Macroeconomic Fundamentals and the Exchange Rate Dynamics: A No-Arbitrage Macro-Finance Approach

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Abstract

This paper investigates relationship between short-run exchange rate dynamics and macroeconomic fundamentals by adopting a no-arbitrage international macro-finance approach, under which the macroeconomic fundamentals enter into the exchange rate dynamics in a nonlinear form. Based on empirical analysis using an enriched dataset including exchange rates, yields of zero-coupon bonds, and macroeconomic variables of the US and the Euro area, the paper finds a close link between macroeconomic fundamentals and the exchange rate dynamics. The model-implied monthly exchange rate changes can explain about 57% variation of the observed data. Having been amplified by the time-varying market prices of risks, the macroeconomic innovations help capture large volatility of exchange rate changes. The foreign exchange risk premium can largely alleviate the forward premium anomaly.

Keywords: Macroeconomic Fundamentals, Exchange Rate Dynamics, Forward Premium Anomaly, Stochastic Discount Factor, Term Structure of Interest Rates, Unscented Kalman Filter

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1. Introduction

The nominal floating exchange rate has often been regarded as an asset price in exchange rate modeling since the 1970s. According to the standard asset pricing theory, its current price should reflect market's expectations concerning present and future economic conditions (Frenkel and Mussa 1985; Obstfeld and Rogoff 1996; Cochrane 2004). However, a long-standing puzzle in international economics and finance is the disconnection between exchange rates movements, especially the short-run movements, and macroeconomic fundamentals such as output, inflation and monetary policy tools.

A variety of models have been proposed to relate exchange rates to macroeconomic fundamentals. Monetary models (Frenkel 1976, 1979; Mussa 1976; Bilson 1978; Dornbusch 1976) states the existence of a long-run equilibrium relationship among relative money supplies, relative income levels and the nominal exchange rate. New open economy macroeconomics models (Obstfeld and Rogoff 2003) attempts to explain exchange rate movements by incorporating imperfect competition and nominal rigidities in a general equilibrium open economy. However, these models can not find empirical evidence on a close relationship between short-run exchange rates movements and macroeconomic fundamentals (Meese 1990; Frankel and Rose 1995; Engel and West 2005). Furthermore, they fail to capture the volatile time-varying foreign exchange risk premium, implied by the well documented forward premium anomaly in foreign exchange markets (Fama 1984; Hodrick 1989; Backus, Gregory, and Telmer 1993; Bansal et al. 1995; Bekaet 1996).

In contrast, this paper investigates interactions between exchange rate dynamics and macroeconomic fundamentals by following a no-arbitrage macro-finance approach. Under a two-country world, the exchange rate of these two economies is governed by the ratio of their stochastic discount factors, which are modeled by a factor representation under the no-arbitrage condition. We take outputs, inflations and short-term interest rates as macroeconomic fundamentals (factors), which are assumed to drive the joint dynamics of two economies. Real output growth directly governs the aggregate consumption of an economy and should be a key element of the stochastic discount factor. Inflation can also enter into the stochastic discount factor via its dynamic interactions with the real production (Piazzesi and Schneider 2006). The short-term interest rate is typically viewed as a macro variable reflecting monetary policy (Duffee 2007). Under specifications of stochastic discount

factors, we extend macro-finance term structure models (Ang and Piazzesi 2003; Diebold, Rudebushch, and Aruoba 2005; Ang, Dong, and Piazzesi 2007) to a two-country framework in order to help identify the time-varying market prices of risks, which in turn amplify roles of macroeconomic innovations on exchange rate changes. This is important since ignoring risk premia or assuming constant market prices of risks may mislead to a conclusion that exchange rates are not linked to macroeconomic fundamentals.

In line with the above modeling settings, the change of exchange rates has a nonlinear relation with macroeconomic fundamentals. In contrast to uncovered interest parity, our model indicates that the expected exchange rate changes are determined by both the interest rate differential of two countries and the foreign exchange risk premium and that the unexpected exchange rate changes are also driven by the fundamental innovations, whose roles are amplified by the time-varying market prices of economic risks.

The paper is related to previous studies on joint dynamics of exchange rates and yield curves (Bansal 1997; Backus, Foresi, and Telmer 2001; Dong 2006; Engel and West 2006; Han, Hammound, and Ramezani 2010; Marcello and Taboga 2009). However, it is different for the following critical points. First, we explicitly introduce macroeconomic fundamentals to modeling exchange rates without relying on any latent factors. As a result, our study is more economically meaningful. Second, we do not impose any restrictions on interdependence between two economies and use a more flexible approach. We do find evidence that real outputs of different economies comove tightly and strongly impact each other. Third, our study sheds new lights on the relationship between macroeconomic fundamentals and the exchange rate dynamics. Macroeconomic fundamentals and innovations enter into the exchange rate dynamics in a nonlinear form. Through the nonlinear transformation, the unexpected macroeconomic innovations play a crucial role in driving exchange rates and in generating large volatility of exchange rate changes. Fourth, we propose an efficient likelihood-based estimation method by using the unscented Kalman filter, which is recently developed in the field of engineering and can efficiently handle highly nonlinear state-space models (Julier and Ulman 1997, 2004).

Using monthly data of the US and the Euro area (EA) ranging from January, 1999 to December, 2008, we find that the model-implied monthly exchange rate changes have a 76% correlation with the observed data and that they can explain 57% variation of the data. We also find that

both economies are highly interdependent and mutually influence each other. These are in stark contrast to previous studies using monetary and new open economy macroeconomics models and to a recent study of Dong (2006), which follows similar modeling approach to this paper. The former finds that those models can only explain at most 10% variation of the data (Lubik and Schorfheide 2005; Engel and West 2005), and the latter, by assuming no influence of the foreign country (Germany) on the home country (the US) and using latent factors, finds that his model can explain 38% variation of exchange rate changes between the US dollar and the German Mark.

The time-varying foreign exchange risk premium plays an important role in explaining the forward premium anomaly and in remedying the failure of uncovered interest parity. For example, if we run a regression of exchange rate changes only on the interest rate differential, the estimated coefficient is negative and far away from unity, and the R^2 is very tiny. However, if we introduce the risk premium term, as suggested by our model and Fama (1984), this estimate is positive and close to unity, and the parameter estimate of the risk premium term is positive and highly significant. The R^2 is improved dramatically.

We find a close link between macroeconomic fundamentals and the exchange rate dynamics. Macroeconomic fundamentals indirectly impact exchange rate movements through time-varying market prices of risks and their shocks has time-varying effects on the exchange rate dynamics. If we impose constant market prices of risks on the model and hence the dynamic of exchange rate changes become time-homogeneous, the model performance is deteriorated dramatically. For example, the exchange rate changes implied from the model with the constant market prices of risks can only explain 25% variation of the observed data, less than half of the explanation power of our model. By decomposing the exchange rate changes into three parts, the interest rate differential, the foreign exchange risk premium and the macro shock driving part, we find that the first two components are much smoother and that a large fraction of variation of exchange rate changes is explained by the last component. For example, the last component, which is the product of the differential of market prices of risks and fundamental innovations, can explain 37% variation of the observed data, taking 76% of the total explanation power of the model. To deeply explore sources of the explanation power, we also investigate whether it is enough to explain the exchange rate dynamics only using the yield curve information under the same modeling framework without relying on any macro variables. We find that in this case, the resulted model-implied exchange rate

changes only explain 13% variation of the data.

The rest of paper is organized as follows. Section 2 presents the data and implements a preliminary analysis. Section 3 introduces a no-arbitrage macro-finance modeling approach for the exchange rate dynamics. Section 4 proposes a likelihood-based estimation relying on the unscented Kalman filter. Section 5 presents the empirical results and discusses their economic implications. Section 6 concludes the paper.

2. Data and Preliminary Analysis

2.1. Data

Different from most studies using quarterly data, we use monthly data of the United State (US) and the Euro area (EA) and focus on the short-run movements of exchange rates. The US is taken as the home country, and the EA as the foreign country. There are three types of data: the macroeconomic data (outputs and inflations), the yields data, and the exchange rate data. Data range from January 1999 to December 2008 in monthly frequency, in total, 120 months.

The home and foreign output gaps and inflations are proxied by their own Industrial Production Index's and CPI's, respectively. The US data are downloaded from the Federal Reserve, St. Louis, and the EA data are downloaded from the European Central Bank (ECB). The raw macroeconomic data are seasonally adjusted. Inflation rates are measured by the 12-month changes of the log CPI's, and output gaps are constructed by applying the HP filter (Hodrick and Prescott 1997) to Industrial Production Index's with the smoothing parameter being set to 129,600 (Ravn and Uhlig 2002).

The US yields data are those with maturity 1, 3, 12, 24, 36, 48, and 60 months. The 1-month and 3-month rates are directly from the CRSP Fama-Bliss rate file. While the yields of 12-month and longer maturity are derived from the respective bond prices obtained from the CRSP Fama-Bliss discount bond file using the formula $y_t^{(n)} = -\log(P_t^{(n)}/100)/n$, where $P_t^{(n)}$ is the bond price at time t with maturity n-year, and $y_t^{(n)}$ is the corresponding yield. Since the EA yields data from the ECB are available only from December 2006 and are not long enough to implement estimation, they are proxied by German data, which are obtained from BBK Statistics-Deutsche Bundesbank. The EA yields have maturity 1-month, 3-month, 12-month, 36-month, and 60-month.

The exchange rate data are those of the-last-day US-Dollar/Euro spot exchange rates of each

month, downloaded from the Federal Reserve, St. Louis.

The upper panels of Figure 1 plots the annualized macroeconomic data used in estimation. We can see that two inflation rates are very persistent. Output gaps are more volatile, and the comovement between them is very clear. During the sample period, the US output gap reaches a high level in the mid of 2000, just before the collapse of the dot-com bubble, and has a dramatic decline during the 2008-2009 financial crisis. Similar movements can also be observed in the EA output gap. The lower panels of Figure 1 presents the annualized yields of the US and of the EA. The US yields are more volatile than the EA ones. The high levels of the US and the EA yields happen around the same time as those of their output gaps.

— Figure 1 around here —

2.2. A Counter-Factual Study

In this subsection, we implement a counter-factual study on interactions between the exchange rate changes innovations and the macroeconomic variables fluctuations. This study provides an empirical support to our modeling framework in the next section.

Let Z_t be a vector of macroeconomic variables with the last element being the change of exchange rate

$$Z_t = [g_t^{(h)},\ \pi_t^{(h)},\ y_{1t}^{(h)},\ g_t^{(f)},\ \pi_t^{(f)},\ y_{1t}^{(f)},\ \Delta s_t]'. \tag{1}$$

where the home variables are denoted with superscripts (h), and the foreign ones are denoted with superscripts (f). $g_t's$ denote output gaps, $\pi_t's$ inflation rates, $y_{1t}'s$ yields of the zero-coupon bond with maturity one month and represent the short-term interest rates, and Δs_t is the log difference of the spot exchange rate (the exchange rate change).

We use data of the US and the Euro Area (EA) described in subsection 2.1. Since the the exchange rate changes are in one-month change, we divide macroeconomic variables by twelve and transform them into monthly values. Assume that Z_t follows a vector autoregression process. According to both the Akaike information criterion (AIC) and the Bayesian information criterion (BIC), VAR(1) is the best model for these data. We estimate this VAR(1) and identify innovations by the Cholesky decomposition, which implies that innovations of the exchange rate changes have the lagged impacts on the macroeconomic variables, whereas the first six macroeconomic innova-

tions have both the contemporaneous and the lagged impacts on the exchange rate changes.

Using parameter estimates and identified innovations from the VAR(1) model, we implement a counter-factual simulation to construct artificial time series for all variables in (1) by simulating the VAR(1) model over the sample period conditionally upon setting innovations of the exchange rate changes to zero. Figure 2 plots the actual and the simulated time series of the first six macro variables in Z_t; and the upper panel of Figure 4 plots the actual and the simulated time series of the exchange rate changes.

— Figure 2 around here —

It thus becomes clear that the exchange rate change is the only variable strongly affected by its own innovations and that the idiosyncratic fluctuations in exchange rate changes are not significant at all in explaining fluctuations of macroeconomic variables. This finding is consistent with Lubik and Schorfheide (2005), Pericoli and Taboga (2009) and De Santis and Favero (2009). Lubik and Schorfheide (2005) empirically find that there is no role of the exchange rate change on monetary policy rules. Pericoli and Taboga (2009) find that the exogenous shocks to the exchange rate have a negligible impact on the yield curve. De Santis and Favero (2009) find that the exchange rate is irrelevant to determine co-movements of macroeconomic variables.

We find that the simulated exchange rate changes can only explain a small proportion of variation of the data (18%). We note that our findings here are very robust with different orders of the first six macroeconomic variables in the VAR specification. However, the contemporaneous macroeconomic innovations enter into the exchange rate dynamics in a linear form under this VAR specification. This study by the reduced-form VAR model implies that innovations of the exchange rate changes are irrelevant to determine the dynamic of macroeconomic variables and that the contemporaneous macroeconomic innovations may have potential power to explain exchange rate movements through a nonlinear form.

3. The Modeling Framework

Consider a two-country world, a home country and a foreign country, each with its own currency. Under the absence of arbitrage, the exchange rate between these two currencies is governed by the ratio of their stochastic discount factors. In this section, we firstly discuss how to model stochastic

discount factors and how to relate them to macroeconomic fundamentals in subsection 3.1. We then proceed to model the exchange rate dynamics in subsection 3.2. Subsection 3.3 introduces a two-country affine term structure model for pricing zero-coupon bonds.

3.1. Macroeconomic Fundamentals and Stochastic Discount Factors

In each country, we take the output gap $\tilde{g}$, the inflation $\tilde{\pi}$ and the short-term interest rate $\tilde{r}$ as main macroeconomic fundamentals. Putting the home and foreign factors together, we have a state vector X_t in a two-country open economy,

$$X_t = \left[\tilde{g}_t^{(h)},\ \tilde{\pi}_t^{(h)},\ \tilde{r}_t^{(h)},\ \tilde{g}_t^{(f)},\ \tilde{\pi}_t^{(f)},\ \tilde{r}_t^{(f)}\right]', \tag{2}$$

where the home factors are denoted with superscripts (h), and the foreign factors with superscripts (f). Tildes are used upon factors to distinguish them from the market observed macroeconomic variables, which are assumed to be collected with measurement errors. We assume that the state X_t determines the dynamics of two-country open economy and follows a Gaussian vector autoregression process,

$$X_t = \mu + \Phi X_{t-1} + \Sigma\varepsilon_t, \tag{3}$$

where μ is a constant 6×1 vector, Φ a constant 6×6 matrix, ε_t an i.i.d Gaussian white noise $N(0, I_6)$, and Σ a low-triangular matrix such that $\Sigma'\Sigma$ captures the variance-covariance of ε_t.

In this two-country world, assume that no-arbitrage holds. Then, in each country, there exists at least one almost surely positive process M_t with $M_0 = 1$ such that the discounted gains process associated with any admissible trading strategy is a martingale (Harrison and Kreps 1979). M_t is called the stochastic discount factor (SDF). In a Lucas-type exchange economy (Lucas 1982), the stochastic discount factor is also often interpreted as the representative agent's intertemporal marginal rate of substitution. We denote the home SDF as $M_t^{(h)}$ and the foreign one as $M_t^{(f)}$. In the following and throughout the text, whenever a relation holds both for the home and the foreign countries, we suppress the superscript (h) or (f) unless otherwise specified.

For absence of a generally accepted equilibrium model for asset pricing, many studies use flexible factor models under the no-arbitrage condition (Cochrane 2004). In this paper, we also use a factor representation for the SDF's, based on which the exchange rate and the term structure of interest

rates are modeled. For each of the home and foreign stochastic discount factors ($M_t^{(h)}$ and $M_t^{(f)}$), assume that it has an exponential form

$$\begin{aligned} M_{t+1} &= \exp(m_{t+1}) \\ &= \exp\Big(- \tilde{r}_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\Big), \end{aligned} \tag{4}$$

where $\tilde{r}_t$ is the short-term interest rate of that country, λ_t is the time-varying market prices of risks assigned by investors in that country, and ε_t is the shock to the state X_t, which is the only common term for both the home and foreign SDF's. Of course, we have $\lambda_t = \Big(\lambda_t^{\tilde{g}^{(h)}}, \lambda_t^{\tilde{\pi}^{(h)}}, \lambda_t^{\tilde{r}^{(h)}}, \lambda_t^{\tilde{g}^{(f)}}, \lambda_t^{\tilde{\pi}^{(f)}}, \lambda_t^{\tilde{r}^{(f)}}\Big)'$, the market price of risk for each factor in the state vector X_t, respectively.

Denote the market prices of risks assigned in the home country as $\lambda_t^{(h)}$ and those assigned in the foreign country as $\lambda_t^{(f)}$. We use the state X_t to summarize uncertainties in this two-economy world and assume that market prices of risks assigned in each country are affine functions of X_t (Dai and Singleton 2002; Duffee 2002)

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{5}$$

where λ_0 is a constant 6×1 vector, and λ_1 a constant 6×6 matrix. The specification (5) implies that investors of each country may assign different market prices for these risks contained in the state X_t if λ_0 and λ_1 are different across these two countries and that if $\lambda_t^{(h)}$ and $\lambda_t^{(f)}$ comove tightly, the two SDF's could be highly correlated.

3.2. Exchange Rate Dynamics and Forward Premium Anomaly

Define the nominal spot exchange rate $\mathcal{S}_t$ at time t as the domestic currency price of one unit of the foreign currency. No-arbitrage and law of one price dictate that the ratio of the stochastic discount factors between the home and foreign economies determines the dynamics of their exchange rate (Backus et al. 2001; Bekaert 1996; Brandt and Santa-Clara 2002; Brandt, Cochrane, and Santa-Clara 2006). We thus have

$$\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} = \frac{M_{t+1}^{(f)}}{M_{t+1}^{(h)}}. \tag{6}$$

The above relation formally defines the link between the stochastic discount factors of two economies and exchange rate movements between them. In complete markets, the stochastic discount factors in both economies are unique, and they uniquely determine the dynamics of their exchange rate. When markets become incomplete, there may exist many different stochastic discount factors that can guarantee the absence of arbitrage. In this case, introduction of extra instruments, i.e. zero-coupon bonds, can help identify market prices of risks.

Taking natural logarithms for both sides of equation (6) and using specifications of the SDF's (4), we obtain the following exchange rate dynamic equation

$$\begin{aligned} \Delta s_{t+1} &\equiv s_{t+1} - s_t = m_{t+1}^{(f)} - m_{t+1}^{(h)} \\ &= \left(\tilde{r}_t^{(h)} - \tilde{r}_t^{(f)}\right) + \frac{1}{2}\left(\lambda_t^{(h)\prime}\lambda_t^{(h)} - \lambda_t^{(f)\prime}\lambda_t^{(f)}\right) + \left(\lambda_t^{(h)} - \lambda_t^{(f)}\right)'\varepsilon_{t+1}, \end{aligned} \tag{7}$$

which shows that macroeconomic fundamentals X_t are imparted to the exchange rate dynamic via market prices of risk in a nonlinear form and that shocks to output gaps, inflations and interest rates also drive variation of the exchange rate changes. This is in contrast to the traditional models that often assume linear relation between the exchange rate and macroeconomic fundamentals or/and that only use latent factors and do not have this economically meaningful interpretations.

The time-varying conditional mean, $\mu_t^s \equiv (\tilde{r}_t^{(h)} - \tilde{r}_t^{(f)}) + \frac{1}{2}(\lambda_t^{(h)\ \prime}\lambda_t^{(h)} - \lambda_t^{(f)\ \prime}\lambda_t^{(f)})$, captures predictable variation of returns in foreign exchange markets. We can see that market prices of risks not only are important in determining the conditional mean of exchange rate changes, but also directly affect the conditional volatility of exchange rate changes through $\sigma_t^s \equiv \lambda_t^{(h)} - \lambda_t^{(f)}$. However, the weak predictability of exchange rate changes implies that μ_t^s can only explain a small fraction of variation of the exchange rate changes. Shocks to macroeconomic factors may be important in explaining large variation of exchange rate changes. Exposure of the exchange rate to macroeconomic innovations is amplified by the difference of the time-varying market prices of risks between two economies, and hence exchange rate changes are heteroskedastic in our model.

The above expression clearly shows that uncovered interest parity does not hold in our model. Uncovered interest parity states that the currency with higher interest rate is expected to depreciate against the one with lower interest rate and thus the expected change of exchange rate is equal to the interest rate differential of two countries. However, in our model, the expected change of

exchange rate is composed of two parts, the interest rate differential and a term called the foreign exchange risk premium rp_t,

$$rp_t \equiv \frac{1}{2}(\lambda_t^{(h)\prime}\lambda_t^{(h)} - \lambda_t^{(f)\prime}\lambda_t^{(f)}). \tag{8}$$

The importance of the time-varying foreign risk premium is also argued by Fama (1984) who points out that the departure from uncovered interest parity should be attributed to a time-varying risk premium.

3.3. Term Structure of Interest Rates

Having specified the stochastic discount factors for the home and foreign countries, we can model the short rates and price zero-coupon bonds. Introduction of bonds in our modeling framework is important for identifying market prices of risks. Because short rates of the home and foreign countries have been included in the state X_t as factors, the affine short rate equations can be easily specified as

$$\tilde{r}_t = \delta_0 + \delta_1' X_t, \tag{9}$$

where $\delta_0 = 0$, and $\delta_1^{(h)} = (0,0,1,0,0,0)'$ for the home country and $\delta_1^{(f)} = (0,0,0,0,0,1)'$ for the foreign country.

No-arbitrage guarantees that a zero-coupon bond with maturity n-year in each country can be priced at time t by using the following Euler equation

$$\tilde{P}_t^{(n)} = E_t[M_{t+1}\tilde{P}_{t+1}^{(n-1)}] \tag{10}$$

with the initial condition $\tilde{P}_t^{(0)} = 1$. Again, tilde indicates the true value. Under specifications of the state (3), the short rate (9) and the SDF (4), we can show that the bond price is an exponential linear function of the state X_t

$$\tilde{P}_t^{(n)} = \exp(A_n + B_n' X_t), \tag{11}$$

where A_n and B_n solve the following difference equations

$$A_{n+1} = A_n + B_n'(\mu - \Sigma\lambda_0) + \frac{1}{2}B_n'\Sigma\Sigma' B_n - \delta_0, \tag{12}$$

$$B_{n+1} = (\Phi - \Sigma\lambda_1)' B_n - \delta_1, \tag{13}$$

with $A_1 = -\delta_0$ and $B_1 = -\delta_1$ being the initial conditions. Accordingly, the yield is also an affine function of the state

$$\tilde{y}_t^{(n)} \equiv -\frac{\log P_t^{(n)}}{n} = a_n + b_n' X_t, \tag{14}$$

where $a_n = -A_n/n$ and $b_n = -B_n/n$.

From the difference equations (12) and (13), we can see that the constant market price of risk parameter λ_0 only affects the constant yield coefficient a_n, whereas the parameter λ_1 affects the factor loading b_n. This implies that the parameter λ_0 affects average term spreads and average expected bond returns, whereas the parameter λ_1 governs time variation in term spreads and expected bond returns.

4. Econometric Methodology

Since we assume that the real macroeconomic factors are unobservable and that the econometrician observed macroeconomic variables are contaminated with measurement errors, we first transform the model into a state-space representation and then use a Bayesian filtering approach to estimate the model.

At each period t, we can observe the exchange rate change Δs_t, the yields of zero-coupon bonds in the home and foreign countries ($y_t^{(h)}$ and $y_t^{(f)}$), and the output gaps and inflations of the home and foreign countries ($V_t^{(h)} = (g_t^{(h)}, \pi_t^{(h)})'$ and $V_t^{(f)} = (g_t^{(f)}, \pi_t^{(f)})'$). We assume that each of these variables is collected with the normal i.i.d measurement errors. Thus, we have the following measurement equations

$$\begin{aligned}
\Delta s_t &= (\tilde{r}_{t-1}^{(h)} - \tilde{r}_{t-1}^{(f)}) + \frac{1}{2}(\lambda_{t-1}^{(h)\prime}\lambda_{t-1}^{(h)} - \lambda_{t-1}^{(f)\prime}\lambda_{t-1}^{(f)}) \\
&\quad + (\lambda_{t-1}^{(h)\prime} - \lambda_{t-1}^{(f)\prime})\Sigma^{-1}(X_t - \mu - \Phi X_{t-1}) + \eta_t^{\Delta s} \\
y_t^{(h)} &= a^{(h)} + b^{(h)\prime} X_t + \eta_t^{y^{(h)}}, \\
y_t^{(f)} &= a^{(f)} + b^{(f)\prime} X_t + \eta_t^{y^{(f)}}, \\
V_t^{(h)} &= (I_2 \;\; 0_{2\times 4}) X_t + \eta_t^{V^{(h)}}, \\
V_t^{(f)} &= (0_{2\times 3} \;\; I_2 \;\; 0_{2\times 1}) X_t + \eta_t^{V^{(f)}},
\end{aligned} \tag{15}$$

where we use $\varepsilon_t = \Sigma^{-1}(X_t - \mu - \Phi X_{t-1})$ in the exchange rate dynamic equation, $y_t^{(h)}$ is a 7×1 vector

containing yields of all maturity considered in the home country, $y_t^{(f)}$ is a 5×1 vector containing yields of all maturity considered in the foreign country, and η_t's capture measurement errors with distinct variances for different variables/series and are assumed to be mutually independent.

We have the latent factor X_t, which follows a first-order VAR with its dynamic (3). From the measurement equations, we note that observations depend on the current and lagged macroeconomic factors X_t and X_{t-1}, both of which should be taken as states and can be written in the following form

$$\begin{pmatrix} X_t \\ X_{t-1} \end{pmatrix} = \begin{pmatrix} \mu \\ 0_{6\times 1} \end{pmatrix} + \begin{pmatrix} \Phi & 0_{6\times 6} \\ I_6 & 0_{6\times 6} \end{pmatrix} \begin{pmatrix} X_{t-1} \\ X_{t-2} \end{pmatrix} + \begin{pmatrix} \Sigma \\ 0_{6\times 6} \end{pmatrix} \varepsilon_t. \tag{16}$$

Given the state-space model representation (15) and (16) with Gaussian noises, we can implement model estimation using Bayesian filtering approaches. We have noted that the exchange rate dynamic equation is a highly non-linear function of states, which makes the standard Kalman filter inapplicable. Instead, we can use the nonlinear Kalman filters. The usually used nonlinear Kalman filter is the extended Kalman filter, which linearizes the nonlinear system around the current state estimate using a Taylor approximation. However, for the highly nonlinear system, the extended Kalman filter is computationally demanding and performs very poorly. An alternative is the unscented Kalman filter (UKF), recently developed in the field of engineering (Julier and Ulman 1997, 2004). The idea behind this approach is that in order to estimate the state information after a nonlinear transformation, it is favorable to approximate the probability distribution directly instead of linearizing the nonlinear functions. The unscented Kalman filter overcomes to a large extent pitfalls inherent to the extended Kalman filter and improves estimation accuracy and robustness without increasing computational cost.

To implement the unscented Kalman filter, we firstly concatenate the state variables $x_{t-1} = [X_{t-1}, X_{t-2}]'$, the observation noises η_{t-1} and the state noises ε_{t-1} at time $t-1$

$$x_{t-1}^e = \begin{bmatrix} x_{t-1}' & \eta_{t-1}' & \varepsilon_{t-1}' \end{bmatrix}', \tag{17}$$

whose dimension is $L = L_x + L_\eta + L_\varepsilon$ and whose mean and covariance are

$$\hat{x}^e_{t-1} = \begin{bmatrix} E[x_{t-1}] & 0 & 0 \end{bmatrix}', \qquad P^e_{t-1} = \begin{bmatrix} P^x_{t-1} & 0 & 0 \\ 0 & \Sigma^2_\eta & 0 \\ 0 & 0 & I_6 \end{bmatrix}.$$

We then form a set of $2L+1$ sigma points

$$\chi^e_{t-1} = \begin{bmatrix} \hat{x}^e_{t-1} & \hat{x}^e_{t-1} + \sqrt{(L+\lambda)P^e_{t-1}} & \hat{x}^e_{t-1} - \sqrt{(L+\lambda)P^e_{t-1}} \end{bmatrix} \tag{18}$$

and the corresponding weights

$$w^{(m)}_0 = \frac{\lambda}{L+\lambda}, \qquad w^{(c)}_0 = \frac{\lambda}{L+\lambda} + (1-\alpha^2+\beta), \tag{19}$$

$$w^{(m)}_i = w^{(c)}_i = \frac{1}{2(L+\lambda)}, \quad i = 1, 2, \ldots, 2L, \tag{20}$$

where superscripts (m) and (c) indicate that weights are for construction of the posterior mean and covariance, respectively, $\lambda = \alpha^2(L+\bar{\kappa}) - L$ is a scaling parameter, the constant α determines the spread of sigma points around $\bar{x}$ and is usually set to be a small positive value, $\bar{\kappa}$ is a second scaling parameter with value set to 0 or $3-L$, and β is a covariance correction parameter and is used to incorporate prior knowledge of the distribution of x.

With these sigma points, we implement the UKF as follows: for the time update

$$\begin{aligned} \chi^x_{t|t-1} &= F(\chi^x_{t-1}, \chi^\varepsilon_{t-1}), \qquad \hat{x}^-_t = \sum_{i=0}^{2L} w^{(m)}_i \chi^x_{i,t|t-1}, \\ P^-_{x_t} &= \sum_{i=0}^{2L} w^{(c)}_i (\chi^x_{i,t|t-1} - \hat{x}^-_t)(\chi^x_{i,t|t-1} - \hat{x}^-_t)', \end{aligned}$$

and for the measurement update

$$\begin{aligned} \mathcal{Y}_{t|t-1} &= H(\chi^x_{t|t-1}, \chi^\eta_{t|t-1}), \qquad \hat{Y}^-_t = \sum_{i=0}^{2L} w^{(m)}_i \mathcal{Y}_{i,t|t-1}, \\ P^-_{Y_t} &= \sum_{i=0}^{2L} w^{(c)}_i (\mathcal{Y}_{i,t|t-1} - \hat{Y}^-_t)(\mathcal{Y}_{i,t|t-1} - \hat{Y}^-_t)', \end{aligned}$$

$$
\begin{aligned}
P_{x_t Y_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\chi_{i,t|t-1}^{x} - \hat{x}_t^{-})(\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^{-})', \\
\hat{x}_t &= \hat{x}_t^{-} + P_{x_t Y_t} (P_{Y_t}^{-})^{-1} (Y_t - \hat{Y}_t^{-}), \\
P_{x_t} &= P_{x_t}^{-} - (P_{x_t Y_t} (P_{Y_t}^{-})^{-1}) P_{Y_t}^{-} (P_{x_t Y_t} (P_{Y_t}^{-})^{-1})',
\end{aligned}
$$

where Y_t is the observation vector containing all the observed variables, $\hat{Y}_t^{-}$ its predicted values, $P_{Y_t}^{-}$ its conditional variance-covariance matrix, $\hat{x}_t$ the filtered state vector, and P_{x_t} its variance-covariance matrix.

Assuming that the predictive errors are normally distributed, we can construct the log likelihood function at time t as follows

$$
\mathcal{L}_t(\Theta) = -\frac{1}{2} \ln |P_{Y_t}^{-}| - \frac{1}{2} (Y_t - \hat{Y}_t^{-})' (P_{Y_t}^{-})^{-1} (Y_t - \hat{Y}_t^{-}), \tag{21}
$$

where Θ is a vector of model parameters. Parameter estimates can be obtained by maximizing the joint log likelihood

$$
\hat{\Theta} = \arg\max_{\Theta \in \Xi} \sum_{t=1}^{T} \mathcal{L}_t(\Theta), \tag{22}
$$

where Ξ is a compact parameter space, and T is the length of total observations of the data. Because the log likelihood function is misspecified for the non-Gaussian model, a robust estimate of the variance-covariance matrix of parameter estimates can be obtained using the approach proposed by White (1982)

$$
\hat{\Sigma}_{\Theta} = \frac{1}{T} \left[A B^{-1} A \right]^{-1}, \tag{23}
$$

where

$$
A = -\frac{1}{T} \sum_{t=1}^{T} \frac{\partial^2 \mathcal{L}_t(\hat{\Theta})}{\partial \Theta \partial \Theta'}, \qquad B = \frac{1}{T} \sum_{t=1}^{T} \frac{\partial \mathcal{L}_t(\hat{\Theta})}{\partial \Theta} \frac{\partial \mathcal{L}_t(\hat{\Theta})}{\partial \Theta'}. \tag{24}
$$

With these parameter estimates $\hat{\Theta}$, the latent macroeconomic factors $\hat{X}_t$ can be extracted using the unscented Kalman filter.

The number of parameters in our model is huge. Maximization of the likelihood (21) may involve a large number of likelihood evaluations. Therefore, we adopt a sophisticated quasi-Newton

approach with the inverse Hessian of the likelihood function updated by the BFGS algorithm. The initial values are carefully selected by the following way. We first run the Nelder-Mead optimization algorithm for 100 feasible sets of starting values and stop them after 100 iterations. Then the best 10 parameter estimate sets (in terms of the likelihood) are selected among these 100 runnings as the initial values for the quasi-Newton algorithm. The parameter estimates are those resulting in the largest likelihood among these 10 runnings of the quasi-Newton method.

5. Empirical Results and Discussions

5.1. State Dynamics and Macroeconomic Factors

We firstly take look at parameters governing the state dynamics. Table 1 presents their estimates and corresponding t-ratios (in brackets). The diagonal elements of the matrix Φ determine the persistence of the macroeconomic factors, and the off-diagonal ones of Φ govern their dynamic interactions. We note that the diagonal estimates are all larger than 0.77 and highly statistically significant, indicating that macroeconomic factors are very persistent. In particular, for both the US and the EA, the inflation rate is more persistent than the output gap. This can also be observed from estimates of Σ, where the output gap estimates are larger than the inflation rate estimates and in Figure 1, where the inflation rate is much smoother than the output gap. The US output gap is more persistent than the EA one, whereas its inflation rate is slightly less persistent than that of the EA. The short-term interest rates in both economies are also very persistent.

— Table 1 around here —

From the off-diagonal estimates, we observe a weak link between output gap and inflation in both countries, consistent with previous studies. For each country, we find that the short rate responds positively to shocks to output gap and inflation, suggesting that the short-term interest rate increases with both inflation and real output growth. We split the matrix Φ into four 3×3 sub-matrices. The off-diagonal sub-matrices control the interdependence between two countries. We can see that both economies are mutually dependent since a number of elements in the off-diagonal sub-matrices are statistically significant. In particular, the US output gap has a positively significant impact on the EA output gap, and the EA output gap also positively significantly impacts the US output gap. This implies a co-movement of the business cycles in these two countries. This finding

is in contrast to previous studies that assume that the US economy has an impact on the foreign economy, but not vice-versa.

The dark solid lines in Figure 3 plot the macroeconomic factors extracted from the data using the unscented Kalman filter. The left panels are for the US factors, and the right panels for the EA factors. To compare with the observed data, the estimated factors evolve similarly to the observed ones, but they have smaller variations, indicating that the real data are noisy and really contaminated by measurement errors.

— Figure 3 around here —

5.2. Market Prices of Economic Risks

Table 2 reports parameter estimates of market prices of macroeconomic risks. Most of estimates in $\lambda_0^{(h)}$ and $\lambda_0^{(f)}$ and in $\lambda_1^{(h)}$ and $\lambda_1^{(f)}$ not only have the same signs, but also are very close each other. This implies that the SDF's of two countries should be highly correlated. Indeed, the correlation between the model-implied SDF's is as high as 99%. Brandt et al. (2006) show that volatility of the exchange rate and volatility of the SDF's based on asset markets imply that SDF's must be highly correlated across countries. They report a correlation of about 98% between the US SDF and the SDF of UK, Germany, or Japan. Dong (2006) also finds a very high correlation between the US SDF and the German SDF. Figure 3 also plots the market price of risk of each macroeconomic factor assigned by the home and foreign investors (dashed line and bold line, respectively). We can see that market prices between these two markets are almost indistinguishable and highly correlated. If we think of risks as goods, the prices of these goods are highly equalized in these two markets, indicating that the transaction cost in these markets is pretty small. This can be expected since the US and EA financial markets are maturely developed.

— Table 2 around here —

Since the estimate of μ in the state dynamics is very small, the estimate of λ_0 captures the average market prices on the macroeconomic factors. The matrix λ_1 measures how the market price varies with respect to the risk level. All estimates in $\lambda_0^{(h)}$ and $\lambda_0^{(f)}$ are negative, but some of them are not statistically significant. We note that all the diagonal estimates of $\lambda_1^{(h)}$ and $\lambda_1^{(f)}$ are highly statistically significant and that a number of off-diagonal estimates there are not statistically

different from zero, indicating that in each country, the market price on each macroeconomic factor varies mainly with its own risk level.

For each country, the diagonal output gap estimates in λ_1 are positive, whereas those for inflations and interest rates are negative, indicating that market prices of output gaps become less negative when the real outputs are high, but market prices of inflation factors and short-rate factors become more negative when the inflation rates and the short-rates are high. From Figure 3, we can clearly see these tendencies. The figure also shows that all the market prices of risks in these two economies are negative. Consistent to parameter estimates in λ_1, the market price of each risk is highly correlated with the corresponding macroeconomic factor. Investors demand higher compensation during the recession and during the time when inflation is high. When the short interest rate goes higher, its market price becomes even negatively smaller. Previous studies attribute the upward-sloping mean interest rate term structure to the negative market price of the interest rate risk (Backus et al, 1998).

5.3. Model Performance Analysis

Table 3 presents estimates of standard deviations of the observation measurement errors. For both the US yields and the EA yields, their standard deviations of the measurement errors are very small, ranging from 0.6 basis point to 5.8 basis point. The standard deviations of the measurement errors for the macroeconomic data are also small and are from 0.7 basis point to 3.5 basis point. These results indicate that the model can effectively capture the term structure of interest rates and the dynamics of macroeconomic fundamentals.

— Table 3 around here —

Table 4 reports summary statistics of the observed data and the model-implied values. Consistent with observations in Table 3, we find that the first four moments (mean, standard deviation, skewness and kurtosis) and the autocorrelation of the yield data are very close to the model-implied statistics for both the US and the EA. The difference of summary statistics between the data and the model-implied for macroeconomic variables is also fairly small.

— Table 4 around here —

Even though the standard deviation of the exchange rate measurement errors is large (238 basis point) with comparison to those of yields and macro variables as shown in Table 3, the model-implied mean of the exchange rate changes is very close to that of the observed data (0.15 vs. 0.17). The standard deviation of the model-implied exchange rate changes is smaller than that of the sample (1.77 vs. 2.88). The model-implied skewness and kurtosis are 0.80 and 7.57, respectively, whereas the sample counterparts are -0.06 and 4.37, respectively. However, the correlation between the data and the model-implied exchange rate changes is as high as 76%! If we run a regression of the data (Δs) on the model-implied values ($\Delta \hat{s}$) with a constant, we have the following result

$$\Delta s_t = \underset{(0.0769)}{-0.0001} + \underset{(12.4744)}{1.2289}\ \Delta \hat{s}_t + e_t, \tag{25}$$

where in the brackets are the absolute values of t-ratios. The constant term is very small and not statistically significant, and the coefficient of $\Delta \hat{s}$ is about 1.2 and highly statistically significant. The resulted R^2 of this regression is 57%! Therefore, a reasonable proportion of exchange rate movements can be explained by our model. In contrast, empirical studies based on monetary models and/or new open economy macroeconomics models can only explain at most 10% variation of exchange rate changes. For example, Lubik and Schorfheide (2005) investigate the USD/Euro exchange rate and find that their estimated model explains 10% variation of the one-quarter exchange rate changes in data. A recent study by Dong (2006), which follows a simular approach to this paper, finds that 38% variation of the data can be explained.

The lower Panel of Figure 4 plots the model-implied exchange rate changes $\hat{\Delta s}$ and the observed data. We can see that the data is more volatile than the model-implied exchange rate changes, but the estimated values by our model can capture exchange rate movements reasonably well. In contrast, the linear VAR model of Section 2 is much less capable to capture the volatile exchange rate changes as shown in the upper panel of Figure 4.

— Figure 4 around here —

As comparisons, I also investigate the other two nested models. One assumes that the market price of risk parameter λ_1 is diagonal, and the other assumes that it is zero. Table 5 presents explained variances by the model-implied exchange rate changes and correlations between the observed and model-implied values. We note that our general model has the largest explained variance

and correlation, whereas the constant case has the smallest explained variance and correlation. The likelihood ratio tests reject these two nested models.

— Table 5 around here —

5.4. Foreign Exchange Risk Premium and Forward Premium Anomaly

One of the most notable puzzles in foreign exchange markets is the forward premium anomaly, which finds the tendency for high interest rate currencies to appreciate. Fama (1984) attributes this departure from uncovered interest parity (UIP) to a time-varying risk premium. Our model also suggests that the expected exchange rate change is equal to the sum of the interest rate differential and the risk premium. Table 2 shows that the diagonal estimates of $\lambda^{(h)}$ and $\lambda^{(f)}$ are huge, and this results in a substantial foreign exchange risk premium, which can also been seen in Figure 5. Here I study an augmented UIP, which takes into account not only the interest rate differential but also the foreign exchange risk premium,

$$\Delta s_{t+1} = \alpha_0 + \alpha_1(r_t^{(h)} - r_t^{(f)}) + \alpha_2 rp_t + e_{t+1}, \tag{26}$$

where rp_t is the foreign exchange risk premium defined in equation (8), and e_{t+1} is a noise term. Using our data and the estimated risk premium, we obtain an estimate of α_1 0.63 with t-ratio 0.17 and an estimate of α_2 0.81 with t-ratio 1.97. The resulted R^2 is 0.097. α_2 is statistically significant and its value is not far away from unity, and although α_1 is not significant, its value is positive. If we fix the coefficient of foreign risk premium $\alpha_2 = 1$, the estimate of α_1 is 1.45 with t-ratio 0.97. However, if we impose zero on α_2 and estimate the UIP regression, the estimated α_1 is negative and far away from unity (-1.88 with t-ratio 0.97) and R^2 is only 0.008. The above regressions indicate that the UIP puzzle can be (partially) solved by introducing the foreign exchange risk premium term.

Fama (1984) also argues that the implied risk premium should be negatively correlated with and have larger variance than the interest rate differential. They are always termed as Fama's conditions. Our model implied risk premium (rp_t) does negatively correlate with the interest rate differential ($r^{(h)} - r^{(f)}$) with a correlation about -46% and have a larger variance (0.996 vs. 0.02). The top panel of Figure 5 plots the foreign exchange risk premium and the interest rate differential.

It clearly shows a negative correlation between them and a greater variance of the risk premium.

The middle panel of Figure 5 presents the output gap differential ($g^{(f)} - g^{(h)}$). We find that the risk premium is positively correlated to the output gap differential with a correlation about 31%. This positive correlation implies that when the foreign output gap is higher than the domestic one, people in the market anticipate the foreign currency to appreciate while the domestic currency to depreciate. When one country is in a better economic situation than the other, the market becomes more confident to that country's currency and thus people would like to hold it, leading to its currency to appreciate. The lower panel depicts the risk premium and the inflation rate differential ($\pi^{(f)} - \pi^{(h)}$). They also have a positive correlation (80%). If the current inflation of the foreign country is high, people may expect the central bank to increase its insterest rate in the future. This results in a decreased interest rate differential and an increased risk premium.

— Figure 5 around here —

5.5. Macroeconomic Shocks and the Exchange Rate Dynamics

We have seen that macroeconomic fundamentals such as output gaps, inflation rates, and short term interest rates play important roles in driving the time-varying expected excess returns and in producing high volatility of exchange rate changes. Previous studies find that exchange rate movements are largely disconnected to macroeconomic fundamentals. In monetary models and/or new open economy macroeconomic models, the exchange rate is a linear function of contemporaneous macroeconomic variables. Since the residuals are usually serially correlated in these models, the estimation is always implemented using the first-order differences of relevant variables

$$\Delta s_t = \beta_0 + \beta_1^{(h)} \Delta r_t^{(h)} + \beta_1^{(f)} \Delta r_t^{(f)} + \beta_2^{(h)} \Delta g_t^{(h)} + \beta_2^{(f)} \Delta g_t^{(f)} + \beta_3^{(h)} \Delta \pi_t + \beta_3^{(f)} \Delta \pi_t^{(f)} + v_t, \qquad (27)$$

where v_t is a noise term. In these models, coefficients are typically constrained by $\beta_i^{(h)} = -\beta_i^{(f)}$, for $i = 1, 2, 3$. When estimating this linear model on the data used in this paper, we find a R^2 of 4.7% for the unconstrained regression and a R^2 of 2.8% for the constrained regression. Thus, although macroeconomic factors in our model can account for 57% variation of exchange rate movements, the linear model (27) cannot capture this link between macroeconomic factors and exchange rates.

What exact roles do macroeconomic fundamentals play in our model? We rewrite the exchange

rate dynamic equation (7) as follows

$$\begin{aligned}\Delta\hat{s}_{t+1} &= (\hat{r}_t^{(h)} - \hat{r}_t^{(f)}) + \frac{1}{2}(\hat{\lambda}_t^{(h)\prime}\hat{\lambda}_t^{(h)} - \hat{\lambda}_t^{(f)\prime}\hat{\lambda}_t^{(f)}) + (\hat{\lambda}_t^{(h)\prime} - \hat{\lambda}_t^{(f)\prime})\hat{\varepsilon}_{t+1} \\ &\equiv \Delta\hat{s}_{1,t+1} + \Delta\hat{s}_{2,t+1} + \Delta\hat{s}_{3,t+1}, \end{aligned} \tag{28}$$

where $\Delta\hat{s}_{1,t+1} = \hat{r}_t^{(h)} - \hat{r}_t^{(f)}$ is the estimated differential of short term interest rates between the US and the EA, $\Delta\hat{s}_{2,t+1} = \frac{1}{2}(\hat{\lambda}_t^{(h)\prime}\hat{\lambda}_t^{(h)} - \hat{\lambda}_t^{(f)\prime}\hat{\lambda}_t^{(f)})$ is the estimated foreign exchange rate risk premium, and $\Delta\hat{s}_{3,t+1} = (\hat{\lambda}_t^{(h)\prime} - \hat{\lambda}_t^{(f)\prime})\hat{\varepsilon}_{t+1}$ is the estimated unexpected exchange rate changes related to macroeconomic shocks.

Figure 6 presents these three components of the exchange rate changes. The first component ($\Delta\hat{s}_{1,t+1}$) is very smooth. The second one ($\Delta\hat{s}_{2,t+1}$) becomes volatile in comparison to the first one, but it still has much smaller variation than the model-implied exchange rate changes. This implies that the third component ($\Delta\hat{s}_{3,t+1}$) must be more volatile and should play more important role in explaining exchange rate movements. This is true from the figure that $\Delta\hat{s}_{3,t+1}$ is very volatile and mimics fluctuations of exchange rate changes. The regression of the data on the unexpected exchange rate changes ($\Delta\hat{s}_{3,t+1}$) and a constant results in a R^2 of 37%, taking 76% of the total explained variance.

— Figure 6 around here —

The unexpected exchange rate change $\Delta\hat{s}_{3,t+1}$ is a product of the differential of market prices of risks ($\hat{\lambda}_t^{(h)\prime} - \hat{\lambda}_t^{(f)\prime}$) and the macro innovations ($\hat{\varepsilon}_{t+1}$), both of which are macro-dependent. When we regress the data on the model-implied macroeconomic innovations $\hat{\varepsilon}_{t+1}$ with a constant, the R^2 is 15%. This is close to that obtained in Section 2. However, in VAR approach, macroeconomic shocks on exchange rate changes are time-homogeneous. In our model, the role of the macro innovations is further amplified by the time-varying differential of market prices of risks, and hence the exchange rate dynamic is heteroskedastic. The importance of heteroskedasticity can also be see from the third row in Table 5, where by setting the market prices of risks constant, only 25% variation of the data can be explained. Macroeconomic innovations are always regarded as "news" to macroeconomic fundamentals. Their importance has also been investigated by Engel, Mark and West (2007) and Andersen et al. (2003).

5.6. Is Yield Curve Information Enough?

We have noticed that in our model, macroeconomic fundamentals enter into the exchange rate dynamics in a nonlinear form and that information contained in the term structure of interest rates play very important role in identifying market prices of risks. This modeling approach results in a big explanation improvement with comparison to the previous linear approach. However, people may think that this success of modeling exchange rates may be largely from nonlinear transformation of information contained in the yield data but not from introducing macroeconomic fundamentals. In this subsection, we investigate this issue by keeping the same modeling framework of jointly investigating exchange rate dynamic and two-country interest rate term structures but shutting down macroeconomic information.

The term structure of interest rates can be empirically captured by the "level", the "slope" and the "curvature" (Nelson and Siegel 1987). We construct the empirical yield curve factors as follows. The "slope" (s_t) is defined as the spread between yields with longest and shortest maturity ($y_t^{(60)} - y_t^{(1)}$). The "curvature" (c_t) is defined as two times yields with medium maturity minus the sum of yields with longest and shortest maturity, $2y_t^{(24)} - (y_t^{(60)} + y_t^{(1)})$ for the US and $2y_t^{(12)} - (y_t^{(60)} + y_t^{(1)})$ for the EA. And the empirical "level" factor (r_t) is simply proxied by yields with shortest maturity.

Under this construction, we have a new state vector that includes the home and foreign yield curve factors only $X_t = \left(\tilde{s}_t^{(h)},\ \tilde{c}_t^{(h)},\ \tilde{r}_t^{(h)}, \tilde{s}_t^{(f)},\ \tilde{c}_t^{(f)},\ \tilde{r}_t^{(f)}\right)$. We estimate this model ("L-S-C" model) using the same econometric method discussed before. The last row of Table 5 reports the explained variance and the correlation between the data and the model-implied exchange rate changes under the "L-S-C" model. This model can capture the yield curve dynamics pretty well, consistent with previous studies. For exchange rate changes, we find that it still performs better than the linear model since it can explain 13% variation of the data and the model-implied exchange rate changes has 36% correlation with the observe values. However, its explained variance and correlation are much lower than those implied by our model and its nested models, indicating that yield curve information is not enought to explain the exchange rate dynamics and that macroeconomic fundamentals do really play important roles.

6. Conclusion

This paper investigates relationship between the short-run nominal exchange rates changes and macroeconomic fundamentals by adopting a no-arbitrage macro-finance approach under a two-country framework, where macroeconomic information enters into the exchange rate dynamics through different channels in a non-linear form. Based on empirical analysis using an enriched dataset including exchange rates, yields of zero-coupon bonds, and macroeconomic variables of the US and the Euro area, the paper finds a close link between macroeconomic fundamentals and exchange rate dynamics. The model-implied exchange rate changes can explain about 57% variation of the observed data. This is in stark constrast to previous studies using monetary and new open economy macroeconomics models, which can explain only around 10% variation of exchange rates. Having been amplified by the time-varying market prices of risks, the innovations of macroeconomic fundamentals are the driving engine for generating large volatility of exchange rate changes.

The model in this paper has a fairly good fit to monthly exchange rate changes. However, there is still nearly 40% variation that cannot be explained. This is because there may be other missing factors such as current account (Hooper and Morton 1978), market incompleteness (Brandt and Santa-Clara 2002), government deficit, "news" from the stock market, default risk and so on. Therefore, it would be interesting to investigate these factors and to see whether they can help explain the exchange rate dynamics in the future.

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Table 1: **Parameter Estimates of the State Dynamics**

		$\mathbf{\Phi}$						$\mathbf{\Sigma}(\times 10^3)$					
	$\mu(\times 10^3)$	$\tilde{g}^{(h)}$	$\tilde{\pi}^{(h)}$	$\tilde{r}^{(h)}$	$\tilde{g}^{(f)}$	$\tilde{\pi}^{(f)}$	$\tilde{r}^{(f)}$	$\tilde{g}^{(h)}$	$\tilde{\pi}^{(h)}$	$\tilde{r}^{(h)}$	$\tilde{g}^{(f)}$	$\tilde{\pi}^{(f)}$	$\tilde{r}^{(f)}$
$\tilde{g}^{(h)}$	-0.298	0.834	0.000	0.003	0.010	0.016	-0.002	0.637	0	0	0	0	0
	(0.94)	(3.49)	(0.65)	(0.50)	(2.19)	(0.83)	(0.53)	(3.77)	–	–	–	–	–
$\tilde{\pi}^{(h)}$	0.149	-0.001	0.918	0.023	-0.007	0.001	0.011	-0.018	0.142	0	0	0	0
	(2.50)	(1.24)	(4.48)	(1.81)	(1.16)	(1.75)	(0.38)	(1.43)	(1.94)	–	–	–	–
$\tilde{r}^{(h)}$	-0.083	0.048	0.051	0.941	-0.043	0.007	0.002	0.101	-0.241	0.344	0	0	0
	(0.35)	(2.05)	(1.98)	(2.56)	(1.02)	(0.61)	(0.95)	(1.21)	(0.44)	(1.96)	–	–	–
$\tilde{g}^{(f)}$	-0.331	0.025	0.014	-0.008	0.776	0.008	0.036	0.424	-0.166	0.014	0.796	0	0
	(2.14)	(2.03)	(1.07)	(1.41)	(2.28)	(1.92)	(0.40)	(1.99)	(0.48)	(1.95)	(2.03)	–	–
$\tilde{\pi}^{(f)}$	0.369	0.006	0.015	0.018	0.008	0.930	0.002	-0.008	-0.005	-0.041	-0.060	0.146	0
	(0.83)	(1.15)	(1.06)	(1.11)	(1.65)	(2.67)	(2.02)	(0.81)	(0.54)	(2.16)	(2.15)	(2.27)	–
$\tilde{r}^{(f)}$	-0.029	-0.008	0.011	0.022	0.050	0.095	0.896	-0.113	-0.080	0.252	0.042	-0.192	0.220
	(1.85)	(1.73)	(0.49)	(2.07)	(1.97)	(2.12)	(2.84)	(1.11)	(1.05)	(1.23)	(0.95)	(3.46)	(2.35)

Note: The table reports the parameter estimates for the state dynamics that follow a VAR(1) process. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period for estimation is from February 1999 to December 2008 and the data is in monthly frequency.

Table 2: **Parameter Estimates of Market Prices of Risks**

	US	$\lambda_1{}^{(h)}$						*EA*	$\lambda_1^{(f)}$					
	$\lambda_0^{(h)}$	$\tilde{g}^{(h)}$	$\tilde{\pi}^{(h)}$	$\tilde{r}^{(h)}$	$\tilde{g}^{(f)}$	$\tilde{\pi}^{(f)}$	$\tilde{r}^{(f)}$	$\lambda_0^{(f)}$	$\tilde{g}^{(h)}$	$\tilde{\pi}^{(h)}$	$\tilde{r}^{(h)}$	$\tilde{g}^{(f)}$	$\tilde{\pi}^{(f)}$	$\tilde{r}^{(f)}$
$\tilde{g}^{(h)}$	-0.361	105.1	0.015	-0.001	0.031	-0.012	-0.006	-0.369	95.49	0.004	0.009	0.091	0.012	0.013
	(1.65)	(6.25)	(0.35)	(0.86)	(2.03)	(1.74)	(0.83)	(1.65)	(4.36)	(0.60)	(1.22)	(1.99)	(1.05)	(1.78)
$\tilde{\pi}^{(h)}$	-0.205	0.006	-30.66	0.013	-0.004	0.002	-0.005	-0.223	0.004	-27.10	0.153	0.076	0.005	-0.004
	(1.06)	(0.70)	(3.79)	(3.05)	(0.60)	(1.12)	(0.72)	(2.06)	(0.53)	(2.83)	(1.12)	(0.49)	(1.76)	(0.79)
$\tilde{r}^{(h)}$	-0.313	0.015	-0.047	-106.6	-0.009	-0.018	-0.008	-0.306	0.011	0.098	-107.5	0.050	0.026	0.011
	(2.37)	(1.07)	(1.48)	(3.36)	(1.20)	(1.89)	(2.42)	(2.03)	(2.12)	(0.44)	(3.82)	(0.70)	(1.80)	(0.75)
$\tilde{g}^{(f)}$	-0.211	-0.008	-0.011	0.078	78.25	0.013	0.009	-0.222	0.015	0.035	0.086	84.60	-0.017	0.045
	(0.69)	(0.41)	(0.48)	(1.65)	(4.04)	(1.62)	(0.45)	(0.74)	(2.31)	(0.65)	(1.00)	(3.74)	(0.50)	(1.80)
$\tilde{\pi}^{(f)}$	-0.360	0.022	0.018	-0.022	0.082	-46.49	0.007	-0.414	0.012	-0.101	0.022	-0.029	-6.511	-0.015
	(1.63)	(0.91)	(0.43)	(0.45)	(1.05)	(4.45)	(1.99)	(1.91)	(2.60)	(1.34)	(0.51)	(0.59)	(1.91)	(0.91)
$\tilde{r}^{(f)}$	-0.349	-0.006	-0.004	-0.055	0.011	0.002	-111.5	-0.310	0.012	-0.076	-0.027	-0.008	0.017	-124.4
	(1.74)	(0.42)	(0.99)	(2.04)	(1.87)	(1.99)	(2.87)	(2.04)	(1.53)	(0.80)	(2.04)	(1.11)	(0.60)	(4.48)

Note: The table presents the parameter estimates for affine equation of market prices of risk. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period for estimation is from February 1999 to December 2008 and the data is in monthly frequency.

Table 3: **Standard Deviations of Measurement Errors**

Yields	$n=1$	$n=3$	$n=12$	$n=24$	$n=36$	$n=48$	$n=60$
$\sigma_\eta^{(h)}$	5.76	4.30	2.58	0.58	3.13	2.37	4.63
	(4.83)	(3.10)	(3.33)	(1.95)	(3.44)	(3.08)	(2.03)
$\sigma_\eta^{(f)}$	2.47	3.27	1.11	–	2.26	–	5.27
	(1.91)	(3.07)	(2.02)	–	(2.16)	–	(2.06)
Macro & Exchange Rate	σ_η^{gh}	$\sigma_\eta^{\pi h}$	σ_η^{gf}	$\sigma_\eta^{\pi f}$	–	–	$\sigma_\eta^{\Delta s}$
	3.50	1.19	2.25	0.69	–	–	238
	(3.52)	(1.97)	(2.56)	(2.07)	–	–	(3.07)

Note: The table reports the parameter estimates of standard deviations of measurement errors (in basis point). In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period for estimation is from February 1999 to December 2008 and the data is in monthly frequency

Table 4: **Summary Statistics of Data and Model Implied Data**

	Mean(%)		*Std. Dev.*(%)		*Skewness*		*Kurtosis*		*Autocorr.*	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
A. The US Yields										
$1m$	0.255	0.265	0.144	0.148	0.075	0.169	1.588	1.933	0.963	0.960
$3m$	0.264	0.268	0.148	0.149	0.071	0.152	1.580	1.941	0.972	0.967
$12m$	0.286	0.283	0.143	0.141	0.088	0.113	1.658	1.952	0.970	0.965
$24m$	0.303	0.303	0.128	0.128	0.088	0.092	1.955	1.947	0.960	0.961
$36m$	0.322	0.322	0.115	0.115	0.088	0.080	2.122	1.943	0.951	0.955
$48m$	0.339	0.340	0.104	0.104	0.073	0.072	2.401	1.941	0.938	0.958
$60m$	0.352	0.355	0.093	0.094	0.217	0.066	2.491	1.940	0.935	0.959
B. The EA Yields										
$1m$	0.270	0.268	0.079	0.076	0.216	0.219	1.703	1.938	0.979	0.976
$3m$	0.278	0.267	0.084	0.076	0.223	0.190	1.691	1.861	0.983	0.978
$12m$	0.274	0.274	0.077	0.075	0.156	0.122	1.871	1.787	0.956	0.961
$36m$	0.302	0.302	0.067	0.069	0.111	0.022	2.011	2.219	0.937	0.942
$60m$	0.327	0.322	0.059	0.057	0.141	0.047	2.156	2.330	0.933	0.943
C. The Macro Variables										
$g^{(h)}$	0.000	-0.007	0.177	0.160	-0.849	-0.942	4.592	5.114	0.869	0.870
$\pi^{(h)}$	0.184	0.184	0.034	0.031	-0.804	-0.849	3.353	3.290	0.942	0.966
$g^{(f)}$	0.000	0.002	0.197	0.194	-1.103	-1.150	8.028	7.913	0.794	0.815
$\pi^{(f)}$	0.137	0.137	0.035	0.033	0.002	-0.029	2.386	2.360	0.945	0.959
D. The Exchange Rate										
Δs	0.170	0.149	2.877	1.769	-0.060	0.800	4.368	7.570	0.195	0.155

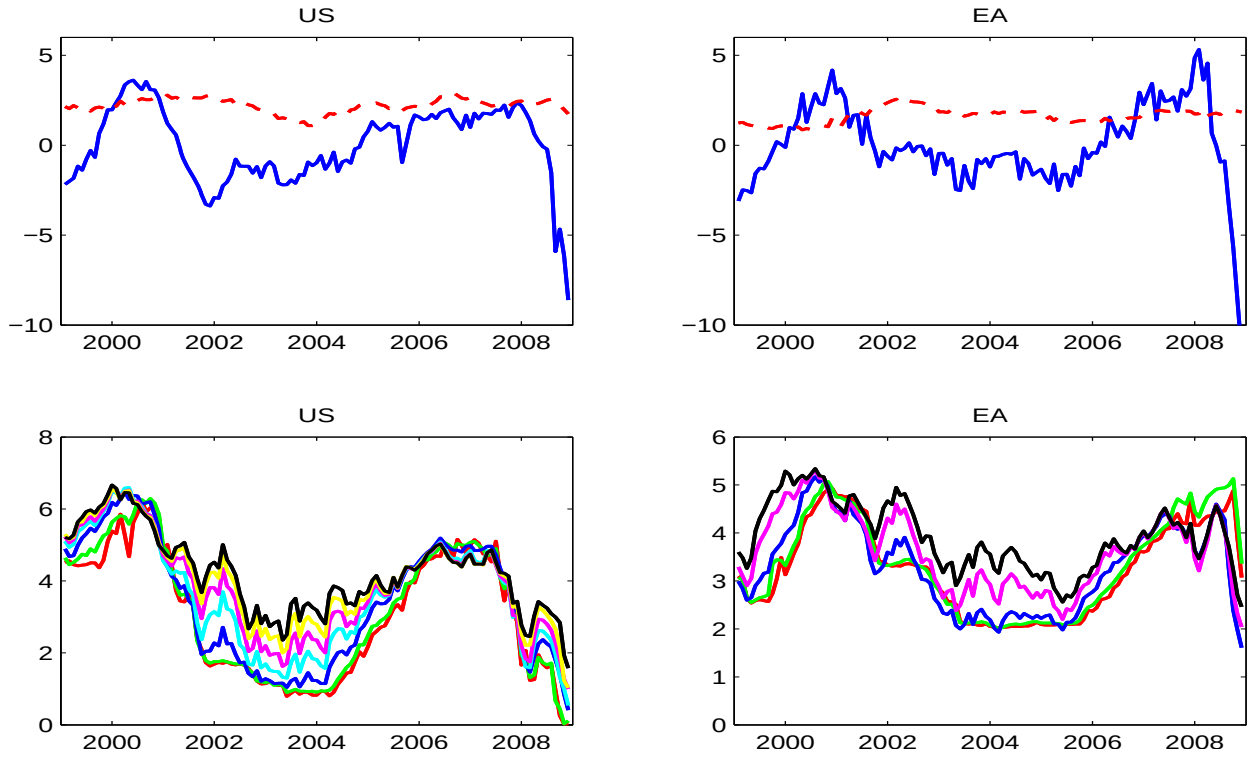
Note: The table reports summary statistics of the data and the model-implied values. Panel A reports moments of the US yields with maturity 1-month, 3-month, 12-month, 24-month, 36-month, 24-month, 48-month, and 60-month. Panel B reports moments of the EA yields with maturity of 1-month, 3-month, 12-month, 36-month, 48-month, and 60-month. Panel C report moments of the US and the EA macroeconomic variables of output gaps and inflation rates. Panel D reports moments of exchange rate changes. The sample period is Febuary 1999 to December 2008 and the data is in monthly frequency.

Table 5: **Model Performance Comparison**

	Explained Variance	Correlation(Δs, $\Delta \hat{s}$)	LR Test
Full	57 %	76 %	–
Diagonal	34 %	58 %	92
Constant	25 %	50 %	173
L-S-C	13 %	36 %	–

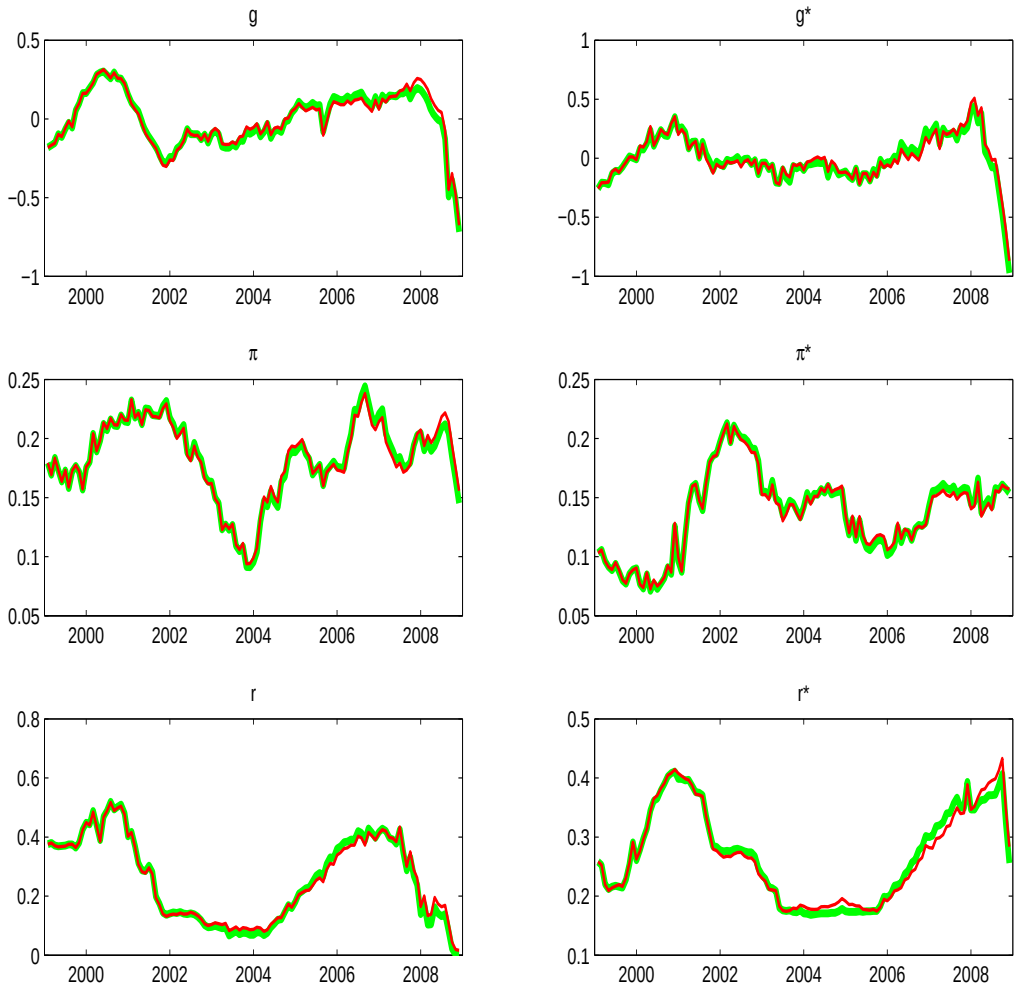
Note: The table reports the explained variance of the model-implied value and correlation between the model-implied values and the observed data for each of four models examined in the paper. The Full refers to our general model where λ_1 is a full matrix. The Diagonal refers to the model where λ_1 is a diagonal matrix. The Constant represents the model where λ_1 is a zero matrix. The L-S-C is the model where only the yield curve information is used without any macroeconomic variables. Explained variance is the adjusted R^2 of a regression of the observed exchange rate changes on the model-implied values with a constant.

Figure 1: Macroeconomic and Yield Data



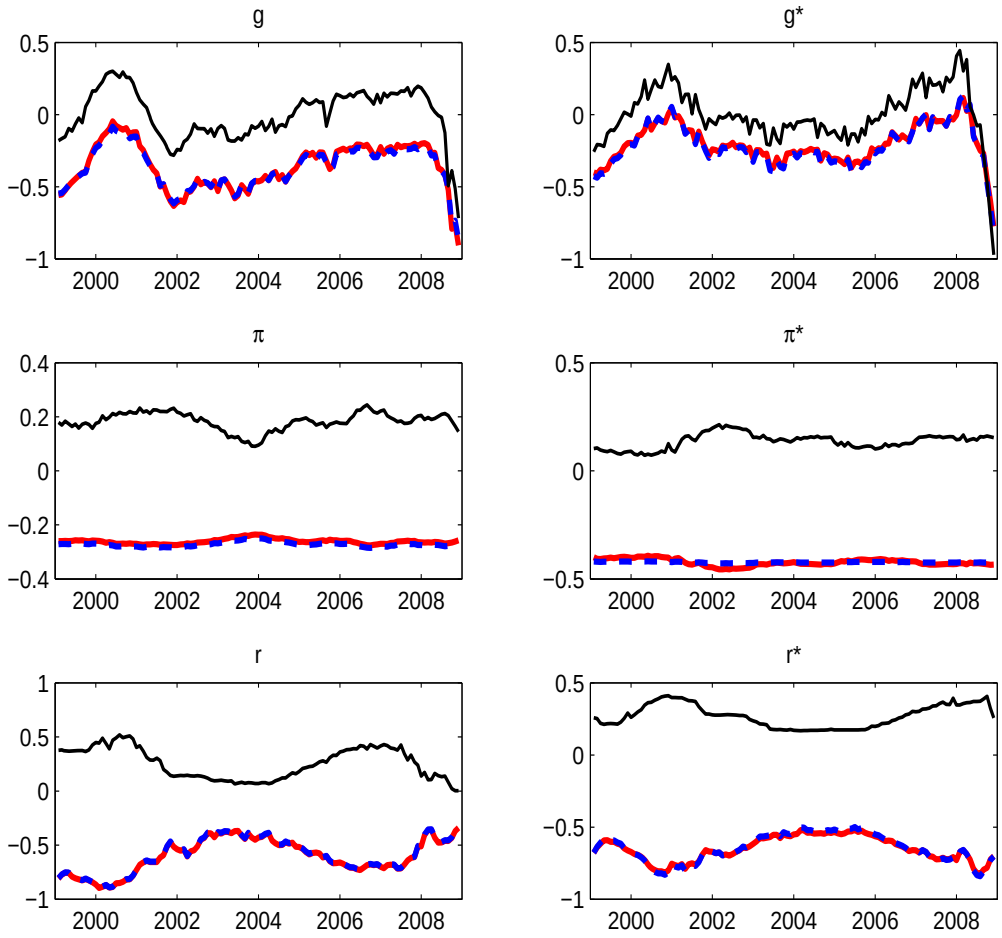
Note: The upper panels of the figure plot the annualized macroeconomic data used in the estimation in the upper panels. The output gaps are in solid lines and inflation rates in dish lines. The lower panels plot the annualized yields. The US yields have maturity 1-month, 3-month, 1-year, 2-year, 3-year, 4-year and 5-year, and the EA yields have maturity 1-month, 3-month, 1-year, 3-year, and 5-year. The left panels are for the US and the right ones for the EA. The unit on the vertical axis is in percentage.

Figure 2: The Role of Exchange Rate Innovations on Macro Variables



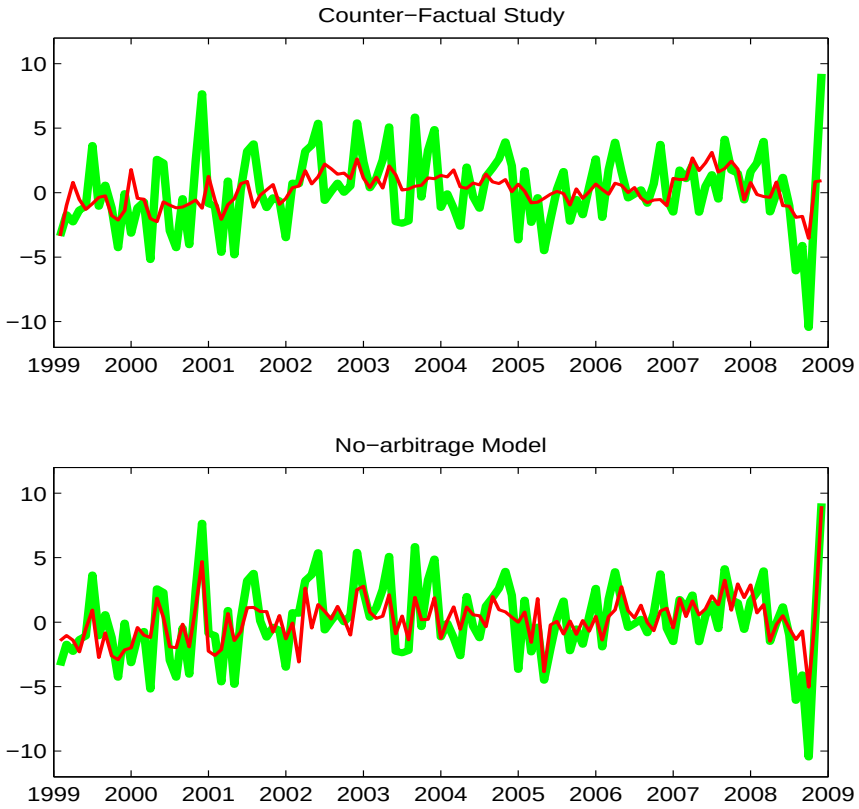
Note: The figure compares the actual and the counter-factual time series $[g_t^{(h)}\ \pi_t^{(h)}\ y_{1t}^{(h)}\ g_t^{(f)}\ \pi_t^{(f)}\ y_{1t}^{(f)}]'$. The counter-factual time series is simulated from the VAR(1) model using the estimated parameter in Section 2 by setting innovations of exchange rate changes to zero.

Figure 3: Macro Factors and Market Prices of Risks



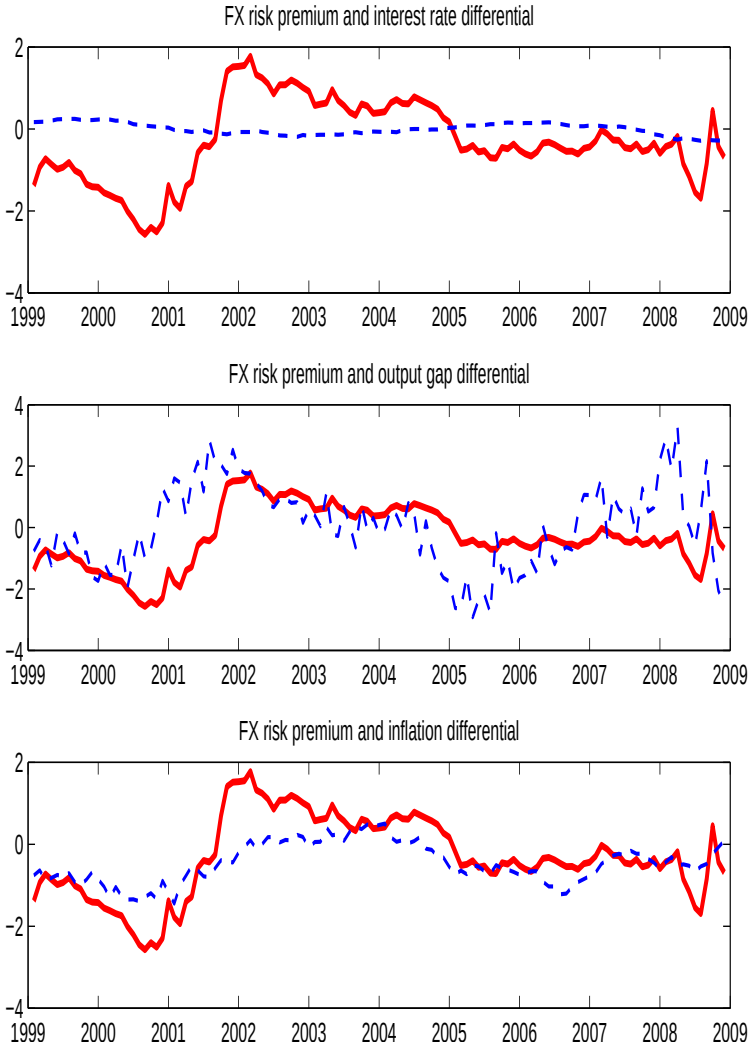
Note: Each panel of the figure plots the extracted macroeconomic factor (dark solid line) and the corresponding time-varying market price of risk assigned by the US market (dashed line) and the EA market (bold line).

Figure 4: Exchange Rate Dynamics



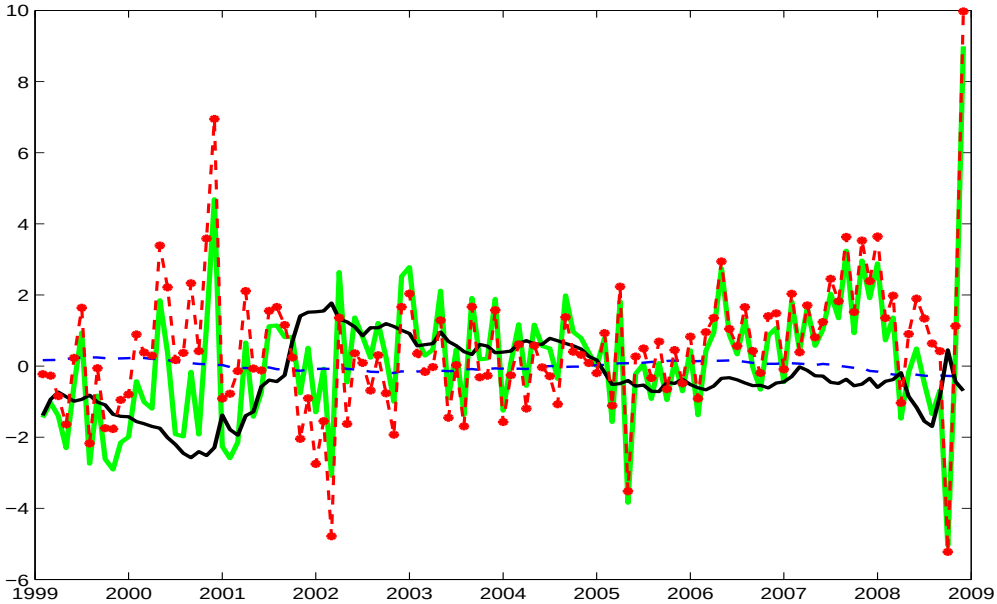
Note: The upper panel plots the simulated exchange rate changes (thin line) from the linear VAR model studied in Section 2.2 and the observe data (bold line). The lower Panel plots the model-implied exchange rate changes (thin line) from the no-arbitrage model and the observe data (bold line). The correlation between the model-implied exchange rate changes from the no-arbitrage model and the observed data is 76%, and the model-implied exchange rate changes can explain 57% variation of the data. The correlation between the simulated exchange rate changes and the observed data is 42%, and the model-implied values only explain 18% variation of the data. The unit on the vertical axis is in percentage.

Figure 5: The Foreign Exchange Risk Premium



Note: The top panel of the figure plots the foreign exchange risk risk premium and the interest rate differential ($r^{(h)} - r^{(f)}$); The middle panel plots the foreign exchange risk premium and the output gap differential ($g^{(f)} - g^{(h)}$). The bottom panel plots the foreign exchange risk premium and the inflation rate differential ($\pi^{(f)} - \pi^{(h)}$). The risk premium is in dark line and the others are in shallow line.

Figure 6: Components of Model-implied Exchange Rates



Note: The figure plots the model-implied exchange rate changes (bold solid line) and its components: $\Delta\hat{s}_{1,t+1} = (\hat{\tilde{r}}_t^{(h)} - (\hat{\tilde{r}})_t^{(f)})$, $\Delta\hat{s}_{2,t+1} = \frac{1}{2}(\hat{\lambda}_t^{(h)\prime}\hat{\lambda}_t^{(h)} - \hat{\lambda}_t^{(f)\prime}\hat{\lambda}_t^{(f)})$, and $\Delta\hat{s}_{3,t+1} = (\hat{\lambda}_t^{(h)\prime} - \hat{\lambda}_t^{(f)\prime})\hat{\varepsilon}_{t+1}$. The first component is in dashed line, the second in dark solid line and the third in dashed line with mark.