi_t(.) K_t} \right) \] (36)

Technology shock process:
\[ A_t = A_{t-1}^\rho e^{\varepsilon_t^A} \]  
(37)

with \( \varepsilon_t^A \sim N (0, \sigma_A^2) \). **Rollover shock process:**

\[ N_t = N_{t-1}^\rho e^{\varepsilon_t^N} \]  
(38)

with \( \varepsilon_t^N \sim N (0, \sigma_N^2) \).

### 9.2 Non-stochastic steady state

This section lists the conditions for the non-stochastic steady state of the economy. From equation (27):

\[ 1 = \beta R \]  
(39)

From equation (28):

\[ (1 - \alpha) \frac{Y}{L} = \theta CL^\varepsilon \]  
(40)

From equation (29):

\[ \frac{Y}{\phi(.)K} = \frac{1}{\alpha} \left( R^k - (1 - \delta) \right) \]  
(41)

where:

\[ \phi(.) \equiv \{ E (\omega \mid \omega > \omega^* (\overline{\omega})) \Pr (\omega > \omega^* (\overline{\omega})) + \gamma \lambda \Pr (\omega \leq \omega^* (\overline{\omega})) \} \]
\[ \equiv 1 - \Phi (z^* (\overline{\omega}) - \sigma) + \gamma \lambda \Phi (z^* (\overline{\omega})) \]

where:

\[ z^* (\overline{\omega}) \equiv (\ln \omega^* (\overline{\omega}) + \sigma^2 / 2) / \sigma \]

From equation (30):

\[ Y = C + I \]  
(42)

From equation (31):

\[ Y = \phi(.) K^\alpha L^{1-\alpha} \]  
(43)

From equation (32):
\[ \frac{I}{K} = 1 - \phi(.) (1 - \delta) \]  
(44)

From equations (33) and (34):

\[ \frac{R^k}{R} = \chi \left( \frac{K}{N} \right) \]  
(45)

From equation (35):

\[ N = \frac{v}{1 - vR} (\phi(.) R^K - R) K \]  
(46)

From equation (36):

\[ Q = 1 \]  
(47)

9.3 Linearized system

This section lists the equilibrium conditions, with variables expressed in terms of log-deviations from their respective non-stochastic steady states.

Consumption savings:

\[-c_t = -E_t c_{t+1} + r_{t+1} \]  
(48)

Labour market:

\[ y_t = c_t + (1 + \varepsilon) l_t \]  
(49)

Expected real rate of return on capital:

\[ E_t r^k_{t+1} = \frac{\alpha Y}{\alpha \phi K + (1 - \delta)} \left( E_t y_{t+1} - k_{t+1} - b_1 \tilde{w}_{t+1} - b_2 E_t \tilde{N}_{t+1} \right) + \frac{1 - \delta}{\alpha \phi K + (1 - \delta)} E_t q_{t+1} - q_t \]

where \( b_1 = \frac{\phi}{\phi} \) and \( b_2 = \frac{\phi}{\phi} \). Aggregate resource constraint:

\[ y_t = \frac{C}{V^t} c_t + \frac{I}{V^t} i_t \]  
(50)

Production function:

\[ y_t = a_t + \alpha k_t + (1 - \alpha) l_t + b_1 \tilde{w}_t + b_2 \tilde{N}_t \]  
(51)

Capital accumulation:
\[ k_{t+1} = \phi (1 - \delta) \left( k_t + b_1 \tilde{\omega}_t + b_2 \tilde{\eta}_t \right) + (1 - \phi (1 - \delta)) i_t \] (52)

External risk premium:

\[ E_t r^k_{t+1} - r_{t+1} = \frac{s_R k}{R^k / \bar{R}} \tilde{\omega}_{t+1} + \frac{s_R}{R^k / \bar{R}} E_t \tilde{\eta}_{t+1} \] (53)

\[ k_{t+1} - n_{t+1} = \frac{k_R}{K/N} \tilde{\omega}_{t+1} + \frac{k_R}{K/N} E_t \tilde{\eta}_{t+1} \] (54)

\[ E_t r^k_{t+1} - r_{t+1} = \frac{k_R}{s_R R^k / \bar{R}} (k_{t+1} - n_{t+1}) + \frac{1}{R^k / \bar{R}} \left( s_R - \frac{k_R}{s_R} \right) E_t \tilde{\eta}_{t+1} \] (55)

Net worth:

\[ n_{t+1} = v \left( \phi R^k K \frac{K}{N} \left( r^k_t + b_1 \tilde{\omega}_t + b_2 \tilde{\eta}_t \right) - \frac{R (K - N)}{N} r_t + \frac{(\phi R^k - R) K}{N} (q_{t-1} + k_t) + Rn_t \right) \] (56)

Investment-Q:

\[ q_t = \varphi \left( i_t - k_t - b_1 \tilde{\omega}_t - b_2 \tilde{\eta}_t \right) \]

Technology shock process:

\[ a_t = \rho_a a_{t-1} + \varepsilon^a_t \] (57)

Rollover shock process:

\[ \tilde{\eta}_t = \rho_a \tilde{\eta}_{t-1} + \varepsilon^N_t \] (58)

### 9.4 Model solution

This section describes the solution to the model. The system of log-linear expectational difference equations set out in Section above can be written in the following matrix form:

\[ E_t [a_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0 \]

where \( z_t \equiv (y_t, c_t, i_t, k_{t+1}, n_{t+1}, r^k_t, \tilde{\omega}_{t+1}, x^1_t, x^2_t, q_t)' \), \( x^1_t = E_t r^k_{t+1} \), \( x^2_t = E_t \tilde{\eta}_{t+1} \), and \( s_t \equiv (a_t, \tilde{\eta}_t)' \). Using the method of Christiano (2002), a solution to the model of the following form is derived:

\[ z_t = A z_{t-1} + B s_t \]
9.5 Simulation estimation

The parameters of the illiquidity shock process are estimated using the SMM estimator proposed by Lee and Ingram (1991). Let \( m_t \) be a \( p \times 1 \) vector of empirical moments from US aggregate data, and let \( m_t(\theta_1, \theta_2) \) be the counterpart of \( m \), computed from simulated data generated by the model presented in this paper, using deep parameters, \( \theta_1 \) and the parameters of the illiquidity shock process, \( \theta_2 \). The SMM estimator, \( \hat{\theta}_2^{SMM} \) is the value of \( \theta_2 \) that solves:

\[
\min_{\theta_2} G(\theta_1, \theta_2)' W G(\theta_1, \theta_2)
\]

where:

\[
G(\theta_1, \theta_2) = (1/T) \sum_{t=1}^{T} m_t - (1/\tau T) \sum_{t=1}^{\tau T} m_t(\theta_1, \theta_2)
\]

is a \( p \times 1 \) vector and \( W \) is the optimal weighting matrix:

\[
W = \lim_{T \to \infty} Var \left( \frac{1}{\sqrt{T}} \sum_{t=1}^{T} m_t \right)^{-1}
\]