On Financial Equilibrium with Intermediation Costs

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Abstract

This paper analyses the set of competitive equilibria in economies with intermediation costs. Non-existence of equilibrium in economies without intermediation costs with price dependent assets is well-known: the price-dependence of return permits a drop in rank of the income-transfer mapping. This paper introduces an intermediation cost on trading assets which endogenously induces a boundary on asset trades and hence we obtain competitive equilibrium in the presence of options even though no financial equilibrium exists without intermediation costs. We show that the equilibrium correspondence is upper hemi continuous. Asset trades are unbounded when the intermediation cost function tends to zero and when there exists no financial equilibrium without intermediation costs. Further the competitive equilibra without intermediation costs can be obtained by a suitable choice of a non-zero intermediation costfunction.

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1 Introduction

This paper analyses the properties of the set of competitive equilibria when intermediation costs are present. We allow the asset structure to include non-linear dependence on spot market prices. The main result is that every economy has a competitive equilibrium and that the equilibrium correspondence parameterized by the cost function and endowments is upper hemi continuous. Further when the intermediation costs go to zero and there exists no equilibrium without intermediation costs in the economy, the asset trades are unbounded. The results are due to a boundary on asset trades induced by the intermediation cost which is assumed to go to infinity if the volume of asset trade goes to infinity. Further we show that the image of the equilibrium correspondence contains the set of equilibria without any intermediation costs.

The model is the standard: we consider a 2 period model with S different states. Here the information sets are trivial increasing in the two periods. In each period-state, spot trading of goods takes place and asset trading takes place in the first period before uncertainty is revealed. Trading in asset markets is costly due to intermediation costs, and the

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revenue from these trades are redistributed to the consumers in a non-specified manner[1]. This could either be in proportion to some pre-specified ownership or proportional to the trading of assets.

The global existence property arises from the presence of intermediation costs: Non-existence when such costs are not present arises from the discontinuity of demand for assets when commodity prices converge to prices for which the dividend matrix drops in rank. The presence of intermediation cost prevents that the drop in rank induces discontinuity of the budget correspondence.

This paper extends the existence result in [Préchac(1996)] to a more general asset structure and intermediation cost function. The result in Préchac concerns real assets which depend linearly on the commodity prices, and dividends are positive; this however exclude important classes of assets, namely options, futures etc. We obtain the stronger result by imposing more restrictions on the cost function such that if the volume of asset trade goes to infinity then the costs go to infinity. In Préchac this is not satisfied since they depend on the *value* of the trades and not the *volume* of trade; hence if the asset price is zero, then the cost is zero. When an asset yields dividends with different signs in different states, the price might end up as zero in equilibrium. The existence result then goes through by assuming that the dividends are positive. The question of continuity of the equilibrium correspondence is not addressed by Préchac.

Firstly, existence results in financial economies with incomplete markets was established by [Radner(1972)] when an exogenous boundary on asset trades was imposed. Later [Hart(1975)] showed that this boundary assumption was essential for the existence result, providing an example in which no financial equilibrium existed when the boundary assumption was dropped. Both contributions considered an asset structure which depended linearly on commodity spot prices. In relation to these results [Duffie and Shafer(1985)] showed that the non-existence of equilibrium is non-generic: a small perturbation of the economy will re-establish existence of equilibrium. However, this result was due to the linearity of the asset dividends dependence on spot market prices as shown in [Polemarchakis and Ku (1990)], in which an example involving options gives rise to a robust non-existence of financial equilibrium. As showed in [Busch and Govindan(2004)] robust non-existence of competitive equilibrium also arise in economies when preferences are not strictly convex.

It is in this line that the existence result of this paper should be viewed: Préchac shows existence when asset structure is linearly dependent on spot market prices and positive dividends, while in this paper we only assume continuity. This comes at the expense of more restrictions on the cost function. However, our result allows asset structures with nominal-, real assets including options. Our result covers both economies considered by Polemarchakis and Ku and Busch and Govindan.

If we consider the competitive equilibrium a reasonable description of the state of the economy, then non-existence of equilibrium implies that prices cannot coordinate actions of the agents. With robust examples of non-existence such situations occur with nonzero probability. The results of this paper imply that these problems disappear when trade on financial markets is costly, even though the revenue from such costs is transferred back to the agents. Intuitively, the upper hemi continuity implies that, even though the agents make small mistakes in their assessment of the characteristics of the economy, the error

[1]Therefore it is an intermediation cost rather than transaction cost; we prefer to distinguish between the two, since the latter is real costs arising in the exchange of commodities, while the first is essentially just a transfer of income between agents

in the expectation of prices and allocation will be small. An interpretation of the result of the unboundness of volume of trade could be that, lowering costs of capital movement between countries could increase the volatility of the volume of trade on the financial markets. The result shows that the introduction of a Tobin tax could induce a higher degree of stability of the global financial markets. However the Tobin tax should be on the volume of trade rather than the value of the trade. Note however that the results make no welfare judgment. In order to state such results one should take into account the "costs" of nonexistence. Further we do not know if reducing the volatility increases the welfare when financial markets are incomplete. The interpretation of the last result implies that by constructing the intermediation costs appropriately the government could, given any economy which posses an equilibrium without intermediation, guarantee that the set of equilibria is the same as without intermediation costs. Moreover the informational requirements of such a policy is quite sparse, they only need to know the equilibrium trading volume.

The paper will proceed as follows: In section (2) we formalize the model and present the main results: existence and upper hemi continuity of the equilibrium set. Section (3) contains the proofs of the main results and the final section (4) extends the result of existence to intermediation costs including fixed fees by considering an economy with a continuum of consumers.

2 The Model

Let H be the set of consumers, there are 2 periods: in the first there is uncertainty on the state of the second in which there are S of states. We denote by s a generic state and take, due to ease of notation, $s = 0$ as the first period. In each state and period there are L perishable goods and hence the commodity space is $\mathbb{L} = \mathbb{R}^{L(S+1)}$. Denote by $P \subset \mathbb{R}_+^{L(S+1)}$[2] the space of spot market prices. Assume that there are J assets and consider an asset structure $V\colon P \to L(J,S)$ where $L(J,S)$ is the space of linear mappings $\mathbb{R}^J \to \mathbb{R}^S$ representing the space of income transfers. A generic portfolio is denoted by $z \in \mathbb{R}^J$ and $q \in Q \subset \mathbb{R}^J$ denotes an asset price vector. Denote by (u^h, ω^h, X^h) a consumer with utility function u^h, initial endowment of commodities $\omega^h \in \mathbb{L}$ and consumption possibility set $X^h \subset \mathbb{L}$. Denote by $\Omega \subset \mathbb{L}^H$ the set of endowments.

Consider a function $c\colon \mathbb{R}^J \times \mathbb{R}^J \to \mathbb{R}$ such that $c(q,z) \geq 0$ is the intermediation cost of obtaining the portfolio $z = (z_1, ..., z_J) = (z_j)_{j=1}^J \in \mathbb{R}^J$ given asset prices $q = (q_1, ..., q_J) = (q_j)_{j=1}^J \in \mathbb{R}^J$. Denote by $\mathcal{C}$ the set of intermediation cost functions, i.e. costs by trading in the asset market additional to the linear cost given by the price per unit.

Given spot market prices $p = (p_0, p_1, ..., p_S) \in P$ such that $p_s \in \mathbb{R}^L$ is the spot market prices in state $s = 0, 1, ..., S$, asset prices $q \in \mathbb{R}^J$ and a transfer $w \in \mathbb{R}$ the budget set $B_h(p,q,w)$ of consumer h is the set of commodity bundles $x = (x_0, x_1, ..., x_S) \in X^h$ and

[2]Notation: Given $x, y \in \mathbb{R}^n$ we write $x \geq y$ iff. $x_i \geq y_i$ for $i = 1, ..., n$, $x > y$ iff. $x \geq y$ and $x \neq y$. This further gives us the sets

$$\begin{aligned} \mathbb{R}_+^n &= \{r \in \mathbb{R}^n \mid r \geq 0\} \\ \mathbb{R}_{++}^n &= \{r \in \mathbb{R}^n \mid r \gg 0\} \end{aligned}$$

Given a normed vector space X we denote by $\mathcal{O}_\epsilon(x)$ the open ball with radius $\epsilon > 0$ and center in $x \in X$.

The set of all subsets of a set X is denoted 2^X and if $X = \mathbb{R}^N$ then we write 2^N

portfolios $z = (z_1, ..., z_J) \in \mathbb{R}^J$ such that

$$p_0 \cdot x_0 \leq p_0 \cdot \omega_0^h - q \cdot z - c(q, z) + w$$

and for every $s = 1, ..., S$

$$p_s \cdot x_s \leq p_s \cdot \omega_s^h + V_s(p) \cdot z$$

Denote by

$$\phi^h(p, q, w) = (\phi_x^h, \phi_z^h)(p, q, w) = \arg\sup\{u_h(x) \mid (x, z) \in B_h(p, q, w)\}$$

the demand correspondence of consumer h. We shall sometimes denote by $\phi^h(p, q, w; \omega, c)$ the demand correspondence of h when we want to emphasize the underlying economy $(\omega, c) \in \Omega \times \mathcal{C}$.

We now define our equilibrium of this economy $(\omega, c) \in \Omega \times \mathcal{C}$, taking the intermediation cost function as given, to be a pair of commodity and asset prices, allocation of commodities and portfolios which are maximal for each consumer given prices and transfers, commodity and asset markets are cleared and the revenue from asset trades are distributed to the consumers by lump-sum transfers:

Definition 1 *A competitive equilibrium of (ω, c) is a tuple (p, q, x, z) such that there exists $w = (w_h)_{h \in H}$ satisfying the following conditions*

1. $(x_h, z_h) \in \phi^h(p, q, w_h)$ *for every* $h \in H$
2. $\sum_{h \in H} x_h - \omega_h = 0$
3. $\sum_{h \in H} z_h = 0$
4. $\sum_{h \in H} w_h = \sum_{h \in H} c(q, z_h)$

Denote by $E(\omega, c)$ the set of such tuples (p, q, x, z) where there exists a competitive equilibrium.

Let $Z = \mathbb{R}^{L(S+1)} \times \mathbb{R}^J \times (\mathbb{R}^{L(S+1)} \times \mathbb{R}^J)^H$ then $E(\omega, c) \subset Z$ for every $(\omega, c) \in \Omega \times \mathcal{C}$ and we shall refer to $E \colon \Omega \times \mathcal{C} \to 2^Z$ as the **Equilibrium correspondence**.

Assume next that the following conditions are satisfied

Assumption. Given the economy $(I, \mathbb{L}, (u^h, \omega_h, X^h)_{h \in H}, V, c)$ we assume that

1. $\omega^h \in \operatorname{int} X^h$ for every $h \in H$
2. $X^h = \mathbb{R}_+^{L(S+1)}$ is closed, convex and bounded from below
3. $u^h \colon X^h \to \mathbb{R}$ is continuous, quasi-concave and strictly monotone
4. $V \colon P \to L(J, S)$ is continuous
5. $c \colon Q \times \mathbb{R}^J \to \mathbb{R}_+$ is continuous and $c(q, 0) = 0$
6. if $\lambda > 1$ then $c(q, \lambda z) \geq c(q, z)$ for every $z \in \mathbb{R}^J$
7. for any $\lambda \in [0, 1]$ and $z, z' \in \mathbb{R}^J$ we have $c(q, \lambda z + (1-\lambda) z') \leq \lambda c(q, z) + (1-\lambda) c(q, z')$
8. $c(q, e^j \lambda) \to \infty$ whenever $\lambda \to \infty$ for every $q \in Q$ and $j = 1, ..., J$

Denote by $\mathcal{C}$ the subset of $C(\mathbb{R}^J \times Q)$ with the weak topology[3] which satisfies the assumptions. Note that by the subspace topology the space $\mathcal{C}$ is not complete since $0 \in \overline{\mathcal{C}} \setminus \mathcal{C}$. In particular this implies that any convergent sequence in the subspace $\mathcal{C}$ must have as a limit point a cost function which is different from the zero function. Hence $\mathcal{C}$ is a convex cone pointed at 0 which is not closed.

The assumptions on the asset structures allow the following types:

1. If $V(p) = V(p')$ for every $p, p' \in P$.

2. Consider the matrix $R \in \mathbb{R}^{SL}$ such that the dividend matrix $V(p)$ is given by

$$V_s(p) = p_s \cdot R_s = V_s(p_s)$$

   where $p_s \in \mathbb{R}^L_+$ is the commodity price vector of state s while $R_s \in \mathbb{R}^L$ is a commodity bundle.

3. As in the above example but letting $\bar{p}_s = (\frac{p_{ls}}{p_{l0}})_{l=1}^L$ (when $p \gg 0$) we have that

$$V_s(p) = \bar{p}_s \cdot R_s$$

4. Considering an asset $r^1 = (r_s(p_s))_{s=1}^S$ with $r(\cdot)$ continuous then letting an asset having the dividend vector given by $r^2(p) = (\max\{k - r_s(p), 0\})_{s=1}^S$ for some $k \geq 0$ we have that $V(p) = (r^1(p), r^2(p))$ satisfies the assumptions.

Thus the dividend structure includes securities such as nominal assets with positive dividends (i.e. bonds and Arrow securities), contingent contracts on commodity bundles, options and equity contracts with limited liabilities.

However remember that we are considering assets or securities, that is, financial commodities which are *tradable* and hence standardized terms in the contracts. Insurance such as house and car insurance and partnerships are *not* tradable but are personal.

The last assumptions concern the properties of the intermediation costs: We assume that the costs are non-negative, increasing and zero if the asset trades are zero. Further if the volume of asset trades goes to infinity, the costs go to infinity. Finally, the costs behave continuously in both asset prices and trades. An example of a cost function which satisfies these assumptions is the following: Let $c_j, k_j > 0$ for $j = 1, ..., J$ then

$$c(q, z) = \alpha \sum_{j=1}^{J} c_j |q^j z^j|^n + \beta \sum_{j=1}^{J} k_j |z^j|^m$$

for every $n, m \in \mathbb{N}$ satisfies the assumptions when $\alpha, \beta > 0$. A cost function which does *not* satisfy the assumptions is

$$c(q, z) = \begin{cases} 0 & z = 0 \\ \alpha q \cdot |z| + F & z \neq 0 \end{cases}$$

for some $\alpha, F > 0$ since it violates the continuity and convexity property in $z = 0$.[4]

Our main result is that under our maintained assumptions every economy possess a competitive equilibrium and that the equilibrium set behaves (upper hemi) continuously:

[3]Hence we say that a sequence $\{f_n\}$ converge to f if and only if for every $K \subset \mathbb{R}^J$ compact and every $\epsilon > 0$ there exists $N \geq 1$ such that for every $n \geq N$ we have that $\sup_{x \in K} |f_n(x) - f(x)| < \epsilon$. By [Mas-Colell(1985)] this space is metrizable, separable and complete

[4]here we have that $|z| = (|z_1|, ..., |z_J|)$ is the point wise absolute value of a vector

Theorem 1 *For every* $(\omega, c) \in \Omega \times \mathcal{C}$ *we have that* $E(\omega, c) \neq \emptyset$

Theorem 2 $E\colon \Omega \times \mathcal{C} \to 2^Z$ *is upper hemi continuous*

Further we can say something about equilibria when we consider a sequence $c^n \in \mathcal{C}$ with $c^n \to 0$ and $E(\omega, 0) = \emptyset$. First of all, the non-existence of equilibrium must imply the need for income transfer when costs approach zero and hence a positive demand for assets; the discounting of future state income must differ across consumers. But then as the cost function is ϵ-distance from the zero function for $\epsilon > 0$ small enough there must exist some positive amount of trade and as ϵ tends to zero the asset trades go to infinity:

Theorem 3 *Assume that* $E(\omega, 0) = \emptyset$*,* $c_n \in \mathcal{C}$*,* $c_n \to 0$ *and* $(p_n, q_n, x_n, z_n) \in E(\omega, c_n)$ *then for every* $M \geq 0$ *there exists* $N \geq 1$ *such that if* $n \geq N$ *then* $\|z_n\| > M$

In the example of [Polemarchakis and Ku (1990)] these results implies that when intermediation costs on asset trade are positive there exists a competitive equilibrium. However as this cost goes to zero the volume of asset trade goes to infinity.

Remark 1 *To see that the requirement that the intermediation costs depend on the volume, rather than the value, of asset trade is necessary, consider the economy with no uncertainty, atleast two spot commodities and the dividend map* $V(p) = p_1 - p_2$*. Construct the economy such that with complete markets the spot prices satisfy that* $p_1 = p_2$*. Considering an intermediation costfunction* $c(q, z) = c|qz|$ *and the no-arbitrage condition* $(1 - c)\lambda V(p) \leq q(1 + c)\lambda V(p)$ *for some* $\lambda > 0$ *we have that* $q = 0$ *in equilibrium and whence follows that* $c(q, z) = 0$ *for every* $z \in \mathbb{R}$*.*

Theorem 4 $E(\omega, 0) \subset E(\omega, \mathcal{C})$ *for every* $\omega \in \Omega$

This theorem simply states that it is possible to construct intermediation cost functions which satisfy the conditions of continuity, convexity etc. while still obtaining the competitive equilibrium without intermediation costs. The point is to tax only those asset trades whose volume exceeds the equilibrium asset trade. The inclusion holds for every $\omega \in \Omega$ by the convention $\emptyset \subset A$ for any set A.

3 Proofs of the Main Theorems

Before giving the proofs we define the sets used in this section: Denote by

$$\begin{aligned} Q &= \{q \in \mathbb{R}^J \mid \max_j |q_j| \leq 1\} \\ P_s &= \{p_s \in \mathbb{R}^L_+ \mid \sum_{l=1}^{L} p_{sl} = 1\} \\ D &= \prod_{s=0}^{S} P_s \times Q \end{aligned}$$

the price space and

$$M(\omega) = \{x \in X \mid \sum_{h \in H} x^h - \omega^h = 0\}$$

the set of market equilibria of commodity allocations. A summary of the sets is stated below

$\mathbb{L} = \mathbb{R}^{L(S+1)}$	The commodity space
$\Omega \subset \mathbb{R}^{HL(S+1)}$	The set of endowments
$X = \prod_{h \in H} X^h$	The set of allocations
$M(\omega)$	The set of spot market clearing allocations
$D \subset \mathbb{R}^{L(S+1)} \times \mathbb{R}^J$	The set of normalized spot and asset prices
$Z = D \times X \times \mathbb{R}^{HJ}$	The set of prices, allocations and portfolios
$\mathcal{C} \subset C(Q \times \mathbb{R}^J, \mathbb{R})$	The set of intermediation cost functions

For every $m \in \mathbb{N}$ define $Z^m = D \times M \times [-m, m]^J$ denote by $\mathcal{E}^m$ the economy restricted to Z^m and let

$$\begin{aligned} B_h^m(p, q, w) &= B_h(p, q, w) \cap M \times [-m, m]^J \\ \phi_h^m(p, q, w) &= \arg\sup\{u^h(x^h) \mid (x^h, z^h) \in B_h^m(p, q, w)\} \end{aligned}$$

be the restricted budget and demand correspondences.

3.1 Proof of Theorem 1

The outline of the proof is as follows:

1. show that every bounded economy (given by demand functions) has an equilibrium
2. show that the asset trade is bounded
3. show that the economies considered satisfy the stated conditions

We now state and prove a theorem similarly to [Radner(1972)] stating that every bounded economy contains an equilibrium.

Lemma 1 *Assume that ϕ_h^m is upper hemi continuous, non-empty- and convex-valued for every $h \in H$. Then there exists $(p^m, q^m, x^m, z^m) \in E(\mathcal{E}^m)$.*

Proof. Define μ_0 by

$$\mu_0(\bar{x}, \bar{z}) = \arg \max_{(p,q) \in D} \sum_{h \in H} p_0 \cdot (\bar{x}_0^h - \omega_0^h) + q \cdot \bar{z}^h$$

and let μ_1 be given by

$$\mu_1(\bar{x}, \bar{z}) = \left(\arg \max_{(p,q) \in D} p_s \cdot \sum_{h \in H} \bar{x}_s^h - \omega_s^h \right)_{s=1}^{S}$$

Note that the correspondence is well-defined since the maximum used in μ_0 and μ_1 are independent. Let then μ be given by

$$\mu(\bar{x}, \bar{z}, \bar{p}, \bar{q}) = \mu_0(\bar{x}, \bar{z}) \times \mu_1(\bar{x}, \bar{z}) \times \left(\prod_{h \in H} \phi_h^m(\bar{p}, \bar{q}, \bar{w}^h) - (\omega^h, 0) \right)$$

with $\bar{w}^h = \pi^h \sum_{k \in H} c(q, \bar{z}^k)$ for some $\pi^h \geq 0$ and $\sum_{h \in H} \pi^h = 1$.

Assume that $(\bar{p},\bar{q},\bar{x},\bar{z})\in\mu(\bar{p},\bar{q},\bar{x},\bar{z})$ then we have that

$$\sum_{h\in H}\bar{p}_0\cdot(\bar{x}_0^h-\omega_0^h)+\bar{q}\cdot\bar{z}^h\geq\sum_{h\in H}p_0\cdot(\bar{x}_0^h-\omega_0^h)+q\cdot\bar{z}^h$$

for every $(p_0,q)\in P_0\times Q$ and hence since

$$\bar{p}_0\cdot(\bar{x}_0^h-\omega_0^h)+\bar{q}\cdot\bar{z}^h=w^h-c(\bar{q},\bar{z}^h)$$

implying that

$$\sum_{h\in H}\bar{p}_0\cdot(\bar{x}_0^h-\omega_0^h)+\bar{q}\cdot\bar{z}^h=0 \tag{1}$$

then

$$\sum_{h\in H}p_0\cdot(\bar{x}_0^h-\omega_0^h)+q\cdot\bar{z}^h\leq 0$$

Since $\delta_l\in D$ we have that $\sum_{h\in H}\bar{x}_0^h-\omega_0^h\leq 0$ and $\sum_{h\in H}\bar{z}^h\leq 0$. Also we have that $-\delta_j\in Q$ such that $\sum_{h\in H}\bar{z}^h\geq 0$ implying that $\sum_{h\in H}\bar{z}^h=0$. Further we have that

$$\sum_{h\in H}\bar{p}_s\cdot(\bar{x}_s^h-\omega_s^h)\geq\sum_{h\in H}p_s\cdot(\bar{x}_s^h-\omega_s^h)$$

for every $p_s\in P_s$ but then since $\bar{p}_s\cdot(\bar{x}_s^h-\omega_s^h)=V_s(\bar{p})\cdot\bar{z}^h$ and $\sum_{h\in H}\bar{z}^h=0$ we have that $\sum_{h\in H}\bar{p}_s\cdot(\bar{x}_s^h-\omega_s^h)\leq 0$. Again this implies that $\sum_{h\in H}\bar{x}_s^h-\omega_s^h\leq 0$ for every $s=1,...,S$. By monotonicity arguments we have that $\sum_{h\in H}\bar{x}_s^h-\omega_s^h=0$ for every $s=0,1,...,S$.

By standard arguments the correspondence μ is upper-hemi continuous, non-empty-, convex valued mapping a non-empty, convex and compact subset of Euclidean space into itself; Thus by Kakutani(1941) there exists some $y\in\mu(y)$. ■

Remark 2 *Note that we only need $L(S+1)-(S+1)$ price variables in obtaining an equilibrium, and not $L(S+1)-2$ which is standard when the asset structure is not real. Hence we can restrict ourselves to price systems which have positive spot prices in each state. This prevents examples of budget correspondences which are not lower hemi continuous as was the case with the example given in [Geanakoplos(1990)], section 4.*

Remark 3 *Note that we actually only need the aggregate excess demand function*

$$\Phi(p,q,w;\omega,c)=\begin{pmatrix}\sum_{h\in H}\phi_h(p,q,w^h;\omega^h,c)-(\omega^h,0)\\ \sum_{h\in H}w^h-c(q,\phi_z^h(p,q,w^h;\omega^h))\end{pmatrix}$$

to be upper hemi continuous and convex valued and not each individuals in order to establish equilibrium.

After having proved this result that for every bounded m-economy there exists an competitive equilibrium we can now show that there exists an equilibrium for the original economy, i.e. that the optimal asset trades must be bounded:

Lemma 2 *Let $(p^m,q^m,x^m,z^m)\in E(\mathcal{E}^m)$ be an equilibrium for every $m\in\mathbb{N}$ then there exists some $(p,q,x,z)\in E(\omega,c)$ which is cluster point of (p^m,q^m,x^m,z^m)*

Proof. Consider a sequence of equilibria (p^m, q^m, x^m, z^m) with $w_h^m = \pi^h \sum_{h\in H} c(q^m, z_h^m)$ where $\pi \in \mathbb{R}_{++}^H$ and $\sum_{h\in H} \pi^h = 1$. By the budget constraints we have that

$$q^m \cdot z_h^m + c(q^m, z_h^m) = p_0^m \cdot (\omega_0^h - x_0^m) + w_h^m \tag{2}$$

Since $(p^m, q^m, x^m)_{m\in\mathbb{N}} \subset D\times M$ there exists a convergent subsequence hence by passing to this subsequence we can assume wlog. that $(p^m, q^m, x^m) \to (p, q, x) \in D \times M$.

Assume that $\|z_{h_1}^m\| \to \infty$ when $m \to \infty$ for some $h_1 \in H$ then there must be a $h_2 \neq h_1$ such that $\|z_{h_2}^m\| \to \infty$ when $m \to \infty$. But then $\pi^{h_1} \sum_{h\neq h_1} c(q^m, z_h^m) \to \infty$ but then $x_{h_1}^m$ which is bounded can't be optimal for some m large enough by monotonicity of preferences.

Since the sequence (p^m, q^m, x^m, z^m) is bounded we have that there exists some $\bar{m} \in \mathbb{N}$ such that for every $h \in H$

$$\arg\max\{u_h(x_h) \mid (x_h, z_h) \in B_h^{\bar{m}}(p, q, w_h)\} = \arg\max\{u_h(x_h) \mid (x_h, z_h) \in B_h(p, q, w_h)\}$$

since $m_1 > m_2$ implies that $B_h^{m_2}(p, q, w) \subset B_h^{m_2}(p, q, w)$ for every (p, q, w).

Hence the sequence (p^m, q^m, x^m, z^m) must contain a convergent subsequence but then the cluster point (p, q, x, z) of this sequence is an equilibrium of $\mathcal{E}$: The market clearing conditions are obviously satisfied and the constraint on the revenue from intermediation costs also. Thus we need to show that $(x_h, z_h) \in \phi_h(p, q, w^h)$; assume otherwise that there exists some $(x_h', z_h') \in B_h^{\bar{m}}(p, q, w^h)$ such that $u_h(x_h') > u_h(x_h)$. Then given $(p^m, q^m, w_h^m) \to (p, q, w_h)$ there exists by lower hemi continuity of $B_h^{\bar{m}}(\cdot)$ a sequence $(\bar{x}_h^m, \bar{z}_h^m) \in B_h^{\bar{m}}(p^m, q^m, w_h^m)$ such that $(\bar{x}_h^m, \bar{z}_h^m) \to (x_h', z_h')$; but then there exists some $N \geq 1$ such that for every $m \geq N$ we have that $u_h(\bar{x}_h^m) > u_h(x_h^m)$ by continuity of u_h; a contradiction. ■

Remark 4 *We note that actually we have proved a stronger theorem than the statement of theorem (1) since the theorem states that we can find some set of transfers of the intermediation cost which gives market clearing and consumer utility maximization. This gives many degrees of freedom and hence is more likely to exist than some pre-specified distribution rule. The lemma however shows that any distribution with positive transfer will do the trick.*

Remark 5 *Note that any sequence of equilibrium prices and allocations $(p^n, q^n, x^n, z^n) \in E(\omega^n, c^n)$ induces a sequence of transfers (w^n) given by*

$$w_h^n = p_0^n \cdot (x_h^n - \omega_h^n) + q^n \cdot z_h^n + c^n(q^n, z_h^n)$$

for every $h \in H$ and $n \in \mathbb{N}$. And when the sequences of price/allocations and economies are convergent then the induced sequence of transfers are also convergent.

To obtain the result of upper-hemi continuity of ϕ_h we use the maximum theorem (see [Ellickson(1993)]) extended to general topological spaces due to the fact that $\mathcal{C}$ is not a subspace of any Euclidean space. In order to apply the maximum theorem we need the following result that the budget correspondence B_h^m for every consumer h and every $m \in \mathbb{N}$ is continuous, non-empty- and convex-valued:

Lemma 3 *The correspondence $B_h^m(\omega, c)\colon D \times \mathbb{R}_+ \to 2^X \times 2^J$ given $(\omega, c) \in \Omega \times \mathcal{C}$ is continuous, non-empty- and convex-valued for every $m \in \mathbb{N}$*

In the proof we omit sub- and superscripts indices of the consumers h, and we show the non-emptyness, convex-valuedness and lower hemi continuity of the original correspondence B_h.

Proof. Since $\omega \in X$ then $(\omega, 0) \in B(p, q, w)$ for every $w > 0$ and $(p, q) \in P \times Q$ we have that $B(p, q, w) \neq \emptyset$.

Consider given (p, q, w) the elements $(\bar{x}, \bar{z}), (\bar{\bar{x}}, \bar{\bar{z}}) \in B(p, q, w)$ and some $\alpha \in [0, 1]$ then we have that

$$\begin{aligned} p_0 \cdot (\alpha\bar{x}_0 + (1-\alpha)\bar{\bar{x}}_0) &= \alpha p_0 \cdot \bar{x}_0 + (1-\alpha) p_0 \cdot \bar{\bar{x}}_0 \\ &\leq \alpha(p_0 \cdot \omega_0 - q \cdot \bar{z} - c(q, \bar{z}) + w) + (1-\alpha)(p_0 \cdot \omega_0 - q \cdot \bar{\bar{z}} - c(q, \bar{\bar{z}}) + w) \\ &\leq p_0 \cdot \omega_0 - q \cdot (\alpha\bar{z} + (1-\alpha)\bar{\bar{z}}) - c(q, \alpha\bar{z} + (1-\alpha)\bar{\bar{z}}) + w \end{aligned}$$

and also

$$\begin{aligned} p_s \cdot \omega_s + V_s(p) \cdot (\alpha\bar{z} + (1-\alpha)\bar{\bar{z}}) &= \alpha(p_s \cdot \omega_s + V_s(p) \cdot \bar{z}) + (1-\alpha)(p_s \cdot \omega_s + V_s(p) \cdot \bar{\bar{z}}) \\ &\geq p_s \cdot (\alpha\bar{x}_s + (1-\alpha)\bar{\bar{x}}_s) \end{aligned}$$

hence $\alpha(\bar{x}, \bar{z}) + (1-\alpha)(\bar{\bar{x}}, \bar{\bar{z}}) \in B(p, q, w)$.

Upper-hemi Continuity: Since $B^m(p, q, w)$ have values in a compact space we consider a sequence $(p^n, q^n, w^n) \to (p, q, w)$ and $(x_n, z_n) \in B^m(p^n, q^n, w^n)$ such that (x^n, z^n) converge; But then we have that

$$\begin{aligned} p_0^n \cdot x_0^n + q^n \cdot z^n + c(q^n, z^n) &\leq p_0^n \cdot \omega_0 + w^n \\ p_s^n \cdot x_s^n &\leq p_s^n \cdot \omega_s + V_s(p^n) \cdot z^n \end{aligned}$$

for every $n \in \mathbb{N}$ and by continuity of the functions we have that as we take limits these inequalities are preserved and hence $(x, z) \in B^m(q, p, w)$.

Lower-hemi continuity: Consider some element $(x, z) \in B(p, q, w)$ and a sequence (p^n, q^n, w^n) with limit point (p, q, w). We look for $(x^n, z^n) \in B(p^n, q^n, w^n)$ such that $(x^n, z^n) \to (x, z)$.

If we have that

$$p_0 \cdot x_0 + q \cdot z + c(q, z) < p_0 \cdot \omega_0 + w$$

then there exists some $N \geq 1$ such that for every $n \geq N$ we have that

$$p_0^n \cdot x_0 + q^n \cdot z + c(q^n, z) < p_0^n \cdot \omega_0 + w^n$$

and hence considering the sequence

$$(x^n, z^n) = \begin{cases} (\omega, 0) & n \leq N \\ (x, z) & n > N \end{cases}$$

we are done.

If on the other hand

$$p_0 \cdot x_0 + q \cdot z + c(q, z) = p_0 \cdot \omega_0 + w$$

then since $p_0 \cdot \omega_0 + w > 0$ there exists $(x', 0)$ such that for some N we have that $n \geq N$ implies

$$\begin{aligned} p_0^n \cdot x' &< p_0^n \cdot \omega_0 + w^n \\ p_0^n \cdot x' &< p_0^n \cdot x_0 + q^n \cdot z + c(q^n, z) \end{aligned}$$

but since the mapping $(x_0, z) \mapsto p_0^n \cdot x_0 + q^n \cdot z + c(q^n, z)$ is continuous there exists (x_0^n, z^n) such that

$$p_0^n \cdot x_0^n + q^n \cdot z^n + c(q^n, z^n) = p_0^n \cdot \omega_0 + w^n$$

which is unique by convexity of c and hence must converge to (x_0, z).

By the $s = 1, ..., S$ budget constraints we let

$$t_s^n = \frac{p_s^n \cdot \omega_s + V_s(p^n) \cdot z^n}{p^n \cdot x_s}$$

such that as $n \to \infty$ we have that $t_s^n \to 1$ by continuity of V. Hence we have found a convergent sequence $(x^n, z^n) \in B(p^n, q^n, w^n)$ with $(x^n, z^n) \to (x, z) \in B(p, q, w)$. ■

This concludes the proof of theorem (1): by lemma (1) there exists for every bounded economy market clearing prices, allocations and portfolios maximizing utility; by lemma (2) there exists a boundary on allocations and portfolios due to intermediation costs; by lemma (3) the budget correspondence is continuous, non-empty and convex valued, which implies that the demand correspondence is upper hemi continuous and convex-valued and thus the conditions of lemma (1) are satisfied and hence the result of theorem (1) is proved.

3.2 Proof of Theorem 2 and 3

In order to apply the maximum theorem to the demand correspondence we need to know the continuity property of the budget correspondence wrt. the space of cost functions:

Lemma 4 *Given $(p, q, w) \in D \times \mathbb{R}_+$ the correspondence $B_h(p, q, w; \omega, \cdot) \colon \mathcal{C} \to 2^X \times 2^J$ is continuous*

In the following we denote $B(c) = B(p, q, w; \omega, c)$ for every $(p, q, w) \in D \times \mathbb{R}_+$ and $(\omega, c) \in \Omega \times \mathcal{C}$ and omitting the prescript h.

Proof. It is easy to see that B have closed graph: since if $c_n \to c$ and $(q_n, z_n) \to (q, z)$ then $c_n(q_n, z_n) \to c(q, z)$ by uniform convergence. Hence if $B(0)$ is compact we are done since $B(c) \subset B(0)$ for every $c \in \mathcal{C}$. Else it follows since given $c_n \to c$ we have that if $(x, z) \in B(c)$ then $\lim_{n\to\infty} p_0 \cdot (x_0 - \omega_0) + q \cdot z + c_n(q, z) - w = 0$ and hence for every $\epsilon > 0$ there exists $\delta > 0$ such that if $c' \in \mathcal{O}_\delta(c) \cap \mathcal{C}$ then $B(c') \subset \mathcal{O}_\epsilon(B(c))$.

Consider now an element $(x, z) \in B(c)$ and a sequence (c^n) such that $c^n \to c$. We look for a sequence $(x^n, z^n) \in B(c^n)$ such that $(x^n, z^n) \to (x, z)$. Take $(x^n, z^n) = \arg d((x, z), B(c^n))$[5]; but since weak convergence implies point wise convergence in every compact subset taking the compact set containing $(\omega, 0)$ and the sequence (x^n, z^n) we have that $d((x, z), B(c^n)) \to 0$ and hence $\lim_{n\to\infty}(x^n, z^n) = (x, z)$. ■

Note that as a corollary to lemma (3) and (4) the correspondence $B \colon D \times \mathbb{R}_+ \times \Omega \times \mathcal{C} \to 2^X \times 2^J$ is continuous in the product topology.

As a corollary to the proof in lemma (2) we have that the set of equilibria is bounded given any intermediation cost function

Proposition 5 *Let $\omega^n \to \omega$ be a sequence then $\bigcup_{n\in\mathbb{N}} E(\omega^n, c)$ is bounded for every $c \in \mathcal{C}$*

[5] where we have that $d(x, A) = \inf_{a\in A} \rho(x, a)$ in any metric space with metric ρ

Proof. Consider the equation

$$q^n \cdot z_h^n + c(q^n, z_h^n) = p_0^n \cdot ((\omega_0^h)^n - x_0^n) + w_h^n$$

where the only change compared with (2) is that ω^n is a sequence; again the right hand side is convergent and the argument in the proof of lemma (2) is repeated. ■

Denote by $\Phi\colon P \times Q \times W \times \Omega \times \mathcal{C} \to 2^{\mathbb{L}} \times 2^{\mathbb{R}}$ the aggregate excess demand Correspondence

$$\Phi(p, q, w; \omega, c) = \begin{pmatrix} \sum_{h \in H} \phi_h(p, q, w^h; \omega^h, c) - (\omega^h, 0) \\ \sum_{h \in H} w^h - c(q, \phi_z^h(p, q, w^h; \omega^h)) \end{pmatrix}$$

where $W = \{(w^h)_{h \in H} \mid w^h \geq 0\}$. Hence we have that the equilibrium correspondence $E\colon \Omega \times \mathcal{C} \to 2^Z$ is given by

$$E(\omega, c) = \{(p, q, x, z) \in Z \mid \exists w \in W : \Phi(p, q, w; \omega, c) = 0\}$$

The equilibrium set is bounded:

Lemma 5 *$E(\omega, c)$ is compact for every $(\omega, c) \in \Omega \times \mathcal{C}$*

Proof. If $(p^n, q^n, x^n, z^n) \in \Phi_{\omega,c}^{-1}(\{0\})$ is a sequence with $(p^n, q^n, x^n, z^n) \to (p, q, x, z)$ then by upper hemi continuity of Φ then $(p, q, x, z) \in \Phi_{\omega,c}^{-1}(\{0\})$. The boundary on the asset trade is a special case of proposition (5). ■

Proposition 6 *$E(c)$ have closed graph in Ω whenever Φ is upper hemi-continuous for every $c \in \mathcal{C}$*

Proof. Let $\{\omega_n\}_{n \in \mathbb{N}}$ be a sequence $\omega_n \to \omega$ and $(p^n, q^n, x^n, z^n) \in E(\omega_n, c)$ for every $n \in \mathbb{N}$ such that $(p^n, q^n, x^n, z^n) \to (p, q, x, z)$; since $w_h^n \to w_h$ by upper hemi continuity of Φ we have that $\Phi(p, q, w; \omega) = 0$ thus $(p, q, x, z) \in E(\omega, c)$. ■

This gives us that the equilibrium correspondence is upper hemi continuous in the endowment space since given any sequence the equilibrium set $\bigcup_{n \in \mathbb{N}} E(\omega_n, c)$ is contained in a compact subspace and hence the closed graph property is equivalent with upper hemi continuity.

Proposition 7 *Consider some given economy $\omega \in \Omega$ then the correspondence $E(\omega)\colon \mathcal{C} \to 2^Z$ is upper hemi continuous*

Proof. Since $\{E(\omega, c^n)\}_{n \in \mathbb{N}}$ is contained in a bounded set of Z there exists a convergent subsequence (p^n, q^n, x^n, z^n) of selections in $E(\omega, c^n)$, but then by upper hemi-continuity of $\phi_h(\cdot; \cdot, c)$ we have that $(x^h, z^h) \in \phi_h(p, q, w; \omega, c)$ and hence $(p, q, x, z) \in E(\omega, c)$.

To see that $\{E(\omega, c^n)\}_{n \in \mathbb{N}}$ is contained in a bounded set: since $c_n \to c \in \mathcal{C}$ we have that $\overline{z} = \sup_n\{z_n\} < \infty$, $\underline{z} = \inf_n\{z_n\} > -\infty$ and $\underline{z} \leq z_n \leq \overline{z}$. This follows by the same argument as in the proof of lemma (2): Assume that we obtain an unbounded sequence of trades, then the cost of at least one trade must go to infinity. By the lower boundary on consumption this is not possible. ■

This gives us the result of theorem (2): by lemma (4) the budget correspondence is continuous in $\Omega \times \mathcal{C}$ hence by the maximum theorem the demand correspondence is upper hemi continuous; the result then follows from proposition (6) and (7).

Next we show that if no financial equilibrium exists and the cost of asset trade goes to zero then the asset trades must be unbounded:

Proof of Theorem 3. By the result of [Radner(1972)] we have that $\|z\| \leq L$ for all z then $E(\omega, 0) \neq \emptyset$, thus implying that if $E(\omega, 0) = \emptyset$ then for any allocations of commodities and portfolios (x, z) such that $\sum_{h \in H} x^h - \omega^h = 0$ and $\sum_{h \in H} z^h = 0$ and prices (p, q) with $(x^h, z^h) \in B_h(p, q, w^h; \omega^h, 0)$ there exist (x', z') and h with $u_h(x'_h) > u_h(x_h)$ and $(x'_h, z'_h) \in B_h(p, q, w^h; \omega^h, 0)$. Assume that there exist $M \geq 0$ such that $\sup_n \|z_n\| \leq M$; let $(p_n, q_n, x_n, z_n) \in E(\omega, c_n)$ be a convergent (sub)sequence with limit (p, q, x, z) then we have that

$$\begin{aligned} 0 &= \sum_{h \in H} x^h - \omega^h \\ 0 &= \sum_{h \in H} z^h \\ (x^h, z^h) &\in B_h(p, q, w^h; \omega^h, 0) \end{aligned}$$

(the last inclusion follows from the closed graph property of the budget set)but then there exists (x', z') and $h \in H$ such that $u_h(x'_h) > u_h(x_h)$ and $(x'_h, z'_h) \in B_h(p, q, w^h; \omega^h, 0)$; then by continuity of u_h there exists some $\delta > 0$ such that for every $\bar{x}_h \in X^h$ with $\|\bar{x}_h - x'_h\| < \delta$ implies that $u_h(\bar{x}_h) > u_h(x_h)$. Since $c_n \to 0$ there exists some $N \geq 1$ such that for all $n \geq N$ we have that $B_h(p_n, q_n, w_n^h; \omega^h, c_n) \cap O_\delta(x'_h, z'_h) \neq \emptyset$ but then (p_n, q_n, x_n, z_n) can't be an equilibrium since h can obtain higher utility and hence does not maximize utility. ■

Finally we show that any equilibrium which is obtained with no intermediation costs can be obtained by an appropriate intermediation cost function in the set $\mathcal{C}$:

Proof of Theorem 4. Let $(z_h)_{h \in H}$ be the equilibrium asset trade of some element $e = (p, q, x, z) \in E(\omega, 0)$. Consider the function $c(q, z) = \operatorname{dist}(z, K) = \inf_{\xi \in K} \|z - \xi\|$ where $K = \mathcal{O}_r(0)$ and $r = \sup_h \|z_h\|$. Then this function clearly satisfies $c \in \mathcal{C}$ and $z_h \in B_h(p, q, w; \omega, c)$ for every $h \in H$ and also (x_h, z_h) maximizes utility on $B_h(p, q, w; \omega, c)$ since it maximizes utility on $B_h(p, q, w; \omega, 0)$ and $B_h(p, q, w; \omega, c) \subset B_h(p, q, w; \omega, 0)$. ■

4 Non-convex Intermediation Costs

In this section we extend the set of intermediation cost functions and indicate how to extend the existence result of theorem (1) to this set. In order to obtain convex-valued aggregate excess demand correspondences we introduce a continuum of consumers: Let $(H, \mathfrak{M}, \lambda)$ be a measure space: H the set of consumers, $\mathfrak{M}$ the set of measurable set of consumers and λ an atomless probability measure[6] on H. The extension of equilibrium is straightforward with substitution of summation with integrals, for all with for almost all etc. We must show that the budget correspondence is continuous:

Lemma 6 *Assume that c is lower semi continuous then B have closed graph*

[6] Remember that a measure $\lambda\colon \mathfrak{M} \to \mathbb{R}_+$ is atomless if for every $E \in \mathfrak{M}$ such that $\lambda(E) > 0$ there exists $F \in \mathfrak{M}$ such that $\lambda(E) > \lambda(F) > 0$

Proof. Let $(p^n, q^n, w^n) \to (p, q, w)$, $(x^n, z^n) \in B(p^n, q^n, w^n)$ and $(x^n, z^n) \to (x, z)$ then by lower semi continuity of c we have that $\limsup_n -c(q, z_n) \leq -c(q, z)$ and hence we obtain

$$\begin{aligned} p_0^n \cdot (x_0^n - \omega_0) &\to p_0 \cdot (x_0 - \omega_0) \\ \lim_{n\to\infty} (-q^n \cdot z^n - c(q^n, z^n)) &\leq \lim_{n\to\infty} \sup(-q^n \cdot z^n - c(q^n, z^n)) \\ &\leq \lim_{n\to\infty} \sup(-q^n \cdot z^n) + \lim_{n\to\infty} \sup(-c(q^n, z^n)) \\ &= -q \cdot z + \lim_{n\to\infty} \sup(-c(q^n, z^n)) \leq -q \cdot z - c(q, z) \end{aligned}$$

hence $(x, z) \in B(p, q, w)$. ■

Lemma 7 *Assume that c is lower semi continuous and continuous if and only if $z \neq 0$ then B is lower hemi continuous*

Proof. Obviously ■

These results combined with the maximum theorem show us that the individual demand correspondences are upper hemi continuous; by using Liapunov's theorem[7] we obtain that the aggregate demand correspondence is convex-valued and hence the result is obtained by using the same arguments as in section (3.1). Substituting "for all" with "almost every" $h \in H$ in the proofs.

5 Final remarks

In this paper we have extended the existence theorem on financial economies when intermediation costs are present due to [Préchac(1996)]. Further we have given examples of equilibria in different economies illustrating some properties of the equilibrium set of prices, allocations and portfolios. Finally we have shown that the equilibrium correspondence mapping cost functions into prices, allocations and portfolios is upper hemi-continuous. Also we show that if the cost function converges to the zero function then the portfolios of the agents must diverge and hence are unbounded. We have further extended the existence result to cost functions which are non-convex and semi-continuous when considering large economies.

We note that the results can be interpreted as a theory of volatility in the financial markets and liberalization of the capital markets. However what we mean by volatility is excessive fluctuations in trades on the capital market to the changes in the fundamental characteristics of the economy. This could be an interpretation of theorem (3): when the economy passes an endowment with no equilibrium without intermediation costs and at the same we have decreased the intermediation costs - then we should observe excessive trade in the asset market. However a note of caution: this interpretation could be criticized since we consider a finite horizon model in which we assume that the fundamentals of the economy are given and modeled, and hence also variations in endowments over time and uncertainty.

It is obvious that we could parameterize the economy by an endowment of assets and the results would still be valid. The new equilibrium correspondence is also continuous in this parameter. Likewise we could parameterize by the set of dividend asset structure.

[7]The point is to consider the measure $E \mapsto \int_E \Phi(p)\, d\lambda$ which is atomless and then use the Liapunov theorem which states that the range is convex, that is that $\int_H \Phi(p)\, d\lambda$ is a convex set

In this paper we have considered intermediation costs on the net asset trade. We note however that the result is easily extendable to the case where intermediation costs are on sale and purchase of the same asset.

An important further study would endogenize the intermediation costs. One could imagine the following model: given the resulting liquidity demand and supply by spot market trade, the agents participate in a game on the finance market in which the outcome is a distribution of asset and hence induces an implicit cost. The asset price in the model is then the decision price, i.e. the price on which the agent chose the relevant portfolio. The task is then to determine conditions under which this game induces a cost function which satisfy the required assumptions of this paper.

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