Inattentive Firms and the Mode of Competition*

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Abstract

We analyze a model in which it is costly for firms to absorb and process information. Therefore firms choose to be rationally inattentive towards changes in demand for some time period. We show that the choices of the strategy variable (price vs. quantity) and the inattentiveness period are interrelated. Prices adapt more flexible to demand shocks but yield lower profits in the short run. Therefore high information costs result in price competition. In this case firms adjust simultaneously while adjustment times differ and alternate under quantity competition. We show how the choice of the strategy variable depends on the adjustment costs and discuss possible extensions.

Keywords: Inattentiveness, Adjustment costs, Imperfect competition

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1 Introduction

It is a common observation in real world that firms do not react to shocks immediately even if they posses the relevant information. An obvious reason for this is that there are substantial costs of absorbing and processing information. For example, Radner (1992) points out that an important goal of managerial occupations is to only absorb and process the relevant information for making decisions. In this paper we address this phenomenon by modeling and analyzing the rational inattentiveness of firms. We show that the duration of inattentiveness is highly affected by the firm's choice of their strategy variable (price or quantity). The strategy variable, in turn, is related to the costs of information processing and with the duration of inattention.

Inattentiveness only recently gained some attention in the economics literature and there are different approaches to model agents who behave rationally inattentive.[1] Here we closely follow Reis' (2006) approach by assuming that firms have to incur costs to find out and process the relevant information. This makes them stay inattentive for some periods in which they do not learn any new information.[2] After a while uncertainty has accumulated and firms plan afresh and adjust optimally to the new state of the world. We use this modeling approach to make predictions about adjustment dates, the choice of the strategy variable in different industries and a comparison between monopoly and duopoly.

More specifically, we consider competition between two firms who produce differentiated goods in a stochastic environment. Market demand is stochastic in the sense that in every instant firms' demand curves are hit by a shock. If the competitors want to adapt their plans to changes in their competitive environment they have to incur adjustment costs.

We find that with quantity competition firms choose to adjust their plans at different dates. The intuition behind this result is the following: if one firm adjusts she reduces the incorrectness of the non-adjusting firm's plan. The reason is that with a positive (negative) shock the adjusting firm produces less thereby keeping the price of the non adjusting firm closer to its expected value. This reduces the gain from adjusting for the other firm. Thus the other firm chooses to postpone her adjustment to a later date. Under price competition exactly the opposite result holds true. Here an adjusting

[1]For example, Sims (2003, 2005) proposes an approach in which agents are attentive in all periods but can only absorb parts of the incoming information. This approach is used e.g. by Moscarini (2004).

[2]For a model of rational inattention on the consumer's side, see Reis (2005).

firm amplifies the effect of a shock and therefore makes it more profitable for the other firm to adjust. The reason is that with price competition the optimal reaction of a firm to a negative (positive) shock is to decrease (increase) her price thereby making the other firm worse off in comparison to non-adjustment. As a consequence both firms adjust simultaneously.

Concerning the interrelation between the costs of planning and the mode of competition we find the following results: Since the length of the inattentiveness period rises with increasing adjustment costs there exists a critical level of planning costs above which both firms choose price competition. This result obtains because a longer duration of inattentiveness implies a higher degree of uncertainty with which the competitors have to cope. Since with fixed prices quantities adjust flexibly to changes in market demand expected profits are higher in price competition compared to quantity competition. If planning costs and therewith the inattentiveness period are sufficiently small, then the higher short-run profits of quantity competition outweigh the previously discussed comparative advantage of price competition. That is why below this distinct second critical value of planning costs both firms choose quantity competition. For intermediate values of planning costs we get the interesting result that asymmetric equilibria can arise with one firm choosing prices and the other one choosing quantities. Here a substantial amount of uncertainty accumulates so that it is optimal for one firm to set prices to adapt flexibly to uncertainty. Yet, for the other firm it is optimal to set quantities and to get higher profits in the short run.

This paper adds to two strands of literature. Firstly, it incorporates strategic aspects in a model of inattention. We show that despite the complication that arise due to strategic interaction similar techniques as in a single player decision framework can be used to solve the game. Moreover, the interaction quite frequently yields asymmetric equilibrium outcomes in a completely symmetric framework.

Secondly, the paper analyzes the tension between price and quantity competition in a dynamic framework. The papers that are closely related to ours are the ones by Singh and Vives (1984) and by Klemperer and Meyer (1986). Singh and Vives analyze a model without uncertainty in which firms choose first their strategy variable and then compete according to their choices. They show that choosing quantities is a dominant strategy. Our model has a similar structure to theirs but in contrast we introduce uncertainty and demonstrate that prices are more flexible in the long run. Klemperer and Meyer also introduce uncertainty and provide different cases for which prices or quantities arise as the choice of the strategic variable. In their paper the choice of the variable and setting its explicit value is a simultaneous decision which is different to our paper where it is sequential. Moreover, none of these papers considers a dynamic framework where firms set complete paths of their chosen strategy variable for a certain time period and where uncertainty accumulates over time.[3]

The rest of the paper is organized as follows. Section 2 sets out the model. In Section 3 we determine the adjustment dates under different forms of competition. The solution to the full game is provided in Section 4. Section 5 discusses possible extensions and Section 6 concludes.

2 The Model

There are two firms denoted by $i = 1, 2$. Each firm i produces a differentiated perishable good at constant marginal cost c. Each firm faces an inverse demand curve

$$p_i = \alpha\theta_t + \gamma p_{-i} - \frac{\delta}{\theta_t} q_i, \tag{1}$$

at date t, with $\alpha, \gamma, \delta > 0$. Since $1 > \gamma > 0$ the products of the firms are substitutes. Moreover we assume that c is sufficiently small such that profits are positive at each instant. Fluctuations in market demand are represented by θ_t whose evolution is governed by a geometric Brownian motion

$$d\theta_t = \sigma\theta_t dz_t, \text{ with } \theta_0 = 1, \tag{2}$$

with drift rate zero and dz_t being a standard Wiener process. Therefore θ_t has an expected value of $\theta_0 e^t$ and a variance of $\theta_0^2(e^{\sigma^2 t} - 1)$, with $0 < \sigma^2 < \infty$. The chosen representation of the inverse demand curve reflects the effects that fluctuations of the market sentiment have: An increase in θ increases market size (increases the intercept) and consumers' willingness to pay (it flattens the demand curve) and vice versa. That these effects are perfectly positively correlated simplifies the following analysis and is not crucial to our results.

In $t = 0$ firms enter a two stage game. In stage 1 firms simultaneously choose their strategy variable, namely either prices, $s_i = p_i$, or quantities, $s_i = q_i$. Each firm observes the other firm's choice and then decides on the path of its chosen strategy. Whereas we assume that the choice of the strategy variable in stage 1 is a once and for all decision and cannot be reversed in the sequel of the game each firm can alter its stage 2 decision each planning time.[4]

[3] For a recent treatment about the issues of price and quantity competition, see Maggi (1996).

[4] In section 5 we discuss how our results would change if we relax this assumption.

As mentioned before it is costly to absorb and process information and so a firm rationally decides to remain uninformed about the true state of the world θ_t in some periods t. Whenever a firm updates her information and adjusts her price or quantity to the new information she faces a finite adjustment cost of $K \geq 0$. A firm decides at which time periods she plans to adjust her price or quantity. We denote this adjustment dates by $D^i(k)$, with $D^i(k) : \mathbf{N_0} \rightarrow \mathbf{R}$. If at date $D^i(k-1)$ a firm plans then the time that elapses before its next adjusting date is $d^i(k) = D^i(k) - D^i(k-1)$ and we call $d^i(k)$ the inattentiveness period of firm i.

If the last adjustment date of a price firm i was $D^i(k)$ she therefore obtains a profit at time $t \in [D^i(k), D^i(k+1)]$ of

$$\pi^i(p_t^i, p_t^{-i}) = \max_{p_t^i} E\Big[(p_t^i - c)(\frac{\theta_t(\alpha\theta_t - p_t^i + \gamma p_t^{-i})}{\delta}) \mid \theta_{D^i(k)}, \{p_t^{-i}\}_{t=D^i(k)}^{D^i(k+1)}\Big], \tag{3}$$

if the other firm has also chosen to set prices while she obtains

$$\pi^i(p_t^i, q_t^{-i}) = \max_{p_t^i} E\Big[(p_t^i - c)(\frac{\theta_t(\alpha\theta_t(1+\gamma) - p_t^i(1-\gamma^2) - \gamma\delta q_t^{-i}}{\delta}) \mid \theta_{D^i(k)}, \{q_t^{-i}\}_{t=D^i(k)}^{D^i(k+1)}\Big], \tag{4}$$

if the other firm has chosen to set quantities. Conversely, if firm i sets quantities she obtains

$$\pi^i(q_t^i, p_t^{-i}) = \max_{q_t^i} E\Big[q_t^i(\frac{\alpha\theta_t^2 - \delta q_t^i + \theta\gamma p_t^{-i}}{\theta}) \mid \theta_{D^i(k)}, \{p_t^{-i}\}_{t=D^i(k)}^{D^i(k+1)}\Big], \tag{5}$$

if firm $-i$ sets prices and

$$\pi^i(q_t^i, q_t^{-i}) = \max_{q_t^i} E\Big[q_t^i(\frac{\alpha\theta_t^2(1+\gamma) - \delta(q_t^i + \gamma q_t^{-i})}{\theta(1-\gamma^2)}) \mid \theta_{D^i(k)}, \{q_t^{-i}\}_{t=D^i(k)}^{D^i(k+1)}\Big], \tag{6}$$

if firm $-i$ sets quantities as well.

Firm i maximizes its expected present discounted (at the rate $r > 0$) value of profits net of planning costs

$$J_1^i(\theta_0, D, \{s_t^{-i}\}_{t=0}^{\infty}, \{s_t^i\}_{t=0}^{\infty}) = E_{\theta_0}\Big[\sum_{k=0}^{\infty}\Big(\int_{D(k)}^{D(k+1)} e^{-rt}\pi_i(\theta_{D(k)}, t - D^i(k), s_t^i, s_t^{-i})dt - e^{-rD(k+1)}K\Big)\Big], \tag{7}$$

via choosing a sequence of adjustment dates $D^i = \{D^i(k)\}_{k=1}^{\infty}$.

The solution concept we employ is Markov perfect equilibrium. Maximizing the expected discounted profits is equivalent to choosing a sequence of planning dates such that at each adjustment date the expected gain that a firm receives from writing the new plan is equal to the cost that she incurs. In order to formalize this we proceed by defining the following function:

$$G^i(\theta_t, t, s_t^{-i}) = E_{\theta_{D^i(k)}}\Big[\max_{s_t^i}\{\pi^i(\theta_t, t, s_t^{-i}, s_t^i)\} - \pi^i(\theta_{D(k)}, t, s_t^{-i}, s_t^i)\Big] \quad (8)$$

The first term on the right hand side represents the expected full information profit that firm i can grasp this instant by responding optimally to her competitor's action and the prevailing state. The second term corresponds to the expected profit of firm i if she instead follows the plan specified at the last adjustment date. Therefore $G^i(\cdot)$ is a measure for the expected value of planning this instant. Subtracting the discounted profits obtained under full information from equation (7) allows us to restate the objective function in the following way:

$$J_2^i(\theta_0, D, \{s_t^{-i}\}_{t=0}^{\infty}, \{s_t^i\}_{t=0}^{\infty}) = E_{\theta_0}\{\sum_{k=0}^{\infty}\big(-\int_{D(k)}^{D(k+1)} e^{-rt}G^i(\theta_t, t, s_t^{-i})dt - e^{-rD(k+1)}K\big)\}. \quad (9)$$

Representation (9), the fact that θ_t evolves according to a geometric Brownian motion and the recursive structure of the problem between adjustment dates allow us to consider the duopolists decision problem as a problem of regulating the amount of uncertainty that builds up in the system. We now turn to the problem's value function.

As usual, the value function is defined as

$$V^i(\theta, s^{-i}) = \max_D J_2^i(\theta, D, s^{-i}, s^i) \quad (10)$$

In equation (10) we used that the instantaneous profit function is defined on a convex set, bounded, continuously differentiable, and strictly concave. Therefore standard results imply, that there is a unique value function solving the problem that is is bounded and at least twice continuously differentiable with a globally non-positive second derivative. It follows from the principle of optimality, that $V^i(\theta, s^{-j})$ satisfies:

$$V^i(\theta, s^{-i}) = \max_d \Big\{ -\int_0^d e^{-rt}G^i(\theta, t, s^{-i})dt + e^{-rd}E[-K + V^i(\theta_d, s_d^{-j})]\Big\}. \quad (11)$$

The representation in (11) points out one important implication of inattentiveness. In between adjustment dates firm i rationally refrains from processing new information. Therefore she decides about the next adjustment date at the instant at which she planned for the last time. As a consequence adjustment is not state-contingent in the sense that each firm observes the true state of the world at each instant and decides whether she wants to plan. Planning is rather recursively state-contingent, which means that it is the latest state of the world that the firm observed while planning that determines the date at which the firm wants to plan for the next time.

3 Analysis

3.1 A general approximate solution

Optimal inattentiveness solves the Bellman equation (11). The necessary first-order condition for optimality is:

$$-G^i(\theta_d, d, s_d^{-i}) = E_{\theta_0}\Big[r(V^i(\theta_d, s_d^{-i}) - K) - V^i_{\theta_d}\frac{\partial \theta_d}{\partial d} - V^i_{s_d^{-i}}\frac{\partial s_d^{-i}}{\partial d}\Big] \tag{12}$$

On the left hand side is the expected flow reward from not planning which is represented by the difference between the profits that the firm gets by following the pre-specified plan and the full information profit. The right hand side captures the value from planning. The first term is the flow value from planning, which is the difference between the value of having a fresh plan and the cost of writing it. The second term is the cost from postponing planning for another instant in which the value of a new plan may change.

The envelope theorem conditions with respect to θ and s^{-i} are:

$$V^i_\theta(\theta, s^{-i}) = -\int_0^d e^{-rt}G^i_\theta(\theta, t, s^{-i})dt + e^{-rd}E\Big[-V^i_{\theta_d}(\theta_d, s_d^{-j})\frac{\partial \theta_d}{\partial \theta} - V^i_{s_d^{-i}}(\theta_d, s_d^{-i})\frac{\partial s_d^{-i}}{\partial \theta}\Big], \tag{13}$$

$$V^i_{s^{-i}}(\theta, s^{-i}) = -\int_0^d e^{-rt}G^i_{s_t^{-i}}(\theta, t, s^{-i})dt + e^{-rd}E\Big[-V^i_{s_d^{-j}}(\theta_d, s_d^{-j})\frac{\partial s_d^{-j}}{\partial s^{-j}}\Big]. \tag{14}$$

Equations (11) to (14) characterize the value function and optimal inattentiveness interval $d^i(\theta, s^{-i})$. We can derive an approximate solution of the dynamic problem by perturbing the system around the point where the cost of planning are zero. This approach requires only that $V(\theta, s^{-i})$ and $d^i(\theta, s^{-i})$ are locally differentiable with respect to the cost of planning. Then:

Proposition 1. *A perturbation approximation of the optimal inattentiveness around the situation when planning is costless is:*

$$d^{i*}(\theta, s^{-i}) = \sqrt{\frac{2K}{G_t^i(\theta, 0, s^{-i})}}.$$

Inattentiveness is positively related to adjustment costs and it decreases with rising $G_t^i(\cdot)$. Whereas the first determinant is obvious the second warrants some discussion. $G_t^i(\cdot)$ measures the velocity with which the losses from being inattentive accumulate. This is clearly influenced by the elasticity of market demand or by the volatility of production. But apart from these factors it is the mode of competition and the effect that $-i'$s action exerts on i's perceived correctness of her plan that influence $G_t^i(\cdot)$. We will focus on the later aspects in the following analysis.

3.2 Adjustment Dates

In this section we analyze the second stage of the game. We restrict the analysis to the cases in which firms either both choose prices or both choose quantities. The results will later be used to determine the equilibrium of the full game.[5]

First, look at the case in which both firms set prices. The profit function of firm i at date 0 in this case is

$$\pi_0^i = \frac{\theta_0}{\delta}(p_0^i - c)(\alpha\theta_0 - p_0^i + \gamma p_0^{-i}).$$

This gives an equilibrium price of $p_0^{i*} = \frac{\alpha\theta_0 + c}{2-\gamma}$. The resulting path for the strategy variable during the first inattentiveness period is flat since $d\theta_t$ follows a geometric Brownian motion with drift zero. Therefore the expected value of θ_t is equal to θ_0.[6] The question is now when the firm chooses to adjust its price for the next time. To determine this period $D^i(1)$ we determine the instantaneous gain from adjusting.

We start with the case in which firm $-i$ does not adjust.[7] In this case the expected profit of firm i in period t if she does not adjust is

$$E_{\theta_0}[\pi^i(p_{na}^i, p_{na}^{-i})] = E_{\theta_0}\Big[\frac{(\alpha\,\theta_t - c + c\gamma)\,(-\alpha\,\theta_0 - c + 2\,\alpha\,\theta_t - \alpha\,\theta_t\,\gamma + \gamma\,\alpha\,\theta_0 + c\gamma)\,\theta_t}{(2-\gamma)^2\delta}\Big].$$

[5]In many industries strategy variables are not subjects of choice for the firms. For example, farmers usually must choose quantities while service suppliers usually set prices.

[6]Due to this result we suppress time indices in the following analysis if they do not denote adjustment dates.

[7]In the following the subscript na denotes not-adjusted variables, whereas the subscript a denotes contemporaneously adjusted variables.

The expected profit if firm i adjusts

$$E_{\theta_0}[\pi^i(p_a^i, p_{na}^{-i})] = E_{\theta_0}\Big[\frac{1}{4}\frac{(2\,\alpha\,\theta_t - \alpha\,\theta_t\,\gamma + \gamma\,\alpha\,\theta_0 - 2\,c + 2\,c\gamma)^2\,\theta_t}{(2-\gamma)^2\,\delta}\Big].$$

As a consequence the expected instantaneous gain from adjusting is approximately[8]

$$G^i(\theta_0, t, p_{na}^{-i}) = E[\pi^i(p_a^i, p_{na}^{-i})] - E[\pi^i(p_{na}^i, p_{na}^{-i})] = 1/4\,\frac{\alpha^2\left(e^{\sigma^2 t}-1\right)}{\delta}, \quad (15)$$

where we have used the fact that $E[\theta_t] = \theta_0 = 1$. Now suppose that firm $-i$ adjusts exactly at date t then by proceeding in the same way as above yields that expected instantaneous gain from adjusting at date t is given by

$$G(\theta_0, t, p_a^{-i}) = E_{\theta_0}[\pi^i(p_a^i, p_a^{-i})] - E_{\theta_0}[\pi^i(p_{na}^i, p_a^{-i})] = 1/16\,\frac{(\gamma+2)^2\,\alpha^2\left(e^{\sigma^2 t}-1\right)}{\delta}. \quad (16)$$

This shows that the expected gain from adjusting increases in the time that elapsed since the last planning date. This is due to the fact that the expected incorrectness of $i's$ plan that is captured by the variance of the demand curve and therefore the variance of profits increases with time. Comparing (16) with (15) yields that the difference in the expected instantaneous gains is

$$1/16\,\frac{\alpha^2\gamma\,(\gamma+4)\left(e^{\sigma^2 t}-1\right)}{\delta},$$

which is strictly positive.

This shows that the expected gain from adjusting is higher if the other firm adjusts in this instant. The intuition behind this result is the following: if there is a negative demand shock and firm $-i$ adjusts she sets a lower price. This intensifies the negative consequence on firm i since she sells even less than without an adjustment of firm $-i$. In the case of a positive demand shock firm $-i$ would amplify the positive consequences on i. In total, however, by adjusting firm $-i$ increases the expected incorrectness of firm $i's$ plan. This in turn implies, that the losses from being inattentive for firm i increase. Therefore she has an higher incentive to plan at date t if firm $-i$ also plans at this instant.

[8] We derived this and the following expressions that involve expected profits by applying the δ-method.

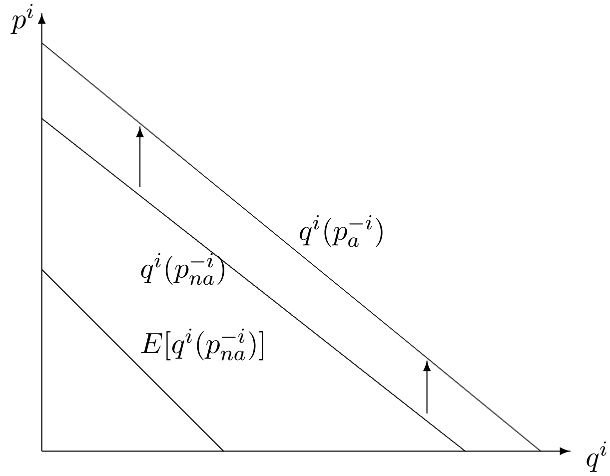

Figure 1: Positive Demand Shock under Price Competition

Now let us turn to the case in which both firms set quantities. The profit function of firm i at date 0 in this case is

$$\pi_0^i = \left(\frac{\alpha\,{\theta_0}^2 + \gamma\,\alpha\,{\theta_0}^2 - \gamma\,\delta\,{q_0}^{-i} - \delta\,q_0^i}{t\,(1-\gamma^2)} - c \right) q_0^i.$$

This gives an equilibrium quantity of $q_0^{i*} = \frac{(1+\gamma)(c\gamma + \alpha\,\theta_0 - c)\theta_0}{\delta\,(\gamma+2)}$. As before we now determine the expected instantaneous gain from adjusting at date t for firm i. By proceeding in the same way as before we get that this gain is given by

$$\begin{aligned} G(\theta_0, t, q_{na}^{-i}) &= E_{\theta_0}[\pi^i(q_a^i, q_{na}^{-i})] - E_{\theta_0}[\pi^i(q_{na}^i, q_{na}^{-i})] = \\ &= \frac{(1+\gamma)\alpha\left(e^{\sigma^2 t}-1\right)(3\,\alpha - 2c(1-\gamma))}{4(1-\gamma)\delta}, \end{aligned} \tag{17}$$

if firm $-i$ does not adjust at time t, while the expected instantaneous gain from adjusting if firm $-i$ does also adjust at date t is given by

$$\begin{aligned} G(\theta_0, t, {q_a}^{-i}) &= E_{\theta_0}[\pi^i(q_i^a, q_{-i}^a)] - E_{\theta_0}[\pi^i(q_i^{na}, q_{-i}^a)] = \\ &= \frac{(1+\gamma)\alpha\,(\gamma-2)^2\left(e^{\sigma^2 t}-1\right)(3\,\alpha - 2c(1-\gamma))}{16(1-\gamma)\delta}. \end{aligned} \tag{18}$$

Substracting (17) from (18) gives

$$-\frac{(1+\gamma)\,\alpha\,\left(e^{\sigma^2 t}-1\right)(3\,\alpha - 2c(1-\gamma))\,\gamma\,(4-\gamma)}{16\,(1-\gamma)\,\delta},$$

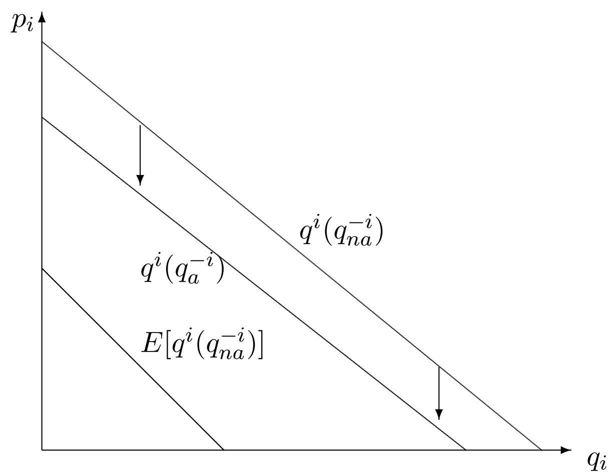

Figure 2: Positive Demand Shock under Quantity Competition

which is negative. Therefore the expected instantaneous gain from adjusting is greater if the other firm has not adjusted.[9] This is opposite to the former case. The intuition is the following: Suppose for example, that there was a negative shock and demand decreased. If firm $-i$ adjusts its quantity then it chooses to produce a smaller quantity than without adjusting. As a consequence prices increase compared to the non adjusting case and profit of firm i does not decrease by such a large amount as in case of non adjustment of firm $-i$. Thus the incentive to adjust for firm i is reduced. Firm $-i$ by adjusting herself stabilizes the price of firm i.

These results can now be used to analyze the adjustment dates of the firms.

Proposition 2. *If both firms set prices, they adjust at simultaneous dates. If both firms choose quantities they adjust sequentially and at alternating dates.*

Proof: From Proposition 1 we know that $d^{i*}(\theta, s^{-i}) = \sqrt{\frac{2K}{G_t(\theta,0,s^{-i})}}$.

Let us first look at the case in which both firms set prices. From (15)

[9]We have shown this result here only for the first adjustment time, starting with q_i^*. But it is easy to show that this results holds true for any possible quantity pair, independent from the non adjustment quantities.

we get that

$$G_t(\theta_0, 0, p_{na}^{-i}) = \frac{\sigma^2 \alpha^2}{4\delta},$$

while from (16) we get

$$G_t(\theta_0, 0, p_a^{-i}) = \frac{\sigma^2 (\gamma + 2)^2 \alpha^2}{16\delta}.$$

Comparing these two equations yields that the second one is bigger than the first one and thus that the denominator of $d^{i*}(\theta_0, p_a^{-i})$ is bigger than $d^{i*}(\theta_0, p_{na}^{-i})$. Therefore

$$d^{i*}(\theta_0, p_a^{-i}) < d_i^*(\theta_0, p_{na}^{-i}).$$

This shows that both firms adjusts at date $d^{i*}(\theta_0, p_{na}^{-i})$ because if only firm $-i$ would adjust then firm i would like to adjust even before which is not possible anymore. So it also adjusts exactly at the same date. This does not only hold for $\theta = \theta_0$ but for any θ_t and so firms always adjust simultaneously.

Now, let us turn to the case in which both firms set quantities. From (17),

$$G_t(\theta_0, 0, q_{na}^{-i}) = \frac{(1+\gamma)\, \alpha\sigma^2 \,(3\,\alpha - 2c(1-\gamma))}{4\,(1-\gamma)\,\delta},$$

and from (18) we get

$$G_t(\theta_0, 0, q_a^{-i}) = \frac{(1+\gamma)\, \alpha\, (\gamma - 2)^2 \,\sigma^2 \,(3\,\alpha - 2c(1-\gamma))}{16\,(1-\gamma)\,\delta}.$$

Here it is easy to see that $G_t(\theta_0, 0, q_{na}^{-i}) > G_t(\theta_0, 0, q_a^{-i})$ and so $d^{i*}(\theta_0, q_{na}^{-i}) < d^{i*}(\theta_0, q_a^{-i})$. So no firm adjust till date $d^{i*}(\theta_0, q_{na}^{-i})$ at which date one of them adjusts, while the other one then waits and adjusts only at date $d^{i*}(\theta_0, q_a^{-i})$. Again, a similar argument holds for all θ_t which proves that firms adjust sequentially. Finally, it is also easy to check that a firm whose last adjustment date is further away in time has a higher $G_t(.)$ and therefore adjusts earlier. So adjustment dates are alternating. ■

It should be mentioned that this result is not related to the fact that prices are strategic complements or that quantities are strategic substitutes. The reason is that we consider here the optimal reaction of a firm if the other one sticks to her pre-specified plan. Then the best reaction of the acting firm on e.g. a lower demand is then to decrease prices thereby making the other firm worse off or respectively to decrease quantity thereby making the other firm better off.

A straightforward result following from the previous analysis is that the time length between adjustment dates is longer in price competition than under quantity competition. For example, if firms set prices then starting from period 0 the first adjustment date is at

$$d^{i*}(\theta_0, p_{na}^{-i}) = \sqrt{\frac{8\delta K}{\sigma^2 \alpha^2}}.$$

On the other hand, under quantity competition the first adjustment date is at

$$d^{i*}(\theta_0, q_{na}^{-i}) = \sqrt{\frac{8\,(1-\gamma)\,\delta K}{(1+\gamma)\,\alpha\sigma^2\,(3\,\alpha - 2c(1-\gamma))}}.$$

Comparing the two values yields that the firm adjusting first under quantity competition adjusts before the first adjusting date in price competition, $d^{i*}(\theta_0, q_{na}^{-i}) < d^{i*}(\theta_0, p_{na}^{-i})$.

The firm that adjusts second under quantity competition adjusts at

$$d^{i*}(\theta_0, q_{a}^{-i}) = \sqrt{\frac{32\,(1-\gamma)\,\delta K}{(1+\gamma)\,\alpha\,\left(\gamma - 2\right)^2\sigma^2\,(3\,\alpha - 2c(1-\gamma))}}.$$

Comparing this with $d^{i*}(\theta_0, p_{na}^{-i})$ gives that $d^{i*}(\theta_0, q_{a}^{-i}) > d^{i*}(\theta_0, p_{na}^{-i})$ if

$$\frac{\alpha\,\sigma^2\left(\left(-8\,\gamma^3 + 6\,\gamma^2 + 8\,\gamma - 8 + 2\,\gamma^4\right)c + \left(4\,\gamma + 8 - 9\,\gamma^2 + 3\,\gamma^3\right)\alpha\right)}{16\,(1-\gamma)\,\delta} > 0.$$

But since $\alpha > c$ this must always hold true. So the time that elapses under price competition before adjusting first is longer than the one under quantity competition when both firms have adjusted. This is a consequence of the fact that prices adjust more flexible to demand shocks. This result is a prerequisite of the analysis of the full game that follows in the next section.

4 Equilibria of the Full Game

In order to analyze the equilibria of the full game we consider the interrelation between the costs of planning and the mode of competition. Since the length of the inattentiveness period rises with increasing adjustment costs there exists a critical level of planning costs above which both firms choose price competition. This result obtains because a longer duration of inattentiveness implies a higher degree of uncertainty with which the competitors have to cope. Since with fixed prices quantities adjust flexibly to changes in market demand expected profits are higher in price competition compared to quantity competition. If planning costs and therewith the inattentiveness period are sufficiently small, then the higher short-run profits of quantity competition outweigh the previously discussed comparative advantage of price competition.

The trade-off between high short-run profits and low flexibility under quantity competition and low short-run profits and high flexibility under price competition leads to the following Proposition:

Proposition 3: *There exists a $K^\star$ below which both firms set quantities in the unique equilibrium. There exists a $K^{\star\star} > K^\star$ above which both firms set prices in the unique equilibrium. For values of K in between $K^\star$ and $K^{\star\star}$ there exist two equilibria in pure strategies, in which one firm set prices and the other firm quantities.*

Sketch of a proof: *In the following we give a sketch of the proof. The full proof still remains to be done!* We first start with the case when both firms set quantities. The instantaneous profit in period 0 is given by

$$\pi^i(q_0^{i*}, q_0^{-i*}) = \frac{(c\gamma + \alpha - c)^2 (\gamma+1)\,\theta_0}{\delta\,(1-\gamma)\,(\gamma+2)^2}. \tag{19}$$

The expected instanteneous profit in any future period t in this case is also given by

$$E_{\theta_0}[\pi^i(q_0^{i*}, q_0^{-i*})] = \frac{(c\gamma + \alpha - c)^2 (\gamma+1)\,\theta_0}{\delta\,(1-\gamma)\,(\gamma+2)^2}. \tag{20}$$

Now look at the case in which the firms set different strategy variables. In this case the instantaneous profit of the price firm is

$$\pi^i(p_0^{i*}, q_0^{-i*}) = \frac{(\gamma+2)^2 (c\gamma + \alpha - c)^2 (1-\gamma)(\gamma+1)}{(4-3\,\gamma^2)^2\,\delta}, \tag{21}$$

while the expected profit in any future period t as long as no firm adjusts is

$$E_{\theta_0}[\pi^i(p_0^{i*}, q_0^{-i*})] = -\frac{(\gamma+2)(c\gamma+\alpha-c)(\gamma+1)\left(c\gamma^3+3\,e^{\sigma^2 t}\gamma^2\alpha-2\,\gamma^2\alpha+\gamma\,\alpha-3\,c\gamma+2\,\alpha+2\,c-4\,e^{\sigma^2 t}\alpha\right)}{(4-3\,\gamma^2)^2\delta}. \tag{22}$$

Comparing (19) and (21) obviously yields that the instantaneous profit if both firms set quantities is higher than if one firms deviates and sets prices. On the other hand, (22) is increasing in t while (20) is independent of t. If $t = 0$ then (22) is equal to (21). But the time of adjusting is increasing in K and so there exists a t below which (22) is smaller than (20) and so for all K smaller than some value K' quantities are preferred to prices. Taking into account adjustment dates there must exist a value of K, call it $K^\star$, such that for all values of $K < K^\star$ both firms set quantities.

Now let us turn to the case when both firms set prices. The instantaneous profit in period 0 is given by

$$\pi^i(p_0^{i*}, p_0^{-i*}) = \frac{(\alpha - c + c\gamma)^2}{(2-\gamma)^2\,\delta}. \tag{23}$$

The expected instantaneous profit in any future period t in which no firm has adjusted is given by

$$E_{\theta_0}[\pi^i(p_0^{i*}, p_0^{-i*})] = \frac{(c\gamma + \alpha - c)\left(e^{\sigma 2t}\alpha(2-\gamma) - (\alpha + c)(1-\gamma)\right)}{(2-\gamma)^2\,\delta}. \tag{24}$$

One can clearly see that (24) is increasing in t while (23) is, of course, independent of t. In case where the two firms set different strategy variables the instantaneous profit of the quantity firm is

$$\pi^i(q_0^{i*}, p_0^{-i*}) = \frac{\left(2\,c - 2\,c\gamma^2 - \gamma\,\alpha - 2\,\alpha + \gamma^2\alpha - c\gamma + c\gamma^3\right)^2}{(4-3\,\gamma^2)^2\,\delta}, \tag{25}$$

while the expected profit in any future period t as long as no firm adjusts is also

$$E_{\theta_0}[\pi^i(q_0^{i*}, p_0^{-i*})] = \frac{\left(2\,c - 2\,c\gamma^2 - \gamma\,\alpha - 2\,\alpha + \gamma^2\alpha - c\gamma + c\gamma^3\right)^2}{(4-3\,\gamma^2)^2\,\delta}. \tag{26}$$

Now (25) is bigger than (23), while (26) is smaller than (24) for high enough t. If K is higher then the time that elapses till the next adjustemnt is higher. So there must exist a value K'' such that for $K > K''$, t is so high that (24) is bigger than (26). Taking into account different adjustment periods we can also find a $K^{\star\star}$ such that for all $K > K^{\star\star}$ both firms choose prices.

It remains to show that $K'' > K'$. Comparing (22) with (20) gives that the two values are equal if

$$e^{\sigma^2 t} = \frac{\left(8-6\,\gamma^2-\gamma^3\right)\gamma^3(\gamma-1)c+\left(-24\,\gamma^3+32-48\,\gamma^2+16\,\gamma+9\,\gamma^5+14\,\gamma^4+2\,\gamma^6\right)\alpha}{(4-3\,\gamma^2)(1-\gamma)(\gamma+2)^3\alpha} \tag{27}$$

On the other hand, comparing (26) with (24) shows that they are equal at

$$e^{\sigma^2 t} = \frac{\gamma^3(\gamma-1)\left(\gamma^3-6\,\gamma^2+8\right)c+\left(32\,\gamma^3+18\,\gamma^4-15\,\gamma^5-48\,\gamma^2-16\,\gamma+32+\gamma^6\right)\alpha}{\alpha\left(-4+3\,\gamma^2\right)^2(2-\gamma)}. \tag{28}$$

Now (28) is bigger than (27) if

$$\frac{\gamma^4\left(\gamma^6-4\,\gamma^5-36\,\gamma^4+8\,\gamma^3-64+96\,\gamma^2\right)(c\gamma+\alpha-c)}{\left(-4+3\,\gamma^2\right)^2(1-\gamma)\,(\gamma+2)^3\,\alpha\,(2-\gamma)} < 0.$$

But since $\gamma < 1$ this always holds true. Therefore $K'' > K'$ and this also translates into $K^{\star\star}$ and $K^{\star}$.

It still remains to be checked that $K^{\star\star}$ is indeed bigger than $K^{\star}$ when accounting for different adjustment dates. ■

The result that for low (high) values of K firms choose quantities (prices) is straightforward. The interesting case arises when K is in some middle range. In this case there can be asymmetric equilibria in a completely symmetric game. The intuition behind this outcome is that the merits and disadvantages of each strategy variable are balanced in this case. The adjustment cost are high enough such that some variance accumulates and it pays off for one firm to set prices, thereby sacrificing short term profits for being more flexible in adapting. On the other hand, it does not pay for the other firm to switch to prices as well, because in this case her short term profits would be lower than if she chooses quantities and the accumulation of variance is not too high. In other words, the benefit from switching to prices from a quantity-quantity equilibrium is higher than the benefit from switching to prices from a quantity-price equilibrium.

4.1 Adjustment frequency and Market structure

The analysis in this section connects with previous work on the effects of inattentiveness in a monopolistic environment (Reis (2006)). In the following we want to address the question whether the duopolists' incentive to process information is unambiguously lower in duopoly than in a monopolistic setting.

Due to our specification of the demand curve a monopolist chooses to set prices for every value of planning costs, since he appreciates their flexible nature the most and is not concerned with competitive issues. We find that the inattentiveness period of a monopolist is equal to:

$$d^{m*}(\theta) = \sqrt{\frac{2K\delta(1-\gamma)}{\alpha^2\sigma^2}}. \tag{29}$$

Now we turn to duopoly. Suppose that $K > K^{**}$ such that both firms choose prices. Then the inattentiveness period of firm i is equal to:

$$d^{i*}(\theta, s^{-i}) = \sqrt{\frac{8K\delta}{\sigma^2\alpha^2}}. \tag{30}$$

Since the numerator in (29) is always smaller than the numerator in (30) we see, that the planning frequency of each duopolist in price competition is lower than that of the monopolist.

If $K < K^*$ the first adjustment dates of the duopolists under quantity competition are governed by the time derivatives of (17) and (18). It is straightforward to show, that if marginal costs are small the monopolist adjusts later than the second firm under quantity competition. These results are summarized in the following Proposition:

Proposition 4. *The planning frequency is lower in price competition than in monopoly. If*

$$0 < c < \frac{\alpha(4 - 3\gamma^2(3-\gamma))}{8 - 2\gamma(\gamma^3 - 4\gamma^2 + 3\gamma + 4)},$$

then the planning frequency of both firms in quantity competition is higher than in monopoly.

5 Extensions

One restrictive assumption in the above analysis is that firms have to make a once and for all choice of their strategy variable in the first stage of the game in period zero. We think, that in most cases firms would stick to their choice in period zero even if we allowed them to change their strategy variable at each adjustment date.

Suppose that both firms chose to set quantities at date zero and let firm i be the first one to adjust. Firm i could be tempted to switch to prices at its first adjustment date since it expects firm $-i$ to stay rationally inattentive for some time period after i adjusted. Therefore it could safely steal part of $-i'$s market share by setting prices that better adjusts to shocks than the quantity of firm $-i$. However, in equilibrium $-i$ foresees i's action and would react optimally in period zero. In particular, she would set a quantity path that changes at the adjustment date of firm i to react optimally to the price choice of firm i. Thereby it would eliminate i's incentive to choose prices because in period zero in this region firms do not choose different strategy variables. On the other hand, if both firms chose to set prices at date zero. Since adjustment is simultaneous in that case it is obvious that no firm would ever have an incentive to respond by setting quantities. In the third case, in which firms set different strategic variables the analysis might get complicated and the equilibrium might change.

Another possible extension would be to consider a situation with more than two competitors. Whereas we expect the adjustment frequency per firm to remain unchanged if all firms chose prices we think that we could get interesting results in the case where firms chose quantities as their strategy variable.

If there were n firms in the market it is suggestive that the adjustment frequency of all firms increases whereas the inattentiveness period of each single firm increases compared to the duopoly case. Then it could be, that there is some finite n at which it would pay for all firms to choose prices in period zero. This result can arise, because the competitive advantage of quantity competition diminishes the more firms there are in the market.

6 Conclusion

We have studied a model of imperfect competition in continuous time in which it is costly for firms to absorb and process information. As a consequence firms remain rationally inattentive for some time period and optimally choose a sequence of adjustment dates.

We can show that the length of the inattentiveness period is affected by the mode of competition. If both firms compete in prices then they remain inattentive for a longer time span as compared to quantity competition. Moreover, firms adjust simultaneously in price competition and sequentially and if they compete in quantities.

We solved for the equilibria of the full game by building on the fact that the duration of inattentiveness is positively related to the cost of processing information. If this cost is sufficiently large, then firms compete in prices whereas they prefer quantity competition if processing cost are small enough. In between these critical values an equilibrium arises in which firms compete in different strategy variables.

Furthermore there is a interrelation between the industry structure and the mode of competition. A monopolist would prefer to set prices no matter how large adjustment costs are. Duopolists instead choose their strategy variable depending on the level of planning costs and their adjustment frequency can be higher or lower dependent on the chosen variable.

The next steps that are to be taken in the analysis of this model comprise of inter alia an explicit solution for the value function. This step is necessary in order to determine the exact length of the inattentiveness interval for all values of adjustment costs. Furthermore this step is crucial for a thorough analysis of the equilibria of the full game.

It seems to be reasonable that firms incorporate information about extraordinary events that influence market demand more quickly than fluctuations that are caused e.g. by randomly changing tastes of their consumers. Since the model currently does not capture the effect of how firms react to large or anticipated shocks such as a interest rate decision by the central bank or a terrorist attack, one can think about introducing a second stochastic component that follows a Poisson process. This measure could lend more realism to the model. It can be helpful in confronting the theoretical implications that inattentiveness has on the competitive behavior of firms with the data in order to test their empirical validity.

7 Appendix

7.1 Proof of Proposition 1

The proof of Proposition 1 follows Reis (2006).[10] We rewrite the cost of planning as $K = \kappa^2 \tilde{K}$, where κ is a non-negative scalar. The solution is approximated around $\kappa = 0$. Then:

$$V^i(\theta, s^{-j}) = \max_d \Big\{ -\int_0^d e^{-rt} G^i(\theta, t, s^{-i}) dt + e^{-rd} E_\theta[-\kappa^2 \tilde{K} + V^i(\theta_d, s_d^{-j})] \Big\}. \tag{31}$$

The optimality conditions are only slightly different:

$$-G^i(\theta_d, d, s_d^{-i}) = E_\theta \Big[r(V^i(\theta_d, s_d^{-i}) - \kappa^2 \tilde{K}) - V^i_{\theta_d} \frac{\partial \theta_d}{\partial d} - V^i_{s_d^{-i}} \frac{\partial s_d^{-i}}{\partial d} \Big], \tag{32}$$

$$V^i_\theta(\theta, s^{-i}) = -\int_0^d e^{-rt} G^i_\theta(\theta, t, s^{-i}) dt + e^{-rd} E_\theta \Big[-V^i_{\theta_d}(\theta_d, s_d^{-j}) \frac{\partial \theta_d}{\partial \theta} - V^i_{s_d^{-i}}(\theta_d, s_d^{-i}) \frac{\partial s_d^{-i}}{\partial \theta} \Big], \tag{33}$$

$$V^i_{s^{-i}}(\theta_0, s_t^{-i}) = -\int_0^d e^{-rt} G^i_{s^{-i}}(\theta, t, s^{-i}) dt + e^{-rd} E_\theta \Big[-V^i_{s_d^{-j}}(\theta_d, s_d^{-j}) \frac{\partial s_d^{-j}}{\partial s^{-j}} \Big], \tag{34}$$

$$V^i_\kappa(\theta, s^{-i}) = e^{-rd} E_\theta \Big[-2\kappa \tilde{K} + V^i_\kappa(\theta_d, s_d^{-i}) \Big]. \tag{35}$$

The last condition is the envelope theorem condition with respect to κ. The system of equations (31) to (35) defines the optimum. When $\kappa = 0$, the solution to the system is $d^* = 0$ and $V^i(\theta, s^{-i}) = 0$. At this optimum $G^i(\theta, 0, s^{-i}) = 0$ for all θ and s^i. Moreover the n^{th}-order derivatives of V^i and G^i with respect to θ and s^i are all zero. Perturbing the system (31) to (35) by differentiating with respect to κ and evaluating at $\kappa = 0$ yields:

$$\begin{aligned}
V^i_\kappa &= V^i_\kappa \\
-G^i_t d^i_\kappa &= r V^i_\kappa - \frac{d}{d\kappa}\Big(\frac{1}{dt} E_\theta(dV^i)\Big) \\
V^i_{\theta\kappa} &= E_\theta \Big[V^i_{\theta_d \kappa} \frac{\partial \theta_d}{\partial \theta} + V^i_{s_d^{-i}\kappa} \frac{\partial s_d^{-i}}{\partial \theta} \Big] \\
V^i_{s^{-i}\kappa} &= E_\theta \Big[V^i_{s_d^{-i}\kappa} \frac{\partial s_d^{-i}}{\partial s^{-i}} \Big] \\
0 &= -r V^i_\kappa d_\kappa - 2\tilde{K} + \frac{d}{d\kappa}\Big(\frac{1}{dt} E_\theta(dV^i)\Big)
\end{aligned}$$

[10] The method we use can also be looked up in Judd (1999).

All the functions are evaluated at θ and s^i and $t = 0$. The first, third and fourth equation do not contain information regarding d^{i*}. But the second and the fifth form a system of equations that we can use to solve for d_κ by substituting for $E_\theta(dV^i)$. This yields:

$$d^i_\kappa = \sqrt{\frac{2\tilde{K}}{G^i_t}}.$$

Since the approximation to d^{i*} is $d^{i*} = d^{i*}_\kappa \kappa$, and since $\sqrt{K} = \kappa\sqrt{\tilde{K}}$, the expression for d^{i*} follows. ■

References

- K.L. Judd (1999): *Numerical Methods in Economics*, The MIT Press, Cambridge, Massachusetts.

- P. Klemperer and M. Meyer (1986): "Price Competition vs. Quantity Competition: The Role of Uncertainty," *RAND Journal of Economics*, 17, 618-638.

- G. Maggi (1996): "Strategic Trade Policies with Endogeneous Mode of Competition," *American Economic Review*, 86, 237-258.

- G. Moscarini (2004): "Limited Information Capacity as a Source of Inertia," *Journal of Economic Dynamics and Control*, 28, 2003-2035.

- R. Radner (1992): "Hierarchy: The Economics of Managing," *Journal of Political Economy*, 30, 1382-1415.

- R. Reis (2006): "Inattentive Producers," *Review of Economic Studies*, 73, 793-821.

- R. Reis (2006): "Inattentive Consumers," *Journal of Monetary Economics*, forthcoming.

- C. Sims (2003): "Implications of Rational Inattention," *Journal of Monetary Economics*, 50, 665-690.

- C. Sims (2005): "Rational Inattention: A Research Agenda,"Working Paper, Princeton University.

- N. Sing and X. Vives (1984): "Price and Quantity Competition in a Differentiated Duopoly ," *RAND Journal of Economics*, 15, 546-545.

- M.L. Weitzman (1974): "Prices vs. Quantities," *Review of Economic Studies*, 41, 477-491.