On the spike in hazard rates at unemployment benefit expiration: The signalling hypothesis revisited*

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Abstract: We revisit the signalling hypothesis, according to which potential employers use the duration of unemployment as a signal on the productivity of applicants. We suggest that the quality of such a signal is very low when the unemployed get unemployment benefits: individuals have good reasons to stay unemployed. Conversely, the signal becomes much more efficient once benefits are elapsed: skilled workers should not stay unemployed in such a case. Therefore, the potential duration of unemployment benefits should drive employers’ expectations and their recruitment practices. This mechanism can explain why hazards fall after benefit expiration, and why unemployment duration responds more to the potential duration of benefits than to replacement rates.

Keywords: Worker heterogeneity; Signalling; Hazard rate; Unemployment compensation; Moral hazard

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Introduction

This paper is a theoretical contribution to the literature on job search and unemployment insurance. We revisit the signalling hypothesis, according to which potential employers use the duration of unemployment as a signal on the productivity of applicants. We suggest that the quality of such a signal is very low when the unemployed get unemployment benefits: individuals have good reasons to stay unemployed. Conversely, the signal becomes much more efficient once benefits are elapsed: skilled workers should not stay unemployed in such a case. Therefore, the potential duration of unemployment benefits should drive employers’ expectations and their recruitment practices. This mechanism can explain why hazards fall after benefit expiration, and why unemployment duration responds more to the potential duration of benefits than to replacement rates.

Why is this important? Our paper is mostly theoretical. However, it can also be used to address two empirical puzzles that the standard job search theory hardly explains.

On the one hand, hazard rates increase prior to the exhaustion date, and strongly decline afterwards. Most of the unemployment compensation systems of the OECD countries deliver declining benefits with the unemployment spell. Benefits are proportional to the pre-unemployment wage for short unemployment spells, while they fall down to a common standard determined by the public assistance system for longer durations. Since the late 1980s early 1990s, a number of contributions have shown that the probability of leaving unemployment dramatically rises just prior to benefits lapse (see e.g. Moffitt, 1985, Meyer, 1990, and Katz and Meyer, 1990, for the US, Ham and Rhea, 1987, for Canada, Carling et al, 1996, for Sweden, Joutard and Ruggiero, 1996, and Dormont et al, 2006, for France; Van Ours and Vodopivec, 2006, for Slovenia; see also Card et al, 2007, for a different perspective). However, hazard rates fall shortly after they peak around the exhaustion date. The rise in hazard is predicted by the standard job search theory: reservation wages go down and search efforts go up as the exhaustion date becomes closer (see for instance Mortensen, 1977, 1986, and Van den Berg, 1990). However, the theory also predicts that hazard rates should stay constant afterwards, while they typically fall.

On the other hand, estimates show that the duration of unemployment positively responds to the various components of unemployment compensation generosity. However, it is much more responsive to changes in potential duration than to changes in replacement rate. Well-known studies find positive and significant effects on unemployment duration from higher benefits (see for instance Narendranathan et al, 1985, Katz and Meyer, 1990, Van den Berg, 1990, who obtain an elasticity of duration to benefit typically lower than one). However, there is a wide dispersion in estimates, and several studies do not find any effects (see Nickell, 1979, for the UK, Lynch, 1989, for the US, Hujer and Schneider, 1989, for Germany, Groot, 1990, for the Netherlands), or even a negative impact (see Jones, 1996, for Canada). Fewer studies examine the elasticity of average duration to potential duration. However, they conclude that this elasticity is indubitably positive (see Moffitt, 1985, and Katz and Meyer, 1990, for the US, Ham and Rea, 1987, for Canada, Ham et al, 1998, for the Czech and Slovak Republics). This asymmetric response of unemployment duration to benefit level and benefit duration is nicely illustrated by Katz and Meyer (1990), also quoted in Atkinson and Micklewright (1991). They perform simulations from their estimates and show that a given UI expenditure cut achieved via reducing the length of entitlement has twice the effect of one coming via a cut in benefit levels. This asymmetry is usually downplayed on the basis of its supposed inconsistency. If the potential duration of benefits plays a role, this must be
so because the unemployed lose some income. As the magnitude of the loss is governed by replacement rates, replacement rates should also be part of the story\(^1\).

In this paper, equilibrium hazard rates result from the interplay between workers’ job search strategies and employers’ hiring strategies. We elaborate on Lockwood (1991) who examines the argument according to which the duration of unemployment conveys a signal on worker’s ability which leads employers to discriminate against the long-term unemployed. We argue that moral hazard effects induced by unemployment compensation alter the value of the signal in a way that is consistent with the two empirical regularities highlighted previously.

A key aspect of our contribution relies on its ability to feature a realistic pattern of hazard rates as an equilibrium outcome of a model with worker heterogeneity, imperfect information, signalling and moral hazard effects. Our model closely follows Lockwood (1991) to the noticeable exceptions that workers are allowed to set their search effort and there is duration-dependent unemployment compensation. There are two types of workers, good and bad, and firms are only willing to hire the good workers. At the beginning of unemployment episode, all workers are fully entitled to unemployment benefits. After a fixed interval of time, benefits fall to a lower level. Workers can set a low search effort or a high one. At the time of interview, firms have a positive probability of detecting a bad worker. This assumption ensures that exit rates are lower for the bad workers than for the good ones at given search effort.

In this environment, agents have to select unemployment duration-contingent strategies that are mutually consistent in equilibrium. Workers set the pace of search effort, while firms set their hiring policy. We focus on a particular equilibrium configuration that is empirically relevant. In this configuration, (i) bad workers always choose a low search effort, (ii) good agents start seeking jobs with a low effort, then set a high effort prior to the potential duration of benefits, and (iii) firms set a larger than the exhaustion date duration above which applicants always get rejected. We provide the set of necessary and sufficient conditions that leads to the existence and uniqueness of such an equilibrium.

Then, we analyze how hazard rates respond to changes in the institutional environment. All the different components of unemployment insurance (potential duration and benefit levels before and after the exhaustion date) originate moral hazard effects and as such weaken the signal conveyed by unemployment duration. Therefore, both the duration at which good workers start searching a job with a high effort, and the cut-off duration above which employers start rejecting applicants, are increasing in unemployment insurance generosity. However, the magnitude of the impact of each component depends on one key parameter: the return to search of a high effort. When this parameter is large, good agents wait very long before setting a high search investment. In the non-frictional case where this parameter tends to infinity, agents set a high effort once they have reached the exhaustion date and immediately get a job offer. Firms rationally expect they will only meet bad agents once the exhaustion date is elapsed and systematically reject all the applicants. More generally, the higher the return to search of a high effort, the closer the cutoff duration to the exhaustion date. Interestingly, benefit levels before and after the exhaustion date only marginally affect this statement. This may explain why hazard rates respond a lot to the potential duration of benefits and not that much to benefit levels.

Very few papers explain the decline in hazard after the exhaustion date. Two types of arguments have been put forward. First, seeking a job may require lowering the leisure time. Mortensen (1977)\(^1\) For instance, the entitlement effect put forward by Mortensen (1977) can explain why benefit levels may have a negative impact on average duration. However, the potential duration of benefits should also have a negative effect in such a case.
shows that if leisure and consumption are substitutes, then a fall in benefits raises the opportunity cost of searching and hazard rates go down. Second, benefits can be used to improve job search efficiency. This argument is due to Tannery (1983). In such a case, the opportunity cost of seeking a job is the marginal utility of consumption. When benefits go down, the marginal utility of consumption goes up and search spendings are reduced (see Ben-Horim and Zuckerman, 1988, and Decreuse, 2002).

Our paper is related to contributions that emphasize the role of employers’ beliefs and hiring strategies to explain duration dependence in hazard rates. In Blanchard and Diamond (1994), employers can meet several applicants at a time and marginally prefer workers with a short duration. Resulting hazard rates display negative duration dependence. In Coles and Masters (2000), skills depreciate during the unemployment episode. Owing to recruitment costs, employers set a cut-off duration very similar to Lockwood’s and ours above which employers reject all applications. Our paper goes a step forward by highlighting the interaction between job search and recruitment strategies. It also argues that the design of unemployment compensation is a key variable affecting the outcome of such an interaction.

The rest of the paper is organized as follows. Section 2 introduces our model and examines individual strategies. Section 3 is devoted to the analysis of equilibrium. Section 4 discusses some empirical implications. Section 5 concludes.

All proofs are gathered in the Appendix.

2 The model

This section introduces the main assumptions of the model and considers the microeconomic choices made by individuals and firms.

2.1 Environment

We follow Lockwood (1991). We depart from Lockwood in three ways. First, there are unemployment benefits, which potential duration is finite. Second, workers make search efforts. Third, the matching technology does not depend on the number of vacancies.2

We are interested in the steady-state of a continuous time economy populated by a continuum of firms and workers. All agents have the same felicity function \( v \) which positively depends on consumption. They discount time at rate \( \rho > 0 \).

At each instant, a new cohort of individuals of total size \( n > 0 \) enters unemployment. This cohort is composed of \( \pi_0 n \) good individuals, and \( (1 - \pi_0) n \) bad individuals, \( \pi_0 \in (0, 1) \). Workers only differ in their productivity, with \( y_g > y_b \). Workers can either be employed, unemployed, or non-participant. To ensure the existence of a steady-state number of unemployed, we assume that agents die/retire at rate \( n \). When unemployed, agents have to seek a job, which means choosing a search effort \( e \). There are two levels of effort: either effort is high and \( e = h > 0 \), or effort is low and \( e = 0 \). The cost of effort is \( ce \), \( c > 0 \). This implies that the cost of a low effort is normalized to zero. The probability of contacting a vacant position in the time interval \( dt \) is \( m (1 + e) \, dt \).

\(^2\)This assumption allows us to neglect the labor demand side of the model, while focusing on the novel aspects that we stress.
Each firm is endowed with a single job slot, which can either be filled or vacant. Firms endowed with a vacant position must incur the cost \( \gamma > 0 \). Filled jobs produce either \( y_g \) or \( y_b \), depending on worker’s type. There is a single wage \( w \in (y_b, y_g) \), which is set exogenously. This departs from Lockwood in which firms pay the monopsony wage. This is so in our model to ensure that there are gains to seek jobs. As a result of this wage, firms do not want to hire bad workers who generate negative profits. However, worker’s type is imperfectly observable. Firms receive a private signal on worker’s type at the time of interview. If the worker is good, the signal is good with probability \( \phi_g = 1 \). If the worker is bad, the signal is good with probability \( \phi_b = \phi \in (0, 1) \). It is bad with the complementary probability \( 1 - \phi \). Once a worker has found a job, he/she leaves the search market forever and enjoys the utility level \( W = v(w)/r \).

The flow number of matches only depends on the number and efforts of job-seekers\(^4\). Let \( u \) denote the mass-number of active unemployed, and let \( \bar{e} \) denote their average search effort. The total number of matches is \( um(1 + \bar{e}) \). Matching is random, which means that meetings are equiprobably distributed between the two sides of the markets, i.e. good and bad workers have the same probability of contacting a vacant job. The contact rate for a vacant position is \( m/\theta \), where \( \theta = v/[u(1 + \bar{e})] \) is the labor market tightness.

Finally, the unemployment compensation scheme runs as follows. Unemployed workers receive unemployment benefits \( b(s) \) contingent on unemployment duration \( s \). Let \( T \) be the potential duration of unemployment benefits. We have:

\[
b(s) = \begin{cases} 
  b_{\text{max}} > 0 & \text{if } s \leq T \\
  b_{\text{min}} < b_{\text{max}} & \text{else}
\end{cases}
\]

To ensure that agents do not refuse job offers, \( b_{\text{max}} < w \).

### 2.2 Job-seeker behavior

In this sub-section, we examine the job-seeking behavior of unemployed individuals at given firms’ hiring policy.

Let \( U_i(s) \) denote the value function of a type-\( i \) unemployed whose unemployment duration is \( s \). We have

\[
r U_i(s) = \max_{e=h,0} \{ v(b(s)) + m(1+e)\phi_i(s)[W-U_i(s)] - ce + U'_i(s) \}
\]

where \( r = \rho + \delta \) is the effective discount rate, and \( \phi_i(s) \) is the firms’ hiring policy that the workers expect.

**Assumption A1** Firms’ hiring policy is given by

\[
\phi_i(s) = \begin{cases} 
  \phi_i & \text{if } s \leq \Delta, \text{ with } \Delta > T \\
  0 & \text{else}
\end{cases}
\]

This policy means that employers discriminate against the long-term unemployed. The cut-off duration \( \Delta \) above which applications are always rejected will be called the duration of employability. It

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3. This is a reduced form of Lockwood, who assumes that firms can choose whether to test workers prior to hiring them. The parameter \( \phi_i \) is then the probability of passing the test. Good workers are always successful and \( \phi_g = 1 \), while bad workers may fail and \( \phi_b \in (0,1) \).

4. There are no congestion effects between the job-seekers.
is supposed common to all the firms. We also restrict our attention to the empirically plausible case where \( \Delta \geq T \). In the sequel, we will refer to this particular configuration as the baseline equilibrium configuration. We will provide additional restrictions later to ensure that such an equilibrium exists. In subsection ??, we briefly discuss the other types of equilibrium that our model may admit.

The optimization problem \((*)\) can be written as follows

\[
ru_i(s) = \max_{e=0}^{e(h)} \{v(b(s)) + m(1 + e)\phi_i[W - U_i(s)] - ce + U_i'(s)\} \quad (2)
\]

The resulting value function \(U_i(s)\) is strictly decreasing for all \(s \in [0, \Delta]\).

Let \(e_i(s)\) denote the optimal trajectory of effort. This trajectory satisfies \(e_i(s) = h\) if and only if

\[
\phi_i m [W - U_i(s)] \geq c \quad (4)
\]

This implies that either \(e_i(s) = 0\) for all \(s \geq 0\), or there exists a unique duration \(\sigma_i \in [0, \Delta]\) such that \(e_i(s) = h\) iff \(s \geq \sigma_i\).

The problem must be solved backward. In the remaining, \(a_i = \phi_i m (1 + h)\).

**Step 1.** If \(\phi_i m [v(w) - v(b_{\min})] < rc\), then \(e_i(s) = 0\) for all \(s \geq 0\). If not, go to Step 2.

**Step 2.** Solve the following Cauchy problem for all \(s \leq \Delta\)

\[
rx_i^1(s) = v(b_{\min}) + a_i [W - x_i^1(s)] - ch + x_i^1'(s) \quad (5)
\]

\[
r x_i^1(\Delta) = v(b_{\min}) \quad (6)
\]

This yields

\[
x_i^1(s) = \frac{v(b_{\min}) + a_i W - ch}{r + a_i} \left[1 - e^{-(r + a_i)(\Delta - s)}\right] + \frac{v(b_{\min}) e^{-(r + a_i)(\Delta - s)}}{r}
\]

If \(\phi_i m [W - x_i^1(T)] < c\), then \(\sigma_i\) is such that \(\phi_i m [W - x_i^1(\sigma_i)] = c\). If not, go to Step 3.

**Step 3.** Solve the following Cauchy problem for all \(s \leq T\)

\[
r x_i^2(s) = v(b_{\max}) + a_i [W - x_i^2(s)] - ch + x_i^2'(s) \quad (7)
\]

\[
x_i^2(T) = x_i^1(T) \quad (8)
\]

This yields

\[
x_i^2(s) = \frac{v(b_{\max}) + a_i W - ch}{r + a_i} \left[1 - e^{-(r + a_i)(T - s)}\right] + x_i^1(T) e^{-(r + a_i)(T - s)}
\]

If \(\phi_i m [W - x_i^2(0)] < c\), then \(\sigma_i\) is such that \(\phi_i m [W - x_i^2(\sigma_i)] = c\). If not, \(e_i(s) = h\) for all \(s \leq \Delta\).

Note that \(e_h(\Delta) = h\) implies that \(e_h(\Delta) = h\). This property allows us to focus on the configuration where \(e_h(\Delta) = h\) and \(e_b(\Delta) = 0\).

**Assumption A2** \(\phi m [v(w) - v(b_{\min})] < rc\)

**Assumption A3** \(m [W - x_b^2(0)] < c < m [W - x_b^1(T)]\)

The following Proposition summarizes our results.
Proposition 1  Job-seekers’ efforts

Let

$$\sigma = T + \ln \left[ \frac{v(b_{\text{max}}) + e^{\frac{c-r}{m}} - v(w)}{v(b_{\text{max}}) - v(b_{\text{min}}) + m (1 + h) \frac{v(w) - v(b_{\text{min}})}{r} - c} \right] e^{-r(m(1+h))(\Delta - T)}$$  \hspace{1cm} (9)

Under Assumptions A1 to A3,

(i) Bad workers set $e_b(s) = 0$ for all $s \geq 0$;

(ii) Good workers set $e_g(s) = h$ if $s \in [\sigma, \Delta]$ and $e_g(s) = 0$ else

Assumption A2 ensures that bad workers always set the low level of effort. Assumptions A3 guarantees that good workers set the high level of effort before they have reached the potential duration of benefits $T$, but after some time spent in unemployment – that is $\sigma \in (0, T)$. Note that Assumptions A1 to A3 are necessary and sufficient conditions.

Figure 1 depicts the resulting patterns of hazard rates.

[Insert Figure 1]

The hazard rate of good workers features the typical spike prior to losing benefit entitlement. The spike in hazard lasts until firms discriminate against the long-term unemployed. At the same time, the hazard rate of bad workers is flat throughout the spell of unemployment.

We focus on this particular configuration for simplicity. We discuss some empirical implications in Subsection 4.

2.3  The composition of unemployment

The previous Subsection carefully examines job-seekers’ behavior and resulting hazards. In this Subsection, we analyze the implications of such hazard rates on the distribution of unemployment spells, the distribution of type by unemployment duration, and the distribution of contact by type and unemployment duration.

Let $u_i(s, t)$ denote the size of the cohort of type-$i$ unemployed whose unemployment duration is $s$ as of time $t$. It evolves according to the following partial differential equation:

$$\frac{\partial u_i(s, t)}{\partial s} + \frac{\partial u_i(s, t)}{\partial t} = - [\phi_i(s) m (1 + e_i(s)) + u_i(s, t)] \hspace{1cm} (10)$$
$$u_i(0, t) = n \pi_i(0) \hspace{1cm} (11)$$

The total size of the cohort of duration $s$ unemployed is $u(s, t) = u_g(s, t) + u_b(s, t)$. Finally, the number of job-seekers is $U(t) = \int_0^\infty u(s, t) ds$. Given that we only focus on a steady-state, $\partial u_i(s, t) / \partial t = 0$ and the dependence vis-à-vis time $t$ will be neglected.

Let $\psi(s)$ denote the pdf of the distribution of unemployment duration, while $\pi_i(s)$ denote the proportion of type-$i$ unemployed conditional on duration $s$. By construction

$$\psi(s) = \frac{u(s)}{U} \hspace{1cm} (12)$$
$$\pi_i(s) = \frac{u_i(s)}{u_g(s) + u_b(s)} \hspace{1cm} (13)$$
Finally, let \( p_i(s) \) denote the probability of contacting a type-\( i \) worker conditional on contacting a worker whose unemployment duration is \( s \). Random matching implies that

\[
    p_i(s) = \frac{m \left[ 1 + e_i(s) \right] u_i(s)}{m \left[ 1 + e_g(s) \right] u_g(s) + m \left[ 1 + e_b(s) \right] u_b(s)} \quad (14)
\]

**Proposition 2  DISTRIBUTION OF TYPES AND UNEMPLOYMENT SPELLS**

*Under Assumptions A1 to A3,*

\[
    \pi_g(s) = \begin{cases} 
        \frac{\pi_0}{\pi_0 + (1-\pi_0)e^{-m(1-\phi)\sigma}} & \text{if } s < \sigma \\
        \frac{\pi_0}{\pi_0 + (1-\pi_0)e^{-mkz + m(1+h-\phi)\sigma}} & \text{if } s \in [\sigma, \Delta]
    \end{cases} \quad (15)
\]

\[
    p_g(s) = \begin{cases} 
        \frac{\pi_0}{\pi_0 + (1-\pi_0)e^{-m(1-\phi)\sigma}} & \text{if } s < \sigma \\
        \frac{\pi_0}{(1+h)\pi_0 + (1-\pi_0)e^{-mkz + m(1+h-\phi)\sigma}} & \text{if } s \in [\sigma, \Delta]
    \end{cases} \quad (16)
\]

\[
    \psi(s) = \begin{cases} 
        \frac{\sum_{i=g,b} n \pi_i(0) \exp\left(-\int_0^s \left[m\phi_i(z)(1+e_i(z)) + n \right] dz \right)}{\int_0^s \sum_{i=g,b} n \pi_i(0) \exp\left(-\int_0^z \left[m\phi_i(z)(1+e_i(z)) + n \right] dz \right) dz} & \text{if } s \in [0, \Delta] \\
        0 & \text{else}
    \end{cases} \quad (17)
\]

where \( \sigma, e_g(s) \) and \( e_b(s) \) are defined in Proposition 1.

Figure 2 depicts the pattern of the proportion of good workers by unemployment duration.

[Insert Figure 2]

As the hazard rate of good workers is always higher than the hazard rate of bad workers, the proportion of good workers falls with the duration of unemployment. Indeed,

\[
    \pi_g'(s) = -m \left[ 1 + e_g(s) - \phi \right] \pi_g(s) [1 - \pi_g(s)] < 0 \quad (18)
\]

Note that the function \( \pi_g \) is continuous on \([0, \Delta]\), while its derivative is not continuous in \( \sigma \), when good workers start searching stronger.

Figure 3 depicts the pattern of the proportion of good agents by contact among individuals who have spent the duration \( s \) in unemployment.

[Insert Figure 3]

The function \( p_g \) is strictly decreasing on each interval where it is continuous. Indeed,

\[
    p_g'(s) = -m \left[ 1 + e_g(s) - \phi \right] p_g(s) [1 - p_g(s)] < 0 \quad (19)
\]

However, the function \( p_g \) jumps upwards in \( \sigma \), as good workers suddenly become overrepresented among the applicants.

This discontinuity is important because the function \( p_g \) will shape employers’ beliefs on the composition of applicants by duration. The resulting hiring policy must be consistent with the one that has been postulated in Assumption A1. Typically, the existence of an equilibrium will require that

\[
    \lim_{s \to \sigma} p_g(s) > p_g(\Delta). \quad \text{In words, the probability of contacting a good agent conditional on contacting an unemployed of duration } \sigma \text{ must be larger than the probability of contacting a good agent conditional on contacting an unemployed of duration } \Delta. \quad \text{Figure 3 has been drawn assuming that this restriction holds.}
\]

Finally, note that the pdf of the unemployment duration distribution has three properties: it is continuous on the support \([0, \Delta]\), it is strictly decreasing in \( s \), reflecting the fact that none can enter the distribution at some positive duration, and its derivative is discontinuous in \( s = \sigma \), as good workers start finding jobs at a faster rate.
2.4 Firm behavior

Let $V$ denote the value of a vacant position, while $\Pi_i$ is the value of a job filled by a worker of type $i$. We have:

\[
\begin{align*}
\rho V &= -\gamma + \frac{m}{\theta} \mathbb{E}_s \max \{ \mathbb{E}_i [\Pi_i - V \mid s], 0 \} \\
r \Pi_i &= y_i - w
\end{align*}
\]

(20)

(21)

Firms endowed with a vacant slot have to expect the type of the workers they may meet. They do so on the basis of (i) the unemployment duration $s$ that they observe, (ii) their expectation on the trajectory of individual search efforts, and (iii) the probability $\phi \in (0, 1)$ of not detecting a bad worker at the time of interview.

It follows that

\[
\mathbb{E}_i [\Pi_i - V \mid s] = \Pr(\text{signal is good}) \Pr(\text{worker is good} \mid \text{signal is good}) \times (\Pi_g - V) \\
+ \Pr(\text{signal is good}) \Pr(\text{worker is bad} \mid \text{signal is good}) \times (\Pi_b - V) \\
+ \Pr(\text{signal is bad}) \times 0
\]

Firms’ beliefs on workers’ types obey the Bayes rule. Therefore,

\[
\Pr(\text{worker is good} \mid \text{signal is good}) = \frac{\Pr(\text{worker is good} \cap \text{signal is good})}{\Pr(\text{signal is good})}
\]

\[
= \frac{p_g(s)}{p_g(s) + p_b(s) \phi}
\]

and

\[
\Pr(\text{worker is bad} \mid \text{signal is good}) = \frac{\Pr(\text{worker is bad} \cap \text{signal is good})}{\Pr(\text{signal is good})}
\]

\[
= \frac{p_b(s) \phi}{p_g(s) + p_b(s) \phi}
\]

For a particular firm, we have

\[
\rho V = -\gamma + \frac{m}{\theta} \int_0^\infty \psi(s) \max \{ [p_g(s)(\Pi_g - V) + (1 - p_g(s)) \phi (\Pi_b - V)], 0 \} \, ds
\]

(22)

Expected profits depend on two distributions: the distribution of unemployment durations, and the distribution of workers’ type conditional on contact and unemployment duration. Firm’s optimal hiring strategy $\tilde{\phi}_i(s)$ follows.

**Proposition 3** Firms’ hiring strategy

The policy function $\tilde{\phi}_i(s)$ is such that

\[
\tilde{\phi}_i(s) = \begin{cases} 
\phi_i & \text{if } p_g(s) \geq -\frac{\Pi_g \Pi_b - V(1 - \phi)V}{\Pi_g - \phi \Pi_b - (1 - \phi)V} \\
0 & \text{else}
\end{cases}
\]

\textsuperscript{5}Equation (21) assumes that the job is destroyed whenever the worker retires. This assumption is innocuous, because the value of a vacancy $V$ is driven to 0 in equilibrium.

\textsuperscript{6}Note that it is implicitly assumed that unemployable workers participate in the search market. This results from the cost structure of search efforts: given zero cost for a low effort, agents are marginally indifferent between searching a job or not. Assuming an $\varepsilon$-cost would suffice to prevent non-employable workers to search. In such a case, one would have to consider the distribution of unemployment duration conditional on the fact that $\phi_i(s) > 0$. 

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Finally, there is free entry of new firms on the search market. This drives the value $V$ of vacancy to zero.

## 3 Equilibrium time-dependence in hazard rates

This section considers the equilibrium of the model. We proceed in two steps. First, we study the existence and uniqueness of equilibrium. Second, we make comparative exercises and study how hazard rates respond to changes in the design of unemployment compensation.

### 3.1 Equilibrium

In a symmetric equilibrium, all firms have the same hiring policy $\phi_i(s)$, all workers of the same type have the same job search behavior $e_i(s)$, agents maximize their gains, and expectations are compatible with equilibrium outcomes. We focus on the particular type of equilibrium that we have highlighted so far. Given that our framework may feature other types of equilibrium, we need to differentiate this equilibrium from other types. In the sequel, we will call it the baseline equilibrium.

**Definition** Baseline equilibrium

A baseline equilibrium is a vector $(\sigma, \Delta, \theta)$ and a set of four functions $(e_g, e_b, \psi, p_g)$ such that

(i) $e_b(s) = 0$ for all $s \geq 0$ and $e_g(s) = h$ iff $s \in [\sigma, \Delta]$, where $\sigma$ is defined in Proposition 1

(ii) $\psi$ and $p_g$ are defined in Proposition 2

(iii) For $i = g, b, \phi_i(s) = \phi_i(s) = \phi_i$ iff $s \in [0, \Delta]$, where $\phi_i$ is defined in Proposition 3

(iv) $V = 0$

(v) $0 < \sigma < T < \Delta$

(i) states that the postulated job-seeking behavior is optimal for both types of workers. (ii) reminds that the distribution of unemployment duration and the proportion of good workers by duration are implied by individual strategies. (iii) states that the postulated hiring policy is optimal for the firms. (iv) is the free-entry condition. Finally, (v) makes clear that good workers set a high search intensity before benefits are elapsed and potential employers discriminate against the long-term unemployed after the loss of benefit entitlement.

**Proposition 4 Existence and uniqueness of a baseline equilibrium**

(i) In a baseline equilibrium, we have

$$
\sigma = T + \ln \left[ \frac{v(b_{\max}) + c \frac{T}{m} - v(w)}{v(b_{\max}) - v(b_{\min}) + \left( m(1 + h) \frac{2(w - v(b_{\min}) - ch)}{r} e^{-(r+m(1+h))(\Delta-T)} \right)} \right]^{\frac{1}{m+1}}
$$

(JS)

$$
\Delta = \frac{h}{1+h-h_{\phi}} + \frac{1}{m(1+h-h_{\phi})} \ln \left[ \frac{y_g - w - \pi_0 (1 + h)}{\phi (w-y_b) \left( 1 - \pi_0 \right)} \right]
$$

(HS)

$$
\theta = \frac{m}{\gamma} \int_0^\Delta \psi(s) [p_g(s) \Pi_g + (1 - p_g(s)) \phi \Pi_b] ds
$$

(FE)
(ii) There may exist a baseline equilibrium

(iii) If a baseline equilibrium exists, it is unique

(i) The (JS) locus results from good workers’ equilibrium job search strategy. The (HS) locus results from firms’ equilibrium hiring strategy. This strategy involves that \( p_g(\Delta) = -\phi \Pi_b / (\Pi_g - \phi \Pi_b) \), which yields (HS). Finally, tightness results from the free-entry condition. This yields (FE). Given that we assume that job search efforts do not create congestion effects, tightness determination has no feed-back effects on individual choices. The solving of equilibrium reduces to find a couple \((\sigma, \Delta)\) that satisfies (JS) and (HS).

(ii) We provide during the proof of Proposition 4 the set of necessary and sufficient conditions leading to the existence of a baseline equilibrium. These conditions are not particularly appealing. Indeed, we must check that (JS) and (HS) intersect at least once. We must also check that \( 0 < \sigma^* < T < \Delta^* \). Finally, we must check that \( \lim_{s \to \sigma^*} p_g(s) \geq p_g(\Delta) \). This leads to four inequalities that define the parameter space compatible with the existence of a baseline equilibrium. Of course, we show that this parameter space is nonempty.

(iii) The (JS) locus and the (HS) are both strictly increasing. On the one hand, a longer cut-off duration raises the value of search, and good workers delay the moment at which they start searching with a high intensity. On the other hand, an increase in \( \sigma \) raises the proportion of bad workers at all duration. In turn, this leads firms to delay the duration above which they reject all applications. However, the slope of the (JS) curve is always lower than the slope of the (HS) curve. This establishes the uniqueness of equilibrium.

Note that there is a single equilibrium in our model, while there may be multiple equilibria in Lockwood’s. This is so because Lockwood assumes that the matching rate \( m \) depends on the market tightness \( \theta \). This originates a feed-back effect from job creation to the composition of unemployment. Namely, job profitability increases with the mean productivity of the job-seekers. But, tightness raises such a mean productivity. Job profitability may increase with tightness as a result, which explains multiple equilibria. Abstracting from such feed-back effects allows us to focus on the novelty of our paper: the interaction between workers’ search and employers’ hiring strategies.

Figure 4 depicts the equilibrium.

[Insert Figure 4]

Proposition 4 allows us to interpret the duration-dependence in hazard rates observed in the data as an equilibrium outcome. This results from the interplay between job search and hiring strategies in an environment characterized by matching frictions, worker heterogeneity and asymmetric information on workers’ types. Employers have no reasons to discriminate against the unemployed before the exhaustion date because good workers have strong reasons to stay unemployed, and are therefore likely to meet. Things become different after the exhaustion date because good workers should manage to exit unemployment around the exhaustion date. This explains the fall in hazards after the exhaustion date: employers reject the applications of unemployed who have no reasons to stay unemployed unless they are of the bad type.
3.2 Changes in unemployment compensation scheme

In this Subsection, we examine the impacts of unemployment compensation on equilibrium hazard rates.

Proposition 5 Properties of the baseline equilibrium

Assume that there exists a baseline equilibrium. Then,

(i) $\sigma^*$ and $\Delta^*$ are strictly increasing in $b_{\text{min}}$, $b_{\text{max}}$ and $T$

(ii) as $h$ tends to infinity, $\sigma^*$ and $\Delta^*$ tend to $T$

(i) standardly shows that the various components of unemployment compensation generosity originate moral hazard effects that are detrimental to search efforts. Confronted to a more generous scheme, good workers wait longer to make high search efforts and $\sigma^*$ increases. However, such moral hazard effects alter the signalling value of unemployment duration. The probability of recruiting a good worker increases at all duration. In turn, employers are less reluctant to hire long-term unemployed and the cut-off duration $\Delta^*$ increases too.

Figure 4 depicts these effects of unemployment insurance. The components of unemployment insurance only affect the (JS) locus that shifts rightward. The equilibrium moves along the (HS) locus that is positively sloped. $\sigma^*$ and $\Delta^*$ increase as a result.

(ii) shows that $\sigma^*$ and $\Delta^*$ tend to $T$ in the non-frictional case where $h$ becomes arbitrarily large. Indeed, good workers wait the exhaustion date to set the high search effort. They immediately exit unemployment and no good workers remain among the cohort of unemployed. Employers expect this and do not hire the workers who have overtaken the exhaustion date in unemployment.

This result has a major implication. According to (i), all the different components of unemployment insurance originate moral hazard effects. But it says nothing about the magnitude of the different effects. (ii) tells us that the magnitude depends on parameter $h$. In the non-frictional case where $h$ tends to infinity, the potential duration of benefits governs equilibrium hazard rates, while replacement rates $b_{\text{max}}$ and $b_{\text{min}}$ have no impacts.

This provides a simple explanation to the fact that estimated hazard rates respond much less to changes in benefit levels than to changes in potential duration. If good workers can activate a sufficiently efficient technology, they massively exit the unemployment state around the exhaustion date. This drives employers’ beliefs who reject all the applicants of a cohort whose unemployment duration is larger than $T$.

Proposition 5 tells a general lesson. The design of unemployment insurance affects the signalling value of unemployment duration. This should be taken into account by policy makers.

4 Empirical relevance

In this Section, we consider two empirically relevant aspects of our model. First, our model makes predictions on individual hazards that may be confronted to data. Second, our model makes particular assumptions on employers’ information set that deserve to be discussed.
4.1 From theoretical to empirical hazards

We highlight the coexistence of two subpopulations in our paper. They broadly recover the general divide between movers and stayers highlighted by the empirical literature on unobserved heterogeneity. In our paper as in Lockwood, the distinction between movers and stayers is an equilibrium outcome. A key feature of this approach is that ex-ante heterogeneity shapes employers’ and workers’ beliefs in a way that originates true duration dependence in hazard rates.

Hazard rates have three main properties in our model: (i) they are piecewise continuous, (ii) unobserved good workers benefit from a higher exit rate at all durations than unobserved bad workers, (iii) good workers’ and bad workers’ hazards differently respond to benefit exhaustion.

(i) is consistent with piecewise constant hazard models introduced by Lancaster (1990). In such models, the hazard is typically written as follows

\[ \mu (s \mid x) = \mu_0 (s) h (x) \]  
\[ \mu_0 (s) = \begin{cases} 
\mu_1 & \text{if } s \in [0, \sigma_1] \\
\mu_2 & \text{if } s \in (\sigma_1, \sigma_2] \\
... & \\
\mu_M & \text{if } s \in (\sigma_{M-1}, \infty) 
\end{cases} \]  

where \( \mu_k \) are constant and \( \sigma_k \) are define points in time, \( 0 < \sigma_1 < \sigma_2 < ... < \sigma_{M-1} < \infty \).

In our model, workers have all the same observed characteristics. However, the analysis can be generalized to any homogenous group of workers, with the assumption that search markets are segmented by observable characteristic. In the case of good workers, our model predicts three time intervals (\( M = 3 \)), with \( \sigma_1 = \sigma, \sigma_2 = \Delta, \mu_1 = m, \mu_2 = m (1 + h) \) and \( \mu_3 = 0 \).

(ii) Our model hinges on the fact that there are two groups of workers on each market segment, the good and the bad, that cannot be differentiated easily by employers. These proportions may or may not differ between observable groups, yet the intuition suggests that the initial proportion of good workers should increase with skill level. This means that there is unobserved heterogeneity, and the econometrician should account for it.

(iii) There are several ways to modelize unobserved heterogeneity to capture the mover-stayer dichotomy. However, they are not all compatible with our model. The simplest way to deal with unobserved heterogeneity is to assume that there is an individual specific component \( \varepsilon \) in hazard that is independent of both \( s \) and \( x \). Formally,

\[ \mu (s \mid x, \varepsilon) = \mu_0 (s) h (x) \varepsilon \]  

It is then usual to assume a simple functional form for the distribution of the error term, like the Gamma distribution, or, closer to our model, a discrete distribution (see Lancaster, 1979).

However, this hazard function implies that the two sub-populations must experience similar qualitative patterns in baseline hazard rates. By contrast, our model predicts distinctive qualitative patterns across the two groups of workers. In other words, the individual component \( \varepsilon \) should not be independent of \( s \) – and probably of \( x \) as well. This requirement may be too strong to be compatible with identification, yet it is an essential feature of our model.
4.2 Further evidence

Dormont et al (2006) provide another type of evidence. They estimate hazard rates by pre-unemployment earnings. They show that the hazard rate of formerly higher-paid workers features a spike, while the hazard rate of formerly lower-paid workers is fairly smooth around the exhaustion date. Of course, the distinction between lower-paid and higher-paid workers does not really fit with our model in which there is a single wage. However, this suggests that ability to respond to benefit exhaustion should be positively correlated to skill level. In addition, our model provides a simple explanation to Dormont et al finding. Suppose that there is a separate search market for each skill, and that formerly higher-paid workers compete on the high-skill segment, while formerly lower-paid workers compete on the low-skill segment. Suppose also that for a reason or another, estimation techniques used by Dormont et al fail to identify the two groups of workers in each segment. In such a case, they estimate the average hazard among each group of workers. Provided that the proportion of good workers increases with skill level, our model predicts that the mean average rate and the mean peak in such a rate should be higher among the formerly higher-paid workers than among the formerly lower-paid workers.

4.3 Employers’ information set

Our model hinges on the assumption that employers can observe both one’s unemployment duration and one’s potential duration of benefits. Several other papers assume that employers observe the unemployment duration (see e.g. Blanchard and Diamond, 1994, Coles and Masters, 2000). They rely on the fact that cvs implicitly display this information, or that employers should be able to get it during the interview. The potential duration of benefits may or may not be difficult to observe. There are two main obstacles.

First, eligibility rules may be very complicated. They can vary with former job duration or demographic characteristics. Nevertheless, employers should always be able to evaluate the applicant’s situation vis-à-vis the benefit system using the information displayed by the applicant’s cv (former job duration and current unemployment duration). In addition, there are certain regularities that depend on the type of advertised job that should be known by employers. For instance, a senior position should mostly attract relatively experimented workers who entered unemployment with full coverage and maximum potential duration7 (or, similarly, workers benefiting from a longer duration because of their age as in France, Germany or Sweden). Conversely, a very junior position should attract individuals who are mostly non-entitled to unemployment benefits.

Second the legislation may be volatile. For instance, the French unemployment insurance system has changed several times across the past twenty years, going from two-step duration-contingent compensation (1986-1992), then almost continuous decrease in benefit level over the unemployment episode (1993-1996), then two-step again (1997-2003), plus change in the maximum duration (2004-). One interesting question concerns the impact of such a volatility on employers’ beliefs vis-à-vis the long-term unemployed. Intuitively, risk-averse workers facing institutional uncertainty should be more prompt to

7One may argue that signalling problems are less relevant for such workers who have a long work record. This perspective abstracts from human capital transferability problems. For instance, human capital accumulated during the previous job could either be purely specific (in such a case, the worker is bad) or fully transferable to a new job (in such a case the worker is good).
exit the unemployment state. Unless labor market skills and risk aversion are too negatively correlated, this effect should be stronger for the good workers. Overall, the quality of the signal conveyed by unemployment duration should increase with institutional volatility, strengthening employers’ discrimination vis-à-vis the long-term unemployed. The exact formalization of this argument is left for future work.

However, employers are likely to know whether one is covered by unemployment benefits or not. Indeed, it is in the interest of covered workers to say and prove that they are covered. A worker who is not covered can easily be discovered in such a case.

5 Conclusion

This paper is a theoretical contribution to the literature on job search and unemployment. We revisit the signalling hypothesis, according to which potential employers use the duration of unemployment as a signal on the productivity of applicants. We suggest that the quality of such a signal is very low when the unemployed get unemployment benefits: individuals have good reasons to stay unemployed. Conversely, the signal becomes much more efficient once benefits are elapsed: skilled workers should not stay unemployed in such a case. Therefore, the potential duration of unemployment benefits should drive employers’ expectations and their recruitment practices. This mechanism can explain why hazards fall after benefit expiration, and why unemployment duration responds much more to the potential duration of benefits than to replacement rates.

Beyond its focus on hazard rates, our paper delivers a major lesson: the design of unemployment compensation alters the signalling value of unemployment duration. This should be taken into account by policy makers. We plan to address this issue in future work.
APPENDIX

A Proof of Proposition 1

Given Assumption A1, the solving of the maximization problem \((*)\) is given in the main text that precedes Proposition 1.

(i) results from Step 1 of the solving and Assumption A2.
(ii) results from Step 2, Step 3, and Assumption A3.

By solving, Assumptions A1 to A3 are not only sufficient but also necessary conditions.

B Proof of Proposition 2

Assumption A1 to A4 imply that Proposition 1 holds. Therefore,

\[ e_b(s) = 0 \text{ for all } s \geq 0 \]  

and

\[ e_g(s) = \begin{cases} h \text{ if } s \in [\sigma, \Delta] \\ 0 \text{ else} \end{cases} \]  

Solving the Cauchy problem (10)-(11) leads to

\[ u_i(s) = n\pi_i(0) \exp \left\{ -\int_0^s [\phi_i(z) m(1 + e_i(z)) + n] dz \right\} \]

One can use these different equations and the definitions of \(\psi, \pi_g\) and \(p_g\) given in the main text to show (i) to (iii).

C Proof of Proposition 3

The main text makes clear that \(\bar{\phi}_i(s) = \bar{\phi}_i\) iff

\[ p_g(s)(\Pi_g - V) + (1 - p_g(s))\phi(\Pi_b - V) \geq 0 \]

The result follows.

D Proof of Proposition 4

(i) follows from the definition of a baseline equilibrium. The (JS) locus is implied by Proposition 1. The (HS) locus results from Proposition 2, Proposition 3 and the free-entry assumption \(V = 0\). From Proposition 2 and free entry, \(\phi_i(s) = \bar{\phi}_i\) iff \(s \in [0, \Delta]\) implies that \(p_g(\Delta) = -\phi\Pi_b/(\Pi_g - \phi\Pi_b)\). From Proposition 2, the function \(p_g\) is continuous and strictly decreasing on \((\sigma, \Delta]\). It follows that \(\Delta = p_g^{-1}(-\phi\Pi_b/(\Pi_g - \phi\Pi_b))\). The computation leads to (HS). Finally, the (FE) locus results from imposing \(V = 0\) in equation (22) that defines the value of a vacancy.

(ii) The definition of baseline equilibrium involves to find a positive vector \((\sigma, \Delta, \theta)\) that solves (JS), (HS) and (FE), and satisfies

\[ \phi m [v(w) - v(b_{\min})] < rc \]
\[ 0 < \sigma < T < \Delta \]

\[
\lim_{s \to \sigma^-} p_g(s) > -\phi \Pi_b / (\Pi_g - \phi \Pi_b)
\]

(31)

(32)

\(\sigma\) and \(\Delta\) are jointly determined by (JS) and (HS). Then \(\theta\) follows from (FE). Therefore, the solving reduces to find \(\sigma\) and \(\Delta\) that solve (JS) and (HS) and satisfy conditions (30) to (32).

We now provide necessary and sufficient conditions for the existence of a baseline equilibrium.

Lemma 1. Let

\[
A_1 = v(b_{\text{max}}) + e^{\frac{r+m}{m}} - v(w)
\]

\[
A_2 = v(b_{\text{max}}) - v(b_{\text{min}})
\]

\[
A_3 = m (1 + h) \frac{v(w) - v(b_{\text{min}})}{r} - ch
\]

\[
A_4 = \ln \left[ \frac{y_g - w}{\phi(w - y_b)} \frac{\pi_0}{1 - \pi_0} (1 + h) \right]
\]

Let also

\[
a_1 = r + m (1 + h)
\]

\[
a_2 = m (1 + h - \phi)
\]

Consider the function \(P: \mathbb{R} \to \mathbb{R}\) be such that

\[
P(x) = A_2 e^{-a_1 T} e^{a_1 x} + A_3 (e^{a_1 x}) \frac{1 + \phi}{1 - \phi} e^{-\frac{a_1}{a_2} A_4} - A_1 = 0
\]

There exists a baseline equilibrium iff

\[
\phi m [v(w) - v(b_{\text{min}})] < rc \quad (C1)
\]

\[
P(\text{max} \{0, Z_1\}) < 0 \quad (C2)
\]

\[
P(\text{min} \{T, Z_2\}) > 0 \quad (C3)
\]

where

\[
Z_1 = \frac{1 + h - \phi}{h} (T - A_4/a_2)
\]

\[
Z_2 = \frac{1}{m (1 - \phi)} \ln \left[ \frac{y_g - w}{\phi(w - y_b)} \frac{\pi_0}{1 - \pi_0} \right]
\]

Proof. Note first that bad agents never set the high effort iff condition (C1) holds. Using (HS), one can replace \(\Delta\) in (JS). After simple computations, one obtains

\[
P(\sigma) = 0
\]

The function \(P\) is strictly increasing, with \(P(-\infty) = -A_1\) and \(P(\infty) = \infty\). It follows that there is a unique \(\sigma\) such that \(P(\sigma) = 0\). This \(\sigma\) and associated \(\Delta\) given by (HS) is an equilibrium candidate. This candidate must satisfy constraints (30) to (32). First, \(\sigma > 0\) iff \(P(0) < 0\). Second, \(\sigma < T\) iff \(P(T) > 0\). Third, using (HS), one can see that \(\Delta > T\) is equivalent to \(\sigma > Z_1\). Therefore, \(\Delta > T\) iff \(P(Z_1) < 0\). Fourth, \(\lim_{s \to \sigma^-} p_g(s) > -\phi \Pi_b / (\Pi_g - \phi \Pi_b)\) is equivalent to \(\sigma < p_g^{-1} (-\phi \Pi_b / (\Pi_g - \phi \Pi_b))\). In turn, this is equivalent to \(P(Z_2) > 0\). Conditions C2 and C3 result from these four cases.
To conclude the proof, one need to show that the parameter space defined by conditions C1 to C3 is nonempty. The following example checks all the conditions: $\rho = 4\%, \delta = 2.6\%, \pi_0 = 0.8, m = 0.8, c = 0.2; \phi = 0.3, b_{\min} = 0.3, b = 0.6, T = 1, w = 1.2, y_g = 1.3, y_h = 0.9, v(x) = 0.1x^{0.5}, h = 5$ [To check].

(iii) Uniqueness is a by-product of the former proof.

E Proof of Proposition 5

(i) The (HS) locus does not depend on the parameters that shape the unemployment compensation scheme. Therefore, we only need to know how $b_{\min}, b_{\max}$ and $T$ affect the (JS) locus. This locus results from

$$m \left[ W - x_i^2 (\sigma, \Delta; b_{\min}, b_{\max}, T) \right] = c \quad (33)$$

The function $x_i^2$ is strictly increasing in $b_{\min}, b_{\max}$ (for $\sigma < T$) and $T$. It is also strictly decreasing in $\sigma$. It follows that $\partial \sigma (\Delta^*, b_{\min}, b_{\max}, T) / \partial b_{\min} > 0$, $\partial \sigma (\Delta^*, b_{\min}, b_{\max}, T) / \partial b_{\max} > 0$, and $\partial \sigma (\Delta^*, b_{\min}, b_{\max}, T) / \partial T > 0$. In each case, the (JS) locus shifts rightward in Figure 4. The result follows.

(ii) As $h$ tends to infinity, the (HS) locus tends to the 45-degree line so that $\Delta = \sigma$, while the (JS) locus tends to the vertical line $\sigma = T$. The result follows.
References


Fig. 1: Hazard rates of good and bad workers – Baseline equilibrium
Fig. 2: Proportion of good workers by unemployment duration – Baseline equilibrium
Fig. 3: Probability of contacting a good worker by unemployment duration Baseline equilibrium
Fig. 4: Existence and uniqueness of a baseline equilibrium