The relation between private consumption and government spending: theory and some evidence from a DSGE approach FIRST DRAFT

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14-08-2007

Abstract

This paper reconsiders the effect of fiscal policy using a New-Keynesian dynamic stochastic general equilibrium model. Specifically, we focus on the relation between private consumption and government spending. First, we want to see if government consumption either substitutes, complements or is not related at all with private consumption. Second, we analyse how this complementarity/substitutability issue affects the response of private consumption to a government spending shock. Thirdly, allowing for the presence of a fraction of credit constrained households in the model, we want to study the interaction between the complementarity/substitutability issue and the presence of non-optimizing households. Preliminary estimates, using US data, show that substitutability between government and private consumption emerges. This substitutability, together with the *so-called* negative wealth effect, makes consumption to fall after a government spending shock

1 Introduction

This paper reconsiders the effect of fiscal policy using a New-Keynesian dynamic stochastic general equilibrium (N-DSGE) model. We try to better investigate the relation between private and government consumption to shed light on the response of private consumption to a government spending shock. Specifically, we allow preferences to depend both on private and government consumption in order to discover some form of complementarity/substitutability between private and government consumption. So far, this issue has been hardly studied within a N-DSGE model, however, it can play a central role for identifying the relation between private and government consumption. Indeed, the two main components of government consumption, i.e. spending for health and education, can produce, ceteris paribus, some externalities for private consumption. For example, on the one hand, public health services can reduce the need for private hospitals, and, public schools can reduce the need for private tutors. On the other hand, education can act as a complement for other components of private consumption, that is, better educated people can contribute to an increase in the demand for books or for visiting museums. Even other items of government spending can act as substitutes or complements for private consumption: spending for public order can reduce the need for a private policeman, subsidies for purchasing computers can increase private expenditures for internet services and so on. These possible relations between private consumption and different items of public spending makes government consumption, in aggregate, to be either substitute or complement for private consumption. Thus, omitting *a priori* this channel of substitutability/complementarity can produce a bias in the estimate of the response of private consumption to a government consumption shock.

There is no general consensus on the qualitative response that private consumption should have in response to a government spending shock, both theoretically and empirically. From a theoretical point of view, both standard neoclassical and New-Keynesian models, e.g. Baxter and King (1993), predict that private consumption falls following a positive shock on government spending because the negative wealth effect lowers the households' permanent income. Recently, there have been some attempts to build models which generate a positive response of private consumption to a government spending shock. Linnemann (2006) builds a neoclassical model in which leisure and consumption enter not separable in the utility function. As leisure falls following the negative wealth effect, the substitutability between consumption and leisure[1] implies the marginal utility of consumption must increase, making the agent want to consume more. Linnemann and Schabert (2003) formulate a New-Keynesian model in which government consumption enters non separable in the households' utility function. They find that government spending causes positively private consumption for sufficiently low values of the elasticity of substitution between private and government consumption. Interestingly, Galì, Lòpez-Salido and Vallès (2007), henceforth GLV (2007), build a N-DSGE model in which they introduce a market imperfection, namely, a share of the population cannot borrow or lend. Here,

[1] The reader can also think to a complementarity betrween hours worked and consumption.

because of price rigidity, not only labor supply, but also labor demand increases due to a government consumption shock. If the shift in labor demand is sufficiently large, real wage increases. Since the consumption of liquidity constrained individuals depends positively on the real wage, their consumption will increase. For sufficiently large share of liquidity constrained individuals, aggregate consumption can increase after a government spending shock. From an empirical point of view, one of the recent interesting finding using vector autoregressions, states that shocks to government spending seem to be associated with increases in private consumption (Blanchard and Perotti (2002), Canzoneri et al.(2002), Fatas and Mihov (2002), Mountford and Uhlig (2005))[2]. However, Ramey and Shapiro (1998), using the so called narrative approach within a vector autoregressions framework, find that government consumption crowds out private consumption. Recently, there have been some attempts to estimate N-DSGE models with a role for fiscal policy, but the evidence is not clear-cut. On the one hand, Smets and Wouters (2003) estimate a standard N-DSGE model, and, find that government spending crowds-out private consumption. Moreover, Conen and Straub (2005) estimate a N-DSGE model similar to GLV(2007), and, find the share of credit constrained consumers to be relatively small, such that, it is not sufficient to make aggregate consumption to increase after a government spending shock. On the other hand, Lopez-Salido and Rabanal (2006) estimate a N-DSGE in which they find that the form of complementarity between consumption and hours worked helps consumption to increase after a government shock. However, their estimated share of credit constrained individuals is quite low. Interestingly, Boukez and Rebei (2003) study the parameter governing complementarity within a real business cycle model, they find a strong complementarity between private and public spending for US so that government spending crowd-in private consumption.

Therefore, the present paper has two main goals. First, we want to estimate, within a N-DSGE model, if government and private consumption are either substitutes or complements in the aggregate. Second, we want to analyse how this estimated relation between government and private consumption affect the response of private consumption to a government spending shock, i.e., if the issue of substitutability/complementarity mitigate or reinforce the typical negative wealth effect produced by a government spending shock. Moreover, following GLV (2007), we allow in our model the presence of a share of consumers which are

[2] It is worth mentioning that some other empirical papers find opposite results with respect to those outlined here. Among others, Ramey and Shapiro (1998), using the so called narrative approach, find that government consumption crowds out private consumption.

liquidity constrained. Hence, we want to study how the interaction between the complementarity/substitutability issue and the presence of non-optimizing households influences the response of private consumption to a government spending shock.

The outline of the paper is as follows. In Section 2 we describe the main features of the model. In section 3, we describe how the model is solved. In section 4, we outline the main implications of the model. In section 5, we provide some preliminary estimates of the model. A conclusion will end the paper.

2 The model

Our model is an extended version of the N-DSGE models developed by Smets and Wouters (2003), henceforth SW (2003), and Schmitt-Grohé and Uribe (2004), henceforth SU (2004). The model features four types of economic agents: households, firms, a monetary authority and a fiscal authority. In our extended version, we follow GLV (2007), in allowing for two different types of households: optimizing households, who can trade in asset markets and thus are able to smooth consumption, and liquidity constrained households, who cannot participate in the asset market and therefore just consume their after-tax disposable income. In addition the model features two sources of nominal frictions and five real rigidities. The nominal frictions include price and wage stickiness à la Calvo (1983) and Yun (1996) with indexation to past inflation. The real rigidities originate from internal habit formation in consumption, monopolistic competition in factor and products markets, investment adjustment costs, and variable costs of adjusting capacity utilization. Finally, we allow government spending to enter the utility function of both types of households. Specifically, we combine private and public consumption in the utility function using a constant-elasticity-of-substitution (CES) aggregator, implying that the two goods could be either substitutes or complements (with different degree of imperfection). In the following, we briefly outline the behaviors of the different types of agents.

2.1 Households

The economy is populated by a continuum of households indexed by $i \in [0; 1]$. A share ω of this continuum of households, referred to as rule of thumb consumers, henceforth ROT consumers, do not trade in assets and simply consume their after-tax disposable income. The remaining share $1-\omega$, called Ricardian households, have access to financial markets, where they buy and sell government bonds, and accumulate physical capital, the services of which they rent out to firms.

It is assumed that the households supply differentiated labor services to a continuum of unions within the household sector, indexed over the same range of the households, $i \in [0;1]$, which act as wage setters in monopolistically competitive markets. The unions pool the wage income of all households and then distribute the aggregate wage income in equal proportions amongst the latter. The households, in turn, are assumed to supply sufficient labor services to satisfy labor demand.

2.1.1 Ricardian Households

Let $N_t^o$and $\tilde{C}_t^o$ represent the labor supply expressed in hours worked and effective consumption for optimizing households. Effective consumption is assumed to be a CES aggregator of private consumption C_t^o and government consumption G_t:

$$\tilde{C}_t^o = \left[\phi \left(C_t^o\right)^{\frac{v-1}{v}} + (1-\phi)\, G_t^{\frac{v-1}{v}}\right]^{\frac{v}{v-1}} \tag{1}$$

where ϕ is the weight of private consumption in the effective consumption aggregator, and $\nu \in (0;\infty)$ is the elasticity of substitution between private consumption and government spending[3].

Each Ricardian households maximizes its lifetime utility function by choosing consumption, next period's financial wealth in form of one period government bonds, B_{t+1}^o, next period's physical capital stock, K_{t+1}^o, and the intensity with which the installed capital stock is utilized,u_t, given the following lifetime utility function:

$$E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[\frac{\left(\tilde{C}_{t+k}^o - \gamma \tilde{C}_{t+k-1}^o\right)^{1-\sigma_c}}{1-\sigma_c} - \frac{\psi}{1+\sigma_n} \left(N_{t+k}^o\right)^{1+\sigma_n} \right] \right\} \tag{2}$$

where σ_c denotes the degree of relative risk aversion, σ_n is the Frisch elasticity, the elasticity of wages with respect to hours worked (i.e. the inverse of the elasticity of work effort with respect to wage). The parameter $\gamma \in (0;1)$ measures the degree of habit

[3] When $\nu = 0$, we have a "Leontief" aggregator, i.e. C_t^o and G_t become perfect complements. When $\nu = 1$, we have a "Cobb-Douglas" aggregator of the form $\tilde{C}_t^o = C_t^{o\phi} G_t^{(1-\phi)}$. As $\nu \to \infty$, we will have a linear aggregator of the form $\tilde{C}_t^o = \phi C_t^o + (1-\phi)\, G_t$, the two goods are perfect substitutes. For intermediate values of ν, as we will see in Section 4.1, even other model parameters contribute to the determination of the degree of substitutability/complementarity between the two goods.

formation in consumption. $\beta \in (0;1)$ is the subjective discount factor. ψ is a positive parameter set such that hours worked are 0.31 in steady state.

The Ricardian household faces the following budget constraint (expressed in real terms):

$$C_t^o+I_t^o+\varphi\left(I_t^o,K_t^o\right)+B_{t+1}^o = r_t^b B_t^o+(1-\tau^w)w_t N_t^o+(1-\tau^k)\left[r_t^k u_t - a\left(u_t\right)\right]K_t^o+D_t^o-T_t^o \quad (3)$$

where the terms on the left-hand side show how the household uses its resources, while the terms on the right-hand side indicate the resources the household has at its disposal. $\varphi\left(I_t^o,K_t^o\right)=\frac{\kappa}{2}\left(\frac{I_t^o}{K_t^o}-\delta\right)^2 K_t^o$ represents the adjustment cost in investment, with κ being a positive parameter. r_t^b is the risk-less return on government bonds. τ^w and τ^k are the pay-roll tax levied on the household's wage income, $w_t N_t^o$, and, the tax rate at which capital is taxed, respectively. $a\left(u_t\right)$ represents the cost of using capital at intensity u_t[4]. D_t^o are the dividends paid by household-owned firms.

The capital stock evolves according to the following capital accumulation equation:

$$K_{t+1}^o = (1-\delta)\,K_t^o + I_t^o \quad (4)$$

where δ is the depreciation rate. Letting λ_t^o the lagrange multipliers associated with the budget constrained 3. First order conditions associated with the optimal choice of $C_t^o, B_{t+1}^o, K_{t+1}^o, N_t^o, u_t$ are:

$$\lambda_t^o = \phi^o\left(\frac{\tilde{C}_t^o}{C_t^o}\right)^{\frac{1}{v}}\left[\left(\tilde{C}_t^o-\gamma\tilde{C}_{t-1}^o\right)^{-\sigma_c^o}-\beta\gamma\left(\tilde{C}_{t+1}^o-\gamma\tilde{C}_t^o\right)^{-\sigma_c^o}\right] \quad (5)$$

$$\beta E_t\left\{\frac{\lambda_{t+1}^o}{\lambda_t^o}r_t^b\right\}=1 \quad (6)$$

$$\lambda_t^o = \frac{N_t^{o\sigma_n}}{(1-\tau)w_t} \quad (7)$$

[4] The adopted functional form for the capacity utilization cost is $a(u_t)=\gamma_1(u-1)+\frac{\gamma_2}{2}(u-1)^2$. The latter was taken by SU(2004).

$$\lambda_t^o = \frac{\beta\lambda_{t+1}^o \left\{1 + (1-\tau_{t+1})\, r_{t+1}^k u_{t+1} - a(u_t) - \delta + \kappa\left(\frac{I_{t+1}^o}{K_{t+1}^o} - \delta\right)\frac{K_{t+2}^o}{K_{t+1}^o} - \frac{\kappa}{2}\left(\frac{I_{t+1}^o}{K_{t+1}^o} - \delta\right)^2\right\}}{\left[1 + \kappa\left(\frac{I_t^o}{K_t^o} - \delta\right)\right]} \tag{8}$$

$$r_t^k = a'(u_t) \tag{9}$$

2.1.2 Non-Ricardian Households

Let $N_t^r$and $\tilde{C}_t^r$ represent the labor supply expressed in hours worked and effective consumption for ROT consumers. As before, effective consumption is assumed to be a CES aggregator of private consumption C_t^r and government consumption G_t:

$$\tilde{C}_t^r = \left[\phi\left(C_t^r\right)^{\frac{v-1}{v}} + (1-\phi)\, G_t^{\frac{v-1}{v}}\right]^{\frac{v}{v-1}} \tag{10}$$

For unmodelled reasons (e.g. myopia, limited access to financial markets, binding borrowing constraint), ROT consumers solve a static problem, i.e., they maximize, at each period, their utility function, $U\left(\tilde{C}_t^r, N_t^r\right) = \left[\frac{\left(\tilde{C}_t^r\right)^{1-\sigma_c}}{1-\sigma_c} - \frac{\psi}{1+\sigma_n}\left(N_t^r\right)^{1+\sigma_n}\right]$, subject to the zero-savings constraint, $C_t^r = (1-\tau_t^w)\, w_t N_t^r - T_t^r$. The associated intra-temporal condition is:

$$\frac{N_t^{r\sigma_n}}{(1-\tau)w_t} = \phi^r\left(\frac{\tilde{C}_t^r}{C_t^r}\right)^{\frac{1}{v}}\left(\tilde{C}_t^r\right)^{-\sigma_c^r} \tag{11}$$

Like for Ricardian households, it is assumed that ROT consumers take the pooled wage income as given and supply sufficient labor services to satisfy labor demand.

2.1.3 Wage Setting

There is a continuum of monopolistically competitive unions within the household sector indexed by $i \in [0;1]$, which act as wage setters for the differentiated labor services supplied by the two types of households. Unions takes the aggregate wage rate, w_t, and aggregate labor demand, N_t, as given.

Following Calvo (1983), unions receive permissions to optimally reset their wage rate in a given period t with probability $(1-\xi_w)$. All unions that receive permission to reset their wage rate choose the same wage rate $\tilde{w}_t$. Those unions that do not receive permission are allowed to adjust their wage rate at least partially according to the following scheme:

$$w_{i,t} = w_{i,t-1} \left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t} \right) \tag{12}$$

where $\tilde{\chi} \in (0; 1)$ measures the degree of indexation to past inflation.

Each union i that receives permission to optimally reset its wage rate in period t, is assumed to maximize household lifetime utility, as represented by equation 2 , taking into account the wage-indexation scheme and the demand for labor services, the latter being given by:

$$N_{i,t} = \left(\frac{w_{i,t}}{w_t} \right)^{-\tilde{\eta}} N_t \tag{13}$$

where $\tilde{\eta} > 1$ is the wage-elasticity of demand for a specific labor variety.

Thus, we obtain the following first-order condition for the union's optimal wage setting decision in period t:

$$0 = E_t \sum_{s=0}^{\infty} (\xi_w \beta) \lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}} \right) \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}} \right)^{\tilde{\eta}} \left[\frac{\tilde{\eta} - 1}{\tilde{\eta}} \frac{\left(1 - \tau_{t+s}^w\right) \tilde{w}_t}{\prod\limits_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}} \right)} - \frac{w_{t+s} \left(1 - \tau_{t+s}^w\right)}{\mu_{t+s}^w} \right] \tag{14}$$

This expression states that in the labor markets in which the wage rate is reoptimized in period t, the real wage is set so as to equate the union's future expected average marginal revenue to the average marginal cost of supplying labor. The union's marginal revenue s period after its last wage reoptimization is given by $\frac{\tilde{\eta}-1}{\tilde{\eta}} \tilde{w}_t \prod\limits_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\tilde{\chi}}}{\pi_{t+k}} \right)$. Here, $\frac{\tilde{\eta}-1}{\tilde{\eta}}$ represents the mark-up of wages over marginal cost of labor that would prevail in absence of wage stickiness. The third factor in the expression for marginal revenue reflects the fact that as time goes by without a chance to reoptimize, the real wage declines as the price level increases when wages are imperfectly indexed to past inflation. In turn, the marginal cost of supplying labor is given by the marginal rate of substitution between consumption and leisure, or $\frac{-U_{N,t+s}}{\lambda_{t+s}} = \frac{w_{t+s}\left(1-\tau_{t+s}^w\right)}{\mu_{t+s}^w}$. The variable μ_{t+s}^w is a wedge between the disutility of labor and the average after-tax real wage prevailing in the economy. Thus, μ_{t+s}^w can be interpreted as the average mark-up that unions impose on the labor market[5]. λ_{t+s} can be calculated as a weighted average between the marginal utility of consumption of ROT consumers and the one of the Ricardians.

Aggregate labor demand, N_t, and aggregate nominal wage,W_t, are determined by the following Dixit-Stiglitz indices:

$$N_t = \left[\int_0^1 (N_{i,t})^{1-\frac{1}{\tilde{\eta}}} \right]^{\frac{1}{1-\frac{1}{\tilde{\eta}}}} \tag{15}$$

$$W_t = \left[\int_0^1 (W_{i,t})^{1-\tilde{\eta}} \right]^{\frac{1}{1-\tilde{\eta}}} \tag{16}$$

The equilibrium real wage reflects the fact that, at each time t, some unions are allowed to reoptimize their wages, some are not. For this reason the aggregate real wage must satisfy the following rule:

$$w_t^{1-\tilde{\eta}} = \xi_w w_t^{1-\tilde{\eta}} \left(\frac{\pi_{t-1}^{\chi}}{\pi_t} \right)^{1-\tilde{\eta}} + (1-\xi_w)\, \tilde{w}_t^{1-\tilde{\eta}} \tag{17}$$

2.1.4 Aggregation

Aggregate consumption and hours worked are given by:

$$C_t = \omega C_t^r + (1-\omega)\, C_t^o \tag{18}$$

$$N_t = \omega N_t^r + (1-\omega)\, N_t^o \tag{19}$$

with the labor-market equilibrium being characterized by $N_t = N_t^r = N_t^o$[6].

Only Ricardians consumers can holds financial assets, accumulate capital, and, receive dividends from firms, so that:

[5] Following SU (2004), we write the wage setting equation in recursive form, i.e., $f_t^1 = (1-\tau_t)\left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\lambda_t \left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} N_t + \xi_w \beta E_t \left(\frac{\pi_t^{\tilde{\chi}}}{\pi_{t+1}}\right)^{1-\tilde{\eta}} \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}}\right)^{1-\tilde{\eta}} f_{t+1}^1$, and, $f_t^2 = (1-\tau_t)\frac{\lambda_t}{\mu_t^w} w_t \left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} N_t + \xi_w \beta E_t \left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{-\eta} \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}}\right)^{-\eta} f_{t+1}^2$, and, finally setting $f_t^1 = f_t^2$.

[6] This is a consequence of the asumptions that unions pool the wage income of both group of households. To ensure that pooling finally results in the same wage income, hours worked in both groups need to be equal in equilibrium.

$$B_{t+1} = (1-\omega) B_{t+1}^{o} \tag{20}$$

$$K_{t+1} = (1-\omega) K_{t+1}^{o} \tag{21}$$

$$I_{t+1} = (1-\omega) I_{t+1}^{o} \tag{22}$$

$$D_{t} = (1-\omega) D_{t+1}^{o} \tag{23}$$

Finally, aggregate lump-sum taxes are given by:

$$T_t = \omega T_t^r + (1-\omega) T_t^o \tag{24}$$

2.2 Firms

We assume that there is a continuum of monopolistically competitive firms indexed by $j \in [0,1]$ each of which produces a single variety of final goods ($Y_{j,t}$), using as inputs capital services ($K_{j,t}$) and labor services ($N_{j,t}$). The production function is given by

$$Y_{j,t} = A_t K_{j,t}^{\alpha} N_{j,t}^{1-\alpha} - \Phi \tag{25}$$

where A_t is a technology shifter common to all firms. $a_t = \log A_t$ follows an AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_t^a$, where ε_t^a is an IID-normal technology shock. The parameter Φ represents fixed cost of production.

The existence of an economy-wide competitive factor market implies that all firms will pay the same rental rate r_t^k and the same nominal wage W_t. Hence, cost minimization subject to the production technology 25 yields first order conditions for inputs which can be expressed as relative factor demands[7] and real marginal cost:

$$\frac{K_t}{N_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k} \tag{26}$$

$$MC_t = \frac{\left(r_t^k\right)^{\alpha} w_t^{1-\alpha}}{A_t \alpha^{\alpha} (1-\alpha)^{1-\alpha}} \tag{27}$$

[7] It is worth noting that, since there is imperfect competition in the goods market, the first order conditions with respect to w_t and r_t^k are affected by the level of the mark-up in the products market, that is, $\frac{\eta}{\eta-1}$.

2.2.1 Price setting

Following Calvo (1983), at each period t a firm receives permission to optimally reset its price with probability $1-\xi_p$. The firms which receive permission to reset their prices chose the same price $\tilde{P}_t$. Those firms, which cannot optimally reset their price, index their price to past price inflation according to the following rule:

$$P_{j,t} = P_{j,t-1}\pi_{t-1}^{\chi} \tag{28}$$

where $\chi \in (0;1)$ measures the degree of indexation to past inflation.

All firms that can reset their price in period t, maximize the expected present discounted value of profits

$$E_t\left\{\sum_{s=0}^{\infty}\xi_p^k\beta_{t,t+s}D_{j,t+s}\right\} \tag{29}$$

subject to the demand for its output $Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\eta}Y_t$ and the price indexation scheme 28, where $\beta_{t,t+s}$ is the stochastic discount factor of the (ricardian) households owning the firm and:

$$D_{j,t} = P_{j,t}Y_{j,t} - MC_t\left(Y_{j,t}+\Phi\right) \tag{30}$$

are period t nominal profits distributed as dividends to (ricardian) households.

The first order condition with respect to $\tilde{P}_t$ is

$$E_t\left\{\sum_{s=0}^{\infty}\xi_p^s\beta_{t,t+s}P_{t+s}\left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta}\prod_{k=1}^{s}\left(\frac{\pi_{t+k+1}^{\chi}}{\pi_{t+k}}\right)y_{t+s}\left[\frac{\eta-1}{\eta}\left(\frac{\tilde{P}_t}{P_t}\right)\prod_{k=1}^{s}\left(\frac{\pi_{t+k+1}^{\chi}}{\pi_{t+k}}-MC_{j,t+s}\right)\right]\right\}=0 \tag{31}$$

where $\eta > 1$ is the price-elasticity of demand for a specific good variety.

According to this expression, optimizing firms set the nominal prices so as to equate average future expected marginal revenues to average future expected marginal costs. Under flexible prices ($\xi_p = 0$), the above optimality condition reduces to a static relation equating marginal costs to marginal revenues period by period[8].

Like the aggregate real wage, the equilibrium price reflects the fact that, at each time t, some firms are allowed to reset their prices, some are not. For this reason the aggregate

[8] Following SU (2004), we wish to write the price setting equation in recursive form, i.e., $x_t^1 = y_t MC\tilde{p}_t^{-\eta} + \alpha E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{-\eta}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta-1}x_{t+1}^1$, and, $x_t^2 = y_{t\tilde{p}t}^{-\eta} + \alpha\beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{1-\eta}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta}x_{t+1}^2$, and, finally setting $\eta x_t^1 = (\eta-1)\,x_t^2$ (where $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$).

price index must satisfy the following rule:

$$P_t^{1-\tilde{\eta}} = \xi_w \left(P_{t-1}\pi_{t-1}^{\chi}\right)^{1-\tilde{\eta}} + (1-\xi_w)\, P_t^{1-\tilde{\eta}9} \tag{32}$$

2.3 Fiscal and Monetary authorities

The fiscal authority purchases final good, G_t, repays the outstanding level of debt, issues bonds, B_{t+1}, and raises taxes, both lump-sum, and, distortionary.

$$r_t^b B_t + G_t = B_{t+1} + T_t + \tau_t^w [w_t N_t] + \tau_t^k \left[r_t^k u_t - a(u_t)\right] K_t - \tau_t \delta K_t \tag{33}$$

Then, following GLV(2007), we assume a fiscal policy rule of the following form:

$$T_t = \phi_b B_t + \phi_g G_t \tag{34}$$

where ϕ_b and ϕ_g are positive constants.

Here, government spending is assumed to evolve exogenously following an AR(1):

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{35}$$

where $g_t = \log(G_t)$, and, ε_t^g is an IID-normal government shock.

Finally, the monetary authority follows the following simple rule:

$$\frac{i_t}{i} = \left(\frac{i_{t-1}}{i}\right)^{\phi r} \left(\frac{\pi_t}{\pi}\right)^{\phi_\pi (1-\phi r)} \varepsilon_t^m \tag{36}$$

where i_t and i are the nominal interest rate and its steady state value, respectively. π_t and π are the inflation rate and its steady state value, respectively. $\phi_r \in (0;1)$ is the parameter stating the share of the adjustment in the nominal interest rate which depends on its past values. ϕ_π is a parameter greater than one.

2.4 Market clearing

The labor market is in equilibrium when the labor demand of firms equals the differentiated labor services supplied by households at the wage rates set by unions, that is:

[9] To express the aggregate price index in terms of π_t and $\tilde{p}_t$,we can divide the equation by $\tilde{P}_t^{1-\eta}$. So, we can obtain $1 = \xi_p \left(\frac{\pi_{t-1}^{\chi}}{\pi_t}\right)^{1-\eta} + (1-\xi_p)\, \tilde{p}_t^{1-\eta}$.

$$N_t = \int_0^1 (N_{i,t}) d_i$$

The market for capital is in equilibrium when the demand for capital services by the firms equals the capital services supplied by households at the market rental rate, that is:

$$K_t = \int_0^1 (K_{i,t}) d_i \tag{37}$$

The market for government bonds is in equilibrium when the outstanding government bonds are held by households at the market interest rate. Finally, the good market is in equilibrium when the supply by firms equals the demand by households and government:

$$Y_t = C_t + I_t + G_t + a\left(u_t\right) K_t \tag{38}$$

3 Solution of the model

To solve the model the Blanchard-Khan algorithm has been used. First order conditions have been log-linearized and then a mapping between the state variables of the model and the control variables has been determined.

To check if our model gives reasonable results, we compare our impulse responses with the ones obtained by a standard N-DSGE, which omits both the presence of ROT consumers and the issue of substitutability/complementarity (e.g. SU(2004)). We use a plausible reparametrization of the model (see Table 1), in particular, we shut down the channel of ROT consumers and the one of substitutability/complementarity between private and government consumption. The impulse response functions are in Figure 1,2 and 3 and fairly replicate the ones of SU(2004).

[insert Figures 1,2,3 and Table 1 here]

4 The model's implications

In this section we want to analyze the two main important issues of our model: the complementarity between private and government consumption and the role of ROT consumers. In particular, we want to stress how these two issues affect the response of private consumption to a government spending shock.

4.1 The complementarity between C and G

Now, we focus our attention on the substitutability/complementarity issue, abstracting from the presence of ROT consumers in the model[10].

For a given level of consumption, we represent the effect of a change in government spending on the marginal utility of consumption (in a log-linearized form):

$$\frac{\partial \lambda_t}{\partial g_t} = (1-\phi)\left(\frac{g}{\tilde{c}}\right)^{\frac{v-1}{v}}\left[\frac{1}{v} - \sigma_c + \sigma_c \gamma \frac{(1-\beta) + (1-\gamma)\,\beta^2\gamma^2\rho_g}{(1-\beta\gamma)}\right] \tag{39}$$

When $\frac{\partial \lambda_t}{\partial g_t}$ is greater than 0, private and governmnet consumption are defined to be complements, whereas, when $\frac{\partial \lambda_t}{\partial g_t}$ is less than 0, private and governmnet consumption are defined to be substitutes. When $\frac{\partial \lambda_t}{\partial g_t}$ is equal to 0, private and governmnet consumption are not related via the channel of the preferences.

It is easy to see that this model completely nests a standard N-DSGE model, where there is no habit persistence ($\gamma = 0$) and the government consumption is assumed to be pure waste ($\phi = 1$). In this case, the right-hand side of equation 39 collapse to zero, so that government spending affects consumption only through the wealth channel. Figure 4 describes the impulse response to a 1 percent government spending shock, generated by this version of the model[11]. It shows that a positive government spending shock decreases consumption, investment and real wage and increases hours worked and output. Intuitively, an increase in government spending means a lower permanent income for the agent. Thus, provided that consumption and leisure are normal goods, the household, who is poorer, decreases both. Labor supply increases for any given level of real wage. This leads to an increase in output and a lower real wage. Owing to consumption smoothing, output decreases less, in absolute value, than output. Thus, the representative agent must dissave and, as a consequence, investment decreases.

[insert Figure 4 here]

Now, we consider a version of the model where we shut-off the channel of habit formation ($\gamma = 0$), but where effective consumption depends on government spending ($\phi < 1$).

[10] It is worth noting that the following exercise fairly replicates the one performed in Boukez Rebei (2003), although the latter is carried-out in a real business cycle framework with a different utility function for households.

[11] The remaining parameters follow the parametrization of Table 1.

In this case, the derivative $\frac{\partial \lambda_t}{\partial g_t}$ has the same sign as the term $\frac{1}{v} - \sigma_c$. When the elasticity of substitution is lower than $\frac{1}{v}$, government spending raises the marginal utility of consumption, ceteris paribus. That is, the complementarity dampens the negative wealth effect, and, for sufficiently low values of ν, the complementarity effect may actually offset the negative wealth effect, causing consumption to increase after a government spending shock. Figure 5 depicts the effect of complementarity and substitutability on the economy's response to a government spending shock[12]. We consider three different scenarios by setting the elasticity of substitution, ν, to 1.5, 0.45, and 0.25, respectively. Figure 5 shows that, when ν is equal to 1.5 a government spending shock produces a larger crowding-out effect on consumption than that predicted by the model without government in the utility function. This is because government consumption lowers the marginal utility of consumption, and, this fact reinforces the negative wealth effect. When ν is equal to 0.45, private and public spending become complements as $\frac{\partial \lambda_t}{\partial g_t}$ is greater than 0, ceteris paribus. Figure 5 shows that the complementarity effect mitigates the negative wealth effect, but the overall effect of the shock on consumption is still negative. Hence, the degree of complementarity is feeble in this case. Then, we consider the case when $\nu = 0.25$. Under this parametrization, the complementarity effect is strong enough to dominate the wealth effect, so that private consumption is crowded-in by government spending. It is worth noting that the smaller is ν, the higher is the degree of complementarity, the higher is the increase of the marginal utility of consumption due to a government spending shock, the higher is the incentive to work more. That is, an higher degree of complementarity makes labor supply to react stronger to a government spending shock. Moreover, the increase in labor supply should amplify the decline in the real wage.

[insert Figure 5 here]

Finally, we consider the model with habit formation in effective consumption ($0 < \gamma < 1$). The sign of the derivative $\frac{\partial \lambda_t}{\partial g_t}$ is given by the right-hand side of equation 39. Having $\sigma_c = 2$, it is straightforward to show that the derivative is decreasing in γ. That is, habit formation dampens the complementarity effect. Indeed, habit-forming households smooth both the absolute value of consumption and its rate of change. As a result, the consumption response to shock is smaller on impact and more gradual under habit formation than under

[12] As stated earlier, for government spending to play a role in the utility function the weight of private consumption in the CES aggregator must be strictly less than 1. Following prevoius papers, as Bouakez and Rebei (2003), and Kwan (2006), we set $\phi = 0.8$.

time-separable preferences. Figure 6 show the impulse responses of the model variables to a government spending shock, for three different values of habit formation (with ν fixed at 0.25). It is visible that, the higher is the habit persistence, the less is the response on impact of consumption.

[insert Figure 6 here]

4.2 The role of ROT consumers

Now, we focus our attention on the role of ROT consumers in the model, shutting down the channel of the complementarity/substitutability between private and government consumption.

As it clears from several papers (e.g. GLV(2007), and, Conen and Straub (2005)) the presence of ROT consumers does not imply, per sè, a positive response of private consumption to a government spending shock. To obtain this positive response, we need several other ingredients which need to interact with the presence ROT consumers. In the following, we list some of these features. First of all, imperfect competition. Imperfect competition generates an aggregate demand externality according to which an increase in output leads to a rise in profits and income. Higher profits and income in turn dampens the negative wealth effect. Secondly, the behavior of ROT consumers. From the budget constraint of the ROT consumers, we see that, given the extent of taxation, they can increase their consumption if their labor income ($w_t N_t$) increases. After a government spending shock, ROT consumers may increase hours worked but they may in fact face a lower real wage. For reasonable values of the labor supply elasticity (GLV(2007) set it at 0.2) consumption of ROT consumers can go up. Consumption of optimizing households still goes down driven by the negative wealth effect, however, if the share of ROT consumers is sufficiently large in the economy, the aggregate consumption can go up. The behavior of the real wage is crucial in this case. If the real wage increases after a government spending shock, the chance that consumption of ROT consumers goes up is higher. Two issues can help reproduce this increase in the real wage: sticky prices and an imperfectly competitive labor market. The presence of sticky prices raises the possibility that labor demand reacts stronger than labour supply, with real wages increases alongside labor supply. Such an increase is possible as long as monetary policy's reaction to future output gap is not too strong (see Linnemann and Schabert (2003) for further details). The non-competive labor market implies that there is a wedge between the marginal product of labor and the real wage. This wedge, in the line of our model, can be interpreted as an average between the mark-up that unions impose on the labor market and the mark-up that firms charge on the products market. If this mark-up is counter-cyclical, then, even if the marginal product of labor goes down (because of the increase in the labor supply) the real wage still increases. Finally, some words on the role of wage rigidity. Completely fixed wages do not help consumption to go up after a government spending shock. Indeed, for very high level of wage stickiness, real wage and so consumption cannot increase after a government shock. This is because the latter assumption introduces inertia in the wage adjustment process, therefore dampening the initial sharp increase in the real wage.

Figure 7 compares two sets of impulse responses. One comes from a standard N-DSGE model (calibrated using Table 1), the other from a model with ROT consumers (calibrated using Table 2). The model with ROT consumers produces impulse responses similar to the ones of GLV(2007). Importantly, in the "ROT" framework, unlike the standard N-DSGE case, both real wage and consumption actually go up.

[insert Figure 7, and Table 2 here]

4.3 Interacting "ROT" with complementarity

One of the necessary condition to reproduce a positive response of consumption in a model with ROT consumers is to set the share of non-optimizing households to be at 0.5, at minimum. However, recent estimates of this share (see Conen and Straub (2005) and Rabanal and Lopez-Salido (2006)) range from 0.1 to 0.25. With these values of ω is difficult to reproduce a positive response of consumption. Indeed, setting $\omega = 0.2$ in our model, the consumption sharply drops after a government spending shock (see Figure 8).

[insert Figure 8 here]

One way to recreate a positive response of consumption, is to interact the presence of ROT consumers with a complementarity effect. We set the share of ROT consumers equal to 0.2 and ν equal to 0.25. In Figure 9 the positive response of consumption to a government spending shock pops up again.

[insert Figure 9 here]

5 Bayesian estimation of the model (...TO BE COMPLETED)

As a starting point of our empirical analysis, we estimate a reduced N-DSGE model using US data. In particular, we rule out from the model described above two ingridients: capital and ROT consumers. We end up having a DSGE model with nominal rigidities, price and wage stickiness, and with government spending in the utility function. The model presents just lump-sum taxation. We consider three different shocks, that is a monetary policy shock, a government spending shock, and a technology shock, modelled as above. Given that this model omits some important features, like the dynamic of investment or the presence of non-optimizing households, the following estimation results needs to be taken with caution.

We estimate this N-DSGE model using bayesian inference methods. In particular, we specify a prior distribution for each parameter to be estimated relying on information from earlier studies. Using priors helps in reducing the numerical difficulties associated with a highly non-linear estimation problem such as ours.

5.1 Methodology

Let $P(\theta)$ be the prior distribution of the parameter vector $\theta \in \Theta$, and let $L(Y_T/\theta)$ be the likelihood function for the observed data $Y_T = \{y_t\}_{t=1}^{T}$, conditional on the parameter vector θ. The likelihood is computed starting from the log-linear state space form representation of the model by means of the Kalman filter. The posterior distribution of the parameter vector θ is then obtained combining the likelihood function for Y_T with the prior distribution of θ, that is:

$$P(\theta/Y_T) \propto L(Y_T/\theta)P(\theta) \tag{40}$$

where "$\propto$ " indicates proportionality.

In order to obtain numerically a sequence from this unknown posterior distribution, we follow An and Schorfeide (2006) and employ the Metropolis-Hastings algorithm, M-H henceforth (see Appendix A for further details).

5.2 Data and prior distributions

We use three variables to estimate the model, that is consumption, output and real wage. Consumption is measured by the private spending on non-durables goods and services, deflated by the implicit GDP deflator. Output is measured by real GDP. The real wage is measured by compensation per hour in the nonfarm business sector, deflated by the implicit GDP deflator. All variables, except the real wage, are converted to per-capita terms by dividing them by the civilian population, age 16 and over. The variables are seasonally adjusted, then are logged, and, finally detrended using the Hodrick-Prescott filter calibrated for quarterly data. Our sample goes from 1955:4 to 2006:3. The raw data are taken by FRED.

Regarding the choice of the prior distributions for the parameters of the model, we follow SW(2007). In particular see Table 3. However, following the existing literature we fix several parameters throughout the estimation. This includes setting β to 0.99, η to 4, and $\tilde{\eta}$ to 21. Again G/Y is equal to 0.2 in steady state, while, N is 0.31 in steady state.

Regarding the parameter which enter in the CES aggregator, i.e. ν and ϕ, the following is worth noting. Boukez and Rebei (2003) and Kwan (2006) find that the parameter ϕ is poorly identifiable, so that they fix it to be 0.8 and then perform some robustness tests around that value. We follow them in this procedure. As far as concern the parameter ν, we reparametrize its parameter space from a bounded parameter space ($\nu \in (0; \infty)$) to an unbounded one ($\nu\_b \in (-\infty; \infty)$), where $\nu = \exp(\nu\_b)$. Given the prior means of the other parameters which affect the derivative $\frac{\partial \lambda_t}{\partial g_t}$, the value of ν_b which makes this derivative equal to zero is around $log(0.9)$. So, to be agnostic regarding the issue of complementarity, we perform the following two exercises. First, we assume that the prior of ν_b follows a normal distribution with mean $log(0.5)$ and standard deviation equal to 1. In this case our prior beliefs are that government consumption and private consumption are complements, on average. Second, we give the same shape for the prior of ν_b as before, but with a prior mean equal to $log(1.8)$. In this latter case, we are clearly assuming that the two goods are substitute, on average.

5.3 Estimation Results

The estimates are obtained from the M-H algorithm with 40.000 iterations, divided in 4 blocks, that is 4 chains. The number of iterations seems to be sufficient to achieve convergence since the within-sequence variance in the chain and the between-sequence across chains variance happen to be quite the same as the number of iterations increase (see Figures 10,11,12,13,14). For more details see Appendix A. The first 40% of the draws are discarded since they can be still affected by the choice of the initial value. The acceptation rate for the following draws is around 35%.

[insert Figure 10,11,12,13,14 here]

Table 3 reports the estimation results and Figure 15,16 depict the priors and posterior distribution for each estimated parameter. These results are obtained by fixing the prior mean of ν_b equal to $log(0.5)$, i.e. imposing complementarity on average. Very similar results boil down if we set the prior mean of ν_b equal to $log(1.8)$, i.e. imposing substitutability on average. For these reason, the second set of results are not reported here. Our main focus is on the estimate of ν_b. The estimated posterior mean of ν_b implies a value of ν around 3. Moreover, looking at the shape of the posterior distribution of ν_b in Figure 16, it seems that the likelihood is quite informative. The sign of $\frac{\partial \lambda_t}{\partial g_t}$ conditional on the estimated parameters is unambiguously negative, that is government consumption and private consumption are estimated to be imperfect substitutes. This finding contrasts with Bouakez and Rebei (2003), who find the two goods being imperfect complements. Regarding the other parameters, the estimation results say that there is a quite large degree of habit persistence, and, a higher degree of price stickiness with respect to the one of wage stickiness. Both a moderate degree of price and wage indexation emerge. The parameters of the Taylor rule says that 40% of the adjustment of the today interest rate takes place via the response to past values of the interest rate, the remaining 60% of the adjustment takes place via a response to inflation. The estimated autoregressive coefficients of the exogenous processes, driving government spending and technology, are quite large.

[insert Figures 15,16 and Table 3 here]

Figures 17,18,19 depicts the estimated impulse response to a government spending shock. The 90% posterior intervals are reported. The main result is the response of private consumption. Here, two effects operate together, the negative wealth effect and the substitutability between private and government consumption. Both contribute to decrease consumption. This finding seems to contrast with the results of Blanchard and Perotti (2002), Mountford and Uligh (2005), however, they seem to replicate the ones of Ramey and Shapiro (1998) and Conen and Straub (2005). Another interesting results is that the real wage increases on impact. This finding supports the theoretical finding of Linnemann and Schabert (2003) described above. The behavior of the other variables is standard. The output increase is driven by the increase in the labor supply. The marginal cost increases due to the increase in the real wage. In turn, the price level is driven by the marginal cost, so inflation increases. The model also ensures that the mark-up of the labor market and the goods market are counter-cyclical, indeed, they decrease. The nominal interest rate tracks the path of inflation since monetary authority tries to dampen the dynamic of inflation.

[insert Figures 17,18 here]

6 Conclusions and policy implications (...TO BE COMPLETED)

The paper seems to have two set of conclusions, the one is theoretical, the other is empirical.

Starting from theory. First, we say that our model is able to replicate, given appropriate reparametrizations, some of the main important findings already presented in the literature. In particular, we can replicate the results of both a standard N-DSGE model, and, a N-DSGE model which embeds the presence of ROT consumers. Second, we describe how the issue of complementarity works in this model through the channel of the marginal utility of consumption. In particular, we show how this issue play a crucial role for making consumption to increase after a government spending shock, whenever the share of ROT consumers is relatively low in the economy.

From an empirical point of view, we estimate a simplified version of our model. The elasticity of substitution between government spending and private consumption is estimated to be around 3. Given this value, the derivative of the marginal utility of consumption with respect to government consumption is unambiguously negative, making the two goods being imperfect substitute. Indeed, the estimated impulse response of consumption with respect to a government spending shock is negative, because both the substitutability effect and the negative wealth effect work in the same direction. The estimated model produces a positive response of the real wage to a government spending shock. As said before, given the small scale of the estimated model, our estimation results has to be taken with caution.

If we take it for granted, one important policy implication emerges from these estimates. Government consumption has a negative multiplier, such that, a tight fiscal policy can be expansionary in this framework.

However, other crucial issues has to be still understood in this literature. Two of these open questions are the following. First, what are the items of government consumption which drives this substitutability? This can be answering by exploring disaggregated data both on government and private consumption. Second, what are the economic intuitions for which an item of government consumption should substitute/complement one of private consumption? A model which investigates the question related to the efficiency/inefficiency of the public goods should shed light on this issue.

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8 Appendix A

The main building blocks of the Bayesian technique used here are the following. First, the linear rational expectations system is solved to obtain the Kalman filter, that is a transition equation and a measurement equation. If a unique stable solution exists, then the Kalman filter is used to evaluate the likelihood function associated to the state space representation of the system. Since the priors are generated from well-known densities, the computation of $P(\theta)$ is straightforward.

The M-H algorithm is comprised of universal algorithms that generate Markov chains with stationary distributions that corresponds to the posterior distribution of interest. In particular we use a particular algorithm called Random-Walk Metropolis (RWM). The RWM works in this way.

1. Use a numerical optimization routine to maximize $P(\theta/Y_T)$. Denote with θ_M the posterior mode

2. Let $\sum_M$ the inverse of the Hessian computed at θ_M

3. Draw $\theta^{(0)}$ from $N(\theta_M, c_o^2 \sum_M)^{13}$

4. For $s = 1, ..., n_{sim}$, draw $\theta^{(s)}$ from the proposal distribution $N(\theta_M, c_o^2 \sum_M)$. The jump from $\theta^{(s-1)}$ is accepted with probability $min\left\{1, r(\theta^{(s-1)}, \theta^{(s)}/Y_T)\right\}$ and rejected otherwise ($\theta^{(s)} = \theta^{(s-1)}$). Here

$$r(\theta^{(s-1)}, \theta^{(s)}/Y_T) = \frac{P(\theta^{(s)}/Y_T)}{P(\theta^{(s-1)}/Y_T)} \tag{41}$$

5. Approximate the posterior expected value of a function $h(\theta)$ by $\frac{1}{n_{sim}} \sum_{s=1}^{n_{sim}} h(\theta^{(s)})$

Under fairly general regularity conditions, e.g. Walker (1969), the posterior distribution of θ will be asymptotically normal. The maximization of the posterior density kernel is carried out with a version of BFGS quasi-Newton algorithm written by Chris Sims. The algorithm uses a fairly simple line search and randomly perturbs the search direction if it reaches a cliff caused by non-existence or non uniqueness of a stable rational expectation solution for the DSGE model.

The RWM generates a sequence of dependent draws from the posterior distribution of θ that can be averaged to approximate posterior moments. Geweke (1999), and Brooks and Gelman (2006) reviews regularity conditions that guarantee the convergence of the Markow Chain generated by the Metropolis Hastings algorithm to the posterior distribution of interest and the convergence of $\frac{1}{n_{sim}} \sum_{s=1}^{n_{sim}} h(\theta^{(s)})$ to the posterior expectations $E\left[h(\theta)/Y_T\right]$.

In our specific case, we use the diagnostics computed by Brooks and Gelman (1998). Their intuition is the following. Let $\theta^{(0,i)}$ for $i = 1, ..., m$ denote m initial values which are taken from very different regions of the parameter space. Let $\theta^{(s,i)}$ for $s = 1, ..., S$ denote S RWM draws from ith starting values. Intuitively, if the effect of the starting value has been removed, each of the m sequences should be the same as one another. Hence, the variance calculated across the sequences should not be too large relative to the variance within a sequence (for the exact expression of these two variances see Koop(2003) or Brooks and Gelman (1998). Using the rule-of-thumb that the two variances are equal, up to a difference of 20%, should ensure convergence. If this is not the case, either the effect of the initial replication has not worn off, or the number of taken draws is too low. Figure 10,11,12 13,14 reports the within and the between variances calculated recursively as the number

[13] In reality, we start from our prior mean.

of draws increase. This calculation is made for all the length of the sequence, and even splitting the sequence in two batches (called m2 and m3 in the figures).