Communication, Decision-making and the Optimal Degree of Transparency of Monetary Policy Committees

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July 2008

Abstract

In this paper we model the dynamic decision-making process of a monetary policy committee with heterogeneous members, which is observed by the financial markets. We show that transparency about the different views among committee members surrounding the economic outlook makes monetary policy more predictable. However, communicating the diversity of views about the monetary policy decision may lead to less predictability in the short term. Communication in the form of voting records is shown to have the greatest effect on market participants’ near term policy expectations. These results support previous empirical findings and have strong implications for how committees should communicate.

JEL Codes: E50, E52, E58

1 Introduction

The conduct of monetary policy has changed markedly since the 1990s. Over the past decade, a number of central banks shifted responsibility for interest rate setting to a monetary policy committee. In addition, central banks have become more independent and now pay close attention to explaining what they do and what underlies their decisions. More transparency and increased use of communication can be seen as a consequence of these developments (de Haan et al, 2007).

This paper provides a theoretical model of a monetary policy committee that meets repeatedly over time and which is observed by the financial markets. It thereby analyses theoretically how committees...
should communicate. The model developed contrasts in several ways with previous work on monetary policy transparency and communication. First, we employ a model in which decisions are set by a monetary policy committee with heterogeneous members rather than assuming that there exists a representative central banker. It is assumed that policy makers themselves face uncertainty in assessing the state of the economy\(^3\) and that there are two sources of heterogeneity between members: Members receive different signals on the state of the economy and they face uncertainty about the precision of these signals. They are furthermore assumed to have different preferences regarding inflation. Second, we introduce asymmetric information between the committee and market participants. This asymmetry of information is due to imperfect knowledge of market participants of the signals on the state received by committee members as well as the precision of these signals and uncertainty about the implicit inflation targets of committee members. As a result of the first source of this asymmetric information, the optimal assessment of the state of the economy by financial markets is inferior to that of the committee\(^4\).

Besides providing a new framework in which to analyse optimal communication strategies of committee members, several interesting insights are obtained. It is shown that communication matters for financial markets’ expectations of the policy decision. Actual and perceived transparency of committee members about their views on the state of the economy is beneficial in that it leads to greater policy predictability. In addition, communication of a divergence of views by committee members regarding the monetary policy decision, as for example the publication of voting records, may in the short run lead to less policy predictability. We also find several empirical predictions of the model. It is shown that predictability of monetary policy decisions increases with the time a committee has been in office. This is because over time some of the initial information asymmetry between the committee and market agents is eliminated as market agents are able to learn about committee members’ preferences and the precision of signals that members receive on the state of the economy. In addition, the impact on the yield curve of communicating diverse views of committee members on monetary policy should be greatest for near-term policy expectations.

This paper is related to the recent literature on the optimal design and communication strategies\(^3\)The fact that central banks do not know the current state of the economy with certainty, has for example been stressed in papers by Orphanides (2003) and Aoki (2006).\(^4\)This feature of the model receives support from an empirical study by Romer and Romer (1996), which compares forecast errors of the Federal Reserve and of commercial forecasters and finds evidence of an informational advantage of the Federal Reserve. Romer and Romer (1996) argue that this advantage might not be due to better data availability but to central bank staff which is better at processing and interpreting information.
of monetary policy committees. Existing theoretical papers on optimal committee design investigate questions such as why it is beneficial to have a committee rather than a single policymaker and what the optimal decision-making procedure and size of such a committee should be (see for example, Gerlach-Kristen (2006), Sibert (2006)). The general consensus has emerged that delegating monetary policy to a committee leads to superior policy for a number of reasons, such as the ability to pool judgements of different members and the possibility of learning from other members. Furthermore, the literature so far has concluded that a committee of limited size is optimal due to the costs of acquiring information about different members and coordination costs (Sibert, 2006).

There has also been a lively debate on the effects of publishing voting records on the behaviour of committee members. This was discussed extensively by Buiter (1999) and Issing (1999) and was subsequently analysed in theoretical papers by Sibert (2003) and Gersbach and Hahn (2005). Sibert (2003) argues, that there may be positive incentive effects for committee members to keep inflation low. However, Gersbach and Hahn (2005) conclude that at least in the case of the ECB, the publication of voting records is likely to exacerbate the so-called small country bias and induces members of the Governing Council to vote for the optimal interest rate of the country they represent.

There has furthermore been an increasing number of papers examines the optimal communication strategies of central banks and their effects on financial markets. Communication of central banks can take various forms, including information on the range of views expressed in committee meetings, by publishing minutes and voting records, as well as speeches and interviews of committee members. The existing theoretical literature on this subject differs in its conclusions. Some economists stress the importance of communication in providing central banks with the means to influence key asset prices in the economy (Blinder (1998), Bernanke (2004))\(^6\). Although policymakers have only direct control of the short term interest rate, they want to shape interest rate expectations along the yield curve and thereby anchor expectations, which is seen as crucial in achieving the central bank’s objectives (Eggertson and Woodford, 2003). Hence, communication may prove useful for central banks because it can be a direct tool to influence markets expectations and it may be used to reduce noise in financial markets and lead to greater predictability of policy decisions. However, Morris and Shin (2002) argue that too much communication may not be desirable, as the private sector may overreact to central bank

\(^5\)An overview of this literature is provided by Blinder (2007).

\(^6\)Bernake (2004) notes: 'Control of the federal funds rate is therefore useful only to the extent that it can be used as a lever to influence more important asset prices and yields-stock prices, government and corporate bond yields, mortgage rates-which in turn allow the Fed to affect the overall course of the economy'.
announcements. Their finding does capture a concern expressed by some policy makers. For example, in discussing the release of FOMC minutes, Yellen(2005) expressed the view that 'Financial markets could misinterpret and overreact to the minutes'.

There is also an emerging empirical literature on communication by central banks. The results show that communication has an effect on policy predictability. Gerlach-Kristen (2004) finds that voting records of committee members can convey information about the views of individual members and their publication may hence lead to greater policy predictability. Papers by Kohn and Sack (2004) and by Reeves and Sawicki (2007) show that communication and in particular the publication of minutes significantly affect near-term interest rate expectations. Ehrmann and Fratzscher (2005) investigate communication by three major central banks (the Bank of England, the Federal Reserve and the European Central Bank) and try to evaluate the effectiveness of central bank communication on financial markets in terms of their ability to anticipate monetary policy decisions. They conclude that a higher degree of communication dispersion among committee members about monetary policy worsens the ability of financial markets to anticipate future monetary policy decisions and hence raises the degree of uncertainty. They also find that communication of risks and diversity of views of the committee surrounding the economic outlook enhances the ability to anticipate future monetary policy decisions. Furthermore they show that a higher frequency of communication helps markets to predict future monetary policy.

The remainder of the paper is organised as follows. Section 2 sets out the basic model. The solution of this basic model is discussed in Section 3. In Section 4, results are reported that examine the degree of policy predictability with different degrees of transparency. We also analyse implications of different degrees of transparency on the yield curve of the economy. Conclusions are summarised in Section 5.

2 The Model

2.1 General Setup

We follow Svensson (1997) in adopting a simple backward looking model of the economy. Key features of this model are that demand and supply shocks have persistent effects and that monetary policy affects the output gap and inflation with a lag.
The model of the economy is structured as follows:

\[ \pi_{t+1} = \pi_t + \alpha_1 y_t + \epsilon_{t+1} \]  

\[ y_{t+1} = \beta_1 y_t - \beta_2 (i_t - \pi_t) + \eta_{t+1} \]  

where \( \epsilon_{t+1} \) and \( \eta_{t+1} \) are normally distributed variables with \( E(\epsilon_{t+1}) = E(\eta_{t+1}) = 0 \), \( Var(\epsilon_{t+1}) = \sigma_y^2 \) and \( Var(\eta_{t+1}) = \sigma_t^2 \). Furthermore \( \pi_t \) is the inflation rate, \( y_t \) the output gap and \( i_t - \pi_t \) denotes the real repo rate. Following Ellingson and Soederstroem (2001), an equation for the term structure of interest rates is added to the above model. Bonds of different maturities are seen as imperfect substitutes and so the interest rate on a discount bond of maturity \( n \) at time \( t \) is set as the average of expected future short term interest rates during the time to maturity plus a term premium\(^7\):

\[ i^n_t = \frac{1}{n} \sum_{s=0}^{n-1} i_{t+s} + \xi^n_t \]  

where \( i_{t+s} \) is the expected short term interest rate \( s \) periods ahead and \( \xi^n_t \) is the term premium at time \( t \) for maturity \( n \). Thus in determining long rates, market participant will form rational expectations about the future path of the short central bank rate.

The policymaker is assumed to have the following loss function:

\[ L_t(\pi_t, y_t) = \frac{1}{2} \left[ (\pi_t - \pi^*)^2 + \lambda y_t^2 \right] \]  

where \( \lambda \) is the weight attached to output stabilization and \( \pi^* \) denotes the inflation target.

The intertemporal loss function is

\[ E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L(\pi_\tau, y_\tau) \]  

with \( \delta \) denoting the discount rate.

In order to find the optimal reaction function, we need to minimise Equation (2.4) with respect to

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\(^7\)Following Ellingsen and Soederstroem (2001), throughout this paper, it will be assumed that the term premium is independent of all relevant variables, that is, that the expectations hypothesis of the term structure holds.
Equations (2.1), (2.2) and (2.5). Svensson (1997) shows that the reaction function follows

\[ i_t = \pi_t + \frac{1 - c}{\beta_2 \alpha_1} (\pi_t - \pi^*) + \frac{1 - c + \beta_1}{\beta_2} y_t = \pi_t + A(\pi_t - \pi^*) + B y_t \]  

(2.6)

where

\[ c = \frac{\lambda}{\lambda + \delta \alpha_1^2 k} \quad \text{and} \quad k = \frac{1}{2} \left( 1 - \frac{\lambda(1 - \delta)}{\delta \alpha_1^2} + \sqrt{\left( 1 + \frac{\lambda(1 - \delta)}{\delta \alpha_1^2} \right)^2 + \frac{4\lambda}{\alpha_1^2}} \right) \]

Thus, the real repo rate \( i_t - \pi_t \) is increasing in the excess of current inflation over the inflation target and in current output. The instrument of the central bank hence depends on current inflation and output, not because current inflation is targeted, but because current inflation and output determine future inflation.

Following Ellingson and Soederstroem (2001), it is possible to find a closed form expression for the economy’s yield curve. This can be done using the reaction function and the inflation and output relationships. As shown in the Appendix the yield curve can be expressed as

\[ i^n_t = \frac{1}{n} \left\{ \pi_t + A(\pi_t - \pi^*) + B y_t + [1 + A(1 - \beta_2 B)] X_n [\pi_t - \pi^* + \alpha_1 y_t] + (n - 1)\pi^* + \xi^n_t \right\} \]  

(2.7)

### 2.2 A Model of a Monetary Policy Committee and Financial Markets under Uncertainty

We assume the existence of a monetary policy committee with \( N \) members. This committee is observed by the financial markets. We assume that there is no heterogeneity among the market agents. However, there are two sources of heterogeneity among the committee members. First, each committee member is assumed to only imperfectly observe the current shock to output. The shock that occurred in the previous period is however assumed to be perfectly known and hence there is no uncertainty with regards to the output gap in the previous period. Each member, \( j \), receives a signal of the current state, the variance of which is unknown to each committee member and has to be estimated.

\[ y_t^{(j)} = y_t + \varepsilon_t^{(j)} \]  

(2.8)

with \( \varepsilon_t^{(j)} \sim N(0, \sigma^2_{y,j}) \) for \( j = 1, 2, ..., N \).

Hence, each committee member \( j \) receives a signal on the state of the economy, which is unbiased
and has a constant variance. This variance is however, unknown to committee members and is estimated by member \( k \) as \( \sigma^2_{y,t}^{(k,j)} \) for \( j = 1, \ldots, N \) and \( k = 1, \ldots, N \).

The second source of heterogeneity comes from the preferences of committee members regarding the inflation target. It will be assumed that committee members have their own implicit inflation targets\(^8\). This may differ among committee members. For each committee member \( j \),

\[
\theta_t^{(j)} = \pi^* + z_t^{(j)} \tag{2.9}
\]

where \( z_{t+1}^{(j)} = z_t^{(j)} = z^{(j)} \). We hence assume that \( z_t^{(j)} \) is constant and the implicit inflation targets of committee members are degenerate\(^9\).

Financial markets know the state equations (2.1) and (2.2) and they are aware of the general process by which interest rates are set, that is they know the general form of the loss function. However, there are two important information asymmetries between the central bank committee and financial markets. First, financial markets do not observe the signal that each committee member receives on the current output gap perfectly. Instead, financial markets receive public information on the signals of committee members:

\[
y_t^{(F,j)} = y_t^{(j)} + \zeta_t^{(F,j)} = y_t + \zeta_t^{(F,j)} = y_t + \zeta_t^{(F,j)} \tag{2.10}
\]

where \( \zeta_t^{(F,j)} \sim N(0, \sigma_{\zeta_t}^{2(F,j)}) \) for \( j = 1, \ldots, N \), and \( \sigma_{\zeta_t}^{2(F,j)} = \sigma_{y_t}^2 + \sigma_{\zeta_t}^2 \).

The noise \( \zeta_t^{(F,j)} \) arises from the fact that committee members may not communicate their signals perfectly to financial markets. When \( \sigma_{\zeta_t}^{2(F,j)} = 0 \), the signals \( y_t^{(F,j)} \) communicate \( y_t^{(j)} \) without any noise and so there is perfect actual transparency about the committee members’ signals on the output gap. We assume that the variance of \( \zeta_t^{(F,j)} \) is unknown to market agents and is estimated as \( \tilde{\sigma}_{\zeta_t}^{2(F,j)} = \tilde{\sigma}_{y_t}^2 + \tilde{\sigma}_{\zeta_t}^2 \). Thus perfect transparency as perceived by financial markets implies that \( \tilde{\sigma}_{\zeta_t}^{2(F,j)} = 0 \).

Transparency as used in this paper hence refers to the absence of asymmetric information between the

\(^8\)This assumption is based on the idea that a number of inflation targeting central banks (such as Australia, Canada, Sweden for example) have inflation target bands. It should also be noted that there exist central banks such as the Bank of Japan, where members of the monetary policy committee are not given any explicit inflation targets. Furthermore, even if a central bank has a point target such as the Bank of England, committee members might disagree as to the speed with which inflation should be brought down or up to the target given that committee members are also interested in output stabilisation. It is hence reasonable to assume that there are small differences between committee members regarding \( \pi^* \), which is interpreted as the point target of committee members. It should be noted that alternatively one could assume that committee members have different preferences regarding the output gap stabilisation. This would be reflected in different values of \( \lambda \) in the loss functions. However, this would lead to very complicated mathematics but intuitively one would expect to obtain the same results as with the assumption of different implicit inflation targets.

\(^9\)It would also be possible to follow Svensson (1997) in assuming that the implicit inflation targets follow AR(1) processes. This would not affect the basic results of this paper.
policymakers and financial markets and hence follows the definition by Geraats (2002). Nevertheless it is also related to Winkler’s (2000) definition, which uses the concepts of openness referring to the amount and precision of information released and common understanding, i.e. the extent to which the public understands the information supplied by the central bank.

The second information asymmetry regards the preferences of committee members about the inflation target. It is assumed that financial markets do not know the implicit inflation targets of committee members with certainty because they do not observe \( z^{(j)} \). It is assumed, that financial markets know that \( z^{(j)} \) is constant. If the committee publishes its individual voting records, those records provide the public with a signal on the inflation targets of members. The accuracy of this signal will depend on the how precisely financial markets are able to assess the optimal nowcast of the state of each committee member after deliberation. Financial markets can use the signals on \( z^{(j)} \) contained in voting records together with the fact that implicit inflation targets are constant to form an optimal assessment of the implicit inflation target of committee members in period \( t \).

The timing of events in any period \( t \) is as follows: First, each committee member receives a signal on the current state of the economy (denote this stage as \( t_{first} \)). Subsequently financial market receive the public information \( y_{t(F,j)} \). Market agents then rationally form a prediction of the interest rate decision of the committee, which will be a function of the best assessment of the current state of the economy and the best assessment of the implicit inflation target of the median member(\( t_{second} \))\(^{10}\). The committee then meets, deliberates, votes on an interest rate by majority and publishes its interest rate decision (\( t_{third} \)). Finally and before the beginning of period \( t + 1 \), the committee publishes its voting records and the true output gap in period \( t \) is learned by the committee and the market agents (\( t_{fourth} \))\(^{11}\).

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\(^{10}\)David Archer pointed out that the timing of the model is not realistic as information on the economic outlook is usually given after the policy decision has been released and not before. A possible solution to this would be to assume that information on the state of the economy in the next period is given in the minutes. This would mean that one time period (\( t_{first} \)) essentially disappears but the model would otherwise be identical and there would be no effect on the results.

\(^{11}\)We hence assume that the publication of voting records occurs at the same time as the true state of the economy of period \( t \) is learned. In the case of the Bank of England, for example, voting records and minutes are published at 9.30 a.m. on the Wednesday 13 days after the monthly committee decision, so that they are available to the public before the committee next meets. The minutes contain information about the views of the committee on the economic outlook. We make the simplifying assumption that committee members know the true state in period \( t \) and this is communicated without error to financial markets through the minutes.
3 Solving the Model

3.1 The Decision-Making Process of the MPC

It is assumed that when the committee meets and deliberates, committee members exchange their signals on the state of the economy. Hence, each committee member is able to combine his signal on the state of the economy with those of other members optimally to form an assessment of the output gap in period $t$. In order to combine the signals of all committee members optimally, committee members have to come up with an estimate of the precision of their own signal and the precision of the signals that their colleagues receive. Committee members are assumed to recursively update their estimates of the underlying variance of the signals that they receive. That is in period $t$, the estimate that committee member $j$ has of his own variance equals

$$
\tilde{\sigma}^2_{t,y}(j,j) = \frac{t-1}{t} \tilde{\sigma}^2_{t-1,y}(j,j) + \frac{1}{t-1} (y^{(j)}_{t-1} - y_{t-1})^2
$$

(3.1)

for $j = 1, 2, ..., N$.

Similarly the estimate that member $j$ forms about the precision of member $k'$s signal can be written as:

$$
\tilde{\sigma}^2_{t,y}(j,k) = \frac{t-1}{t} \tilde{\sigma}^2_{t-1,y}(j,k) + \frac{1}{t-1} (y^{(k)}_{t-1} - y_{t-1})^2
$$

(3.2)

for $j = 1, 2, ..., N$ and $k = 1, 2, ..., N$ and $j \neq k$.

Therefore, in period $t$, the estimates that committee member $j$ forms about the precision of his signal and those of his colleagues are a linear combination of estimates of the precision of signals that $j$ had in period $t-1$ and the squared difference between the signals received in $t-1$ and the actual realisation of the state in period $t-1$. If in the initial period in which the committee meets, committee members have different priors about the precision of signals, then $k'$s evaluation of the variance of his own signal will not equal $j'$s evaluation of $k'$s signal in period $t$ unless $t = \infty$. This very intuitive result can be shown mathematically. When the committee meets for the first time at $t = 1$, members $j$ and $k$ have different priors on $k'$s variance of the signal, i.e. $\tilde{\sigma}^2_{1,y}(j,k) \neq \tilde{\sigma}^2_{1,y}(k,k)$. After the policy decision is made, the true state is revealed and committee member $j$ updates his prior of $k'$s variance according

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\(^{12}\)Signals are combined using the methods developed by Bates and Granger (1969) and Dickinson (1973) on the optimal combination of forecasts.
to:

\[ \sigma_{2,y}^{-2(j,k)} = \frac{1}{2} \sigma_{1,y}^{-2(j,k)} + (y_1^{(k)} - y_1)^2 \]

Hence for period \( t \):

\[
\sigma_{t,y}^{-2(j,k)} = \frac{t - 1}{t} \sigma_{t-1,y}^{-2(j,k)} + \frac{1}{t - 1} (y_t^{(k)} - y_{t-1})^2
\]

\[
= \frac{1}{t} \sigma_{1,y}^{-2(j,k)} + \sum_{k=0}^{t-1} \left[ \frac{t - k}{t} \frac{1}{t - k - 1} \right] (y_{t-k-1}^{(k)} - y_{t-k-1})^2
\]

where we have repeatedly substituted in for \( \sigma_{t-1,y}^{-2(j,k)} \). It can easily be seen from the above expression that the first term \( \frac{1}{t} \sigma_{1,y}^{-2(j,k)} \to 0 \) as \( t \to \infty \). It can be inferred that

\[
\sigma_{t,y}^{-2(k,k)} = \frac{1}{t} \sigma_{1,y}^{-2(k,k)} + \sum_{k=0}^{t-1} \left[ \frac{t - k}{t} \frac{1}{t - k - 1} \right] (y_{t-k-1}^{(k)} - y_{t-k-1})^2
\]

where the first term \( \frac{1}{t} \sigma_{1,y}^{-2(k,k)} \to 0 \) as \( t \to \infty \). Hence, estimates of committee members converge over time as the different initial priors are given less weight as time progresses.

It can be shown that the optimal combination of signals for each committee member \( j \), which is the solution to a signal extraction problem, is a linear combination of all signals with the weights determined by the perceived precision of signals\(^{13}\). Therefore,

\[
\tilde{y}_t^{(j)} = \bar{B}_j \begin{bmatrix} y_t^{(1)} & y_t^{(2)} & \cdots & y_t^{(N)} \end{bmatrix}' \tag{3.3}
\]

where

\[
\bar{B}_j = \begin{bmatrix} \bar{\Omega}_j^{-1} \\ \bar{\Omega}_j^{-1} i \end{bmatrix}
\]

and

\[
\bar{\Omega}_j = \begin{bmatrix} \sigma_{y,t}^{-2(j,1)} & 0 & \cdots & 0 \\ 0 & \sigma_{y,t}^{-2(j,2)} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{y,t}^{-2(j,N)} \end{bmatrix}
\]

Therefore as long as \( \sigma_{y,t}^{-2(j,j)} \neq \sigma_{y,t}^{-2(k,j)} \) for all \( j = 1, \ldots, N \), \( k = 1, \ldots, N \) but \( j \neq k \), combined signals of the state of the economy will differ between committee members. In order to form an optimal assessment

\(^{13}\)The full derivation of optimal weights is provided in the Appendix.
of the output gap for period $t$, committee members are assumed to use the standard Kalman filtering formulae. The state equation is given by

$$y_t = \beta_1 y_{t-1} - \beta_2 (i_{t-1} - \pi_{t-1}) + \eta_t$$  \hspace{1cm} (3.4)$$and the observation equation can be written as:

$$\tilde{y}^j_t = \tilde{B}_j \left[ y^{(1)}_t \ y^{(2)}_t \ \ldots \ y^{(N)}_t \right]'$$  \hspace{1cm} (3.5)$$with $E(\tilde{y}^j_t) = y_t$ and $E(\tilde{y}^j_t - y_t)^2 = \tilde{B}_j \Omega \tilde{B}_j' = H^j_t$. The true variance is unknown to committee members and hence the perceived variance, $\tilde{H}^j_t = \tilde{B}_j \Omega \tilde{B}_j'$, is used.

Using the true state in period $t-1$, the optimal prediction for the output gap in period $t$ is given by

$$y_{t:t-1} = \beta_1 y_{t-1} - \beta_2 (i_{t-1} - \pi_{t-1})$$  \hspace{1cm} (3.6)$$and the variance of this optimal prediction equals $P_{t:t-1} = \sigma^2_{\eta}$. Therefore the perceived mean squared error of the prediction vector is given by

$$\tilde{F}^{(j)}_t = P_{t:t-1} + \tilde{H}^j_t$$  \hspace{1cm} (3.7)$$Using the standard Kalman filtering formulae\footnote{See Hamilton (1994) for a general discussion of the Kalman filter.}, the optimal estimate of the state of the economy in period $t$ and the variance of this estimate are given by:

$$y^{(j)}_{t:t} = y_{t:t-1} + P_{t:t-1} (\tilde{F}^{(j)}_t)^{-1} [\tilde{y}^j_t - y_{t:t-1}]$$

$$= y_{t:t-1} + \tilde{K}^{(j)}_t [\tilde{y}^j_t - y_{t:t-1}]$$  \hspace{1cm} (3.8)$$where $\tilde{K}^{(j)}_t$ denotes the Kalman gain of committee member $j$. Optimal nowcasts of the state hence differ between committee members because of asymmetric information between committee members regarding the precision of signals. The fact that there is disagreement between committee members is confirmed when looking at actual voting data of MPCs (Gerlach-Kristen, 2002).

The perceived variance (i.e. the variance that uses the estimated variances of committee member
j) equals:
\[
\tilde{P}_t^j = P_{t,t-1} - P_{t,t-1}(\tilde{F}_t^j)^{-1}P_{t,t-1}
\] (3.9)

The true variance of the optimal assessment of the output gap, which is unknown to committee member j is given by:
\[
P_t^j = P_{t,t-1} + \left(P_{t,t-1}(\tilde{F}_t^j)^{-1}\right)^2 [H_t^j + P_{t,t-1}] - 2 \left((P_{t,t-1})^2(\tilde{F}_t^j)^{-1}\right)
\] (3.10)

Hence the interest rate that will be voted for by each committee member will equal\(^{15}\)
\[
i_{t,t}^{(j)} = \pi_t + A \left(\pi_t - \pi^* - z^{(j)}\right) + By_{t,t}^{(j)}
\] (3.11)

Given that the committee decides on an interest rate by majority voting, the interest rate set by the committee will correspond to the interest rate recommendation of the median voter. Therefore the interest rate set by the committee will be:
\[
i_{t,t}^{(c)} = median(i_{t,t}^{(j)})
\] (3.12)

### 3.2 Financial Markets

In order to form a prediction of the committee decision, financial markets can combine the public information they receive on the output gap signals of committee members and use this combined estimate as the observation equation. This is again a signal extraction problem and in order to combine the perceived signals optimally, financial markets need to form an estimate of the precision of the public information that they receive from each committee member. For each committee member j, this variance will be the sum of the estimated variance of the signal of that committee member and the estimate of the variance of the noise of the public information:
\[
\tilde{\sigma}_{\xi,t}^2(F_{j}) = \tilde{\sigma}_{y,t}^2(F_{j}) + \tilde{\sigma}_{\xi,t}^2(F_{j})
\] (3.13)

\(^{15}\)It should be noted that we make use of the principle of certainty equivalence here: if there is uncertainty about the state of the economy and the economy is assumed to follow a linear model with a quadratic loss function, the optimal policy is the same as if the state of the economy were fully observable, except that the policymaker should respond to an efficient estimate of the state of the economy rather than to its current value (Svensson and Woodford, 2003). In addition, we make use of the separation principle: This states that the estimation of the current state of the economy (the signal extraction problem) and the determination of the optimal response coefficients (the optimisation problem) can be treated as separate problems (Svensson and Woodford, 2003).
The estimate of market agents of the precision of the public information for each committee member in period \( t \), is a linear combination of the previous estimate and the observed difference between the public information and the true state. Thus for committee member \( j \), the estimate of the precision of the public information received from this member equals:

\[
\tilde{\sigma}^{2(F,j)}_{\zeta,t} = \frac{t - 1}{t} \tilde{\sigma}^{2(F,j)}_{\zeta,t-1} + \frac{1}{t - 1} (y^{(F,j)}_{t-1} - y_{t-1})^2
\]

Financial markets can use their estimates of the precision of public information to come up with an optimal linear combination of the public information received on committee members’ signals:

\[
\tilde{y}^F_t = \tilde{B}_F \left[ \begin{array}{c} y^{(F,1)}_t \\ y^{(F,2)}_t \\ \vdots \\ y^{(F,N)}_t \end{array} \right]'
\]

where

\[
\tilde{B}_F = \begin{bmatrix} \tilde{\Omega}^{-1}_{F} \\ \tilde{\Omega}^{-1}_{F}' \end{bmatrix}
\]

and

\[
\tilde{\Omega}_F = \begin{bmatrix} \tilde{\sigma}^{2(F,1)}_{\zeta,t} & 0 & \ldots & 0 \\ 0 & \tilde{\sigma}^{2(F,2)}_{\zeta,t} & \ldots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & \tilde{\sigma}^{2(F,N)}_{\zeta,t} \end{bmatrix}
\]

Financial markets can use this optimal combination of their assessment of committee members’ signals together with the state equation for output in order to form an optimal assessment of the output gap in period \( t \), which can be used to form an interest rate prediction. This is a standard Kalman filtering problem, similar to Section 3.1.

Using the standard Kalman filtering formulae, the optimal estimate of the state of the economy in period \( t \) is given by:

\[
y^{F}_{t,t} = y_{t|t-1} + P_{t|t-1}(\tilde{F}^{(F)}_t)^{-1} [y^{F}_t - y_{t|t-1}]
\]

where \( \tilde{K}^F_t = P_{t|t-1}(\tilde{F}^{(F)}_t)^{-1} \) denotes the Kalman gain for financial markets and \( \tilde{F}^{(F)}_t = P_{t|t-1} + \tilde{H}^F_t \)

with \( \tilde{H}^F_t = \tilde{B}_F \tilde{\Omega}_F \tilde{B}_F' \).

The perceived variance (i.e. the variance that uses the estimated variances of financial markets)

\[\text{Details of the signal extraction problem can be found in the Appendix.}\]
equals:
\[ P_t^F = P_{tt-1} - P_{tt-1}(\tilde{F}_t^{(F)})^{-1} P_{tt-1} \]  
(3.17)

The true variance of the optimal assessment of the output gap, which is unknown to financial markets is given by:
\[ P_t^F = P_{tt-1} + \left( P_{tt-1}(\tilde{F}_t^{(F)})^{-1} \right)^2 [H_t^F + P_{tt-1}] - 2 \left( (P_{tt-1})^2 (\tilde{F}_t^{(F)})^{-1} \right) \]  
(3.18)

where \( H_t^F = \tilde{B}_F \Omega_F \tilde{B}_F \)

In order to predict the interest rate set by the committee, financial markets need to know the implicit inflation targets of individual members, which are not observed directly. The publication of individual voting records of committee members provides financial markets with signals on these. We will consider the model with and without the publication of voting records.

**The interest rate prediction of financial markets when voting records are published**

When voting records are published, these provide financial markets with a public signal on the implicit inflation targets of committee members. Financial markets know that the interest rate voted for by committee member \( j \) follows:
\[ i_{tt-1}^{(j)} = \pi_{t-1} + A(\pi_{t-1} - \pi^* - z^{(j)}) + B y_{tt-1}^{(j)} \]  
(3.19)

Thus, if they were able to observe \( y_{tt-1}^{(j)} \) perfectly, they would be able to perfectly infer \( z^{(j)} \) from the individual voting records. Unless committee members communicate their optimal assessment of the economy after deliberation has taken place, financial markets only observe \( y_{tt-1}^{(j)} \) imperfectly. If there is no communication of committee members about their optimal assessment of the state after deliberation, financial markets estimate of \( y_{tt-1}^{(j)} \) for all \( j = 1, 2, ..., N \) is equal to \( y_{tt-1}^{(F)} \). Therefore if voting records are published, the signal that financial markets receive of \( z^{(j)} \) in period \( t-1 \) is given by
\[ \xi_{tt-1}^{(F,j)} = 1 + A(\pi_{t-1} - \pi^*) + B y_{tt-1}^{(F)} - \frac{1}{A} i_{tt-1}^{(j)} \]  
(3.20)

where \( E(\xi_{tt-1}^{(F,j)}) = z^{(j)} \) and \( E((\xi_{tt-1}^{(F,j)} - z^{(j)})^2 = (\frac{B}{A})^2 (E(y_{tt-1}^{(F)} - y_{tt-1}^{(j)})^2) \). It should be noted that the true variance of \( \xi_{tt-1}^{(F,j)} \) is unknown to financial markets as this is a function of the variances of signals received by committee members and the variance of the public information on those signals received by financial markets.
Financial markets hence face a signal extraction problem. The timing is as follows: In the initial period, \( t = 0 \), before the committee meets for the first time in period \( t = 1 \), financial markets form a prior on what they think the implicit target of each committee member is (this prior could be based on the reputation of the committee member or some communication of the committee member \( j \) through the media for example). Denote this prior as \( z_0^{(F,j)} \), where

\[
  z_0^{(F,j)} = z_0^{(j)} + \nu^{(F,j)}
\]

(3.21)

with \( \nu^{(F,j)} \sim N(0, \sigma^2_{\nu,(F,j)}) \).

In the first period financial markets use this prior in order to predict the monetary policy decision of the committee. Once, the committee publishes its voting records, financial markets can update their initial prior in order to come up with an optimal assessment of \( z^{(j)} \). This is done using the basic Kalman filtering formulae with \( z_t^{(j)} = z_{t-1}^{(j)} \) as the state equation and

\[
  \xi_t^{(F,j)} = z_{t-1}^{(j)} + \phi_t^{(F,j)}
\]

(3.22)

where \( \phi_t^{(F,j)} = \beta y_t^{(F)} - y_t^{(j)} \) as the measurement equation. Thus in period \( t - 1 \) after voting records have been published, the best assessment of \( z^{(j)} \) equals:

\[
  z_{t-1,t-2}^{(F,j)} = z_{t-1,t-2}^{(F,j)} + \frac{\bar{\text{Var}}(z_{t-1,t-2}^{(F,j)})}{\bar{\text{Var}}(z_{t-1,t-2}^{(F,j)}) + \text{Var}(\xi_t^{(F,j)})} \left[ \xi_t^{(F,j)} - z_{t-1,t-2}^{(F,j)} \right]
\]

(3.23)

where the weights is written in perceived terms because the true variances are unknown to financial markets.

The true variance of the best assessment of \( z^{(j)} \) equals

\[
  \text{Var}(z_{t-1,t-1}^{(F,j)}) = \left\{ \begin{array}{c}
  \text{Var}(z_{t-1,t-2}^{(F,j)}) + \left( \frac{\bar{\text{Var}}(z_{t-1,t-2}^{(F,j)})}{\text{Var}(z_{t-1,t-2}^{(F,j)}) + \text{Var}(\xi_t^{(F,j)})} \right)^2 \\
  \left( \text{Var}(\xi_t^{(F,j)}) + \text{Var}(z_{t-1,t-2}^{(F,j)}) \right) - 2\text{Var}(z_{t-1,t-2}^{(F,j)}) \frac{\bar{\text{Var}}(z_{t-1,t-2}^{(F,j)})}{\text{Var}(z_{t-1,t-2}^{(F,j)}) + \text{Var}(\xi_t^{(F,j)})}
\end{array} \right\}
\]

Hence, the best assessment for \( z^{(j)} \) in period \( t \) before voting records in that period are published is given by

\[
  z_{t-1}^{(F,j)} = z_{t-1,t-1}^{(F,j)}
\]

(3.24)
The prediction of the interest rate that committee member \( j \) will vote for equals

\[ i_{t|t}^{(F,j)} = \pi_t + A(\pi_t - \pi^*_t - z_{t-1}^{(F,j)}) + B y_{t|t}^{(F)} \]  

(3.25)

The prediction of financial markets for the interest rate set by the committee will be:

\[ i_{t|t}^{(F,c)} = \text{median}(i_{t|t}^{(F,j)}) \]  

(3.26)

**The interest rate prediction of financial markets when voting records are not published**

When voting records are not published, financial markets only receive a signal on the inflation target of the median voter once the policy decision is published. However, because in the next period \( t + 1 \), the assessments of output of committee members will have changed and the median voter is thus likely to be a different committee member than in period \( t \), the usefulness of the policy decision for receiving information on the individual preferences of committee members on the inflation target is limited. Financial markets will realise this and do not combine their priors on committee members’ preferences in this case with any signals. The best assessment of \( z_t^{(j)} \) equals

\[ z_{t-1}^{(F,j)} = z_0^{(F,j)} \]  

(3.27)

and

\[ \text{Var}(z_{t-1}^{(F,j)}) = \text{Var}(z_0^{(F,j)}) \]

Thus, the predicted interest rate decision of committee member \( j \) equals:

\[ i_{t|t}^{(F,j)} = \pi_t + A(\pi_t - \pi^*_t - z_0^{(F,j)}) + B y_{t|t}^{(F)} \]  

(3.28)

The prediction of financial markets for the interest rate set by the committee will be:

\[ i_{t|t}^{(F,c)} = \text{median}(i_{t|t}^{(F,j)}) \]  

(3.29)

**Financial markets’ response to the publication of the interest rate decision of the committee** When the interest rate of the committee is announced, financial markets can use the policy decision to adjust their optimal assessment of the state in period \( t \) before individual voting records
are published and the true state becomes known. Financial markets know that the assessment of the output gap of the economy of the committee is more precise than their own and they can use the policy decision, keeping the preference parameter fixed at their prior, to infer something about \(y^{(c)}_{t} \). Financial markets know that

\[
i^{(c)}_{t} = \pi_{t} + A(\pi_{t} - \pi^{*} - z^{(c)}_{t}) + B y^{(c)}_{t}\tag{3.30}
\]

where the true \(z^{(c)}_{t}\) is unknown. However, financial markets can use their assessment of \(z^{(c)}_{t}\) in period \(t\) (which will differ depending on whether or not voting records are published) in order to infer something about \(y^{(c)}_{t}\). The signal on \(y^{(c)}_{t}\) that financial markets receive is given by

\[
y^{(F,c)}_{t} = \frac{i^{(c)}_{t}}{B} - \frac{1}{B} \left[ \pi_{t} + A(\pi_{t} - \pi^{*} - z^{(F,c)}_{t}) \right]\tag{3.31}
\]

where \(E(y^{(F,c)}_{t}) = y^{(c)}_{t}\) and \(E(y^{(F,c)}_{t} - y^{(c)}_{t})^2 = E(z^{(F,c)}_{t} - z^{(c)}_{t})^2\). If voting records are not published and financial markets use their prior as the best estimate of the median member’s implicit inflation target, the variance of \(y^{(F,c)}_{t}\) will be known. If voting records are published, this variance will be unknown as it will then be a function of Eq.(3.23). The best assessment of \(y^{(c)}_{t}\) once the committee decision has been published is:

\[
E_{t}(y^{(c)}_{t} | y^{(F,c)}_{t}) = y^{(F)}_{t} + \frac{\widetilde{\text{Var}}(y^{(F)}_{t})}{\text{Var}(y^{(F,c)}_{t}) + \text{Var}(y^{(F,c)}_{t})} \left[ y^{(F,c)}_{t} - y^{(c)}_{t} \right]\tag{3.32}
\]

Hence, the new assessment of market agents of the median member’s assessment of the state in period \(t\), once the interest rate decision has been published, will be a linear combination of the initial optimal assessment of the state of financial markets and the signal on the optimal assessment of the median member that is received through the publication of interest rates.

4 Results

To fully understand the implications of the two information asymmetries between the monetary policy committee and financial markets, it will be assumed in section 4.1 that there is only asymmetric information about the signals received by committee members on the state of the economy. Preferences on the inflation target will be identical for all committee members and perfectly known to financial
markets. We also distinguish between the model with no intertemporal dimension to decision making (i.e. the committee meets only once) and the model with an intertemporal dimension to decision-making where there will be repeated interaction between committee members. In Section 4.2 imperfect information about implicit inflation targets of committee members is added and the implications of this asymmetry for the desirability of publishing voting records is analysed.

To simulate the model over time, the values used for the parameters of the Svensson (1997) backward looking model are extracted from an empirical paper by Rudebusch and Svensson (1998) in which the model is estimated for US data. Thus, \( \alpha_1 = 0.7, \beta_1 = 1.16, \) and \( \beta_2 = 0.1. \) Furthermore using the empirical results of a paper by Favero and Rovelli (2003), which also estimates the Svensson (197) model for US data, we set \( \lambda = 0.5, \sigma^2_y = 7.8 \) and \( \delta = 0.975. \) It should be noted that the results are not qualitatively sensitive to these parameter assumptions.

For all simulations, we randomly draw the true variances, \( \sigma^2_{y,j} \) and \( \sigma^2_{\varepsilon,F,j} \) for each committee member \( j = 1, \ldots, N \) as well as perceived variances, \( \hat{\sigma}^2_{y,t}, \hat{\sigma}^2_{\varepsilon,t} \) and \( \hat{\sigma}^2_{\varepsilon,j} \) for each committee member \( j = 1, \ldots, N \) using 10000 draws. For example, \( \sigma^2_{y,j} = (u(j))^2 \) where \( u(j) \sim N(0,1). \) We then compute the covariance matrices, \( \Omega, \Omega_F, \tilde{\Omega}_j \) and \( \tilde{\Omega}_F. \) In order to generate error terms for committee members’ signals in each period \( t, \) we use a Cholesky decomposition of the true covariance matrix, \( \Omega, \) to give a lower triangular matrix \( L, \) which is multiplied by a vector of uncorrelated simulated shocks (10000 draws). This produces a shock vector \( LU \) with covariance properties of the system being modelled. The \( j^{th} \) element of this shock vector corresponds to \( \varepsilon_t^{(j)}. \) For financial market, we generate error term for the public information received from each committee member, with the same method as above with the exception that we now use \( \Omega_F \) as the covariance matrix.

### 4.1 Perfect common knowledge of inflation targets

#### 4.1.1 No intertemporal dimension to decision-making

This subsection makes the simplifying assumption of just one round to decision-making. It is assumed that the committee meets only once and that the policy recommendation of each committee member is only based on the optimal combination of signals and not on past values of the output gap. It has been shown in the Appendix that the true mean squared error of the combined signals for committee member \( j \) equals

\[
E \left[ \left( \bar{y}_t - y_t \right) \left( \bar{y}_t - y_t \right)' \right] = \tilde{B}_j \Omega \tilde{B}_j = \frac{i\tilde{\Omega}_j^{-1}}{i\tilde{\Omega}_j^{-1}i'} \Omega \left[ \begin{array}{c} i\tilde{\Omega}_j^{-1} \\ i\tilde{\Omega}_j^{-1} \end{array} \right]'
\]  

(4.1)
It can be shown that the policy error in this simplified version of the model equals

\[
E(i^c_{t,t} - E(i^c_{t,t}))^2 = B^2 E \left[ (\hat{y}^c_t - y_t)(\hat{y}^c_t - y_t) \right]
\]

(4.2)

where \( \hat{y}^c_t \) corresponds to the optimally combined signal of the median policymaker. As figure 1 demonstrates when there is uncertainty about the true variances of signals and committee members have to use estimates or perceived variances, then a committee of size \( N \) will set an interest rate on average further away from the true interest rate than a committee which is not facing such uncertainty. This is because committee members may give too much or too little weight to certain committee members relative to what is optimal.

Another characteristic of the model is that if a committee member is perceived to receive a highly accurate signal of the state, this member’s signal will be given a high weight. This observation can provide some explanation for the fact, why in some central bank committees, the chairman dominates the meeting if he is perceived as very able to the extent that the variance of the his signal of the economy is very low.

In addition, it can be seen from figure 1 that there is marginal benefit of having a larger committee is decreasing with the number of members. This corresponds to the results of the previous literature, which typically argues for a committee of relatively limited size (Erhart et al, 2007 and Sibert, 2006).

As for financial markets, it can easily be verified that the true mean squared error of the combined perceived signals of committee members equals:

\[
E \left[ (\hat{y}^F_t - y_t)(\hat{y}^F_t - y_t) \right] = \frac{i\Omega^{-1}_{F}}{i\Omega^{-1}_{F}} \Omega_{F} \left[ \frac{\Omega^{-1}_{F}}{i\Omega^{-1}_{F}} \right]'
\]

(4.3)

Predictability of monetary policy decisions can be measured in terms of the expected squared
deviation of \( i^{F}_{t,t} \) from \( i^{c}_{t,t} \):

\[
E(i^{F}_{t,t} - i^{c}_{t,t})^2 = B^2 E \left\{ \left[ (y^{F}_{t} - y_{t}) - (y^{c}_{t} - y_{t}) \right] \left[ (y^{F}_{t} - y_{t}) - (y^{c}_{t} - y_{t}) \right]' \right\}
\]

\[
= B^2 \left\{ E \left[ (y^{F}_{t} - y_{t})(y^{F}_{t} - y_{t})' \right] + E \left[ (y^{c}_{t} - y_{t})(y^{c}_{t} - y_{t})' \right] \right. \\
- E \left[ (y^{c}_{t} - y_{t})(y^{F}_{t} - y_{t})' \right] - E \left[ (y^{F}_{t} - y_{t})(y^{c}_{t} - y_{t})' \right] \right\}
\]

\[
= B^2 \left\{ \frac{\hat{\Sigma}}{\hat{\Omega}} \frac{\hat{\Sigma}^{-1}}{\hat{\Omega}^{-1}} + \frac{\hat{\Sigma}}{\hat{\Omega}^{-1}} \frac{\hat{\Sigma}^{-1}}{\hat{\Omega}} \right\}
\]

It can be seen that the above expression for the policy prediction error of financial markets is a direct function of the true variances that financial markets have in observing the public information on committee members’ signals, i.e the values of \( \sigma^2_{\varepsilon}^{2(F,j)} \) for \( j = 1, 2, ..., N \). So, in terms of policy predictability high actual transparency about the economic outlook (i.e. small \( \sigma^2_{\varepsilon}^{2(F,j)} \)) is optimal. Transparency about the central bank’s forecasts or models is also referred to as economic transparency (Geraats, 2002). Simulating the model for \( N = 9 \) confirms that the policy prediction error of financial markets increases linearly with the true communication noise of committee members. This is shown in figure 2. In order to minimise the above expression, we also need high perceived transparency and hence low \( \tilde{\sigma}^2_{t,\varepsilon}^{2(F,j)} \). As demonstrated in figure 3 where we again simulate the model for \( N = 9 \) members, the highest possible predictability given the asymmetry of information about the variances of committee members’ signals is achieved when perceived transparency about signals is equal to actual transparency, which is set to zero. Thus in order to maximise predictability in the static case, it is optimal for to set \( \sigma^2_{\varepsilon}^{2(F,j)} = \tilde{\sigma}^2_{t,\varepsilon}^{2(F,j)} = 0 \). The following proposition summarises the key results.

**Proposition 1** When there is only one round to decision-making and there is asymmetric information about the variances of signals received by committee members,

(i) a committee size \( N \) which is not facing uncertainty about the precision of signals outperforms a committee with members that face this kind of uncertainty,

(ii) actual transparency about signals received by committee members on the output gap improves the predictability of policy decisions. In addition, the smaller the difference between actual and perceived transparency, the more predictable monetary policy will be.
4.1.2 An intertemporal model of decision-making

When committee members meet repeatedly over time, they can observe the error that each committee member makes in estimating the state of the economy because after the committee has met, the true output gap in that period is revealed to members. It is hence possible for each member to observe \( y_t^{(j)} - y_t \) and this error can be used to update the previous estimate of the variance of that committee member. Over time, because each committee member updates his prior with the same errors, estimates converge as the asymmetry in assessing precision matrices disappears.

Figure 4 shows the expected squared deviation of the interest rate that is set by committee member \( j \) and that is set by committee member \( k \), where \( j = 1, \ldots, N \) and \( k = 2, \ldots, N \) and \( N = 9 \). It can be seen that the expected difference between interest rates set by different members decreases over time and there is hence less disagreement over time.

It is also possible to analyse how the ability of financial markets to predict policy decisions changes over time. Financial markets are able to update the estimate they have for the variance of the public signal that they receive from each committee member. Financial markets observe \( y_t^{(F,j)} \) after the policy decision has been made in period \( t \). This means that they can calculate their estimate of \( \sigma_{\zeta,t+1}^2 \) using (3.14). As demonstrated for committee members, over time the initial prior will be given less and less weight and hence the estimated variance of financial markets, \( \sigma_{\zeta,t+1}^2 \), will converge to the true variance, \( \sigma_{\zeta}^2 = \sigma_{y,j}^2 + \sigma_{\varepsilon}^2 \). As in the static case, predictability of monetary policy decisions can be measured in terms of the expected squared deviation of \( i_{t,t}^F \) from \( i_{t,t}^c \):
rates will become perfectly predictable as \( t \to \infty \).

The key results are summarised in proposition 2:

**Proposition 2** When there is asymmetric information about the variance of signals that committee members receive on the output gap and committee members meet repeatedly over time

(i) Nowcasts of committee members converge over time. Hence, the longer a committee with the same committee members has been in office, the less disagreement one should observe between committee members when it comes to making the policy decision.

(ii) Interest rates will become more predictable as financial markets learn the true variance of the public information they receive over time. Hence, the longer a committee has been in office, the less surprise there should be to financial markets when the new interest rate is announced.

(iii) Interest rates will never be fully predictable by financial markets as long as there is no full transparency about the initial nowcasts of committee members.

Hence, as in the static model, there is again a benefit in terms of greater policy predictability of being fully transparent about the economic outlook (in this case the output gap). The fact that communication about the diverse views of economic outlook by committee members increases policy predictability has been confirmed in the empirical literature (Ehrmann and Fratzscher, 2005)).

### 4.2 Asymmetric Information about the Implicit Inflation Targets of Individual Members

When committee members have different implicit inflation targets, interest rate recommendations during the voting procedure differ because of these different implicit targets and because of different assessment of the precision of signals on the state of the economy. Over time the asymmetry in the assessment of the precision of signals is eliminated. However, even when this asymmetry is completely eliminated interest rates that are voted for by members will still differ. This difference will be a direct function of the difference in implicit inflation targets and this can easily be inferred from (3.11). Over time as the asymmetry in estimated precision of committee members’ signals is eliminated, the interest rates will now differ because of different implicit inflation targets. This is illustrated in figure 6. It can be seen that over time the expected squared deviation of interest rates set by members \( j \) and \( k \) will converge to the squared difference in implicit inflation targets.
There are two different information asymmetries between financial markets and the committee:

First, they receive noisy public information on the signals of committee members on the output gap and they are unsure about the precision of this public information. Secondly, financial markets have to form an expectation of the implicit inflation target of each member. As long as financial markets are unsure about the optimal assessment of the output gap of committee members after deliberation, voting records will be a noisy signal on committee members’ preferences. From (3.23) it can be seen that for each committee member \( j \), the weight given to the noisy signal of \( j’ \)'s implicit inflation target depends on the expected perceived squared deviation between the best assessment of output of financial markets and the best assessment of committee member \( j \). Because of the uncertainty about the true variances of the public information, initially, there may be too much weight given to the information in voting records, relative to what is optimal. Over time as we have shown in Section 4.1.2 the true variances will be learned and hence the weight given to the information on \( j \)'s implicit inflation target will become more accurate. The fact that there may be losses in predictability in the short term when voting records are published compared to the case when voting records are not published is illustrated in figure (7). Figure (7) shows that in the initial periods predictability is higher when voting records are not published. Eventually, predictability becomes higher when voting records are published. The reason for this is that over time \( E(\hat{y}_{jt}^F - y_{jt})^2 - E(\hat{y}_{jt}^F - y_{it}^{(j)})^2 \) decreases when financial markets learn the true variances of the signals they receive. Hence, the weight attached to the information contained in voting records becomes more accurate. The more accurate the estimated variance of the signal is to the true variance, the more beneficial it becomes to publish voting records.

From Eq.(3.23) it can be seen that the variance of the updated estimates of implicit inflation targets is a function of \( Var(\varepsilon_{i,t}^{(F,j)}) \) which in turn is a function of \( E \left[ \left( \hat{y}_{jt}^F - y_{jt}^{(j)} \right) \right]^2 \) and hence the actual communication error of the committee. Thus, the more transparent the committee is about the signals it receives on the output gap (i.e. the smaller \( \sigma_{\varepsilon_{i,t}^{(F,j)}}^2 \)), the smaller will be the possible short term loss of publishing voting records.

From the above discussion it directly follows, that if committee members are fully transparent about their optimal assessments of the state after deliberation, it will be optimal to publish voting records and financial markets will learn the true implicit inflation targets of committee members once voting records are published after the first period of decision making. These key results are summarised in proposition 3:
Proposition 3 When there is asymmetric information between committee members and financial markets about the implicit inflation targets of individual members,

(i) interest rate predictability could initially decline when voting records are published as markets may attach too much weight to the information on policy preferences contained in voting records. However, eventually it becomes beneficial to publish voting records in that policy is more predictable than if voting records were not published.

(ii) the more transparent committee members are about the signals received on the state of the economy, the smaller the possible short term loss of publishing voting records.

(iii) if there is full transparency about optimal nowcasts (after signal extraction) it will be optimal to publish voting records and the asymmetry of information about policy preferences disappears once voting records have been published.

Thus, when there are to sources of information asymmetry between the committee and financial markets, it is still optimal to be fully transparent about the economic outlook. Even if there is full transparency about the economic outlook, there is still a short term loss in predictability of publishing voting records resulting from the uncertainty about the precision of public information in the short term. Morris and Shin (2002) result that financial markets may put too much weight on public information (here information contained in voting records) hence holds in the short term. According to the model presented in this paper, in the long term, financial markets learn about the optimal weights that should be given to the information contained in voting records and hence predictability of monetary policy is enhanced. Ehrmann and Fratzscher (2005) analyse communication strategies pursued by the BoE, the ECB and the Fed between January 1999 and May 2004 and show that there seem to be losses in predictability of policy decisions when the committee communicates its diverging views about the monetary policy decision. Our theoretical model only confirms this result in the short run. However, the model in this paper is simulated for a committee, in which members do not change over time. If there are outgoing members of the committee and new members join the committee, financial markets have to start learning again about these new members. This means that the uncertainty about the precision of signals of committee members may never be sufficiently reduced to make the publication of voting records beneficial in terms of greater policy predictability.
4.3 The Yield Curve of the Economy

In the initial stage before the interest rate decision of the committee has been published, the term structure can be written as:

\[ i_{t,\text{second}}^n = \frac{1}{n} \left\{ \pi_t + A(\pi_t - \pi^* - z_{t\pi-1}^{(F,c)}) + B y_t^F + [1 + A(1 - \beta_2 B)] X_n \right\} \]

(4.6)

Ellingson and Soederstroem show that interest rates of all maturities are positively related to supply shocks in period \( t \), which are observed perfectly by both the committee and the market agents, with the magnitude diminishing with maturity. The best assessment of the output gap by financial markets, \( y_t^F \), is a function of output, inflation and the interest rate in period \( t-1 \) as well as the optimal combination of public information on the output gap signals received by committee members in period \( t \). It can be directly inferred from Eq.(3.16) that the current assessment of the output gap is positively related to \( \bar{y}_t^F \), which is a direct function of \( y_t \).

Therefore

\[ \frac{\partial i_{t,\text{second}}^n}{\partial y_t} = \frac{1}{n} \left\{ B(\bar{K}_t^F) + [1 + A(1 - \beta_2 B)] X_n \alpha_1(\bar{K}_t^F) \right\} \]

(4.7)

In order to determine the sign of the above derivative, we need to evaluate the sign of \([1 + A(1 - \beta_2 B)] X_n\). If this is positive, the derivative will be positive. It can be shown that

\[ [1 + A(1 - \beta_2 B)] X_n = X_n + (1 - \beta_1 - \beta_2 \alpha_1 A) A X_n \]

But

\[ 0 < \beta_2 \alpha_1 A = \frac{\alpha_1^2 \delta k}{\lambda + \alpha_1^2 \delta k} \leq 1, \]

which implies that

\[ 0 < \beta_2 \alpha_1 A X_n = 1 - (1 - \beta_2 \alpha_1 A)^{n-1} \leq 1 \]

for all \( n \). Thus

\[ \beta_2 \alpha_1 A^2 X_n \leq A \]

Since \( \beta_1 < 1 \), this implies that

\[ X_n + (1 - \beta_1 - \beta_2 \alpha_1 A) A X_n > 0 \]
It can also be shown that $\frac{\partial i^n_{t,sec\_ond}}{\partial n_t}$ falls with maturity $n$. Using Eq. (7.5),

$$\frac{\partial i^n_{t+st}}{\partial n_{t+st}} = (1 - \beta_2\alpha_1 A) \frac{\partial i^n_{t+s-1,t}}{\partial n_{t+s-1,t}}$$

Since $\beta_2\alpha_1 A \leq 1$, the response of future short term interest rates to a current demand shock is non-increasing over time. These results extend to all three stages in period $t$ as can be easily verified.

When the committee publishes its interest rate decision, this will have a direct implication on the short end of the yield curve as financial markets are able to revise their interest rate prediction for period $t$. In terms of the expectation of future short term interest rates, the interest rate set by the committee contains some information on the optimal assessment of the state of the economy by the median committee member, which financial markets can use to update $y_{t,t}^F$. The expression for the yield curve hence becomes:

$$i^n_{t,third} = \frac{1}{n} \left\{ \hat{i}^c_{t,t} + [1 + A(1 - \beta_2 B)] X_n \left[ \pi_t - \pi^* - z_{t,t-1}^{(F,c)} + \alpha_1 E \left( y_{t,t}^c \mid y_{t,t}^{(F,c)} \right) \right] \right\} + \xi^n_t \quad (4.8)$$

and so the variance of the sum of expected short term interest rates equals

$$E(i^n_{t,third} - i^n_{t,sec\_ond})^2 = \left( \frac{1}{n} \right)^2 \left\{ E(\hat{i}^c_{t,t} - i_t^{(F,c)})^2 + (C\alpha_1)^2 E(E(\hat{i}^c_{t,t} \mid y_{t,t}^{(F,c)}) - y_{t,t}^F)^2 \right\} \quad (4.9)$$

where $C = [1 + A(1 - \beta_2 B)] X_n$

Therefore the variability in future short term interest rates will depend on

$$(C\alpha_1)^2 E \left[ E(y_{t,t}^c, y_{t,t}^{(F,c)}) - y_{t,t}^F \right]^2 = (C\alpha_1)^2 \left( \Gamma^F \right)^2 \left[ E(y_{t,t}^{(F,c)} - y_{t,t}^c)^2 + E(y_{t,t}^F - y_{t,t}^c)^2 \right]$$

But $E(y_{t,t}^{(F,c)} - y_{t,t}^c)^2 = E(z_{t,t-1}^{(F,c)} - z^c)^2$. Hence, the smaller the actual variance of the estimate of markets agents’ of the implicit inflation targets and the smaller the expected squared deviation between market agents’ best assessment of the state and that of the median committee member, the less variability there will in future short term interest rate when the interest rate decision is published. In section 4.1.2, we have shown that over time $E(y_{t,t}^{(F)} - y_{t,t}^c)^2$ decreases. Hence if $E(z_{t,t-1}^{(F,c)} - z^c)^2$ remains unchanged (i.e. no voting records published) or decreases (this eventually happens when voting records are published but not immediately), then there will be less variability in the sum expected short term interest rates over time. In addition, as we have shown in Section 4.1.2, if the committee is fully transparent about
its signals and perceived to be so by financial markets, then \( E(y_{it}^{(F)} - y_{it}^{(c)})^2 \) will eventually converge to zero as \( t \to \infty \). Hence transparency about the economic outlook leads to less variability in expected future short term interest rates.

The impact on the yield curve when the committee publishes its voting records and the true state of the economy is learned can also be assessed. Financial markets will adjust their expectations of future short term interest rates given the true state and they will use the voting records to update their assessment of member \( j \)'s implicit inflation target. The yield curve can now be expressed as:

\[
i^n_{t, fourth} = \frac{1}{n} \left\{ \frac{i^n_{t} + [1 + A(1 - \beta_2 B)] X_n}{\pi_t - \pi^*_t - z^{(F,c)}_{it} + \alpha_1 y_t} \right\} + \xi^n_t \tag{4.10}
\]

and hence

\[
E(i^n_{t, fourth} - i^n_{t, third})^2 = \left( \frac{1}{n} \right)^2 \left\{ E \left[ - (C + n - 1) (z^{(F,c)}_{it} - z^{(F,c)}_{it-1}) - C \alpha_1 (E(y_{it}^{(F)} | y_{it}^{(F,c)} - y_t)) \right] \right\}^2 \tag{4.11}
\]

In order to evaluate the effect of publishing voting records on the yield curve, we need to investigate how \( E(i^n_{t, fourth} - i^n_{t, third})^2 \) depends on \( E(z^{(F,c)}_{it-1} - z^{(F,c)}_{it})^2 \). It can easily be seen that \( E(i^n_{t, stage3} - i^n_{t, stage2})^2 \) is positively related to \( E(z^{(F,c)}_{it-1} - z^{(F,c)}_{it})^2 \). This in turn can be written as

\[
E(z^{(F,c)}_{it-1} - z^{(F,c)}_{it})^2 = \left( \frac{\text{Var}(z^{(F,c)}_{it-1})}{\text{Var}(z^{(F,c)}_{it}) + \text{Var}(\xi^n_t)} \right)^2 \left( \frac{B}{A} \right)^2 \left( E(y_{it}^{(F)} - y_{it}^{(c)})^2 + \text{Var}(z^{(F,c)}_{it-1}) \right)
\]

Thus, the greater the true variance of the signal on \( z^{(c)} \) in period \( t \), \( \left( \frac{B}{A} \right)^2 \left( E(y_{it}^{(F)} - y_{it}^{(c)})^2 \right) \), the greater will be the variability in future short term interest rates. This implies, that the longer a committee has been in office and the more precisely financial markets are able to estimate the precision of committee members signals (i.e. the lower \( E(y_{it}^{(F)} - y_{it}^{(c)})^2 \), the less variability there will be in future expected short term interest rates.

It can also be shown that the effects of the publication of voting records on the variability in future short term interest rates decreases with maturity \( n \). This is because

\[
i^n_{t+s} = (1 - \beta_2 \alpha_1 A) i^n_{t+s-1, t}
\]
and thus
\[ E(i_{t+st, fourth}^n - i_{t+st, third}^n)^2 = (1 - \beta_2\alpha_1 A)^2 E(i_{t+s-1,t, fourth}^n - i_{t+s-1,t, third}^n)^2 \]

Thus the greatest impact of the publication of voting records is observed for near-term interest rate expectations. The key results are summarised in proposition 4:

**Proposition 4**

(i) in all three stages of period \( t \), interest rates of all maturities will be a positive function of demand and supply shocks, with the magnitude diminishing with maturity.

(ii) The variability of future short term interest rates when the policy decision is published depends on the expected squared deviation between the optimal assessment of the output gap of market agents and the optimal assessment of market agents’ of the median member’s assessment of the output gap. Over time, if the variance of market agents’ assessment of the implicit inflation target of the median member stays constant or decreases, there will be less variability in the sum of expected future interest rates.

(iii) the more precise the assessment of output before the true output gap is learned and the more precise the initial estimate of the implicit inflation target of the median member, the less variability there will be in future expected short term interest rates when voting records are published. This also implies, that the longer a committee has been in office and the more precisely financial markets are able to estimate the precision of committee members signals, the less variability there will be in future expected short term interest rates when voting records are published.

(iv) the effect of communication in the form of publication of voting records should be more pronounced for the near-term interest rate expectations.

The above results imply that greater transparency about the economic outlook, leads to less volatility in financial markets. In addition, the impact of communication in the form of publishing voting records should be most visible in near term interest rate expectations. This is indeed confirmed empirically by Reeves and Sawicki (2007) for the Bank of England. The authors conclude that there is a significant response of financial markets to the publication of voting records and that this is particularly true of short term sterling futures implied rates.
5 Conclusion

Recently, there has been a growing number of theoretical and empirical papers investigating the optimal degree of transparency and communication of central banks. Against this background this paper has provided an attempt to address these questions within a model of a monetary policy committee with heterogeneous committee members that is observed by financial markets. It is assumed that there exists asymmetric information between the committee and market agents, which means that the communication strategy of the committee has an important impact on interest rate expectations of financial markets.

The main results of this paper show that it is optimal for the committee to be transparent about information regarding the current state of the economy. This helps financial markets to predict the policy decision and to eliminate the uncertainty regarding the precision of the information of committee members. In addition, whilst there may be short term losses from publishing voting records and thereby revealing possible disagreement within the committee regarding the policy decision, in the long run, policy predictability is increased as financial markets are able to infer member’s preferences from voting records once their estimate of precision of public information has become sufficiently precise. By improving predictability of policy decisions, actual transparency about the committee members’ information regarding the economic outlook furthermore leads to less volatility in expected interest rates along the yield curve.

There are also some implications for the optimal design of monetary policy committees. It is found that a committee of limited size is optimal especially when considering the likely increasing coordination costs as the committee gets larger. In addition, because committee members are able to learn about each other over time and thereby integrate diverse information sets better, it is optimal for members to serve for a relatively long time without staggered terms. In light of these findings, the rotation scheme as used by the Federal Reserve Open Market Committee and proposed for the ECB does not seem optimal. However, it should be noted that the analysis in this paper does not consider the costs of having committee members serving longer terms. In addition, the paper abstracts from any political considerations which arise when determining how to design a central bank committee. Hence care needs to be taken when drawing policy conclusions.

We also find several empirical predictions of the model. It is shown that predictability of monetary policy decisions generally increases with the time a committee has been in office. This is because over
time some of the initial information asymmetry between the committee and market agents is eliminated as market agents are able to learn about committee members’ preferences and the precision of signals that members receive on the state of the economy. In addition, the impact on the yield curve of communicating the diverse views of committee members on monetary policy should be greatest for near-term policy expectations.

Some useful directions for further research should be noted. First, it would be interesting to empirically test some of the predictions of this paper. In addition, it would be possible to make the model more applicable to committees with a chairman that dominates the meetings as is the case for the Federal Reserve by assuming that the chairman first collects all the different signals of committee members and is then influenced by those before voting takes place. The model could also be made more realistic by assuming that financial markets’ expectations have a direct effect on the loss function of the committee. This could be achieved by using a New Keynesian model of the economy. The drawback of this approach is that with forward looking variables and asymmetric information, the separation principle would no longer hold (Svensson and Woodford (2003)) and the mathematics would become more complicated with the results being less intuitive. It would be interesting to explore some of these open research issues in future work.
6 References


7 Appendix

7.1 Derivation of a Closed Form Expression of the Yield Curve

This derivation follows Ellingson and Soederstroem. The only difference is that we assume the existence of a positive inflation target, which slightly alters the result.

The n-period interest rate is set as an average of future short rates plus a term premium:

\[ i^n_t = \frac{1}{n} \sum_{s=0}^{n-1} i_{t+s} + \xi^n_t \]  

(7.1)

Leading the interest rate rule \( s \) periods and taking expectations gives

\[ i_{t+s} = (1 + A)\pi_{t+s} - A\pi^* + By_{t+s} \]  

(7.2)

Similarly leading the output gap and inflation \( s \) periods, taking expectations and using the formula for the \( nth \) term of a geometric series, we find that

\[ y_{t+s} = -\beta_2 A\pi_{t+s} + \beta_2 A\pi^* \]  

(7.3)

and

\[ \pi_{t+s} = (1 - \alpha_1 \beta_2 A)^s - 1 [\pi_t + \alpha_1 y_t] + \pi^* [1 - (1 - \alpha_1 \beta_2 A)^s - 1] \]  

(7.4)

Thus

\[ i_{t+s} = [1 + A(1 - \beta_2 B)] (1 - \alpha_1 \beta_2 A)^s - 1 [(\pi_t - \pi^*) + \alpha_1 y_t] + \pi^* \]  

(7.5)

and

\[ \sum_{s=0}^{n-1} i_{t+s} = [1 + A(1 - \beta_2 B)] X_n [(\pi_t - \pi^*) + \alpha_1 y_t] + (n - 1)\pi^* \]  

(7.6)

where

\[ X_n = \frac{1 - (1 - \alpha_1 \beta_2 A)^n - 1}{\alpha_1 \beta_2 A} \]

Using the interest rate rule together with Eq.(7.6), the market interest rate of maturity \( n \) is given by:

\[ i^n_t = \frac{1}{n} \{ \pi_t + A(\pi_t - \pi^*) + By_t + [1 + A(1 - \beta_2 B)] X_n [(\pi_t - \pi^*) + \alpha_1 y_t] + (n - 1)\pi^* \} + \xi^n_t \]  

(7.7)
7.2 Solution of the Signal Extraction Problem of Committee Members

We assume that each committee member, \( j \), receives an unbiased signal about the output gap in the current period, \( t \).

\[
y_t^{(j)} = y_t + \varepsilon_t^{(j)}
\]  

(7.8)

with \( \varepsilon_t^{(j)} \sim N(0, \sigma_{y,j}^2) \) for \( j = 1, 2, ..., N \) and \( E\left( \varepsilon_t^{(j)}, \varepsilon_t^{(k)} \right) = 0 \) where \( j \neq k \) and \( k = 1, 2, ..., N \).

The variance of the error term is assumed to be unknown and thus has to be estimated by committee member \( j \) for member \( k \) as \( \tilde{\sigma}_{y,t}^{2(j,k)} \) at time \( t \).

Committee members can optimally combine their signals of the state during the deliberation process. The solution of this signal extraction problem will be a linear combination of all estimates:

\[
\tilde{y}_t^{(j)} = b_{j1}y_t^{(1)} + b_{j2}y_t^{(2)} + b_{j3}y_t^{(3)} + \vdots + b_{jN}y_t^{(N)}
\]  

(7.9)

where the weights must add up to one. This can be written in matrix form as

\[
\begin{align*}
\tilde{y}_t^{(j)} &= \tilde{B}_j \begin{bmatrix}
y_t^{(1)} \\
y_t^{(2)} \\
\vdots \\
y_t^{(N)}
\end{bmatrix} \\
&= \begin{bmatrix}
y_t \\
\varepsilon_t^{(1)} \\
\varepsilon_t^{(2)} \\
\vdots \\
\varepsilon_t^{(N)}
\end{bmatrix}
\end{align*}
\]  

(7.10)

(7.11)

where \( \tilde{B}_j = \begin{bmatrix} b_{j1} & b_{j2} & \cdots & b_{jN} \end{bmatrix} \)

Hence the true variance of \( \tilde{y}_t^{(j)} \) is

\[
E(\tilde{y}_t^{(j)} - y_t)^2 = E \left( \begin{bmatrix} \varepsilon_t^{(1)} \\
\varepsilon_t^{(2)} \\
\vdots \\
\varepsilon_t^{(N)} \end{bmatrix} \right) \left( \begin{bmatrix} \varepsilon_t^{(1)} \\
\varepsilon_t^{(2)} \\
\vdots \\
\varepsilon_t^{(N)} \end{bmatrix} \right)^T
\]  

(7.12)

\[
= \tilde{B}_j \Omega \tilde{B}_j^T
\]

where \( \Omega = \begin{bmatrix}
\sigma_{y,1}^2 & 0 & \cdots & 0 \\
0 & \sigma_{y,2}^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_{y,N}^2
\end{bmatrix} \)

However, since the true variance is unknown, committee members need to use the matrix of perceived variances:
Thus for member $j$, $\widetilde{\Omega}_j = \begin{bmatrix} -2(j,1) \sigma_{y,t} & 0 & \cdots & 0 \\ 0 & -2(j,2) \sigma_{y,t} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & -2(j,N) \sigma_{y,t} \end{bmatrix}$

In order to minimise Eq.(7.12) we need to differentiate $\widetilde{B}_j \widetilde{\Omega}_j \widetilde{B}_j'$ with respect to $\widetilde{B}_j'$ remembering that the $j^{th}$ element in $\widetilde{B}_j'$ is a function of the other elements, i.e.

$$b_{jj} = 1 - b_{j,1} - b_{j,2} - ..., b_{j,N}$$

We hence have to minimise $\widetilde{B}_j \widetilde{\Omega}_j \widetilde{B}_j'$ subject to the constraint that $iB_j' = 1$

where

$$i = \begin{bmatrix} 1 & 1 & \ldots & 1 \end{bmatrix}_{Nx1}$$

We write the Lagrangian of this problem as

$$L = \widetilde{B}_j \widetilde{\Omega}_j \widetilde{B}_j' - \lambda \left[iB_j' - 1\right]$$

(7.13)

We get two first order conditions:

$$2\widetilde{B}_j \widetilde{\Omega}_j - \lambda i = 0$$

(7.14)

and

$$\left[iB_j' - 1\right] = 0$$

(7.15)

Solving the first one with respect to $\widetilde{B}_j$, we obtain

$$\widetilde{B}_j = \frac{\lambda}{2} \widetilde{\Omega}_j^{-1}$$

(7.16)

Substituting Eq.(7.16) into Eq. (7.15) :

$$i\frac{\lambda}{2} \widetilde{\Omega}_j^{-1} t' - 1 = 0$$

(7.17)

Hence

$$\frac{\lambda}{2} = \frac{1}{i\widetilde{\Omega}_j^{-1} t'}$$

(7.18)
and therefore
\[
\tilde{B}_j = \frac{i\tilde{\Omega}_j^{-1}}{i\tilde{\Omega}_j^{-1}i'}
\] (7.19)

Thus the true underlying variance of the combined signals equals:
\[
\tilde{B}_j \tilde{\Omega} \tilde{B}_j' = \frac{i\tilde{\Omega}_j^{-1}}{i\tilde{\Omega}_j^{-1}i'} \Omega \left[ \frac{i\tilde{\Omega}_j^{-1}}{i\tilde{\Omega}_j^{-1}i'} \right]'
\] (7.20)

whereas the variance that committee members perceive is equal to
\[
\tilde{B}_j \tilde{\Omega}_j \tilde{B}_j' = \frac{1}{i\tilde{\Omega}_j^{-1}i'}
\] (7.21)

### 7.3 Solution of the Signal Extraction Problem of Financial Markets

We assume that financial markets rationally combine the public information that they receive about the committee members’s signals on the output gap. The public information that financial market receive for each committee member equals
\[
y_t^{(F,j)} = y_t^{(j)} + \varepsilon_t^{(F,j)} = y_t + \varepsilon_t^{(j)} + \varepsilon_t^{(F,j)} = y_t + \zeta_t^{(F,j)}
\] (7.22)

where \(\zeta_t^{(F,j)} \sim N(0, \sigma_{\zeta,t}^2(F,j))\) for \(j = 1, \ldots, N\).

The variance of the error term is assumed to be unknown and it is assumed that financial markets have a prior on it. Furthermore it is assumed that \(E\left(\zeta_t^{(F,j)}, \zeta_t^{(F,k)}\right) = 0\).

Financial markets can optimally combine their estimates of the state for each committee member. The solution of this signal extraction problem will be a linear combination of all estimates:
\[
y_t^{(F,j)} = b_{F1}y_t^{(F,1)} + b_{F2}y_t^{(F,2)} + b_{F3}y_t^{(F,3)} + \ldots + b_{FN}y_t^{(F,N)}
\] (7.23)

where again the weights must add up to one.

The solution is identical to the one provided for the committee. The only difference is that now the
true covariance matrix equals

$$\Omega_F = \begin{bmatrix}
\sigma_{y,1}^2 + \sigma_\varepsilon^2 & 0 & \cdots & 0 \\
0 & \sigma_{y,2}^2 + \sigma_\varepsilon^2 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_{y,N}^2 + \sigma_\varepsilon^2
\end{bmatrix} \quad (7.24)$$

whereas the perceived covariance matrix containing the estimated variances of financial markets can be written as:

$$\tilde{\Omega}_F = \begin{bmatrix}
\tilde{\sigma}_{y,t}^2(F,1) + \tilde{\sigma}_\varepsilon^2(F,1) & 0 & \cdots & 0 \\
0 & \tilde{\sigma}_{y,t}^2(F,2) + \tilde{\sigma}_\varepsilon^2(F,2) & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & \tilde{\sigma}_{y,t}^2(F,N) + \tilde{\sigma}_\varepsilon^2(F,N)
\end{bmatrix} \quad (7.25)$$

Thus the optimally combined estimate of the signals on the output gap equals

$$\tilde{y}_{t}^{(F,j)} = \tilde{B}_F \begin{bmatrix} y_t^{(F,1)} & y_t^{(F,2)} & \cdots & y_t^{(F,N)} \end{bmatrix}'$$

(7.26)

where

$$\tilde{B}_F = \frac{i\tilde{\Omega}_F}{i\tilde{\Omega}_F^{-1}}$$
7.4 Figures

Note: $E(i^c - E(i))^2$ denotes the variance of the committee decision. $E(i^c) = i^*$ corresponds to the interest rate that would be set if there was no uncertainty about the output gap.

Note: $E(i^F - i^c)^2$ denotes the expected squared deviation of the prediction of financial markets of the policy decision and the actual decision by the committee.
Note: $E((i^F - i^c)^2)$ denotes the expected squared deviation of the prediction of financial markets of the policy decision and the actual decision by the committee.

Note: $E((i^j - i^k)^2)$ is the expected squared difference between the interest rate set by member $j$ and the interest rate set by member $k$. In the above figure, $j = 1$ and $k = 2, ..., N$. 
Note: $E(t^T - i^c)^2$ denotes the expected squared deviation of the prediction of financial markets of the policy decision and the actual decision by the committee in period $t$.

Note: $E(i^j - i^k)^2$ is the expected squared difference between the interest rate set by member $j$ and the interest rate set by member $k$ in period $t$. In the above figure, $j = 1$ and $k = 2, ..., N$. 

Figure 5

Figure 6
Figure 7

Note: $E((i^F - i^c)^2)$ denotes the expected squared deviation of the prediction of financial markets of the policy decision and the actual decision by the committee in period $t$. 