Global Liquidity and Asset Prices: Cross-Country Linkages in a GVAR model

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Abstract

The paper investigates the relationship between money (liquidity) and asset prices on a global scale with a view to answering questions such as: To what extent is the concept of global liquidity important? How are asset prices affected by national and global monetary conditions? And ultimately, how does this affect the ability of central banks to control interest rates and inflation? We use the Global VAR (GVAR) approach of Dees, Holly, Pesaran and Smith (2007) as a starting point. We first model the US, the UK, the euro area and Japan separately yet allowing for country-specific foreign variables to influence the domestic long run relations. However, we adapt the GVAR framework in two directions. First, we aggregate differences of national variables using time-varying weights, and second, we use the Autometrics general-to-specific algorithm to locate suitable models for each country/region. This strategy has the advantage that it allows us to use different specifications of the influence of foreign variables for each country. In addition, this approach saves degrees of freedom; this is crucial given the relatively short sample and the high dimension of models and in turn suggests less need to rely on bootstrapping in testing for cointegration. In a second step, we combine the national models to build a GVAR model in order to capture cross-country linkages and thereby second round effects of shocks to the system in an impulse response analysis. We find evidence of a surge in global liquidity beginning in 2001 which has raised inflation rates and house prices; however this has had limited effects on share prices.

Keywords: Global liquidity, inflation control, money demand, asset prices, cointegration

JEL Classification: C32, E31, E41, E44, E52

VERY PRELIMINARY AND INCOMPLETE DRAFT - PLEASE DO NOT QUOTE!

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1 Motivation: G6 model results

Giese and Tuxen (2007) estimated a model based on aggregated G6 data (US, Japan, Germany, France, Italy, UK) from 1982:4 to 2006:4 and found evidence of a shift in the cointegrating relations around 2001. Essentially, this analysis identified excess liquidity and an excessively low short-term rate (relative to steady state levels). Also, we found evidence of a global aggregate demand curve. These issues are to be investigated further in this paper with a special view to take cross-country linkages into account.

2 Theoretical considerations

The aim in this paper is to investigate interactions between a country's domestic variables and its respective rest-of-the-world (ROW) variables using the Global VAR (GVAR) approach due to *inter alia* Dees, Holly, Pesaran, and Smith (2007) (henceforth DHPS (2007)) but modifying the process of linking country models by using the aggregation method suggested by Beyer, Doornik, and Hendry (2001) (henceforth BDH (2001)). This section recalls some fairly standard relations relevant in this context and thus serves as a guide in the identification process of the Johansen procedure. A word of warning: some of the relations discussed below (and/or combinations of these) are subsets of each other and hence all of them could not be identified within the same model.

2.1 Dornbusch model

We start from the Dornbusch model of exchange rate adjustment as it appears in Obstfeld and Rogoff (1997), Chapter 9.2, adjusting it slightly to allow for further domestic/foreign interactions. The following domestic and foreign variables are considered:

$$\Theta^{country} = (m_r, y_r, p, I_s, I_l, e, m_r^*, y_r^*, p^*, I_s^*, I_l^*), \tag{1}$$

where m_r stands for real money, y_r for real GDP, p for the price level, I_k for an interest rate of maturity $k \in \{s, l\}$, i.e. short or long, and e for the nominal exchange rate. Starred variables denote foreign variables, all small variables are in logarithms, and we define $\Delta p = \pi$. In the next section, we consider an extension of the information set to include asset prices.

The equations in the Dornbusch model are familiar from the open-economy IS-LM framework, but prices are assumed to be sticky in the short run, i.e. purchasing power parity (PPP) does not hold. One link between the foreign and domestic economies is provided through uncovered interest rate parity (UIP):

$$I_k = I_k^* + \Delta_k e^e, \tag{2}$$

where superscript e denotes expectations over time horizon k. Furthermore, the real exchange rate enters the aggregate demand (AD) and aggregate supply/Phillips curve (PC) blocks, such that:

$$y_r^D = y_r^P + \tau_1(e + p^* - p) + \tau_2(I_k - \pi) + \tau_3 y_r^* \tag{3}$$

with $\tau_1 > 0$, $\tau_2 < 0$ and $\tau_3 > 0$ (an increase in e denotes a nominal depreciation), and

$$\pi = \delta_1(y_r - y_r^P) + \delta_2 \Delta e + \delta_3 \pi^* \tag{4}$$

where $\delta_1 > 0$, $\delta_2 > 0$ and $\delta_3 > 0$, and superscript P stands for potential output. Both AD and PC are directly dependent on the exchange rate and foreign economic conditions through trade linkages: demand for domestic output may change as a result of changes in ex- and imports, and at the same time domestic inflation is likely to be affected by changes in ex- and import prices. Money demand is described by

$$m_r^D = \lambda_1 y_r + \lambda_2 \pi + \lambda_3(I_l - I_s), \tag{5}$$

where $\lambda_1 > 0$, $\lambda_2 < 0$ and $\lambda_3 < 0$ such that real money demand reacts positively to a rise in the short rate and negatively to increases in the long rate and inflation. Excess liquidity would then be represented by money supply, m_r^S, exceeding the level of money demand as described by (5), possibly in deviation from real output. Foreign money demand is described in the same way with foreign variables replacing domestic ones.

Equations (2) to (5) define the Dornbusch model,[1] but these are unlikely to hold perfectly in the data. Moreover, statistical properties of the variables may vary, especially their order of integration.[2] Therefore, when using the above relations to identify cointegrating relations in Section 4, it may be necessary - and desirable - to combine equations. For example, (Obstfeld and Rogoff 1997) derive an interesting inverse long-run relation between the differential of the domestic and foreign real interest rate, and the real exchange rate by combining equations (2), (4) and (3), assuming $\tau_2 = \tau_3 = 0$ and, for simplicity, $\delta_3 = \delta_2 = 1$:

$$(I_s - \pi) - (I_s^* - \pi^*) = \psi(e + p^* - p), \tag{6}$$

[1] (Obstfeld and Rogoff 1997) assume $\tau_2 = \tau_3 = \delta_3 = \lambda_2 = \lambda_3 = 0$ in their exposition of the model.

[2] As an example we find the nominal exchange rate to be I(2) whereas the real exchange rate is I(1).

where $\psi = \delta_1 \tau_1 < 0$. A similar relation linking UIP and PPP has been found empirically by Juselius and MacDonald (2003). The reasoning is that UIP and PPP are inherently linked through the balance of payments: UIP describes conditions in the external financial market (capital account), PPP in the external goods market (current account). As a result, disequilibrium in one of the markets will likely affect the other and thus relations may need to be combined in order to obtain stationarity.

Similarly, combining equations (2), (5), (4) and (3), through substituting domestic and foreign money demand (assuming the same coefficients for simplicity) for the short-term interest rates, I_s and I_s^*, and assuming $\tau_1 = \tau_2 = \delta_3 = 0$, we get

$$\begin{aligned} (m_r - m_r^*) - \lambda_1(y_r - y_r^*) - \lambda_2(I_l - I_l^*) - \lambda_3(\pi - \pi^*) &= \pi - \delta_1(y_r - y_r^P) \\ (m_r - m_r^*) - \lambda_1(y_r - y_r^*) - \lambda_2(I_l - I_l^*) - (\lambda_3 + 1)\pi + \lambda_3\pi^* &= -\tau_1(e + p^* - p), \quad (7) \end{aligned}$$

such that an appreciation of the real exchange rate is associated with relatively higher domestic real money demand. Central banks may also follow a Taylor rule, in which they may choose to target the exchange rate,

$$I_s = \gamma_1(\pi - \pi^T) + \gamma_2(y_r - y_r^P) + \gamma_3(e + p^* - p), \quad (8)$$

where $1 < \gamma_1 < 2$, $\gamma_2 > 0$ and $\gamma_3 > 0$. Other foreign variables like π^*, i_s^* or $y_r^* - y_r^{P*}$ could also determine central banks' actions (but would not be target variables).

Two other relationships that may be interesting to look for are a term structure relation and the Fisher parity. The term structure of interest rates as suggested by the expectations hypothesis (EH), see Cox, Ingersoll, and Ross (1985) with the k periods to maturity rate being an average of expected future one period rates plus a risk premium (this is essentially a no-arbitrage condition),

$$I_k = \frac{1}{k}\Sigma_{j=1}^k I_{1,j}^e + \rho(k) \quad (9)$$

where superscript e denotes the expected value of the variable in question, again subscript k denotes time to maturity, and $\rho(k)$ is a risk premium which likely depends positively k. The cointegration implication of the EH in its simplest form is that among r interest rates there should exist $(r-1)$ cointegrating relations such that the spread between every two rates of different maturities is stationary, possibly around a constant.

The Fisher parity is essentially a definition of the real rate of interest and given by

$$I_{r,k} \equiv I_k - \pi_k^e \quad (10)$$

where I_r is the real rate.[3]

2.2 Inclusion of additional asset prices

Since we are particularly interested in relationships between liquidity and asset prices, we will also consider prices of equity, housing and oil. We now modify the relationships discussed above to take these three prices into account. There is no obvious reason why asset prices should enter the money demand equation, but central banks may decide to target asset prices. A modified policy rule, excluding foreign variables, takes the form,

$$I_s = \gamma_1(\pi - \pi^T) + \gamma_2(y_r - y_r^P) + \gamma_3(q - q^T) \tag{11}$$

where q is the log of real asset prices and could be a vector including both stock and house prices, i.e. $q = (s_r, h_r)'$, and we expect $\gamma_3 > 0$. Due to wealth effects, q potentially also enters the AD relation, see Smets (1997) and Disyatat (2005),

$$y_r^D = y_r^P + \tau_1(e + p^* - p) + \tau_2(I_k - \pi) + \tau_3 y_r^* + \tau_4 q. \tag{12}$$

with $\tau_4 > 0$. In the New-Keynesian model employed by Goodhart and Hofmann (2007) asset prices do not enter the PC. However, the oil price is viewed as decisive for inflation due to the fact that it represents the price of a crucial raw material and thus is a significant component of the marginal costs of firms. A revised PC takes the form,

$$\pi = \delta_1(y_r - y_r^P) + \delta_2 \Delta e + \delta_3 \pi^* + \delta_4 oil \tag{13}$$

where $\delta_4 > 0$. Yet another relation could be a demand equation for asset prices, see Disyatat (2005):

$$q = \omega_1(y_r - y_r^P) + \omega_2(I_k - \pi) + \omega_3(e + p^* - p) \tag{14}$$

where $\omega_1 > 0$, $\omega_2 < 0$ and $\omega_3 > 0$. Finally, a number of no-arbitrage conditions related to q would be expected to hold. Risk-adjusted returns to investment in different assets should be equalized according to the efficient market hypothesis provided that risk adjustment is done properly, see Fama (1970). Equating *risk-adjusted* expected returns to investment in equity/housing on the one side and bonds on the other, we obtain

$$\Delta_k q^e = (I_k - \pi_k)^e + \kappa(k, q) + \phi_1(m_r - y_r) \tag{15}$$

[3] Notwithstanding the fact that relations (2), (9) and (10) involve expected variables, we only use actual values of the variables in the empirical analysis.

where $\kappa(k, q)$ is a risk premium which is allowed to depend on the investment horizon and the type of asset in question and $(m_r - y_r)$ is assumed to be a time-varying bubble component discussed further below. Note first that in the light of the Mehra and Prescott (1985) equity premium puzzle a constant may be required as well. Furthermore, we need to distinguish between *ex ante* and *ex post* returns: risk-adjusted *expected* returns should be the same for all types of investments; otherwise risk-free arbitrage is expected and efficient markets would instantly eliminate such possibilities. However, *actual* outcomes may differ from expectations and invalidate (15). The potential bubble in (15), $(m_r - y_r)$, is here assumed to be related to the build-up in liquidity. $\phi_1 > 0$ suggests that a liquidity surplus - defined as money in excess of GDP - may initiate an asset price bubble. One implication of this bubble element is that a (sudden) decline in liquidity may cause the bubble to burst, asset prices to decline and thereby lead to a credit crunch which in turn could threaten financial stability. It may also enter equation (14).

3 Econometric methodology

In order to investigate spillovers between countries, we use a CVAR framework on a country level, and then combine the country models according to Pesaran et al.'s global VAR (GVAR) methodology. We extend the latter in two ways:

1. Aggregation of country data: Continue to use the method for aggregation of country level data to ROW variables suggested in Beyer, Doornik, and Hendry (2001), i.e. aggregating log differences and using variable weights. Weights may either be real GDP or trade weights (we focus on the former while Pesaran et al. employ the latter).

2. Include a differing number of country-specifc foreign variables in each country model as suggested by general-to-specific modelling.

Data sources in this study are:[4]

[4] m: For most countries M3 is used except in the case of the UK and Japan where M4 and M2 plus cash deposits is used, respectively. Note also that US M2 growth was used to extrapolate the US M3 series from 2006:1 and onwards when publication of M3 was discontinued. h: BIS calculation based on national sources. Series for the US and UK are quarterly throughout, for France, Italy and Japan semi-annual series were interpolated to create quarterly series, and for Germany annual series were interpolated. s: France: Paris Stock Exchange SBF 250, Italy: ISE MIB Storico Generale, Japan: TSE Topix, UK: FTSE 100, US: NYSE Composite, Germany: CDAX.

Variable	Description	Source
y	Nominal output (GDP)	OECD EOL
m	Broad money stock	National sources
p	GDP deflator (implicit)	OECD EOL
I_s	Short term interest rate (3-month deposit rate)	OECD EOL
I_l	Long term interest rate (10-year government bond rate)	OECD EOL
h	House price index	BIS
s	Share price index (key national indices)	OECD MEI
oil	Crude oil price (F.O.B. spot brent)	OECD EOL

3.1 Construction of the GVAR

In building the GVAR from the country models, we deviate slightly from the approach taken by DHPS (2007) this is because we aggregate series, not in levels, but in differences as suggested by BDH (2001). This complicates the linking of the models slightly but can indeed be done as we describe below.

3.2 Country VARX* models

We first consider the VARX$^*(2,2)$ specification used in modelling each country. However, in order to formulate the hypothesis of cointegration it is convenient to cast the model (17) in error correction model (ECM) form[5],

$$\mathbf{\Delta x}_{it}= \mathbf{h}_{i0}+\mathbf{\Pi}_i\widetilde{\mathbf{x}}_{i,t-1}+\mathbf{\Gamma}_{i1}\mathbf{\Delta x}_{i,t-1}+\mathbf{\Upsilon}_{i0}\mathbf{\Delta x}^*_{i,t}+\mathbf{\Upsilon}_{i1}\mathbf{\Delta x}^*_{i,t-1}+\widetilde{\mathbf{\Theta}}_i\widetilde{\mathbf{D}}_{i,t}+u_{i,t}, \tag{16}$$

where $\mathbf{x}_{it}$ is the $k_i\times 1$ vector of domestic variables, $\mathbf{x}^*_{it}$ is the $k^*_i\times 1$ vector of country specific foreign variables to be discussed further below, $\widetilde{\mathbf{D}}_{it}$ is a matrix of dummy variables, $\mathbf{h}_{i0}$ a constant term and t a linear time trend; $u_{i,t}$ is an error term assumed to be independently and identically distributed $N_{k_i}(0,\Omega_i)$; $i = 0,1,...,N$ is a country index, $t = 1,2,...,T$ a time index and we assume fixed values of $\mathbf{x}_{i,-1}$, $\mathbf{x}_{i,0}$, $\mathbf{x}^*_{i,-2}$, $\mathbf{x}^*_{i,-1}$ and $\mathbf{x}^*_{i,0}$. Finally, $\widetilde{\mathbf{x}}_{i,t}= (\mathbf{x}'_{it},\mathbf{x}^{*\prime}_{it},t)'$ which is $(k_i+k^*_i+1)\times 1$ such that we allow for a trend restricted to the cointegration space (as well as an unrestricted constant captured by $\widetilde{\mathbf{D}}_t$). We now have

Conditional on at least two variable in $\widetilde{\mathbf{x}}_{i,t}$ being integrated of order (at most) 1, the hypothesis of cointegration can be fomulated as a reduced rank condition on $\mathbf{\Pi}_i$, i.e. $rank(\mathbf{\Pi}_i) = r_i < k_i$.[6] In that case we can decompose this as $\mathbf{\Pi}_i= \boldsymbol{\alpha}_i\boldsymbol{\beta}'_i$, where $\boldsymbol{\beta}_i$ provides the $(k_i+k^*_i+1)\times r$ matrix of cointegration vectors and $\boldsymbol{\alpha}_i$ is the $k_i\times r$ matrix of adjustment coefficients, see Johansen (1996).

[5] This is the model form estimated by CATS.

[6] The formal condition for ruling out higher orders of integration is $|\alpha'_\perp\Gamma\beta_\perp|\neq 0$ with $\Gamma = I-\Gamma_1$.

The form of the model considered by DHPS (2007) in deriving impulse responses (IR) is the autogression (AR) form,

$$\mathbf{x}_{it}= \mathbf{h}_{i0} + \mathbf{h}_{i1}t + \mathbf{\Phi}_{i1}\mathbf{x}_{i,t-1}+\mathbf{\Phi}_{i2}\mathbf{x}_{i,t-2}+\mathbf{\Psi}_{i0}\mathbf{x}^*_{\mathbf{i,t}} + \mathbf{\Psi}_{i1}\mathbf{x}^*_{i,t-1}+\mathbf{\Psi}_{i2}\mathbf{x}^*_{i,t-2} + \mathbf{\Theta}_i\mathbf{D}_{i,t}+u_{i,t}, \quad (17)$$

where $\mathbf{D}_t$ now includes the restricted linear trend term as well. The relationships between the coefficients in (16) and (17) are as follows,

$$\begin{aligned}
\mathbf{\Phi}_{i2} &= -\mathbf{\Gamma}_{i1} \\
\mathbf{\Phi}_{i1} &= \mathbf{I}_{k_i} + \mathbf{\Pi}_i^1 - \mathbf{\Phi}_{i2} \\
\mathbf{\Psi}_{i0} &= \mathbf{\Upsilon}_{i0} \\
\mathbf{\Psi}_{i2} &= -\mathbf{\Upsilon}_{i1} \\
\mathbf{\Psi}_{i1} &= \mathbf{\Pi}_i^2-\mathbf{\Psi}_{i0} - \mathbf{\Psi}_{i2}
\end{aligned}$$

with the $k_i \times (k_i + k_i^* + 1)$ matrix $\mathbf{\Pi}_i$ decomposed in the following way,

$$\mathbf{\Pi}_i = (\underbrace{\mathbf{\Phi}_{i1} + \mathbf{\Phi}_{i2} - \mathbf{I}_{k_i}}_{\mathbf{\Pi}_i^1}, \underbrace{\mathbf{\Psi}_{i0} + \mathbf{\Psi}_{i1} + \mathbf{\Psi}_{i2}}_{\mathbf{\Pi}_i^2}, \mathbf{h}_{i1}) \quad (18)$$

such that $\mathbf{\Pi}_i^1$ contains the first k_i columns and $\mathbf{\Pi}_i^2$ columns $k_i + 1$ to $k_i + k_i^*$ of $\mathbf{\Pi}_i$.

Having estimated the models of each country, we need to link these using the the weights used in constructing $\mathbf{x}^*_{i,t}$ in order to do impulse response analysis which take second round effects into account; this is essentially the focus of the GVAR literature.[7]

3.3 DHPS (2007) set-up

DHPS (2007) calculate country-specific variables using fixed weights as

$$\mathbf{z}_{it} = \mathbf{W}_i\mathbf{x}_t, \quad (19)$$

where $\mathbf{z}_{it} = (\mathbf{x}'_{it}, \mathbf{x}^{*'}_{it})'$ is the country-specific data vector of country i of dimension $(k_i + k_i^*) \times 1$, $\mathbf{W}_i$ is the weighting matrix of country i based on trade volumes which is of dimension $(k_i + k_i^*) \times k$, where $k = \sum_{i=0}^N k_i$ denotes the number of all (domestic) variables in the system; finally, $\mathbf{x}_t = (\mathbf{x}'_{0t}, \mathbf{x}'_{1t}, ..., \mathbf{x}'_{Nt},)'$ is a k-dimensional data vector with domestic

[7] Due to the model selection the number of variables in $\Delta\mathbf{x}^*_{i,t}$ and $\Delta\mathbf{x}^*_{i,t-1}$ may differ from k_i^* and hence in practice we need to put in some zeros columns in the $\mathbf{\Upsilon}$'s to make dimensions fit in calculating e.g. (18).

variables from all countries stacked on top of each other. Using (19) we can re-write the country models (17) in terms of $\mathbf{z}_{it}$,

$$\mathbf{A}_{i0}\mathbf{z}_{it} = \mathbf{h}_{i0}+\mathbf{h}_{i1}\mathbf{t} + \mathbf{A}_{i1}\mathbf{z}_{i,t-1}+\mathbf{A}_{i2}\mathbf{z}_{i,t-2}+\mathbf{u}_{it},$$

where the $\mathbf{A}_{ij}$ matrices are of dimension $k_i \times (k_i + k_i^*)$, $j = 0, 1, 2$, $\mathbf{A}_{i0} = (I_{k_i}, -\mathbf{\Psi}_{i0})$, $\mathbf{A}_{i1} = (\mathbf{\Phi}_{i1}, \mathbf{\Psi}_{i1})$ and $\mathbf{A}_{i2} = (\mathbf{\Phi}_{i2}, \mathbf{\Psi}_{i2})$; note that $rank(\mathbf{A}_{i0}) = k_i$ (full row rank). Note that DHPS (2007) work with a VARX$^*(2,1)$ and thus set $\mathbf{\Psi}_{i2} = 0$.

To construct the global model we define - for simplicity of exposition - a new matrix $\mathbf{A}_j$ of dimension $k \times \sum_{i=0}^{N}(k_i + k_i^*)$ matrix with the $\mathbf{A}_{ij}$ terms on the diagonal and zeros elsewhere,[8] i.e.

$$\mathbf{A}_j = \begin{pmatrix} \mathbf{A}_{0j} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{1j} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_{Nj} \end{pmatrix}$$

In order to link the different country model into one single global model we want to stack the different country models in (17). For this purpose define a $\sum_{i=0}^{N}(k_i + k_i^*) \times k$ weight matrix, $\mathbf{W} = (\mathbf{W}_0', \mathbf{W}_1', ..., \mathbf{W}_N')'$ and likewise for $\mathbf{h}_0, \mathbf{h}_1$ and $\mathbf{u}_t$. Again using (19) the global model can be written in terms of $\mathbf{x}_t$,

$$\begin{aligned} \mathbf{A}_0\mathbf{z}_t &= \mathbf{h}_0+\mathbf{h}_1\mathbf{t} + \mathbf{A}_1\mathbf{z}_{t-1}+\mathbf{A}_2\mathbf{z}_{t-2}+\mathbf{u}_t \qquad (20) \\ &\Updownarrow \\ \mathbf{A}_0\mathbf{W}\mathbf{x}_t &= \mathbf{h}_0+\mathbf{h}_1\mathbf{t} + \mathbf{A}_1\mathbf{W}\mathbf{x}_{t-1}+\mathbf{A}_2\mathbf{W}\mathbf{x}_{t-2}+\mathbf{u}_t \end{aligned}$$

Now isolate $\mathbf{x}_t$ to arrive at the model of interest,

$$\mathbf{x}_t = \mathbf{f}_0+\mathbf{f}_1\mathbf{t} + \mathbf{F}_1\mathbf{x}_{t-1}+\mathbf{F}_2\mathbf{x}_{t-2}+\boldsymbol{\upsilon}_t \qquad (21)$$

where $\mathbf{f}_0 = (\mathbf{A}_0\mathbf{W})^{-1}\mathbf{h}_0$ and the other terms are defined in a similar way.

Note that in order to do this $\mathbf{A}_0\mathbf{W}$ must be invertible; a necessary condition for (21) to be well-defined is therefore that $\mathbf{A}_0\mathbf{W}$ is a square matrix, i.e. of dimension k such that it complies with the k-dimensional $\mathbf{x}_t$. In practice, this means that the number of endogenous variables in each country model must to be the same, i.e. $k_i = k_j$ with $i \neq j$. However, the number of country-specific foreign (weakly exogenous) variables, may vary between different country models, i.e. $k_i^* \neq k_j^*$ with $i \neq j$ is allowed. The former

[8] Note that if all country vectors have the same dimension and all foreign variables are used in all country models the dimension of $\mathbf{A}_j$ is simply $k \times 2k$.

implies that we can allow more flexibility in the model selection process as the variables of different country can be allowed to cointegrate with different country specific foreign variables. In turn this imply that we potentially can save degrees of freedom by excluding unneccesary variables and still ensure that our models are econometrically well-specified.

3.4 Modified set-up

The desirable properties of the BDH (2001) aggregation method comes at a cost since it complicates the process of constructing the global model from teh country model for two reasons: i.) time-varying weights, and ii.) aggregation of differences (as opposed to levels as in DHPS (2001)). In our case, the country-specific variables are defined according to

$$\mathbf{\Delta y}_{it} = \mathbf{W}_{it-1}\mathbf{\Delta x}_t,$$

where $\mathbf{\Delta y}_{it} = (\mathbf{\Delta x}'_{it}, \mathbf{\Delta x}^{*\prime}_{it})'$ is the country-specific $(k_i + k_i^*)$-dimensional data vector of country i in differences. In levels,

$$\mathbf{y}_{it} = \mathbf{y}_{i0} + \sum_{l=1}^{t} \mathbf{W}_{il-1}\mathbf{\Delta x}_l,$$

for a fixed value of $\mathbf{y}_{i0}$[9] Stacking $\mathbf{y}_{it}$ for all countries gives the $\sum_{i=0}^{N}(k_i + k_i^*) \times 1$ vector,

$$\mathbf{y}_t = \mathbf{y}_0 + \sum_{l=1}^{t} \mathbf{W}_{l-1}\mathbf{\Delta x}_l, \tag{22}$$

In terms of our country-specific variables, $\mathbf{y}_t$, the stacked model reads

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{h}_0 + \mathbf{h}_1\mathbf{t} + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \mathbf{u}_t, \tag{23}$$

The model in (**??**) is comparable to (20); $\mathbf{A}_0$, $\mathbf{A}_1$ and $\mathbf{A}_2$ defined in the same way as before. Now substitute the expression for $\mathbf{y}_t$ from (22) into (**??**) to obtain

$$\mathbf{A}_0\mathbf{y}_0 + \mathbf{A}_0\sum_{l=1}^{t}\mathbf{W}_{l-1}\mathbf{\Delta x}_l = \mathbf{h}_0 + \mathbf{h}_1\mathbf{t} + \mathbf{A}_1\mathbf{y}_0 + \mathbf{A}_1\sum_{l=1}^{t-1}\mathbf{W}_{l-1}\mathbf{\Delta x}_l + \mathbf{A}_2\mathbf{y}_0 + \mathbf{A}_2\sum_{l=1}^{t-2}\mathbf{W}_{l-1}\mathbf{\Delta x}_l + \mathbf{u}_t \tag{24}$$

Due to the aggregation of differences we do not end up at an expression that is directly comparable to (21) but if we treat the differences in a way similar to the levels in DHPS (2007) we can nevertheless derive the MA representation of the global model. We therefore

[9] The choice of initial value does not make a difference and in practice we define the average of the four quarters in 2000 to equal ln(100).

manipulate (24) to arrive at an autoregression in $\Delta\mathbf{x}_t$. To do so, first take all lagged values and the initial value term to the right hand side (RHS),

$$\mathbf{A}_0\mathbf{W}_{t-1}\mathbf{\Delta x}_t = \mathbf{h}_0 + \mathbf{h}_1\mathbf{t} + (\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_0)\mathbf{y}_0 + (\mathbf{A}_1 - \mathbf{A}_0)\sum_{l=1}^{t-1}\mathbf{W}_{l-1}\mathbf{\Delta x}_l + \mathbf{A}_2\sum_{l=1}^{t-2}\mathbf{W}_{l-1}\mathbf{\Delta x}_l + \mathbf{u}_t$$

Note that $\mathbf{A}_0\mathbf{W}_{t-1}$ is a square matrix of dimension k and hence (potentially) invertible. Next, pre-multiply by its inverse to finally isolate $\Delta\mathbf{x}_t$,

$$\Delta\mathbf{x}_t = \mathbf{b}_0 + \mathbf{b}_1\mathbf{t} + \mathbf{\Lambda}_0\mathbf{y}_0 + \mathbf{\Lambda}_1\sum_{l=1}^{t-1}\mathbf{W}_{l-1}\mathbf{\Delta x}_l + \mathbf{\Lambda}_2\sum_{l=1}^{t-2}\mathbf{W}_{l-1}\mathbf{\Delta x}_l + \boldsymbol{\epsilon}_t \tag{25}$$

where $\boldsymbol{\epsilon}_t = (\mathbf{A}_0\mathbf{W}_{t-1})^{-1}\mathbf{u}_t$ and

$$\begin{aligned}
\mathbf{b}_0(t) &= (\mathbf{A}_0\mathbf{W}_{t-1})^{-1}\mathbf{h}_0 \\
\mathbf{b}_1(t) &= (\mathbf{A}_0\mathbf{W}_{t-1})^{-1}\mathbf{h}_1 \\
\mathbf{\Lambda}_0(t) &= (\mathbf{A}_0\mathbf{W}_{t-1})^{-1}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_0) \\
\mathbf{\Lambda}_1(t) &= (\mathbf{A}_0\mathbf{W}_{t-1})^{-1}(\mathbf{A}_1 - \mathbf{A}_0) \\
\mathbf{\Lambda}_2(t) &= (\mathbf{A}_0\mathbf{W}_{t-1})^{-1}\mathbf{A}_2
\end{aligned}$$

Note that $\mathbf{\Lambda}$'s become functions of t due to the time varying weights. To see that this is indeed an autoregression, write out the sums in (25),

$$\begin{aligned}
\Delta\mathbf{x}_t = {} & \mathbf{b}_0 + \mathbf{b}_1\mathbf{t} + \mathbf{\Lambda_0}(t)\mathbf{y}_0 + \mathbf{\Lambda}_1(t)\mathbf{W}_{t-2}\mathbf{\Delta x}_{t-1} \\
& + (\mathbf{\Lambda}_1(t) + \mathbf{\Lambda}_2(t))(\mathbf{W}_{t-3}\mathbf{\Delta x}_{t-2} + \mathbf{W}_{t-4}\mathbf{\Delta x}_{t-3} + \mathbf{W}_{t-5}\mathbf{\Delta x}_{t-4} + \ldots + \mathbf{W}_0\mathbf{\Delta x}_1) \\
& + \boldsymbol{\epsilon}_t
\end{aligned} \tag{26}$$

One important difference compared to DHPS (2007) is however that there is no "cut-off" of the autoregression, i.e. at time t all previous values of the left hand side variable appear on the RHS. However, we can still derive the moving average (MA) representation needed for impulse response analysis although the derivation of the generalized impulse response functions (GIRF), which were suggested by Pesaran and Shin (1998), is a bit more involved, see below.

3.5 MA representation and impulse responses

In order to do impulse response functions we need to consider the MA representation of the model (26). DHPS (2007) re-write their model (21) in terms of error terms and deterministic components as follows,

$$\mathbf{x}_t = \mathbf{d}_t + \sum_{j=0}^{\infty} \mathbf{K}_j \boldsymbol{\varepsilon}_{t-j}, \tag{27}$$

where $\mathbf{K}_j = \mathbf{F}_1\mathbf{K}_{j-1} + \mathbf{F}_2\mathbf{K}_{j-2}, j = 1, 2, ...$ with $\mathbf{K}_0 = \mathbf{I}_k$, $\mathbf{K}_j = 0$ for $j < 0$. This is then used to generate the GIRF of a one-standard-deviation shock to the ℓ^{th} element of $\Delta\mathbf{x}_T$ (shock) on its $\jmath^{\text{th}}$ element (impact) given by

$$GIRF(\mathbf{x}_t; u_{\ell t}, h) = \frac{e_\jmath' \mathbf{K}_h (\mathbf{A}_0 \mathbf{W})^{-1} \boldsymbol{\Sigma}_u e_\ell}{\sqrt{e_\ell' \boldsymbol{\Sigma}_u e_\ell}}, h = 0, 1, 2, ..., H; \ell, \jmath = 1, 2, ..., k, \tag{28}$$

where h is time horizon index for which IR are to be computed and H is the maximal number of simulations, $\boldsymbol{\Sigma}_u$ is the covariance matrix of equation (20), and e_ℓ and $e_\jmath$ are $k \times 1$ selection (unit) vectors, i.e. with an entry of 1 at the ℓ^{th} and $\jmath^{\text{th}}$ elements, respectively, and zeros elsewhere.

Within our modified framework we cannot use (28) directly. However, by doing recursive substitution based on (26) we can find an MA representation for the differenced process, $\Delta\mathbf{x}_t$,

$$\Delta\mathbf{x}_t = \mathbf{c}_t + \sum_{j=0}^{\infty} \mathbf{C}_j \boldsymbol{\epsilon}_{t-j},$$

where

$$\begin{aligned} \mathbf{C}_j(t) &= \mathbf{G}_1(t)\mathbf{C}_{j-1} + \mathbf{G}_2(t)\mathbf{C}_{j-2} + \mathbf{G}_3(t)\mathbf{C}_{j-3} + \mathbf{G}_4(t)\mathbf{C}_{j-4} + ... + \mathbf{G}_{t-1}(t)\mathbf{C}_{j-(t-1)} \\ &= \boldsymbol{\Lambda}_1(t)\mathbf{W}_{t-2}\mathbf{C}_{j-1} + (\boldsymbol{\Lambda}_1(t) + \boldsymbol{\Lambda}_2(t))\mathbf{W}_{t-3}\mathbf{C}_{j-2} + (\boldsymbol{\Lambda}_1(t) + \boldsymbol{\Lambda}_2(t))\mathbf{W}_{t-4}\mathbf{C}_{j-3} \\ &\quad + (\boldsymbol{\Lambda}_1(t) + \boldsymbol{\Lambda}_2(t))\mathbf{W}_{t-5}\mathbf{C}_{j-4} + ... + (\boldsymbol{\Lambda}_1(t) + \boldsymbol{\Lambda}_2(t))\mathbf{W}_0\mathbf{C}_{j-(t-1)}, \end{aligned} \tag{29}$$

with $\mathbf{C}_0(t) = \mathbf{I}_k$, $\mathbf{C}_j(t) = 0$ for $j < 0$ and $\mathbf{G}_1(t) = \boldsymbol{\Lambda}_1(t)\mathbf{W}_{t-2}$, $\mathbf{G}_2(t) = (\boldsymbol{\Lambda}_1(t) + \boldsymbol{\Lambda}_2(t))\mathbf{W}_{t-3}$, $\mathbf{G}_3(t) = (\boldsymbol{\Lambda}_1(t) + \boldsymbol{\Lambda}_2(t))\mathbf{W}_{t-4}$, etc., are $k \times k$ matrices. Two complications arise in relation to the $\mathbf{C}$'s compared to DHPS (2007): i.) due to the lack of a "cut-off" in the autoregression in (26) the autoregressive structure of $\mathbf{C}_j$ is also not cut off, and ii.) the $\mathbf{C}_j$ is time-dependent due to its dependence on the time varying weights *as well as* the lack of a "cut-off" point. Note that since we generally assume $\mathbf{x}_t \sim I(1)$ and therefore $\Delta\mathbf{x}_t \sim I(0)$ the properties of $\mathbf{C}_j$ are fundamentally different from those of $\mathbf{K}_j$ as the former generates a stationary variable and the latter a non-stationary variable. Note that $\boldsymbol{\Lambda}_1(T + h) = \boldsymbol{\Lambda}_1(T + 1)$ for $h > 1$.

The corresponding GIRFs of a one-standard-deviation shock to the ℓ^{th} element of $\Delta\mathbf{x}_t$ on its $\jmath^{\text{th}}$ element is (in practice, the weights are all equal to $\mathbf{W}_T$ for $h > 0$),

$$GIRF(\Delta\mathbf{x}_t; u_{\ell t}, h) = \frac{e'_\jmath \mathbf{C}_h (\mathbf{A}_0 \mathbf{W}_{t-1})^{-1} \mathbf{\Sigma}_u e_\ell}{\sqrt{e'_\ell \mathbf{\Sigma}_u e_\ell}}, h = 0, 1, 2, ..., H; \ell, \jmath = 1, 2, ..., k, \qquad (30)$$

Note that this gives the GIRFs for the impact on the *differenced* variable in question. However, the *level* impact can be found by cumulation of these which is straightforward in the IR context: since we know the final value, $\mathbf{x}_T$, we can derive expected future values following a shock recursively as $\widehat{\mathbf{x}}_{T+h} = \mathbf{x}_T + \Delta\widehat{\mathbf{x}}_{T+1} + ... + \Delta\widehat{\mathbf{x}}_{T+h}$, where $\Delta\widehat{\mathbf{x}}_{T+h}$, $h = 0, 1, 2, ...$, are generated by (30).

4 Empirical findings

4.1 Model specification

From a statistical point of view one should always start the nominal specification of the model. However, most nominal variables have a tendency to exhibit I(2) behaviour over available sample sizes and due to the complexity of the I(2) model we would like to map the data to the I(1) space in order to work within a simpler statistical framework. In order to make sure that we do not loose information in this step we need to check that the the nominal-to-real transformation (NRT) is indeed valid and this is a testable hypothesis. If the test is accepted, we can estimate the model in real space and then map the variables back to nominal space, or “re-integrate" the series, for simulation and forecasting purposes.

Say, for example, we are interested in the response of nominal money h periods after a shock originating in the system. We then need to “purge" real money off the nominal element, i.e. instead of $m - p$ look at m. Since our model does not include p directly, we need to infer p from Δp. This is straightforward because we know what the response of Δp was in each period leading up to h after the shock given a certain simulation analysis. p_{t+h} is therefore simply given by:

$$p_{t+h} = p_t + \sum_{i=1}^{h} \Delta p_{t+i} \qquad (31)$$

Since we also know $(m - p)_{t+h}$ from the simulation analysis, nominal money in period h is given by $m_{t+h} = (m - p)_{t+h} + p_{t+h}$. Consider therefore the model with nominal money and prices but real GDP (for simplicity) and interest rates,

$$x_t = (m, p, y_r, I_s, I_l)'_t \tag{32}$$

where m is nominal broad money (M4 in the UK case), p is the GDP deflator, y_r is real output (GDP), and finally I_s and I_l is the three months deposit rate and the 10 year government bond rate, respectively. Interest rates have been divided by 400 for comparison with the inflation rate. The test of the NRT is done in the I(2) model based on 32. The I(2) rank test statistics suggest $r = 2$ cointegrating relations and $s_2 = 2$ I(2) trends, leaving only $s_1 = 1$ I(1) trend among the $p = 5$ series. A more economically plausible scenario might nevertheless be $r = 3$, $s_1 = 2$ and $s_2 = 1$. However, the NRT is rejected no matter what choice of r, s_1 and s_2 is employed. This is often found to be the case, see Juselius (2007), and as the unrestricted estimates are not "too far" from supporting a $(1, -1)$ relation of m and p, we believe it is reasonable to work with the real-transformed series but having in mind the fact that there may be a small I(2) component left. This may distort test statistics, i.e. test of restrictions on the β-matrix making it crucial to check graphically the stationarity of the identified relations.

4.2 Modelling strategy issues

Ideally, we would like to estimate a co-integrated VAR (CVAR) in the following endogenous variables,

$$x_t = (m_r, y_r, \Delta p, I_s, I_l, h_r, s_r, ppp)'_t \tag{33}$$

where m_r is real broad money (M4), y_r is real output (GDP), p is the GDP deflator and thus Δp is the inflation rate, I_s and I_l is the three months deposit rate and the 10 year government bond rate, respectively, s_r a real share price index (FTSE 100), h_r a real house price index (source: BIS), and ppp is the real exchange rate ($ppp = e + p^* - p$). However, the dimension of the system increases very quickly with additional variables, especially since we want to consider cross-country linkages and thus include a real exchange rate (PPP deviations) and a number of foreign variables defined in a Rest of the World (ROW) manner,

$$x_t^{ROW\ extension} = (m_r^*, y_r^*, \Delta p^*, I_s^*, I_l^*, h_r^*, s_r^*)'_t \tag{34}$$

where all foreign variables are marked with a star but otherwise have names similar to those used at the country level.

Unfortunately, with a sample size of less than 100 observations it is not possible to set up a model that includes all the variables in (32) and (34) and we need to think of different ways of reducing the number of parameters. The next section considers some possible approaches in this respect.

4.3 Alternative CVAR approaches for investigation of cross-country interdependencies

In order to cope with the problem of dimensionality we consider in the following a number of approaches to circumvent the degrees of freedom problem that arise as a result of the combination of a small sample and a large number of potentially relevant variables.

1. Simultaneous Equations Model (SEM) in differences of country variables with G6 Error Correction Mechanism (ECM) terms among the regressors. Problem(s): no adjustment to national steady state deviations and no interaction between domestic and foreign variables in long run (no cointegration allowed). Advantage(s): simple.

2. SEM in differences of country variables with both ROW and national ECM terms as regressors (but ECMs estimated in separate models). Problem(s): as in 1. Advantage(s): relatively simple and allows adjustment to domestic and foreign relations.

3. a) CVAR of country variables and some (but not all) foreign variables. Could use general-to-specific (GS) modelling to reduce the system. Problem(s): asymmetric treatment of foreign and national variables. Advantages: allows for some interaction between domestic and foreign variables even in the long run.

4. All country models based on the same model specification and foreign variables the mirror image of the country variables (symmetric treatment of foreign and national variables). Problem(s): degrees of freedom problem and a daunting identification process. Advantage(s): allows a sophisticated type of impulse response analysis and very rich dynamics. Only feasible with transformed variables.

During the past week, we experimented with each of the above approaches. While 1.) did not yield interesting results in terms of adjustment, 2.) was more promising. However, both suffer from a major problem, namely that it is not clear how to treat the shift dummy introduced in our global model to reflect a surge in global money around 2001. This will affect adjustment, and ideally one would want to include ECM terms without the shift dummy, but then they may not be meaningful. As mentioned above, both ways are also deviations from Pesaran's GVAR, and in particular do not allow for long-run relations between foreign and domestic variables.

Both 3. and 4. allow for derivation of a GVAR as suggested by Pesaran et al.. Given the greater flexibility of 3. it is preferable to 4. and our choice of strategy.

4.4 A GVAR with asymmetric treatment of domestic and foreign variables

Regarding 3.), one could reduce the number of foreign variables, e.g. only include foreign interest rates on the assumption that they "summarize" the state of the global money market. Another way is to employ GS modelling, where - conditional on domestic variables being fixed - an algorithm selects those foreign regressors that are relevant.

The following model is considered:

$$\begin{pmatrix} \Delta m_r \\ \Delta y_r \\ \Delta^2 p \\ \Delta I_s \\ \Delta I_l \\ \Delta h_r \\ \Delta s_r \\ \Delta ppp \end{pmatrix}_t = \alpha\beta' \begin{pmatrix} m_r \\ y_r \\ \Delta p \\ I_s \\ I_l \\ h_r \\ s_r \\ ppp \\ m_r^* \\ \Delta y_r^* \\ \Delta p^* \\ I_s^* \\ I_l^* \\ h_r^* \\ s_r^* \end{pmatrix}_{t-1} + \Gamma_1 \begin{pmatrix} \Delta m_r \\ \Delta y_r \\ \Delta^2 p \\ \Delta I_s \\ \Delta I_l \\ \Delta h_r \\ \Delta s_r \\ \Delta ppp \end{pmatrix}_{t-1} + \kappa_0 \begin{pmatrix} \Delta m_r^* \\ \Delta y_r^* \\ \Delta^2 p^* \\ \Delta I_s^* \\ \Delta I_l^* \\ \Delta h_r^* \\ \Delta s_r^* \end{pmatrix}_t + \kappa_1 \begin{pmatrix} \Delta m_r^* \\ \Delta y_r^* \\ \Delta^2 p^* \\ \Delta I_s^* \\ \Delta I_l^* \\ \Delta h_r^* \\ \Delta s_r^* \end{pmatrix}_{t-1}$$
$$+\rho_0 + \rho_1 t + \Theta D_t + \varepsilon_t$$

and estimation proceeds in several stages:

1. Test weak exogeneity in CATS: run whole system (including dummies from partial system), check rank, do weak exogeneity test and mark those variables as weakly exogenous whose significance level for the specific rank is above 1%.

2. General to specific modelling in autometrics (p-value 0.0001): run system in autometrics, not including contemporaneous differences from endogenous foreign variables, and fixing domestic lagged levels and differences, constant, trend and centred seasonals.

	m_r^*	y_r^*	Δp^*	I_l^*	I_s^*	s_r^*	h_r^*
US	✓					✓	
EA				✓	✓		
UK	✓	✓	✓	✓	✓	✓	✓
JP	✓			✓	✓	✓	

Table 1: Weakly exogenous foreign variables identified in country models

3. Estimate country models in CATS, excluding variables and setting lagged and contemporaneous differences to zero where suggested by autometrics.

So, to begin with, we need to identify weakly exogenous foreign variables, i.e. those that should not enter with a contemporaneous difference in the model. Necessarily, these will vary from country to country, and it would be expected that the US and euro area have the least weakly exogenous variables as US and euro area variables are more likely to influence ROW variables than the other way round. Erroneously including a contemporaneous effect will result in simultaineity bias (while erroneously excluding it in omitted variables bias). As a gauge of which variables may be weakly exogenous, we use a test of weak exogeneity in CATS. This requires us to determine the rank of the full system (all variables endogenous), which is too large for serious inference, but the only way to look at weak exogeneity, and then accept weak exogeneity where test results are above 1%.

The model excluding some contemporaneous foreign differences is then evaluated in autometrics, where insignificant variables are deleted in order of significance, but a multi-path search strategy ensures that all possibilities are considered. We choose 0.0001 as the significance level at which foreign variables should be retained (while fixing all domestic regressors). autometrics then suggests a final model where all equations include the same variables.

The final step is then to replicate this specification in CATS, excluding those foreign variables that were excluded by autometrics, and restricting the structure of right-hand side differences according to the program's suggestions. In practice we may need to include some more regressors here that we would like to, as it is not possible to exclude both contemporaneous and lagged difference of the same foreign variable in CATS. We are guided here by the initial results regarding weak exogeneity (i.e. excluding contemporaneous differences first where a variable is not weakly exogenous and then turning to lagged differences.[10]

[10]Note that this explains discrepancies between foreign variables listed in Table 2 and the short-term

	m_r^*	y_r^*	Δp^*	I_l^*	I_s^*	s_r^*	h_r^*	Δm_r^*	Δy_r^*	$\Delta^2 p^*$	ΔI_l^*	ΔI_s^*	Δs_r^*	Δh_r^*
US		✓	✓		✓		✓		✓$_{(t-1)}$	✓$_{(t-1)}$			✓$_{(t)}$	
EA	✓	✓	✓	✓			✓		✓$_{(t-1})$			✓$_{(t-1)}$		✓$_{(t-1)}$
UK				✓		✓		✓$_{(t-1)}$	✓$_{(t)}$		✓$_{(t,t-1)}$	✓$_{(t)}$	✓$_{(t)}$	✓$_{(t-1)}$
JP					✓						✓$_{(t)}$	✓$_{(t-1)}$		

Table 2: Foreign variables retained by autometrics in country models

4.5 Cointegration analysis

In this section, we present results from the country cointegration analysis. As outlined above, the CVARs for each country include the same number of domestic variables, but the number of foreign variables is allowed to vary. Foreign variables are treated as weakly exogenous, i.e. a contemporaneous difference of the foreign variables is included on the right hand side of the CVAR, where appropriate. Otherwise, the foreign variables simply enter in lagged levels and lagged differences.

A constant and restricted trend were included for all countries, and all models are well specifed (autometrics ensures that mis-specification tests for non-normality, heteroskedasticity, ARCH effects and auto-correlation are rejected). For all systems a rank of three was found, using evidence from the trace test, the roots and the unrestricted cointegrating relations. However, different relations were identified which are presented in the following section. Short run estimates, trace test results and roots are given in the Appendix.

All models are reasonably stable over the sample judging by results from recursive estimation. Some problems are present with individual coefficients in α and β' but constancy of the likelihood values, eigenvalues, and test results of long-run restrictions is largely not rejected.

4.5.1 UK

Imposing identifying restrictions on the cointegration space, i.e. on the β-matrix, we can obtain the following relations which are accepted with a p-value of 0.562 ($\chi^2(10) = 8.692$).

The first relation is an inflation equation,

$$\Delta p_t = 1.875 m_{rt} + 3.343 h_{rt} + 2.993(s_{rt} - s_{rt}^*) + 2.015 ppp_t - 4.826t + \varepsilon_{1,t}^{UK} \qquad (35)$$

The second relation is a money demand relation,

$$(m_r - y_r)_t = 0.352 I_{lt} + 0.193 I_{st} + 1.682t + \varepsilon_{2,t}^{UK} \qquad (36)$$

coefficient matrices presented in the Appendix.

The third relation is an aggregate demand schedule,

$$y_{rt} = -0.057(I_l - \Delta p)_t + 0.041h_{rt} + 0.075s_{rt} - 0.118ppp_t - 0.037I^*_{lt} + 0.346t + \varepsilon^{UK}_{3,t} \quad (37)$$

Note that excess money does not actually adjust to the second relation, suggesting that money at least in the UK is determined more by supply rather than demand. Lack of error correction is often a useful sign that such imbalances have a tendency to persist and thus this observation in relation to $(m_r - y_r)$ suggests that there might have been self-fulfilling mechanisms at play which could cause an unwarranted build-up of liquidity and result in "bubble-burst behaviour".

The corresponding α and β matrices are given by:

α	α_1	α_2	α_3
Δm_r	0.015	0.046	0.141
	[2.599]	[1.555]	[1.213]
Δy_r	−0.002	0.006	−0.060
	[−0.958]	[0.494]	[−1.219]
$\Delta^2 p$	−0.767	1.333	16.355
	[−2.467]	[0.838]	[2.611]
ΔI_s	−0.323	−1.521	−5.776
	[−4.427]	[−4.072]	[−3.928]
ΔI_l	0.178	1.454	4.563
	[3.000]	[4.800]	[3.826]
Δh_r	0.016	0.083	0.038
	[1.483]	[1.508]	[0.174]
Δs_r	0.090	0.346	1.327
	[4.905]	[3.682]	[3.584]
Δppp	0.034	0.194	0.641
	[1.926]	[2.133]	[1.792]

β'	m_r	y_r	Δp	I_s	I_l	h_r	s_r	ppp	I^*_l	s^*_r	t
β_1	−1.875	0.000	1.000	0.000	0.000	−3.343	−2.993	−2.015	0.000	2.993	4.826
	[−4.856]	[NA]	[NA]	[NA]	[NA]	[−10.382]	[−8.261]	[−4.678]	[NA]	[8.261]	[10.227]
β_2	1.000	−1.000	0.000	−0.193	−0.352	0.000	0.000	0.000	0.000	0.000	−1.682
	[NA]	[NA]	[NA]	[−10.895]	[−11.401]	[NA]	[NA]	[NA]	[NA]	[NA]	[−23.171]
β_3	0.000	1.000	−0.057	0.057	0.000	−0.041	−0.075	0.118	0.037	0.000	−0.346
	[NA]	[NA]	[−22.910]	[22.910]	[NA]	[−2.633]	[−8.967]	[5.112]	[6.765]	[NA]	[−13.699]

4.5.2 Euro area

For the euro area, the following relations are identified with a p-value of 0.474 ($\chi^2(13) = 12.663$):

The first relation is a money demand relation,

$$\begin{aligned}(m_r - y_r)_t &= -0.147\Delta p_t - 0.102I_{lt} - 0.149I^*_{lt} + 0.251I_{st} - 0.157s_{rt} \\ &\quad +0.530(m^*_r - y^*_r)_t + 0.307t + \varepsilon^{EA}_{1,t} \qquad (38)\end{aligned}$$

The second relation is an aggregate demand schedule,

$$\begin{aligned} y_{rt} &= -0.014[(I_l - \Delta p) - (I_l^* - \Delta p^*)]_t + 0.318(h_r - h_r^*)_t + 0.011 s_{rt} \\ &\quad -0.012 ppp_t + 0.145 m_{rt}^* + 0.433 y_{rt}^* + 0.139t + \varepsilon_{2,t}^{EA} \end{aligned} \tag{39}$$

The third relation is a house price equation,

$$\begin{aligned} h_{rt} &= 0.272(m_r - m_r^*)_t + 2.318 y_{rt} + 0.047 I_{lt} + 0.030 s_{rt} \\ &\quad +0.541(h_r^* - y_r^*)_t - 0.042(I_l^* - \Delta p^*)_t - 0.719t + \varepsilon_{3,t}^{EA} \end{aligned} \tag{40}$$

Again money does not adjust to excess money demand. However, the short term interest rate increases with the first relation, suggesting some error correction from the monetary policy instrument. Similarly, output does not react to the second relation. We nevertheless call it an aggregate demand schedule because the long-term real interest rate moves inversely with output, while share and house prices as well as foreign output move together with domestic output.

The corresponding α and β matrices are given by

α	α_1	α_2	α_3
Δm_r	−0.025 [−0.648]	0.796 [1.023]	0.312 [1.195]
Δy_r	0.037 [2.682]	0.297 [1.034]	0.137 [1.426]
$\Delta^2 p$	0.980 [1.261]	88.312 [5.527]	32.785 [6.104]
ΔI_s	1.382 [6.778]	1.876 [0.447]	0.175 [0.124]
ΔI_l	0.441 [1.876]	1.215 [0.252]	0.981 [0.605]
Δh_r	−0.032 [−2.519]	−0.970 [−3.677]	−0.465 [−5.245]
Δs_r	−1.369 [−6.867]	−29.531 [−7.204]	−8.783 [−6.374]
Δppp	−0.449 [−4.744]	−8.698 [−4.466]	−2.038 [−3.113]

β'	m_r	y_r	Δp	I_s	I_l	h_r	s_r	ppp	m_r^*	y_r^*	Δp_t^*	I_l^*	h_r^*	t
β_1	1.000 [NA]	−1.000 [NA]	0.147 [8.315]	−0.251 [−17.366]	0.102 [4.653]	0.000 [NA]	0.157 [9.035]	0.000 [NA]	−0.530 [−4.205]	0.530 [4.205]	0.000 [NA]	0.149 [7.448]	0.000 [NA]	−0.307 [−7.328]
β_2	0.000 [NA]	1.000 [NA]	−0.014 [−14.490]	0.000 [NA]	0.014 [14.490]	−0.318 [−47.675]	−0.011 [−3.336]	0.012 [8.658]	−0.145 [−12.068]	−0.433 [−25.107]	0.014 [14.490]	−0.014 [−14.490]	0.318 [47.675]	−0.139 [−9.687]
β_3	−0.272 [−10.738]	−2.318 [−41.930]	0.000 [NA]	0.000 [NA]	−0.047 [−12.287]	1.000 [NA]	0.030 [3.180]	0.000 [NA]	0.272 [10.738]	0.541 [19.972]	−0.042 [−11.614]	0.042 [11.614]	−0.541 [−19.972]	0.719 [25.713]

4.5.3 US

It proved tricky to find over-identifying restrictions for the US. The restrictions below are not the usual ones, and especially the third is difficult to interpret. The p-value for testing the restrictions is 0.274 ($\chi^2(13) = 15.548$):

The first relation is a share price relation,

$$s_{rt} = 5.071 y_{rt} - 0.106(I_s - \Delta p)_t + 0.428 ppp_t + 0.341(I_s^* - \Delta p^*)_t + 1.624 h_{rt}^* - 2.150t + \varepsilon_{1,t}^{US} \tag{41}$$

The second relation is an inflation relation,

$$\Delta p_t = 3.595 y_{rt} + 0.160 I_{lt} + 0.262 ppp_t - 4.594 y^*_{rt} + 0.491 \Delta p^*_t + \varepsilon^{US}_{2,t} \quad (42)$$

The third relation is relation describing the short-term interest rate,

$$I_{st} = -2.982(m_r - y^*_r)_t + 3.622 \Delta p_t + 3.079 h_{rt} - 1.585 \Delta p^*_t + 4.574 h^*_{rt} - 3.971 t + \varepsilon^{US}_{3,t} \quad (43)$$

Given the lack of evidence for standard relations such as a money demand or aggregate demand equations, we settled for less standard ones. However, these do show error correction in the adjustment coefficients, e.g. share prices react strongly to the first relations which is hence identified as an equity demand schedule. Most unusual is possibly the third relation, which has the short-term interest rate adjusting to it (among others). The short end of the yield curve appears to be influenced by three main components: as liquidity is high the short-term rate is likely to be low (which makes sense from a point of liquidity generation), as the differential between domestic and foreign inflation is high the short-term rate is likely to be high (fitting with an interpretation of a monetary policy rule), and as both domestic and foreign house prices are high the short-term rate tends to be high (again suggesting a policy rule).

The corresponding α and β matrices are given by:

α	α_1	α_2	α_3
Δm_r	0.033 [6.957]	0.047 [3.930]	0.009 [2.959]
Δy_r	0.017 [4.648]	−0.015 [−1.664]	−0.005 [−2.414]
$\Delta^2 p$	−0.394 [−2.913]	−0.768 [−2.254]	−0.055 [−0.678]
ΔI_l	0.175 [2.419]	0.012 [0.064]	−0.112 [−2.565]
ΔI_s	0.176 [2.401]	−0.702 [−3.791]	−0.240 [−5.410]
Δh_r	0.017 [3.416]	−0.042 [−3.413]	−0.012 [−3.936]
Δs_r	−0.096 [−3.306]	0.317 [4.340]	0.094 [5.332]
Δppp	0.068 [2.226]	−0.248 [−3.224]	−0.043 [−2.345]

β'	m_r	y_r	Δp	I_l	I_s	h_r	s_r	ppp	y^*_r	I^*_s	Δp^*	h^*_r	t
β_1	0.000 [NA]	−5.071 [−5.710]	−0.106 [−2.802]	0.000 [NA]	0.106 [2.802]	0.000 [NA]	1.000 [NA]	−0.428 [−4.178]	0.000 [NA]	−0.341 [−5.461]	0.341 [5.461]	−1.624 [−3.829]	2.150 [2.833]
β_2	0.000 [NA]	−3.595 [−9.853]	1.000 [NA]	−0.160 [−5.594]	0.000 [NA]	0.000 [NA]	0.000 [NA]	−0.262 [−3.965]	4.594 [8.609]	0.000 [NA]	−0.491 [−9.818]	0.000 [NA]	0.000 [NA]
β_3	2.982 [5.488]	−2.982 [−5.488]	−3.622 [−20.586]	0.000 [NA]	1.000 [NA]	−3.079 [−5.246]	0.000 [NA]	0.000 [NA]	0.000 [NA]	0.000 [NA]	1.585 [6.890]	−4.574 [−5.561]	3.971 [6.761]

4.5.4 Japan

For Japan, the following relations are identified with a p-value of 0.113 ($\chi^2(10) = 15.561$):

The first relation is a money demand relation,

$$(m_r - y_r)_t = -0.334\Delta p_t - 0.391(I_l - I_s)_t + \varepsilon_{1,t}^{JP} \tag{44}$$

The second relation is an inflation equation,

$$\Delta p_t = 0.375 s_{rt} + 0.718 ppp_t - 1.693t + \varepsilon_{2,t}^{JP} \tag{45}$$

The third relation is a house price equation,

$$h_{rt} = 1.423 m_{rt} - 0.207(I_s - \Delta p)_t + 0.047 I_{lt} - 0.117 s_{rt} - 0.299 ppp_t + 0.064 I_{st}^{*} - 1.582t + \varepsilon_{3,t}^{JP} \tag{46}$$

Compared with the other countries what is possibly most surprising in the case of Japan, is the relative simplicity of the relations, especially the first and second. They are both quite clear in their interpretation, involving only a few variables. Moreover, adjustment to the relations is also as expected.

The corresponding α and β matrices are given by:

α	α_1	α_2	α_3
Δm_r	−0.038 [−3.070]	0.002 [0.457]	−0.076 [−6.480]
Δy_r	0.031 [1.744]	−0.007 [−1.206]	−0.014 [−0.838]
$\Delta^2 p$	1.052 [1.194]	−1.288 [−4.555]	0.980 [1.191]
ΔI_l	−0.507 [−2.026]	0.127 [1.586]	−0.469 [−2.007]
ΔI_s	0.585 [5.578]	−0.206 [−6.128]	−0.327 [−3.336]
Δh_r	−0.041 [−3.095]	0.014 [3.299]	−0.044 [−3.560]
Δs_r	−0.577 [−3.571]	0.077 [1.480]	−0.566 [−3.751]
Δppp	−0.016 [−0.176]	−0.002 [−0.067]	−0.281 [−3.305]

β'	m_r	y_r	Δp	I_l	I_s	h_r	s_r	ppp	I_s^*	t
β_1	1.000 [NA]	−1.000 [NA]	0.334 [19.072]	0.391 [13.741]	−0.391 [−13.741]	0.000 [NA]	0.000 [NA]	0.000 [NA]	0.000 [NA]	0.000 [NA]
β_2	0.000 [NA]	0.000 [NA]	1.000 [NA]	0.000 [NA]	0.000 [NA]	0.000 [NA]	−0.375 [−4.834]	−0.718 [−5.461]	0.000 [NA]	1.693 [18.032]
β_3	−1.423 [−10.361]	0.000 [NA]	−0.207 [−11.736]	0.000 [NA]	0.207 [11.736]	1.000 [NA]	0.117 [4.095]	0.299 [7.623]	−0.064 [−3.608]	1.582 [10.365]

5 Conclusion

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6 Appendix

This appendix includes some additional estimates for each country VARX* model, i.e. estimates of $\mathbf{\Gamma}_{i1}$,$\mathbf{\Upsilon}_{i0}$ and $\mathbf{\Upsilon}_{i1}$.

6.1 Graphs

Graphs of all national and country specific foreign time series to be included!

6.2 UK

6.2.1 I1 Analysis - Rank Test Statistics

Trace test results							
$p-r$	r	Eigenvalue	Trace	Trace*	CV 95%	P-Value	P-Value*
8	0	0.695	348.226	299.166	222.634	0.000	0.000
7	1	0.518	235.504	202.641	181.648	0.000	0.003
6	2	0.502	166.221	128.341	144.639	0.002	0.283
5	3	0.308	99.926	74.861	111.597	0.217	0.908
4	4	0.257	64.973	43.491	82.501	0.462	0.988
3	5	0.232	36.811	31.161	57.316	0.735	0.919
2	6	0.094	11.795	10.238	35.956	0.990	0.997
1	7	0.025	2.432	NA	18.155	0.995	NA

6.2.2 Roots of the Companion Matrix

The Roots of the companion form matrix at preferred rank ($r = 3$)				
	Real	Imaginary	Modulus	Argument
Root1	1.000	0.000	1.000	0.000
Root2	1.000	0.000	1.000	0.000
Root3	1.000	0.000	1.000	0.000
Root4	1.000	0.000	1.000	0.000
Root5	1.000	0.000	1.000	0.000
Root6	0.810	−0.000	0.810	−0.000
Root7	0.589	0.000	0.589	0.000
Root8	−0.354	0.304	0.466	2.432
Root9	−0.354	−0.304	0.466	−2.432
Root10	0.324	0.316	0.452	0.774
Root11	0.324	−0.316	0.452	−0.774
Root12	−0.068	−0.423	0.428	−1.729
Root13	−0.068	0.423	0.428	1.729
Root14	0.366	0.000	0.366	0.000
Root15	0.038	−0.071	0.081	−1.078
Root16	0.038	0.071	0.081	1.078

6.2.3 The Short-Run Matrices

$\mathbf{\Gamma}_1$	$\Delta m_{r,-1}$	$\Delta y_{r,-1}$	$\Delta^2 p_{-1}$	$\Delta I_{s,-1}$	$\Delta I_{l,-1}$	$\Delta h_{r,-1}$	$\Delta s_{r,-1}$	Δppp_{-1}
Δm_r	0.214 [2.289]	0.662 [3.090]	−0.003 [−2.022]	0.003 [0.600]	0.033 [2.643]	−0.102 [−2.051]	0.020 [1.186]	−0.006 [−0.233]
Δy_r	−0.014 [−0.349]	0.136 [1.512]	−0.002 [−2.423]	0.005 [2.272]	0.017 [3.233]	0.008 [0.398]	0.009 [1.330]	−0.009 [−0.870]
$\Delta^2 p$	−9.679 [−1.931]	−20.677 [−1.796]	0.235 [2.814]	−0.164 [−0.578]	−1.213 [−1.836]	5.979 [2.240]	1.522 [1.719]	−2.189 [−1.582]
ΔI_s	0.850 [0.722]	−1.903 [−0.705]	0.029 [1.452]	0.054 [0.813]	0.579 [3.731]	1.254 [2.001]	−0.138 [−0.665]	0.048 [0.148]
ΔI_l	0.440 [0.461]	−5.416 [−2.472]	0.022 [1.400]	0.127 [2.346]	0.227 [1.805]	−1.033 [−2.034]	0.049 [0.289]	−0.000 [−0.002]
Δh_r	0.150 [0.868]	0.652 [1.641]	−0.006 [−2.078]	−0.007 [−0.764]	0.034 [1.500]	0.257 [2.794]	0.001 [0.031]	0.025 [0.515]
Δs_r	−0.212 [−0.715]	−0.316 [−0.465]	−0.014 [−2.747]	0.007 [0.434]	−0.006 [−0.162]	−0.428 [−2.710]	−0.020 [−0.390]	0.169 [2.072]
Δppp	−0.020 [−0.069]	−2.088 [−3.177]	0.002 [0.487]	−0.043 [−2.652]	0.153 [4.044]	0.054 [0.357]	−0.011 [−0.216]	0.256 [3.244]

	ΔI_l^*	Δs_r^*		$\Delta I_{l,-1}^*$	$\Delta s_{r,-1}^*$
Δm_r	−0.064 [−4.618]	0.019 [1.143]	Δm_r	0.006 [0.530]	0.000 [0.000]
Δy_r	−0.015 [−2.578]	0.021 [3.016]	Δy_r	0.014 [3.217]	0.000 [0.000]
$\Delta^2 p$	2.691 [3.621]	0.317 [0.353]	$\Delta^2 p$	0.369 [0.643]	0.000 [0.000]
ΔI_s	−0.839 [−4.812]	0.592 [2.804]	ΔI_s	−0.151 [−1.119]	0.000 [0.000]
ΔI_l	0.487 [3.442]	−0.182 [−1.062]	ΔI_l	−0.066 [−0.606]	0.000 [0.000]
Δh_r	−0.077 [−2.996]	0.002 [0.054]	Δh_r	0.038 [1.941]	0.000 [0.000]
Δs_r	0.084 [1.901]	0.852 [16.022]	Δs_r	−0.028 [−0.820]	0.000 [0.000]
Δppp	−0.133 [−3.139]	0.052 [1.007]	Δppp	−0.005 [−0.161]	0.000 [0.000]

	Δy_r^*	ΔI_s^*	$\Delta m_{r,-1}^*$	$\Delta h_{r,-1}^*$	DP_93_3	DP_92_4	DP_88_3
Δm_r	−0.195 [−0.754]	0.037 [2.771]	0.057 [0.362]	0.668 [3.796]	−0.061 [−7.665]	−0.015 [−1.868]	0.015 [1.836]
Δy_r	0.267 [2.458]	−0.015 [−2.665]	0.141 [2.127]	0.076 [1.024]	−0.003 [−0.938]	0.002 [0.672]	0.004 [1.326]
$\Delta^2 p$	11.978 [0.861]	−0.695 [−0.975]	9.375 [1.103]	−28.522 [−3.016]	0.942 [2.209]	−0.747 [−1.719]	0.687 [1.587]
ΔI_l	2.465 [0.755]	0.966 [5.770]	7.985 [4.002]	−8.459 [−3.811]	0.128 [1.278]	−0.364 [−3.574]	0.553 [5.450]
ΔI_s	13.188 [4.981]	−0.533 [−3.927]	−5.872 [−3.629]	9.231 [5.127]	−0.022 [−0.274]	−0.163 [−1.969]	−0.041 [−0.503]
Δh_r	−0.194 [−0.403]	0.043 [1.754]	0.824 [2.808]	0.505 [1.547]	0.002 [0.169]	−0.019 [−1.270]	0.052 [3.455]
Δs_r	−1.996 [−2.427]	0.065 [1.545]	−0.202 [−0.402]	0.531 [0.949]	−0.017 [−0.670]	0.075 [2.940]	−0.032 [−1.251]
Δppp	0.187 [0.235]	0.059 [1.454]	0.449 [0.925]	0.016 [0.030]	−0.008 [−0.331]	0.182 [7.334]	0.004 [0.164]

	$CSEAS1$	$CSEAS2$	$CSEAS3$		$CONSTANT$
Δm_r	−0.002 [−0.652]	−0.007 [−2.366]	0.001 [0.203]	Δm_r	−2.689 [−0.969]
Δy_r	0.000 [0.283]	0.001 [0.759]	0.001 [0.751]	Δy_r	1.489 [1.278]
$\Delta^2 p$	0.330 [1.724]	0.059 [0.350]	0.069 [0.385]	$\Delta^2 p$	−502.073 [−3.370]
ΔI_l	0.029 [0.654]	0.024 [0.601]	0.045 [1.076]	ΔI_l	131.883 [3.772]
ΔI_s	−0.109 [−2.987]	−0.070 [−2.202]	−0.059 [−1.738]	ΔI_s	−109.325 [−3.855]
Δh_r	0.029 [4.426]	0.022 [3.830]	−0.000 [−0.032]	Δh_r	0.298 [0.058]
Δs_r	−0.017 [−1.544]	0.002 [0.162]	0.004 [0.419]	Δs_r	−29.158 [−3.311]
Δppp	−0.000 [−0.001]	−0.003 [−0.274]	0.003 [0.300]	Δppp	−14.711 [−1.729]

6.3 Euro area

6.3.1 I1 Analysis - Rank Test Statistics

Trace test results							
$p-r$	r	Eigenvalue	Trace	Trace*	CV 95%	P-Value	P-Value*
8	0	0.743	511.188	402.815	275.106	0.000	0.000
7	1	0.715	383.604	292.414	227.970	0.000	0.000
6	2	0.562	265.640	201.860	184.782	0.000	0.006
5	3	0.461	187.974	143.334	145.520	0.000	0.065
4	4	0.353	129.801	102.035	110.150	0.002	0.141
3	5	0.322	88.864	73.714	78.609	0.008	0.106
2	6	0.274	52.394	39.360	50.766	0.036	0.329
1	7	0.211	22.248	11.978	26.245	0.136	0.759

6.3.2 Roots of the Companion Matrix

The Roots of the companion form matrix at preferred rank ($r = 3$)				
	Real	Imaginary	Modulus	Argument
Root1	1.000	0.000	1.000	0.000
Root2	1.000	0.000	1.000	0.000
Root3	1.000	−0.000	1.000	−0.000
Root4	1.000	0.000	1.000	0.000
Root5	1.000	0.000	1.000	0.000
Root6	0.862	0.000	0.862	0.000
Root7	0.664	−0.308	0.732	−0.435
Root8	0.664	0.308	0.732	0.435
Root9	0.044	−0.471	0.473	−1.477
Root10	0.044	0.471	0.473	1.477
Root11	0.415	0.000	0.415	0.000
Root12	−0.362	−0.098	0.375	−2.878
Root13	−0.362	0.098	0.375	2.878
Root14	0.371	0.000	0.371	0.000
Root15	−0.020	−0.113	0.115	−1.747
Root16	−0.020	0.113	0.115	1.747

6.3.3 The Short-Run Matrices

$\mathbf{\Gamma}_1$	$\Delta m_{r,-1}$	$\Delta y_{r,-1}$	$\Delta^2 p_{-1}$	$\Delta I_{s,-1}$	$\Delta I_{l,-1}$	$\Delta h_{r,-1}$	$\Delta s_{r,-1}$	Δppp_{-1}
Δm_r	−0.11 [−1.16]	−0.21 [−0.69]	0.01 [1.13]	0.01 [0.42]	0.00 [0.01]	0.29 [1.32]	0.00 [0.13]	−0.02 [−0.51]
Δy_r	0.00 [0.11]	−0.25 [−2.23]	−0.00 [−0.05]	0.02 [3.53]	0.01 [1.08]	0.26 [3.15]	0.01 [0.96]	0.00 [0.16]
$\Delta^2 p$	6.46 [3.27]	2.58 [0.42]	0.03 [0.28]	−0.13 [−0.45]	0.11 [0.33]	−3.53 [−0.78]	0.63 [1.83]	0.03 [0.04]
ΔI_s	−1.09 [−2.10]	−1.94 [−1.19]	−0.10 [−3.78]	0.26 [3.29]	0.26 [2.87]	6.63 [5.60]	−0.05 [−0.51]	0.38 [2.14]
ΔI_l	0.49 [0.81]	−3.05 [−1.63]	0.01 [0.27]	−0.07 [−0.74]	0.53 [5.19]	5.25 [3.85]	0.03 [0.31]	0.06 [0.28]
Δh_r	−0.06 [−1.76]	−0.13 [−1.26]	−0.00 [−0.84]	−0.00 [−0.49]	0.02 [3.71]	0.45 [6.07]	0.01 [1.26]	0.01 [1.36]
Δs_r	−0.80 [−1.57]	5.52 [3.47]	−0.07 [−2.69]	−0.01 [−0.10]	0.27 [3.11]	0.69 [0.60]	0.10 [1.12]	0.04 [0.24]
Δppp	−0.02 [−0.07]	1.86 [2.46]	0.00 [0.33]	0.02 [0.57]	0.02 [0.52]	1.70 [3.09]	−0.20 [−4.68]	0.29 [3.59]

	Δm_r^*	Δy_r^*	$\Delta^2 p^*$	ΔI_l^*	Δh_r^*
Δm_r	0.00 [0.00]	0.00 [0.00]	0.00 [0.00]	−0.01 [−0.50]	0.00 [0.00]
Δy_r	0.00 [0.00]	0.00 [0.00]	0.00 [0.00]	−0.01 [−2.29]	0.00 [0.00]
$\Delta^2 p$	0.00 [0.00]	0.00 [0.00]	0.00 [0.00]	−0.42 [−1.41]	0.00 [0.00]
ΔI_l	0.00 [0.00]	0.00 [0.00]	0.00 [0.00]	−0.09 [−1.12]	0.00 [0.00]
ΔI_s	0.00 [0.00]	0.00 [0.00]	0.00 [0.00]	−0.23 [−2.55]	0.00 [0.00]
Δh_r	0.00 [0.00]	0.00 [0.00]	0.00 [0.00]	−0.00 [−0.77]	0.00 [0.00]
Δs_r	0.00 [0.00]	0.00 [0.00]	0.00 [0.00]	0.06 [0.83]	0.00 [0.00]
Δppp	0.00 [0.00]	0.00 [0.00]	0.00 [0.00]	0.11 [3.03]	0.00 [0.00]

	$\Delta m_{r,-1}^*$	$\Delta y_{r,-1}^*$	$\Delta^2 p_{-1}^*$	$\Delta I_{l,-1}^*$	$\Delta h_{r,-1}^*$
Δm_r	0.63 [2.58]	0.14 [0.35]	0.00 [0.88]	0.00 [0.00]	−0.33 [−1.23]
Δy_r	0.10 [1.07]	0.26 [1.79]	0.00 [0.42]	0.00 [0.00]	−0.19 [−2.00]
$\Delta^2 p$	7.21 [1.44]	11.77 [1.46]	0.32 [2.89]	0.00 [0.00]	−4.01 [−0.74]
ΔI_l	−2.15 [−1.63]	−0.35 [−0.17]	−0.00 [−0.03]	0.00 [0.00]	−2.71 [−1.91]
ΔI_s	1.89 [1.24]	−1.42 [−0.58]	−0.00 [−0.08]	0.00 [0.00]	−1.85 [−1.13]
Δh_r	0.02 [0.23]	−0.12 [−0.90]	−0.00 [−1.38]	0.00 [0.00]	0.25 [2.75]
Δs_r	−0.72 [−0.56]	−9.55 [−4.62]	0.05 [1.80]	0.00 [0.00]	5.04 [3.63]
Δppp	0.03 [0.05]	−2.90 [−2.96]	0.02 [1.74]	0.00 [0.00]	0.84 [1.27]

	$\Delta I_{s,-1}^*$	DP_85_4	DP_92_1	DP_88_3	DP_90_1
Δm_r	0.01 [0.71]	0.01 [0.98]	−0.00 [−0.30]	0.00 [0.27]	−0.03 [−2.18]
Δy_r	−0.00 [−0.19]	−0.00 [−0.50]	0.01 [3.25]	0.01 [2.82]	0.01 [2.68]
$\Delta^2 p$	−0.72 [−2.51]	0.48 [2.08]	−0.47 [−2.00]	0.11 [0.46]	0.42 [1.70]
ΔI_l	0.11 [1.42]	−0.24 [−3.94]	0.48 [7.87]	0.39 [6.30]	−0.00 [−0.06]
ΔI_s	0.04 [0.51]	−0.27 [−3.87]	0.19 [2.75]	0.27 [3.77]	0.02 [0.25]
Δh_r	0.02 [5.17]	−0.01 [−1.41]	0.00 [0.68]	−0.00 [−1.26]	0.00 [0.68]
Δs_r	0.20 [2.74]	0.09 [1.47]	−0.02 [−0.27]	0.08 [1.35]	−0.01 [−0.21]
Δppp	0.01 [0.29]	−0.04 [−1.55]	−0.00 [−0.03]	0.08 [2.68]	−0.10 [−3.44]

	$CSEAS1$	$CSEAS2$	$CSEAS3$	$CONSTANT$	
Δm_r	0.02 [3.50]	0.01 [2.75]	0.05 [8.91]	Δm_r	5.82 [1.32]
Δy_r	0.00 [0.55]	0.00 [0.99]	0.00 [1.63]	Δy_r	3.19 [1.96]
$\Delta^2 p$	0.08 [0.69]	−0.03 [−0.29]	−0.05 [−0.43]	$\Delta^2 p$	553.02 [6.12]
ΔI_l	−0.03 [−0.83]	−0.03 [−1.39]	−0.02 [−0.76]	ΔI_l	−13.78 [−0.58]
ΔI_s	0.03 [0.81]	0.00 [0.14]	0.08 [2.53]	ΔI_s	33.30 [1.22]
Δh_r	−0.00 [−0.39]	0.00 [0.44]	0.00 [1.39]	Δh_r	−11.17 [−7.49]
Δs_r	−0.06 [−1.87]	−0.08 [−3.55]	−0.07 [−2.62]	Δs_r	−78.84 [−3.40]
Δppp	0.01 [0.76]	−0.00 [−0.11]	−0.01 [−0.57]	Δppp	3.48 [0.32]

6.4 US

6.4.1 I1 Analysis - Rank Test Statistics

Trace test results							
$p-r$	r	Eigenvalue	Trace	Trace*	CV 95%	P-Value	P-Value*
8	0	0.661	470.661	375.401	257.682	0.000	0.000
7	1	0.636	369.072	251.800	212.598	0.000	0.000
6	2	0.578	274.108	178.940	171.470	0.000	0.020
5	3	0.483	193.023	115.310	134.281	0.000	0.351
4	4	0.373	131.022	78.502	101.002	0.000	0.546
3	5	0.360	87.118	55.878	71.576	0.002	0.416
2	6	0.240	45.178	22.271	45.885	0.058	0.916
1	7	0.186	19.346	6.886	23.588	0.153	0.949

6.4.2 Roots of the Companion Matrix

The Roots of the companion form matrix at preferred rank ($r = 3$)				
	Real	Imaginary	Modulus	Argument
Root1	1.000	0.000	1.000	0.000
Root2	1.000	0.000	1.000	0.000
Root3	1.000	0.000	1.000	0.000
Root4	1.000	0.000	1.000	0.000
Root5	1.000	0.000	1.000	0.000
Root6	0.942	0.000	0.942	0.000
Root7	0.803	0.134	0.814	0.166
Root8	0.803	−0.134	0.814	−0.166
Root9	0.562	0.301	0.638	0.492
Root10	0.562	−0.301	0.638	−0.492
Root11	−0.214	0.404	0.457	2.057
Root12	−0.214	−0.404	0.457	−2.057
Root13	−0.335	−0.000	0.335	−3.142
Root14	0.258	−0.000	0.258	−0.000
Root15	−0.210	0.000	0.210	3.142
Root16	0.132	0.000	0.132	0.000

6.4.3 The Short-Run Matrices

$\mathbf{\Gamma}_1$	$\Delta m_{r,-1}$	$\Delta y_{r,-1}$	$\Delta^2 p_{-1}$	$\Delta I_{l,-1}$	$\Delta I_{s,-1}$	$\Delta h_{r,-1}$	$\Delta s_{r,-1}$	Δppp_{-1}
Δm_r	−0.036	−0.156	0.000	−0.034	0.024	0.104	−0.053	0.049
	[−0.397]	[−0.988]	[0.061]	[−4.452]	[4.149]	[1.033]	[−4.406]	[2.829]
Δy_r	−0.112	0.052	−0.003	−0.000	−0.003	−0.158	0.014	0.006
	[−1.622]	[0.433]	[−0.837]	[−0.071]	[−0.774]	[−2.050]	[1.560]	[0.473]
$\Delta^2 p$	4.747	3.360	−0.146	−0.101	0.117	2.812	−0.053	−0.747
	[1.854]	[0.750]	[−1.310]	[−0.469]	[0.719]	[0.983]	[−0.156]	[−1.521]
ΔI_l	2.226	6.170	−0.257	0.337	−0.273	−2.244	0.697	0.534
	[1.620]	[2.569]	[−4.297]	[2.921]	[−3.132]	[−1.462]	[3.784]	[2.023]
ΔI_s	2.486	1.463	−0.135	0.073	0.274	−3.043	0.703	−0.048
	[1.785]	[0.601]	[−2.238]	[0.623]	[3.103]	[−1.956]	[3.767]	[−0.179]
Δh_r	0.042	−0.350	0.007	−0.023	0.007	0.316	−0.023	−0.020
	[0.445]	[−2.142]	[1.644]	[−2.957]	[1.172]	[3.035]	[−1.862]	[−1.090]
Δs_r	−0.750	0.540	0.017	0.080	0.029	1.960	−0.010	0.189
	[−1.364]	[0.562]	[0.720]	[1.731]	[0.833]	[3.190]	[−0.139]	[1.788]
Δppp	−1.772	−2.402	0.041	−0.059	0.018	−0.919	−0.055	0.063
	[−3.063]	[−2.376]	[1.612]	[−1.216]	[0.479]	[−1.422]	[−0.712]	[0.563]

	Δy_r^*	ΔI_s^*	$\Delta^2 p^*$	Δh_r^*
Δm_r	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]
Δy_r	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]
$\Delta^2 p$	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]
ΔI_l	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]
ΔI_s	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]
Δh_r	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]
Δs_r	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]
Δppp	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]	0.000 [0.000]

	$\Delta y_{r,-1}^*$	$\Delta I_{s,-1}^*$	$\Delta^2 p_{-1}^*$	$\Delta h_{r,-1}^*$
Δm_r	−0.080 [−0.550]	0.009 [1.223]	−0.002 [−0.782]	−0.202 [−2.356]
Δy_r	0.326 [2.930]	0.006 [1.128]	−0.001 [−0.457]	−0.099 [−1.507]
$\Delta^2 p$	4.612 [1.116]	−0.333 [−1.605]	−0.131 [−1.988]	3.725 [1.529]
ΔI_l	0.814 [0.367]	−0.169 [−1.517]	0.096 [2.737]	3.694 [2.827]
ΔI_s	4.983 [2.217]	0.029 [0.255]	−0.011 [−0.305]	1.897 [1.432]
Δh_r	−0.102 [−0.679]	0.028 [3.702]	−0.005 [−2.002]	−0.055 [−0.622]
Δs_r	−2.184 [−2.460]	−0.011 [−0.248]	0.009 [0.661]	−0.977 [−1.868]
Δppp	0.803 [0.860]	0.042 [0.896]	−0.040 [−2.689]	0.373 [0.679]

	DP_84_2	DP_84_4	Δs_r^*
Δm_r	0.002 [0.301]	0.004 [0.700]	0.004 [0.393]
Δy_r	0.007 [1.553]	−0.003 [−0.565]	0.017 [2.233]
$\Delta^2 p$	−0.122 [−0.756]	−0.340 [−2.037]	−0.109 [−0.376]
ΔI_l	0.392 [4.515]	−0.078 [−0.871]	0.178 [1.147]
ΔI_s	0.380 [4.311]	−0.519 [−5.723]	0.093 [0.590]
Δh_r	0.001 [0.227]	−0.000 [−0.016]	0.017 [1.605]
Δs_r	−0.068 [−1.941]	−0.005 [−0.139]	0.562 [9.058]
Δppp	0.005 [0.139]	−0.051 [−1.354]	−0.147 [−2.257]

	$CSEAS1$	$CSEAS2$	$CSEAS3$
Δm_r	0.002 [0.867]	0.006 [2.896]	0.013 [7.152]
Δy_r	−0.001 [−0.504]	−0.000 [−0.241]	0.001 [0.783]
$\Delta^2 p$	−0.199 [−3.269]	−0.218 [−3.557]	−0.179 [−3.350]
ΔI_l	−0.003 [−0.098]	−0.079 [−2.410]	0.002 [0.080]
ΔI_s	0.057 [1.704]	0.013 [0.403]	0.068 [2.347]
Δh_r	0.002 [0.790]	0.007 [3.035]	−0.000 [−0.250]
Δs_r	−0.009 [−0.699]	−0.007 [−0.518]	−0.010 [−0.853]
Δppp	−0.007 [−0.500]	−0.011 [−0.819]	−0.007 [−0.614]

	$CONSTANT$
Δm_r	4.021 [5.238]
Δy_r	2.910 [4.956]
$\Delta^2 p$	−40.083 [−1.838]
ΔI_l	22.975 [1.964]
ΔI_s	40.244 [3.394]
Δh_r	3.493 [4.395]
Δs_r	−21.230 [−4.533]
Δppp	16.552 [3.360]

6.5 Japan

6.5.1 I1 Analysis - Rank Test Statistics

Trace test results							
$p-r$	r	Eigenvalue	Trace	Trace*	CV 95%	P-Value	P-Value*
8	0	0.628	372.027	276.076	204.989	0.000	0.000
7	1	0.556	278.971	201.588	166.049	0.000	0.000
6	2	0.494	202.699	152.912	131.097	0.000	0.001
5	3	0.447	138.716	103.627	100.127	0.000	0.028
4	4	0.289	82.979	51.002	73.128	0.007	0.709
3	5	0.244	50.942	36.925	50.075	0.041	0.451
2	6	0.123	24.599	14.748	30.912	0.223	0.837
1	7	0.122	12.238	6.192	15.331	0.144	0.671

6.5.2 Roots of the Companion Matrix

The Roots of the companion form matrix at preferred rank ($r = 3$)				
	Real	Imaginary	Modulus	Argument
Root1	1.000	−0.000	1.000	−0.000
Root2	1.000	0.000	1.000	0.000
Root3	1.000	0.000	1.000	0.000
Root4	1.000	0.000	1.000	0.000
Root5	1.000	0.000	1.000	0.000
Root6	0.861	0.115	0.868	0.133
Root7	0.861	−0.115	0.868	−0.133
Root8	0.717	0.000	0.717	0.000
Root9	0.302	−0.348	0.461	−0.857
Root10	0.302	0.348	0.461	0.857
Root11	−0.453	0.000	0.453	3.142
Root12	0.127	0.335	0.358	1.207
Root13	0.127	−0.335	0.358	−1.207
Root14	−0.324	−0.000	0.324	−3.142
Root15	0.145	−0.124	0.191	−0.708
Root16	0.145	0.124	0.191	0.708

6.5.3 The Short-Run Matrices

$\mathbf{\Gamma}_1$	$\Delta m_{r,-1}$	$\Delta y_{r,-1}$	$\Delta^2 p_{-1}$	$\Delta I_{l,-1}$	$\Delta I_{s,-1}$	$\Delta h_{r,-1}$	$\Delta s_{r,-1}$	Δppp_{-1}
Δm_r	0.114 [0.940]	0.127 [1.866]	0.002 [1.105]	0.008 [1.180]	0.016 [2.124]	0.318 [4.100]	0.011 [1.607]	0.024 [1.924]
Δy_r	−0.010 [−0.057]	0.100 [1.052]	−0.002 [−0.883]	−0.019 [−2.034]	−0.012 [−1.103]	0.405 [3.726]	0.012 [1.173]	−0.031 [−1.762]
$\Delta^2 p$	−9.200 [−1.083]	−9.594 [−2.009]	−0.172 [−1.687]	0.371 [0.780]	0.614 [1.175]	−5.664 [−1.039]	−1.327 [−2.657]	−0.131 [−0.148]
ΔI_l	1.934 [0.802]	−0.439 [−0.324]	0.001 [0.046]	−0.216 [−1.601]	−0.002 [−0.014]	0.648 [0.419]	0.188 [1.324]	0.326 [1.298]
ΔI_s	−3.777 [−3.738]	0.579 [1.019]	−0.039 [−3.247]	0.009 [0.154]	0.372 [5.973]	3.357 [5.176]	−0.113 [−1.896]	0.130 [1.235]
Δh_r	0.062 [0.484]	0.071 [0.990]	0.001 [0.806]	0.008 [1.172]	0.003 [0.396]	0.895 [10.893]	0.021 [2.745]	0.014 [1.030]
Δs_r	−0.312 [−0.200]	0.712 [0.813]	0.031 [1.663]	0.040 [0.456]	−0.036 [−0.375]	−0.712 [−0.713]	0.380 [4.144]	0.076 [0.468]
Δppp	−0.891 [−1.017]	−0.455 [−0.924]	−0.036 [−3.462]	0.024 [0.493]	0.029 [0.537]	0.890 [1.581]	−0.126 [−2.441]	0.154 [1.692]

	ΔI_s^*
Δm_r	0.000 [0.000]
Δy_r	0.000 [0.000]
$\Delta^2 p$	0.000 [0.000]
ΔI_l	0.000 [0.000]
ΔI_s	0.000 [0.000]
Δh_r	0.000 [0.000]
Δs_r	0.000 [0.000]
Δppp	0.000 [0.000]

	$\Delta I_{s,-1}^*$
Δm_r	−0.015 [−2.720]
Δy_r	−0.000 [−0.002]
$\Delta^2 p$	0.595 [1.548]
ΔI_l	−0.078 [−0.714]
ΔI_s	−0.173 [−3.784]
Δh_r	−0.001 [−0.252]
Δs_r	0.028 [0.393]
Δppp	−0.039 [−0.974]

	DP_84_4	DP_95_2	DP_89_2	ΔI_l^*
Δm_r	0.007 [1.433]	0.007 [1.375]	−0.023 [−4.344]	0.017 [2.283]
Δy_r	0.015 [2.064]	0.006 [0.870]	−0.027 [−3.682]	0.005 [0.482]
$\Delta^2 p$	−0.041 [−0.114]	−0.483 [−1.328]	0.817 [2.197]	−0.956 [−1.847]
ΔI_l	−0.246 [−2.407]	−0.085 [−0.823]	−0.069 [−0.653]	0.355 [2.413]
ΔI_s	0.299 [6.990]	−0.117 [−2.711]	0.056 [1.267]	0.044 [0.709]
Δh_r	0.000 [0.049]	0.006 [1.147]	−0.004 [−0.781]	0.017 [2.182]
Δs_r	−0.013 [−0.202]	−0.084 [−1.255]	−0.089 [−1.312]	0.178 [1.878]
Δppp	−0.069 [−1.855]	−0.101 [−2.700]	0.041 [1.060]	0.077 [1.439]

	$CSEAS1$	$CSEAS2$	$CSEAS3$
Δm_r	−0.001 [−0.572]	−0.000 [−0.083]	−0.003 [−1.873]
Δy_r	−0.004 [−1.882]	−0.002 [−0.994]	−0.003 [−1.184]
$\Delta^2 p$	0.178 [1.597]	−0.016 [−0.133]	−0.057 [−0.541]
ΔI_l	0.009 [0.298]	−0.043 [−1.279]	−0.005 [−0.154]
ΔI_s	−0.002 [−0.164]	0.038 [2.700]	0.018 [1.444]
Δh_r	−0.002 [−1.021]	0.001 [0.442]	0.001 [0.482]
Δs_r	0.004 [0.209]	−0.023 [−1.078]	−0.026 [−1.331]
Δppp	−0.012 [−1.001]	−0.005 [−0.408]	−0.016 [−1.451]

	$CONSTANT$
Δm_r	−3.059 [−6.452]
Δy_r	−0.588 [−0.885]
$\Delta^2 p$	34.611 [1.038]
ΔI_l	−18.481 [−1.952]
ΔI_s	−14.215 [−3.584]
Δh_r	−1.732 [−3.445]
Δs_r	−22.565 [−3.692]
Δppp	−11.405 [−3.314]