A DYNAMIC MODEL FOR OCCUPATIONAL CHOICE, SAVINGS AND FORMATION OF ENTREPRENEURIAL HUMAN CAPITAL UNDER LIQUIDITY CONSTRAINTS AND INCOME UNCERTAINTY

BERTEL SCHJERNING

Abstract. In this paper, I develop an intertemporal model for saving, consumption, human capital accumulation and occupational choice in the presence of liquidity constraints, income uncertainty, and entry costs. Perhaps the most prominent feature of the model is that it generates a well-defined transition pattern characterized by continuous cycling in and out of entrepreneurship; a core phenomenon observed in the data.

The paper contributes to the literature on entrepreneurship by delivering a plausible explanation of observed transition patterns and an improved understanding of some of the intertemporal incentives that underlie the behaviour of entrepreneurs. The model also holds quite different implications for optimal consumption and saving behaviour compared to existing papers within the consumption saving literature.

Date: June, 2005.

Key words and phrases. Entrepreneurship, human capital, asset accumulation, credit constraints, uncertainty, duration dependence, state dependence, dynamic programming, approximation methods.

I gratefully acknowledge the comments I have received from Martin Browning and Nikolaj Malchow-Møller. This paper has also benefited from discussions with José Victor Rios-Rull and Christopher Caroll. I also acknowledge the support from the Center for Applied Microeconometrics (CAM) and Centre for Economic and Business Research (CEBR). The activities of CAM are financed from a grant by the Danish National Research Foundation. None of my sponsors are responsible for the conclusions.

1. Introduction

Over the past couple of decades, a substantial literature on entrepreneurship has developed. In this literature, the individual decision to become entrepreneur is probably the issue that has received most attention. While previous studies have enhanced our knowledge about the decision to become entrepreneur, most of this work seems to neglect important dynamic aspects of entrepreneurial behavior.

Economic decisions for entrepreneurs have long-run implications and intertemporal incentives underlie much of the behavior by economic agents. Therefore consideration of intertemporal aspects is crucial for the understanding of who become entrepreneurs, their behavior and therefore also the design of optimal policies regarding, e.g., bankruptcy laws, business start-up schemes, and public loan guarantees, etc.

There is, however, an increasing recognition of the importance of dynamic aspects in entrepreneurial decision making: In Buera (2003) the interaction between savings and the decision to become entrepreneur is analyzed in a multi-period model with credit constraints. Quadrini (2000) develops a dynamic general equilibrium model where entrepreneurs, subject to credit constraints, can save to finance new innovations. To study the importance of intergenerational transfers in relation to entrepreneurship, Cagetti and DeNardi (2002) develops an overlapping generation model with endogenous credit constraints and a specific role for intergenerational transmission of wealth and ability. As opposed to existing static models where credit constrained individuals are doomed to remain workers, in these dynamic models wage workers can save to overcome binding credit constraints.

The main focus in Quadrini (2000) and Cagetti and DeNardi (2002) is to understand the quantitative implications of entrepreneurship for the wealth concentration in the US economy, whereas Buera (2003) gives predictions about individual saving rates, consumption growth and the transition *into* entrepreneurship. However, despite these recent advance in entrepreneurship literature, the theoretical understanding of dynamic behavior of entrepreneurs is still very limited.

The aim of this paper is to address a number of unresolved questions in relation to the dynamic behavior of entrepreneurs: i) How does human capital accumulation interfere with existing liquidity constraints? ii) How can we explain observed transition patterns, characterized by continuous cycling between occupations?, iii) How do business start-up costs influence entrepreneurial saving incentives and decisions to entry and exit?, iv) What are the implications of transition costs for state dependence and duration dependence of the occupational choice? and v) How are entrepreneurial saving incentives and transition patterns affected by the tightness of credit constrains, and how do these effects vary across individuals with different levels of entrepreneurial ability, entrepreneurial human capital, and asset holdings?

To address these questions, I develop an intertemporal model for saving, consumption, human capital accumulation and occupational choice in the presence of liquidity constraints, income uncertainty, and entry costs. More specifically, I assume that an infinitely lived individual maximizes a time-separable utility function by each period choosing between entrepreneurship and wage work, where transitions between occupations are associated with a cost, and by dividing his resources between consumption, savings and transition costs.

Solving the intertemporal model is boils down to finding a fixed point in an equivalent dynamic programming problem. Since the model has no closed form, the solution to the model has to be computed numerically. The fact that the model has both discrete and continuous choice variables, makes the solution procedure non-trivial.

This paper contributes to the literature on entrepreneurship by delivering a plausible explanation of observed transition patterns and an improved understanding of the intertemporal incentives. The model is interesting in its own right as it may provide an explanation for observed transitions between occupations and since it holds quite different implications for optimal consumption and saving behavior than the existing papers within the consumption saving literature (see e.g., Caroll (1997) and Deaton (1991)).

One prominent feature of the model, is that it generates a well-defined transition pattern between entrepreneurship and ordinary wage work with continuous cycling between these occupations.

The rest of the paper is organized as follows. In section 2, I provide a more detailed explanation of how the model fits in with the existing literature. Section 3 describes the dynamic decision problem faced by the individual. In this section, I also give an analytical characterization of the solution and specify the individual optimization problem as a stochastic dynamic programming problem. Section 4 presents the numerical solution of a simple version of the model without human capital accumulation, where I focus on the implications of entry costs and credit constrains. Section 5 presents numerical solutions of the full model. In this section, I discuss the implications of two polar cases of human capital accumulation: i) when accumulation takes place only in entrepreneurship, and ii) when accumulation takes place only in wage work. Section 6 concludes and discuss directions for future research.

2. Related Literature

The purpose of this section is to explain in more detail how the present paper relates to the existing literature. I start by a brief review of existing static models explaining the decision to become entrepreneur. Hereafter, I briefly review some of the main contributions in the literature on intertemporal consumption and saving, in order to position the contributions of the present paper in relation to this literature. Then, I discuss how this work, relates to existing static models of entrepreneurship, the literature on intertemporal saving and consumption, the literature on investment under uncertainty and the literature on human capital formation. Finally, I review existing dynamic models of occupational choice and discuss how the present paper relates to this work.

The existing models explaining the decision to become entrepreneur relative to wage worker have primarily focused on i) individual differences in risk aversion (Kihlstrom and Laffont (1979), Cramer and Praag (2001)); ii) differences in entrepreneurial ability (Brock and Evans (1986), Holmes and Schmitz (1990), and Fonseca, Lopez-Garzia, and

Pissarides (2001)); iii) how differences in initial wealth, risk aversion and entrepreneurial ability interact with the presence of credit constrains (see e.g. Evans and Jovanovic (1989), Holtz-Eakin, Joulfaian, and Rosen (1994), Blanchflower and Oswald (1998), and Dunn and Holtz-Eakin (2000)); iv) the implications of institutional features for the decision to become entrepreneur. Kihlstrom and Laffont (1983) have thus analyzed the importance of various tax schemes, whereas Fonseca, Lopez-Garzia, and Pissarides (2001) analyze the effects of start-up costs, and Malchow-Møller, Markusen, and Skaksen (2005) focus on institutional wage compression.

In the large empirical literature, the correlation between wealth and the transition into entrepreneurship is probably the issue that has achieved most attention. It is argued that the observed concentration of wealth among entrepreneurs is not simply due to higher incomes earned by entrepreneurs. Part of the of the explanation for the higher asset holdings for entrepreneurs is seen as a consequence of the selection of entrepreneurs among richer families due to the presence of binding credit constraints. A positive relationship between wealth and entrepreneurship is thus seen as evidence for the presence of credit constraints.

Evans and Jovanovic (1989) seek to quantify the importance of credit constraints in a static structural model. In this model, liquidity constraints discourage some people from starting up a business and those who become entrepreneurs after all use less capital. Individuals with relatively high entrepreneurial ability are most likely to be credit constrained, since they are assumed to require the highest level of capital for their businesses. However, in the absence of liquidity constraints, the most productive individuals will have the highest probability of becoming entrepreneurs. Therefore policy makers can use business start-up schemes or public loan guaranties to help these relatively productive entrepreneurs to circumvent binding liquidity constraints.

The key empirical finding in Evans and Jovanovic (1989) is that credit constraints are quantitatively important and have very large welfare costs. However, the structural model in Evans and Jovanovic (1989) is static, and thereby ignores the possibility of saving to overcome liquidity constraints and the accumulation of entrepreneurial human capital. This illustrates the importance of allowing for intertemporal incentives when analyzing entrepreneurial behavior. In a dynamic context, high ability individuals facing borrowing constraints will make an effort to overcome these constraints by saving.

Understandably, many researchers have expressed their concern with potential endogeneity of wealth - which is suspected to be jointly determined with unobserved entrepreneurial ability; see Hurst and Lusardi (2004) for a recent contribution and a critical discussion of this literature. To ameliorate the consequences of the potential selection bias associated with the possible endogeneity of wealth, Blanchflower and Oswald (1998) use inheritance as an instrument. A positive and significant effect of wealth on the probability of becoming self-employed is found. These results are supported by evidence from British questionnaires, where individuals report that the main reason for not to start up a business has been a lack of start-up capital. Furthermore, it is noted that the vast majority of those starting up small businesses use own saving or money from family and friends.

Part of these funds may be guaranteed by initial savings or intergenerational transfers. Holtz-Eakin, Joulfaian, and Rosen (1994) and Blanchflower and Oswald (1998) examine how the receipt of inheritances affects an individuals decision to become entrepreneur. Holtz-Eakin, Joulfaian, and Rosen (1994) document that the size of inheritance has a positive effect on both the decision to start as an entrepreneur and the amount of capital invested in the new enterprise. They argue that these findings are explained by the existence of binding credit constraints and not as the result of the offspring taking over a family firm.

Gentry and Hubbard (2000) show that external financing to start or expand a business is very costly. They extend the static model in Evans and Jovanovic (1989) with an explicit modeling of *costly external financing*. Not by a nongeativity constraint, but with an upward sloping schedule for uncollateralized external financing. The model's predictions are consistent with the empirical findings that: i) portfolios of entrepreneurial households are much undiversified - with the bulk of assets in the active business; and ii).wealth income ratios are much higher for entrepreneurs.

While these studies have enhanced our theoretical understanding of the decision to become entrepreneur, most of this work is based on static models. To get a better understanding of the intertemporal incentives that underlie much of the behavior by entrepreneurs, an explicit modeling of dynamic aspects of the occupational choice should be incorporated. I do this by merging the set-up from the existing static models of entrepreneurship with the approach taken in the literature on intertemporal saving and consumption. Furthermore, I incorporate aspects from the literature on investment under uncertainty and the literature on human capital formation. Therefore, to better position the contributions of the present paper, it is instructive to start with a brief review of some of the main contributions in the literature on intertemporal consumption and saving.

2.1. **Intertemporal Models of Consumption and Saving.** Borrowing constraints has played a central role in the literature on intertemporal consumption and saving behavior; see e.g. Deaton (1991), Aiyagari (1994), Caroll and Kimball (1996) and Caroll (1997). This literature is characterized by models assuming that i) individuals maximize expected discounted time-separable utility over an infinite horizon; ii) that income payments are uncertain and exogenous; and iii) individuals are either subject to borrowing constraints or a strictly positive probability of zero future income.

The main insight from these studies is that individuals will hold a level of precautionary savings as a buffer against negative shocks, either because of credit constraints as in Deaton (1991) or because marginal utility of consumption is convex as in Caroll and Kimball (1996) and Caroll (1997). These two approaches yield similar saving patterns for two different reasons: In the first case, consumers are *constrained* from borrowing, whereas in the latter case consumer *choose* not to borrow due to the risk that he will have zero income in the future.[1] Whether precautionary savings originate from liquidity

[1] Recent work by Caroll (2004) gives a theoretical foundation for the observed similarity between the effects of introducing liquidity constraints or income risk when marginal utility is convex.

The basic insight is that borrowing constraint induces a concavity in the policy function around the point where the borrowing constraint becomes binding. This effectively makes marginal utility convex.

constrains or convex marginal utility these models reach the same central findings: i) Since consumers save for precautionary motives, consumption is monotonely increasing and concave in wealth ("cash on hand"); and ii) the level of precautionary savings is increasing in income uncertainty.

Several studies have tried to quantify the importance of precautionary savings (see Browning and Lusardi (1996) for a survey of empirical applications of the intertemporal consumption model). The conclusions from these studies are very heterogeneous and consensus on the importance of precautionary savings has not yet been reached. For example, based on PSID data, Caroll and Samwick (1998) find that precautionary savings can account for as much as 40 pct. of the total wealth accumulation, while others find limited or no evidence for precautionary savings. As pointed out by Hurst, Lusardi, Kennickell, and Torrealba (2005), the observed correlation between wealth and income risk is spurious: Since entrepreneurs generally face higher income risks and may hold larger proportions of wealth for other reasons than precautionary savings, the correlation between wealth and income risk are simply an artifact of pooling together wage workers and entrepreneurs. In fact, controlling for entrepreneurial status, Hurst, Lusardi, Kennickell, and Torrealba (2005) find much lower levels of precautionary savings.

2.2. **This Work.** As mentioned above, this paper builds on the original contributions in the consumption literature. In the spirit of this literature, I assume that i) an infinitely lived individual maximizes a time-separable utility function by each period dividing his resources between consumption and savings; and ii) the individual is subject to liquidity constraints, as in Deaton (1991). As opposed to the standard intertemporal models of consumption, the uncertain income is no longer exogenous.

To make the model a model of occupational choice, individuals can choose between two mutually exclusive work alternatives. They can either engage in an entrepreneurial activity or they can chose to become wage workers. Wage workers are assumed to receive an uncertain and exogenous income, whereas entrepreneurs face higher income risk and will have to use own wealth to finance their investments due to imperfect capital markets. Therefore, income is no longer exogenous, but depends crucially on wealth and occupation.

In the literature on precautionary savings, individuals facing borrowing constraints or income risk will make effort to overcome constraints or reduce risks by building a buffer against negative shocks. As a result, they are able to accomplish a significant amount of consumption smoothing. As we shall see later, the same logic can be used in the context of occupational choice. Due to the higher asset holdings, entrepreneurs have much smoother consumption paths than wage workers, if invested capital is fully reversible.

As a backdrop, it is useful to elaborate a bit on the definition of entrepreneurs implicitly implied by the model. As argued by Knight (1921), profits and uncertainty are closely connected and introduce the principal role of the entrepreneur: a fundamentally risk-bearing individual who will have to accept the uninsurable possibility of failure in exchange for a compensation in the form of expected profits. In the present model, entrepreneurs use own wealth to finance their business and bear the entire risk associated with these investments. Hence, entrepreneurs in the present paper should be viewed in the light of Knight's definition.

The present model also has a specific role for human capital accumulation. While previous (empirical) studies have treated entrepreneurial experience and work experience as exogenously assigned to individuals, see e.g. Hamilton (2000), in this paper, experience (or learning by doing) will be treated as a behaviorally determined investment decision. In the spirit of the human capital literature, see e.g. Ben-Porath (1967), Blinder and Weiss (1976), and Keane and Wolpin (1997), human capital accumulation is determined jointly with occupational choice decisions. Keane and Wolpin (1997) study the career decisions of young men in a finite horizon model, where individuals can choose between schooling, three work alternatives, and retirement. While early contributions for simplicity assumed that human capital is homogeneous, in Keane and Wolpin (1997) skills are assumed to be occupational specific and their returns vary across occupations. In the present context, I think of human capital as being productive only in entrepreneurship, to capture the idea that while individuals can acquire managerial/entrepreneurial skills in both wage work and entrepreneurship activities, they are only useful in the latter activity.

The model also allows for an explicit role for business start-up costs. This feature has previously been analyzed in Fonseca, Lopez-Garzia, and Pissarides (2001) where a standard matching model with matching between workers and managers is used to shed light on the general equilibrium effects of start-up costs on employment and entrepreneurial activity in the economy. To my knowledge, however, this paper is the first to provide an analysis of how entry costs alter the intertemporal incentives that underlie the decision to become entrepreneur.

The incorporation of entry costs in to the model, directly relates this work to the literature on optimal investment behavior under uncertainty (see Dixit and Pendyck (1994) for a comprehensive account of this approach). The three key assumptions in this literature, which is met in the presence of entry costs, are: i) the costs if investment in entry are indivisible and fully irreversibility, ii) returns to the investments are uncertain, and iii) individuals has an option to postpone the investment decision. In literature on optimal investment behavior under uncertainty, it is argued that these assumptions usually generate a considerable value of waiting; see e.g., Dixit and Pendyck (1994), Fafchamps and Pender (1997), and Malchow-Møller and Thorsen (2005). The model in the present paper, show that this option value is potentially very important.

2.3. **Other models of Dynamic Occupational Choice.** Even though initial wealth apparently plays such an important role in the choice to become entrepreneur, early studies of the choice to become entrepreneur as well as models of household saving decisions, have paid surprisingly little attention to the role of entrepreneurial savings among wealthy high income households. One motivation for some of the existing studies on entrepreneurial saving behavior has therefore been to deal with the fact that existing intertemporal models of saving and consumption reproduce the distribution of wealth poorly. Quadrini and Rios-Rull (1997) reviews heterogeneous agent versions of standard neoclassical growth models with uninsurable shocks to earnings. The models endogenously generate differences in asset holdings as a result of the household's desire to smooth consumption while earnings fluctuate. The conclusion is that the two dominant types of models - dynastic and life-cycle models - reproduce the distribution of wealth poorly.

However, as mentioned in the introduction, there is an increasing recognition of the importance of dynamic aspects in entrepreneurial decision making; see Quadrini (2000), Cagetti and DeNardi (2002), Buera (2003), and Tarajima (2004).

Quadrini (2000) develops a infinite-horizon dynamic general equilibrium model where entrepreneurs, subject to credit constraints, can save to finance new innovations. The key factors that explain the saving behavior of (potential) entrepreneurs are: i) The incentive to accumulate the minimum required assets to engage in entrepreneurial activity or to implement larger projects; ii) additional borrowing costs associated with external financing; and iii) an insurance motive induced by the greater uninsurable income risks faced by entrepreneurs. An additional important feature of the model is the specific modeling of a learning process associated with entrepreneurial activity. Through this learning process, the probability of getting better entrepreneurial ideas increases if the entrepreneur runs successful projects.

The key results in Quadrini (2000) are two-fold: First, the implied entrepreneurial asset accumulation in the model is able to explain the large wealth concentration among the rich in the U.S. economy. Secondly, observed transition patterns between wealth classes can be explained by occupational transition behavior. Consistent with the data from PSID, surviving entrepreneurs and individuals who enter into entrepreneurship are associated with upward transitions in wealth classes.

Tarajima (2004) builds on a model of wealth distribution to include education and occupation choices. Tarajima's model is basically an extension of the model in Quadrini (2000), where the main difference is that households can invest in their direct descendant's education.

In terms of analyzing the dynamics of the entrepreneurial choice, the main limitation of these models is that they only analyze the equilibrium around a steady state - not the transition to this equilibrium. To better understand the intertemporal incentives underlying observed transitions between occupations a full solution of an intertemporal model is needed.

Cagetti and DeNardi (2002) study the importance of intergenerational transfers develops an overlapping generation in a model with endogenous credit constraints and a specific role for intergenerational transmission of wealth and ability. They show that relaxing existing borrowing constraints with generate more entrepreneurs and increase the concentration of wealth in the economy.

This is done in Buera (2003) - though in a simplified environment. To get a tractable model of occupational choice with life-cycle savings, Buera (2003) formulate a deterministic infinite horizon model of occupational choice in continuous time. The model generates a well defined transition pattern of individuals moving from wage work to entrepreneurship. However, since there is no income uncertainty or stochastic elements in the model, it is not able to generate continuous cycling between the states; only the transition to entrepreneurship can be analyzed. As a result, either individuals fall into a poverty trap an remain wage workers forever, or else they enter entrepreneurship, which is an absorbing state.

3. A Dynamic Model of Occupational Choice

3.1. **The Model.** We begin with a basic framework that builds on the intertemporal model introduced by Deaton (1991) of saving and consumption under liquidity constraints. As discussed in the previous section, the new feature is that income is not exogenous, but depends crucially on wealth and occupational choice.

In each period, individuals choose a level of consumption that maximizes a time-separable infinite-horizon utility function

$$E_t\left(\sum_{\tau=t}^{\infty}(1+\delta)^{t-\tau}u(c_\tau)\right)$$

where $\delta > 0$ is the subjective discount rate, c_t is consumption at time t, E_t summarizes expectations given the information available at time t and $u(\cdot)$ is an instantaneous utility function defined over current consumption. The instantaneous utility is assumed to be of the CRRA form: $u(c_\tau) = (1-\eta)^{-1} c_\tau^{1-\eta}$, with $\eta > 0$

At the end of period t, the individual has assets a_t and receives income y_t. The sum $x_t = a_t + y_t$, "cash on hand", is then divided between consumption in period t, c_t, savings, $s_t = (1+r)^{-1}a_{t+1}$ and possibly costs of switching occupation, $\phi(i_t, i_{t+1})$ .Savings, s_t earn interest, r, which become assets in the following period.

Hence, the evolution of liquid assets a_t is governed by

$$a_{t+1} = (1+r)(a_t + y_t - c_t - \phi(i_t, i_{t+1}))$$

It is assumed that individuals are liquidity constrained, implying that liquid wealth can never fall below zero

$$a_t \geq 0, \quad \forall\, t \tag{3.1}$$

To make the model a model of occupational choice, individuals choose among two mutually exclusive work alternatives: Entrepreneurship, e, or wage-employment, we. Compared to wage work, entrepreneurship is a fundamentally different occupational choice with respect to the source of income. Wage-workers inelastically supply one unit of labor at an uncertain market wage, $w\varepsilon_t^{we}$, where ε_t^{we} summarizes the uncertainty in wage income and is distributed according to a truncated normal with mean 1 and variance σ_{we}^2. To ensure a bounded state space, ε_t^{we} is defined on the bounded support $[\underline{\varepsilon}^{we}, \bar{\varepsilon}^{we}]$. Entrepreneurs, on the other hand, derive income from production. Hence, the state dependent income is given by:

$$y_t(i_t, h_t, a_t, \varepsilon_t(i_t)) = \begin{cases} \pi(h_t, a_t, \varepsilon_t^e) & \text{if } i_t = we \\ w\varepsilon_t^{we} & \text{if } i_t = w \end{cases}$$

where $\pi(h_t, a_t, \varepsilon_t^e)$ denotes the entrepreneurs profit function given the level of entrepreneurial human capital, h_t, liquid assets a_t and the productivity chock, ε_t^e.The productivity chock, ε_t^e and entrepreneurial human capital,h_t are assumed to affect the productivity of the entrepreneur directly, while liquid assets operate indirectly through possibly binding capital constraints. If entrepreneurs are capital constrained, they must use own wealth to finance their investments. Therefore the wealth the of entrepreneur affects the efficiency scale of the business.

The transition costs, $\phi(i_t, i_{t+1})$, are specified as:

$$\phi(i_t, i_{t+1}) = \phi^{entry}\mathbf{I}(i_t = we, i_{t+1} = e) + \phi^{exit}\mathbf{I}(i_t = e, i_{t+1} = we)$$

where $\mathbf{I}(\cdot)$ is the indicator function. Hence if an individual switches from wage work to entrepreneurship, $\mathbf{I}(i_t = we, i_{t+1} = e) = 1$ and transition costs equals ϕ^{entry}. Conversely, ϕ^{exit} are transition cost associated with closing down a business.

Entrepreneurial human capital, h_t, evolves according to

$$h_{t+1} = \gamma h_t + \Delta(i_t)$$

where $\gamma < 1$ is the depreciation rate of experience and $\Delta(i_t)$ is the amount of entrepreneurial human capital gained in occupation i_t

If individuals choose to run their own business, they must devote their entire labor endowment to operate the business and have to decide how much capital to invest in the business. As soon as the occupational choice is made, the investment decision is purely static. Entrepreneurs derive income from the production of a single homogeneous good according to a Cobb-Douglas production function $f(h_t, k_t) = \theta k_t^{\alpha_k} h_t^{\alpha_h} \varepsilon_t$, defined over two production factors - entrepreneurial human capital, h_t and the amount of capital invested in the business, k_t. Individuals are assumed to differ with respect to their initial level of assets a_0 and their entrepreneurial ability, θ.

Once the investment decision is made, the entrepreneur receives a realization of the stochastic element of production, ε^e. The disturbance ε^ε summarizes the uncertainty in entrepreneurial income. ε^e is assumed to be independent and identically distributed with bounded support $[\underline{\varepsilon}, \overline{\varepsilon}]$, mean, $\mu_\varepsilon = 1$, and variance, σ_ε^2. As we shall see later, the assumption about boundedness is necessary to ensure a compact state space.

If $k_t > a_t$, the entrepreneur is a net borrower and must rent the remaining capital at a fixed interest rate, r. However, in line with Evans and Jovanovic (1989), it is assumed that entrepreneurs can only borrow up to an amount proportional to the stock of liquid assets a_t. Letting the factor of proportion being $\lambda - 1$,where $\lambda \geq 1$, a potential entrepreneur faces the credit constraint

$$k_t \leq (\lambda - 1)\, a_t + a_t, \; \forall \; t \tag{3.2}$$

or:

$$k_t \leq \lambda a_t, \; \forall \; t$$

If $\lambda = 1$, the entrepreneur must finance all activities in the business from the holding of liquid assets, a_t, while there are no liquidity constraints when $\lambda \to \infty$.

This assumption can be motivated by an underlying market friction, where loan contracts are imperfectly enforceable. Cagetti and DeNardi (2002) explicitly model this type market friction, and find that it generates endogenous entrepreneurial borrowing constraints. In this setup, own wealth act as a collateral to reduce the incentive to default: The larger the amount, the entrepreneur is able to finance from own wealth, the larger the amount the creditor is able to recover. Therefore, the amount the entrepreneur is able to borrow increases with liquid asset holdings, a_t

At the tome when the investment is made, the entrepreneur cannot observe or foretell the value of the idiosyncratic income chock ε_t^e. Thus, the investment decision is taken conditional on the level of entrepreneurial ability, θ, human capital, h_t, and liquid assets, a_t. In each period, the entrepreneur therefore derives his optimal investment by solving the following maximization problem

$$E_\varepsilon \pi \left(h_t, a_t, \varepsilon_t^e\right) = \max_{k_t \leq \lambda a_t} \left(\theta k_t^{\alpha_k} h_t^{\alpha_h} - r k_t\right) \tag{3.3}$$

At an interior maximum, the first order condition is

$$\alpha_k \theta h_t^{\alpha_h} k_t^{\alpha_k - 1} - r = 0$$

By the concavity of (3.3), the optimal level of capital can thus be written as

$$k_t^* = \min\left\{\lambda a_t, \left[\frac{\alpha_k \theta}{r} h_t^{\alpha_h}\right]^{1/(1-\alpha_k)}\right\}$$

For entrepreneurs to be unconstrained we must have

$$\begin{aligned} k_t^* &< \lambda a_t \Rightarrow \\ \theta &< (\lambda a_t)^{1-\alpha_k} \frac{r}{\alpha_k h_t^{\alpha_h}} \end{aligned} \tag{3.4}$$

Since marginal productivity of capital is increasing in entrepreneurial ability, θ more able individuals are more likely to be credit constrained.

In sum, the profit function for entrepreneurs can be written as

$$\pi(h_t, a_t) = \min\left\{\theta (\lambda a_t)^{\alpha_k} h_t^{\alpha_h} \varepsilon_t - r\lambda a_t \ , \ \theta \left[\frac{\theta \alpha_k}{r} h_t^{\alpha_h}\right]^{\frac{\alpha_k}{1-\alpha_k}} h_t^{\alpha_h} \varepsilon_t - r\left(\frac{\alpha_k \theta}{r} h_t^{\alpha_h}\right)^{1/(1-\alpha_k)}\right\}$$

If entrepreneurs are credit constrained, entrepreneurial earnings depend on individual wealth, α_t, while earnings is independent of the level of assets if they are not, i.e. if $k_t < \lambda a_t$.

To summarize: Given current occupation, i_t, cash on hand, x_t and entrepreneurial human capital, h_t and the state dependent income function, $y_t(i_t, h_t, a_t, \varepsilon_t(i_t))$, individuals optimally choose i) assets to carry over for the following period, a_{t+1} and ii) future occupation i_{t+1} to maximize a discounted stream of utility.

3.2. **Characterization of the Solution.** I start by characterizing the solution of the model by inspecting the first order conditions for the intertemporal allocation - the Euler equation. Even though it is not possible to derive a complete analytical solution for the model, the Euler equation provide a convenient way to characterize some of the mechanisms in the model - analytically.

Since the occupational choice is discrete, individuals face only one continuous intertemporal choice, the savings decision. Therefore we can only derive one state dependent Euler equation originating from the first order condition with respect to assets in the following period, a_{t+1}

(3.5)

$$u'(c_t) = \frac{1+r}{1+\delta} E_t \left\{ u'(c_{t+1}) \left[1 + \frac{dy_{t+1}(i_{t+1}, h_{t+1}, a_{t+1}, \varepsilon_{t+1}(i_{t+1}))}{da_{t+1}} \right] \right\} + (1+r)\, \mu_{a_{t+1}}$$

where $\mu_{a_{t+1}} \geq 0$ is the Lagrange multiplier associated with the borrowing restriction in (3.1).

The first order condition in (3.5) states that in optimum, it should not be possible to increase utility through a reallocation of consumption via a_{t+1}. Hence, marginal utility of consumption today (the left hand side) should equal the sum of i) the discounted expected marginal utility consumption in the next period, corrected for the change in future income due to the change in assets and the difference between the subjective and the objective discount rate; and ii) the shadow price of the liquidity constraint.

To identify the different savings motives is important to distinguish between the two types of borrowing constraints.

(1) The *liquidity* constraint faced by all individuals, preventing individuals to smooth consumption perfectly if income fluctuations occur.
(2) The *credit* constraint faced by entrepreneurs with a relatively low level of initial assets.

In the absence of these borrowing constraints, individuals would smooth out consumption so that discounted expected marginal utility is equalized across time periods. This results in the well known consumption/income divergence, which can be explained with essentially the same logic Friedman used long time ago (see Friedman (1957)): consumption does not respond one-for-one to transitory shocks to income because assets are used to buffer consumption against such shocks. This is referred to as the life-cycle saving motive.

However, in the presence of borrowing constraints, individuals reduce consumption today to overcome the expected utility loss induced by either of the two borrowing constraints This leads to two additional saving motives in the model: i) a precautionary saving motive and ii) a entrepreneurial saving motive.

As pointed out in Kimball (1990) a key theoretical requirement to produce precautionary savings is prudence of the value function, $V(x)$. Formally, Kimball (1990) defines prudence of the value function as $-V'''(x)/V''(x)$ or equivalently the convexity of the marginal value function at x. The precautionary motive is present only if individuals are prudent,. i.e. if the marginal value function at x_t is convex. This is the case if the marginal instant utility is convex, i.e. $u'''(c) > 0$.

In the present model, $u(.)$ is CRRA, with coefficient $\eta > 1$. Thereby, $E_t u'(c_{t+1}) \geq u'(c_{t+1})$ and therefore individuals reduce consumption today to overcome the expected utility loss induced by the binding liquidity constraint. In other words, individuals save to buffer against future negative income shocks.

Note that in periods where none of the liquidity constraints are binding $\mu_{a_{t+1}} = 0$ and $k_{t+1} > \lambda a_{t+1}$ the first order condition collapses to a standard Euler equation, where discounted expected marginal utility is equalized over time. However, as pointed out by Deaton (1991), even when liquidity constraints do not bind in a given period, this does not imply that the optimal saving policy coincides with the policy function from the problem without liquidity constraints. The reason is that individuals anticipate that liquidity constraints could be binding in the future. This illustrates that the Euler equation is not a sufficient condition for optimal behavior. Rather, it puts restrictions on the allocation of resources between two successive periods.

With respect to the entrepreneurial motive, credit constraints has an additional effect. Due to the presence of credit constraints, a reallocation of current consumption into future assets, a_{t+1} adds additionally to future consumption through an expected increase in future income

$$\frac{dy_{t+1t}\left(i_t, h_t, a_t, \varepsilon_t\left(i_t\right)\right)}{da_{t+1}} = \begin{cases} \frac{d\pi_t(,h_t,a_t,\varepsilon_t^e)}{da_{t+1}} > 0 & \text{if } i_{t+1} = e \text{ and } k_{t+1} = \lambda a_{t+1} \\ 0 & \text{if } i_{t+1} = we \text{ or } k_{t+1} < \lambda a_{t+1} \end{cases}$$

The reason for the additional saving motive is that entrepreneurs who operate at a suboptimal level of capital due to binding credit constraints can expect an increase in profits if they save more, $\frac{d\pi(h_t,a_{t+1})}{da_{t+1}} > 0$. Just like the precautionary motive, if an individual knows that it will ever be optimal to enter entrepreneurship, this savings motive is relevant at all times (due to the recursive nature of the first order condition).

The entrepreneurial saving motive depends crucially on the factor of proportion $\lambda - 1$, at which entrepreneurs can borrow. The following proposition states how saving incentives are affected by changes in λ

Proposition 1. *The entrepreneurial saving motive is affected by λ in a non-monotone way: For small values of λ, relatively productive individuals with relatively low asset holdings will increase their savings when λ increases. On the other hand, for λ large enough, i.e. when credit constraints become less binding, the expected return to increased savings approaches zero.*

Proof. See appendix □

The intuition behind proposition 1 goes as follows: Initially, as λ increases some highly productive individuals will find it optimal to save more as the perspectives for (future) entrepreneurship becomes better. As λ is further increased, credit constraints eventually become irrelevant, thereby lowering the incentive to save.

Proposition 1 has important implications for the understanding the effect of changes in credit policy, e.g. government loan guaranties. According to proposition 1, relatively productive individuals will increase savings, while less productive individuals decrease savings. On the one hand, this kind of policy will increase wealth inequality in the economy and could be associated with increased probability of default. On the other hand, increasing λ also increases the probability of entry relatively more for productive individuals - due to the increased willingness to use savings to overcome credit constraints.

3.3. **The Dynamic Programming Problem.** In specifying this as a dynamic programming problem, note that the state variables x_t, s_t, h_t summarize all information about the past that bears on current and future decisions. Since y_t is assumed to be *iid* distributed conditional of $h_{t,}$ a_t and i_t, only the sum $x_t = a_t + y_t$ "cash on hand" is relevant for current and future saving decisions and occupational choice. Note also that the problem is stationary in the sense that optimal choices do not depend on time *per se*. Hence, time subscripts can be dropped. To discriminate between the current and future periods, I therefore denote next period variables with a prime. The resulting Bellman equation can thus be formulated as

$$V(x,h,i) = \max_{i',a'\in\Lambda(x,i,h)} u(x,i,a',i') + \beta E_t\left[V(x',h',i')\,|x,h,i\right] \tag{3.6}$$

where E_t summarizes expectations give the information available at the time the decision is made and the value function,$V(x,h,i)$, is the maximum expected discounted utility obtainable by the agent in the given state (x,h,i). The Bellman equation express the recursive relationship between the value function in the current period, $V(x,h,i)$, current utility, $u(x,i,a',i')$, and expectations over the value function in the following period, $E_t\left[V(x',h',i')\,|x,h,i\right]$. Hence, individuals choose $i',a'\in\Lambda(x,i,h)$ to maximize the sum of current utility and discounted expected future utility.

Current utility is

$$u(x,i,a',i') = u\left(x-(1+r)^{-1}a'-\phi(i,i')\right) \tag{3.7}$$

and $\Lambda(x,i,h):\mathbb{S}\to\mathbb{D}$ is a correspondence that summarizes the feasible choice set

$$\Lambda(x,i,h) = \left\{ (a',i')\in\mathbb{D}: \begin{array}{l} 0\le a'\le(1+r)(x-\phi(i,i')) \\ h'=\gamma h+\Delta(i) \\ i'\in I=\{e,we\} \end{array} \right\} \tag{3.8}$$

To make notation a bit more compact, let $\mathbb{D}$ denote the set of controls and let $\mathbb{S}$ denote the state space, such that

$$\begin{aligned} d &= (a,i)\in\mathbb{D} \\ s &= (h,x,i)\in\mathbb{S} \end{aligned} \tag{3.9}$$

Furthermore, $\Lambda(s)\subseteq\mathbb{D}$ is the non-epmty set of feasible controls that summarizes the contingent constraints on the controls d' in state s and $u:\mathbb{S}\times\mathbb{D}\to\mathbb{R}$ is the current pay-off function in given the current state $s\in\mathbb{S}$ and given the control $d'\in\mathbb{D}$ is applied in the following period. Finally let $f(s'|s,d')$ be the probability density that $s'\in S$, i.e. the conditional density that future state s' occurs given current state and control d'. We can now express the infinite horizon, discounted, time separable dynamic programming problem in more compact notation

$$V(s) = \sup_{d' \in \Lambda(s)} u(s, d') + \beta \int V(s') f(s'|s, d') \, ds'$$

Solving the model is equivalent to finding a fixed point of the Bellman equation (3.6). Under certain conditions, a unique solution exist and successive iterations on the Bellman equation will guarantee global convergence to this solution. Roughly speaking, these condition will be met if the subjective discount factor is less than unity, $\beta < 1$, the state space, $\mathbb{S}$ is a compact set and the value function is bounded on this set.

I start by formulating two propositions, stating that is possible to restrict attention to a compact subset of the state space $\mathbb{S}$[2].

Proposition 2. *There exists $h_{high} < \infty$ such that if $h_t \leq h_{high}$ then h' satisfies $h' \leq h_{high}$*

Proof. See appendix. □

In other words, there exists an upper level of h, where the depreciation of experience exceeds the human capital gain in any occupation. Therefore h_t is bounded above.

Proposition 3. *Given $h \leq h_{high}$ where h_{high} satisfies the proposition 2, there exists an $a_{high} < \infty$ such that if $a \leq a_{high}$ then the optimal choice of a' satisfies $a' \leq a_{high}$*

Proof. See appendix. □

Hence, there exists some (finite) upper level of assets, a^{high}, where individuals who for some reason own more than this level, will stop saving.

The intuition behind Proposition 3 goes as follows: Since marginal productivity of capital is decreasing and since shocks to production has bounded support, marginal returns to savings will approach the interest rate, r as a increases, and earnings, y will be bounded from above and below. Since individuals are impatient, in the sense that

[2]The compactness of state space for the discrete occupational choice i_t is trivial as it can only take two values.

$\delta > r$, for a large enough, it will be optimal to stop saving as the life-cycle motive will dominate both precautionary and entrepreneurial saving motives.

We are now ready to formulate conditions that guarantee that the considered dynamic programming has the contraction mapping property. I formalize this in the following proposition which is stated without a formal proof

Proposition 4. *Let* $\mathbb{S}$ *be defined by* (3.9) *with values of* h_{high} *and* a_{high} *satisfying propositions (2) and* (3). *Let* $u : \mathbb{S} \times \mathbb{D} \to \mathbb{R}$ *and* $\Lambda : \mathbb{S} \to \mathbb{D}$ *be given by* (3.7) *and* (3.8) *respectively. Furthermore, let* $f(s'|s, d') = f(x'|s, d')$ *be a continuous density function defined on a bounded support* $[x^{low}, x^{high}]$ *such that* $c^{low} = x^{low} - (1+r)^{-1} a^{high} - \phi(i, i') > 0$. *Then the mapping defined by*

$$\Gamma(V)(s) \equiv \sup_{d' \in \Lambda(s)} u(s, d') + \beta \int V(s') f(s'|s, d') ds' \tag{3.10}$$

is a contraction mapping $\Gamma : \mathbb{B} \to \mathbb{B}$ *taking a complete normed vector space (i.e. a Banach space) of functions from* $\mathbb{S} \to \mathbb{R}$. *The nonlinear operator* Γ *has a unique fixed point* $V = \Gamma(V)$ *and for any* $V_0 \in \mathbb{B}$

$$\left\|\Gamma^k V_0 - V\right\| \leq \beta^k \|V_0 - V\|, \quad k = 1, 2, ... \tag{3.11}$$

Under the conditions stated in the proposition above, the dynamic programming problem has a unique fixed point and successive value function iterations will converge to the unique solution.

As mentioned, a rigorous proof will not be given here. Instead I will try to give an intuitive reasoning: First, u is bounded from below as long as $c = x - (1+r)^{-1} a' - \phi(i, i') > 0$ and bounded above if $s = (h, x, i) \in \mathbb{S}$ is bounded. With values of h_{high} and a_{high} satisfying propositions (2) and (3), it is necessarily the case that $s = (x, h, i)$ stays within a compact set. Secondly, since the continuous density $f(x'|s, d')$ has bounded support $[x_{low}, x_{high}]$ with $x_{low} = y_{low} > 0$, consumption can always be sustained above zero.

I.e. for all values of $x \in [x_{low}, x_{high}]$ it is always feasible to chose a' such that $c > 0$ and consequently u is bounded from above and below and the integral $\int V(s') f(s'|s, d') ds'$

is therefore well-defined. Third, the correspondence $\Lambda(s) \subseteq \mathbb{D}$ is non-epmty and compact valued. Finally, the effective discount factor is below one $\beta = 1/(1+\delta) < 1$ (by assumption). To get the intuition clear: it is necessary to bound the support of the distribution of the disturbance, ε from below such that income, and thereby consumption, are bounded from below too. Otherwise the expectation of the value function may not be well-defined. Under these assumptions plus some regularity conditions, it follows that Γ satisfies the contraction mapping theorem, see Stokey and Lucas (1989)[3]

To solve the model, we have to find a fixed point of the functional equation in (3.6). I use chebyshecv polynomials to represent the value function over the continuous state space. Since the value function has discontinuous first derivatives in the swiching point, I use *piecewise* Chebyshev polynomials to approximate the value function with an edogenously determined join point at the kink of the value function. The use of Chebyshev polynomials to approximate the value function has one important spin-off. Once the model has been solved, Chebyshev approximation of the value function can be utilized to express the policy function in any point of the state space, at almost zero computational cost. .

In line with Rust (1987) a combination of successive contraction iterations and the Newton-Kantorowich algorithm will be used. While contraction iterations guarantee convergence due to the contraction mapping property, the procedure slows down when

[3]It actually turns out that one of the conditions is violated in the present context. Since one of the state variables is discrete, the requirement that the state space is a convex set - is obviously not satisfied. However, the conditions stated in Stokey and Lucas (1989) are *sufficient* conditions and thus too restrictive in the present model. A more general version of the theorem is available in Denardo (1967)

TABLE 1. Baseline Parameter Values

r	η	δ	θ	α^h	α^k	σ_ε^e	λ	w	σ_ε^{we}	γ	Δ	ϕ^{exit}	ϕ^{entry}
0.04	1.5	0.07	0.8	0	0.33	0.3	1	1	0.1	–	–	0	0

the approximation errors $\|V_k - V\|$ become small.[4] In contrast, Newton-Kantorowich iterations are not guaranteed to converge, but converge in a quadratic rate in the neighborhood of the solution[5]. The resulting fixed point algorithm known as the *poly-algorithm,* combines these two algorithms in order to balance robustness versus speed of convergence.

4. Numerical Results - The case without Human Capital Accumulation

In this section, I present numerical solutions of the model. For the purpose of exposition, I will first consider a simpler version of the model where I assume that $\Delta(i) = 0$ and $h_t = 1$. The model without human capital accumulation will serve as a useful starting point, when explaining some of the key features of the model: In particular, I will discuss: i) how highly productive potential entrepreneurs can use savings to overcome binding liquidity constraints; ii) how entry and exit costs affects savings decisions and the transition between the two occupations; and iii) how individuals depending on their initial wealth and entrepreneurial productivity approach two different equilibria in the long run.

4.1. **Baseline Calibration.** Rather than trying to calibrate the model to observed data in order to give quantitative predictions about behavior, the baseline parameters are chosen to identify the mechanisms of the model. The baseline values used in the numerical simulations are listed in Table 4.1.

[4]It follows directly from equation (3.11) in proposition 4 that the upper bound on the approximation error $\beta\|V_k - V\|$ dcreases linearly in $\|V_k - V\|$ (making convergence particular slow for β close to 1).

[5]Kantorovich's Theorem guarantees that given a starting point V_0 in a domain of attraction of the fixed point V of Γ the Newton Kantorovich iterations will converge to V at a quadratic rate (see Rust (1996))

Utility parameters: The first two parameters will be set with little controversy. Taking the time period to be one year, I let the real interest rate of $r = 0.05$ reflect the average market return to wealth. I choose $\eta = 1.5$ as a reasonable value of the the inverse of the intertemporal elasticity of substitution (see e.g. Caroll (1997) and Deaton (1991)). Due to the functional form of the instantaneous utility function, the relative risk aversion and the intertemporal rate of substitution are inversely related.[6] A choice of $\eta = 1.5$ therefore implies that agents are risk averse and slightly prudent - which seems empirically sensible. Harrison, Lau, and Rutstrom (2004) estimate individual risk attitudes using controlled field experiments in Denmark. Their results indicate that the average Dane is risk averse, and that risk neutrality is an inappropriate assumption to apply. They also find that risk attitudes vary significantly in the population roughly within a range of $\eta \in [0, 2]$.

I set the time preference rate $\delta = 0.07$ to be larger than the real interest rate to reflect relatively impatient agents. In the empirical savings literature, the rate of time preference has been estimated much higher. This also applies for the recent literature on experimental economics: Harrison, Lau, and Williams (2002) estimate annual individual discount rates with respect to time to be around 0.25[7]. Deaton (1991) used $\delta = 0.1$ in the simulations of his model. Quadrini (2000) used $\delta = 0.9$ in a dynamic model with entrepreneurial savings calibrated to the US wealth distribution. However, if δ is very high relative to r agents become very impatient and the incentives to accumulate assets

[6]Note that this is only the case if we think of household's preferences over consumption gambles in a static context. This interpretation of η has been subject to much criticism (see e.g. Flavin and Nakagawa (2005)) In a dynamic context, it is more relevant to think of relative risk aversion as the change in the curvature of the value function, i.e.

$$RRA = \frac{\partial^2 V(x,h) / \partial x^2}{\partial V(x,h) / \partial x} x > 0$$

Because the household's degree of risk aversion depends on the curvature of the value function, behavior towards income risks will not only depend on the curvature of the instantaneous utility function - also the state variables. In particular, very wealthy individuals will tend to be less risk averse. Therefore η is sometimes referred to as the curvature parameter.

[7]These estimates may reflect attitudes to risk also.

will be almost zero. Therefore, in order to better illustrate the savings incentives in the model, I set $\delta = 0.07$.

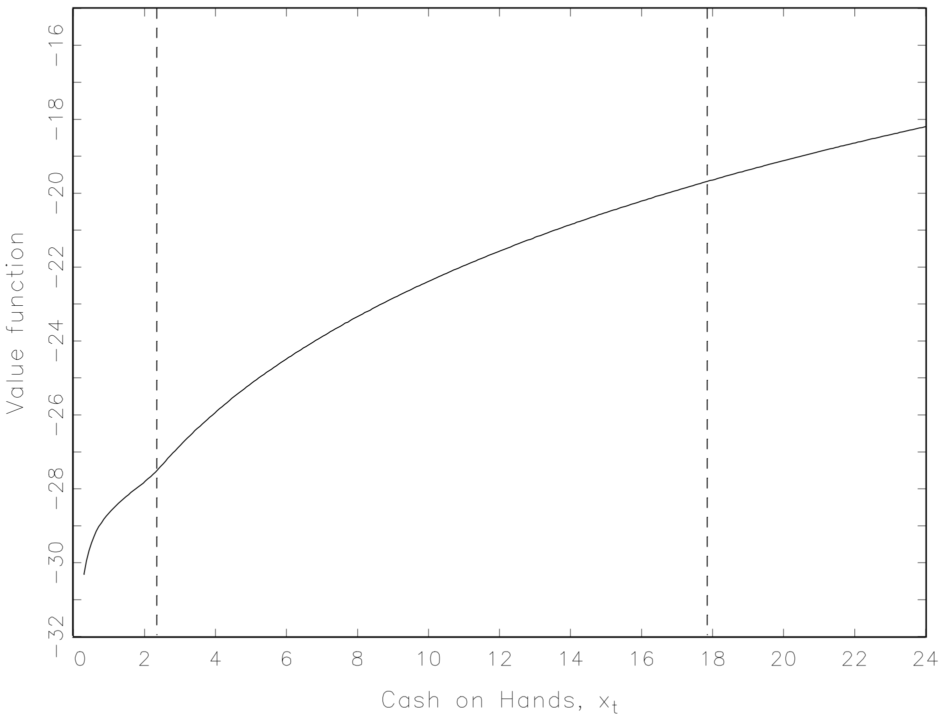
Income parameters: For convenience, mean wages are normalized to one, $w = 1$ with a standard deviation of $\sigma_\varepsilon^{we} = 0.1$. I set $\alpha^k = 0.33$ approximately equal to the structural estimates of return to capital in Evans and Jovanovic (1989) and $\sigma_\varepsilon^e = 0.3$ is chosen in accordance with their structural estimates of the dispersion in entrepreneurial earnings. Contrary to the estimates of Evans and Jovanovic (1989), I choose $\sigma_\varepsilon^{we} < \sigma_\varepsilon^e$ to reflect the very compressed wage structure in Denmark (see e.g. Malchow-Møller, Markusen, and Skaksen (2005)).

Framework conditions: In the baseline scenario, I set $\lambda = 1$ to reflect binding credit constraints. Hence it is not possible to borrow any funds for starting up a business. Entry and exit costs are set equal to zero, $\phi^{entry} = \phi^{exit} = 0$. In what follows, we shall see how changes in these parameters influence savings incentives, occupational choice, income etc.

Figure 1 displays the numerical solution of the value function in the baseline scenario. Two vertical lines mark two threshold levels of cash on hand: The leftmost line marks the reservation value of cash on hand, x_r, where individuals will choose to enter entrepreneurship. The rightmost line marks the level of cash on hands where entrepreneurs will be unconstrained, x_u. To the right of this line, the real interest rate would exceed the marginal product of capital - if all available funds were invested.

It should be apparent from Figure 1, that the value function, does not display the standard properties of concavity and differentiability. The value function is the upper envelope of two underlying value functions associated with each of the two occupational choices. In the crossing point, the indifferent individual switches occupation. The convexity around the kink of the value function is induced by the introduction of the investment opportunities in the model. This investment option ads an extra component to the marginal returns to savings and therefore individuals facing this option will have an entrepreneurial saving motive.

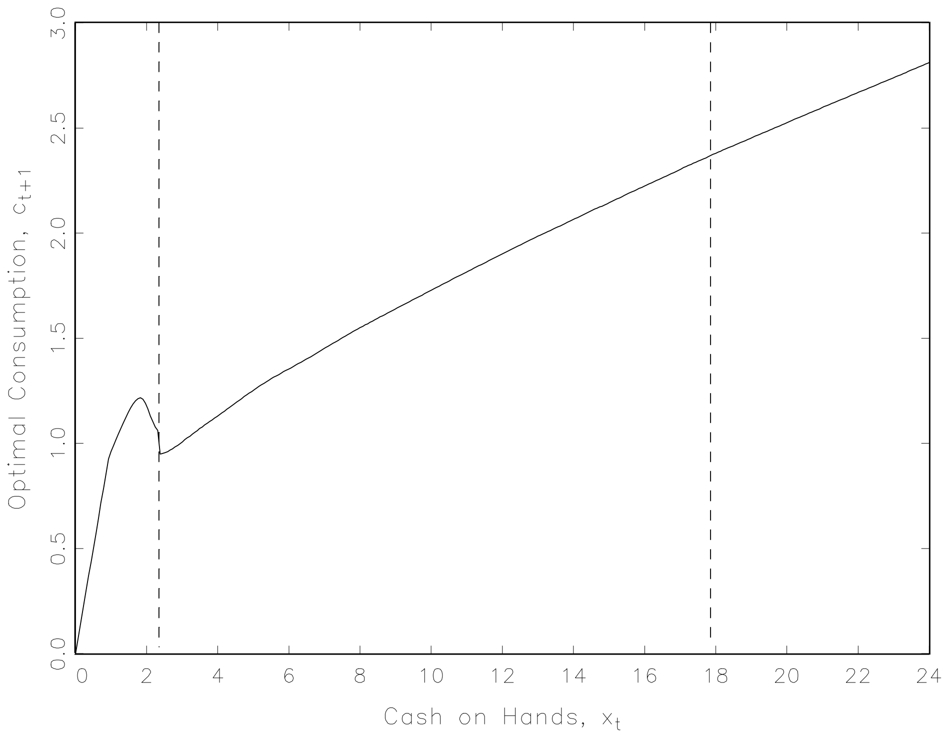
FIGURE 1. VALUE FUNCTION



As we have discussed earlier, the value function has a kink in the crossing point. This is precisely what makes the model solution non–trivial and what causes the Chebyshev approximation method to perform poorly. Therefore I use piecewise Chebyshev polynomials with a single join point in x_r, which is continuously updated at each iteration. This effectively avoids numerically unstable and imprecise solutions with occilirations in the policy functions and occasionally break downs in algorithm.

4.1.1. *Policy functions.* Figure 2 present the policy function for optimal consumption. Again, the two vertical lines mark the threshold values x_r and x_u. Starting from the left, we see that consumption equals cash on hands as long as liquidity constraints are binding i.e. $a_{t+1} = 0$. The individual would actually like to consume more today at the expense of tomorrow's consumption. But since liquid assets can never fall below zero, this is impossible. Due to the precautionary savings motive, the consumption policy function starts to bend off in a slightly concave way around $x_t = 1$. The segment below $x = 1.5$ on the policy function coincides with the saving behavior implied by the model in Deaton

FIGURE 2. OPTIMAL CONSUMPTION



(1991): Precautionary saving arises from the possibility that constraints might bind in the future. Therefore individuals use precautionary savings as an insurance against future negative shocks.

At some point, before the switching point, x_r, optimal consumption drops significantly: Individuals with this level of cash on hands starts to save to become entrepreneurs. In the absence of credit constraints, these individuals would have been entrepreneurs (In fact all individuals would be entrepreneurs at the baseline parameter values). Instead, they use savings to overcome binding liquidity constraints thereby seeking opportunities for higher future income. At the point where the individual chooses to enter entrepreneurship, the consumption policy function drops discretely to a local minimum. Since the entrepreneur is credit constrained and therefore have to operate at a suboptimal level of capital, the effective return to savings will exceed the market interest rate. Therefore, the individual will save more and consume less.

FIGURE 3. EXPECTED FUTURE GROSS INCOME

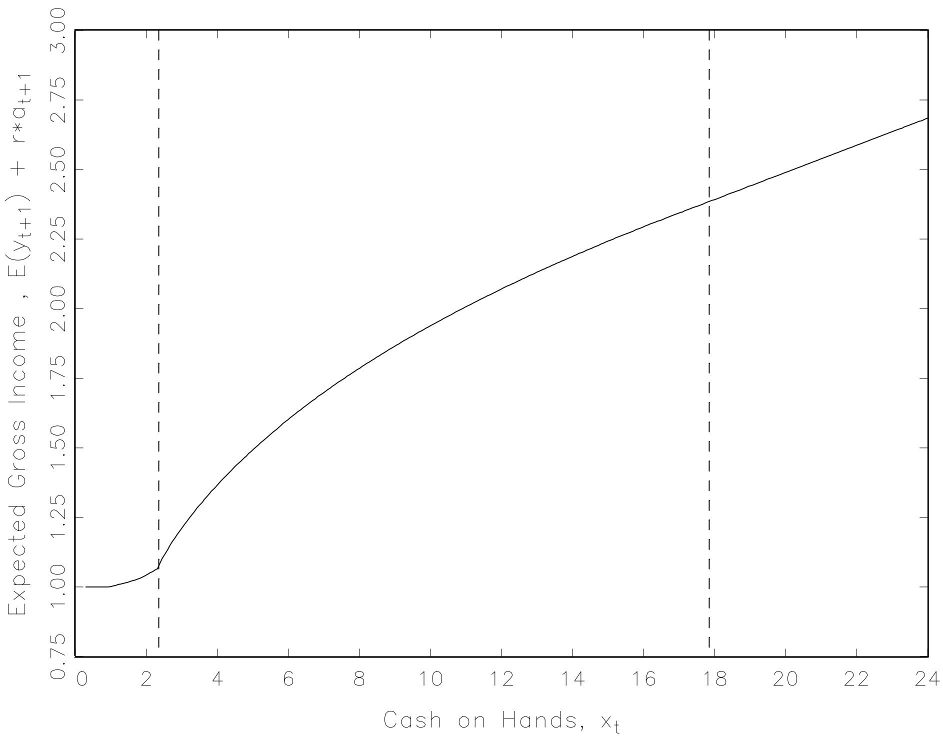


Figure 3 displays the relationship between wealth (cash on hand) and expected future gross income, $E\left[y_{t+1}|x_t\right]+ra_{t+1}$ which is the sum of expected earnings in the next period and interest on liquid assets. The individual earns the fixed wage, $w = 1$ until he chooses to become entrepreneur at the switching point, x_r. For low values of x_t, individuals are liquidity constrained, and will therefore not save any assets for the following period, i.e. $a_{t+1} = 0$. Hence, expected future gross earnings equals $w = 1$. Hereafter, individuals first start to save for precautionary reasons (around $x_t = 1$), then for entrepreneurial reasons (around $x_t = 1.8$). Correspondingly, asset returns, ra_{t+1} starts to increase. Between the two vertical lines, i.e. when $x_t \in [x_r, x_u]$ the entrepreneur is credit constrained and operates at a suboptimal level of capital. As we move towards x_u, the business becomes more capital intensive and the marginal product drops until the business reaches its unconstrained scale, where marginal product equals the real interest rate. Below the point x_u, entrepreneurs chose to invest all available assets in their business, therefore entrepreneurial earnings increase with cash on hand until the entrepreneur is unconstrained with respect to capital, i.e. when $x_t = x_u$. At this point, the entrepreneur is able to self-finance the investments needed to operate the business at the optimal scale and entrepreneurial income (net of interest) is independent of cash on hands.

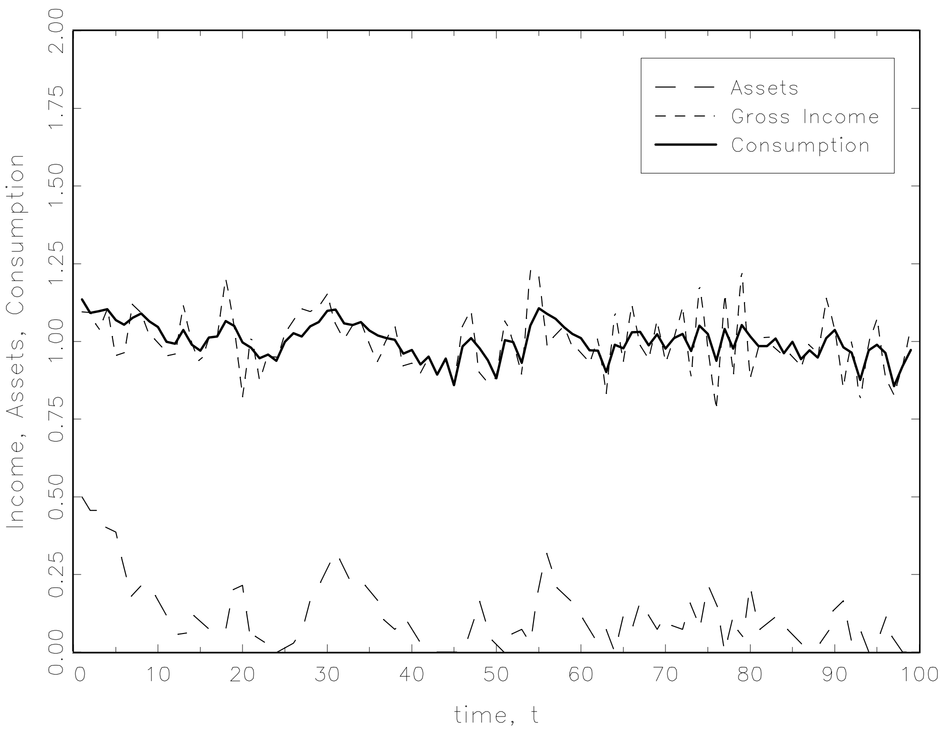
4.1.2. *Simulated Sequences.* Using the same set of baseline parameter values, stochastic model simulations are used to characterize the evolution of the state dependent variables: liquid assets, consumption and gross income. In order to characterize individual saving incentives, first note that there exists a threshold level of cash on hand, such that the individuals with cash below this threshold, $x_t < x_{ns} \in [0, x_r]$ will not save to become entrepreneurs. Unless a sequence of unanticipated positive income chocks occurs, these individuals will instead follow a path that converges to a stationary equilibrium, where the individual remains a wage worker and keeps small levels of precautionary savings as a buffer against negative income shocks. Thus, the entrepreneurial saving motive is dominated by the incentive to smooth consumption over time and relative impatience induced by the relatively high discounting of utility, $\delta > r$.

Individuals with cash on hand above this level, i.e. $x_t \geq x_{ns}$, expect to become entrepreneurs at some point in their carrier. Depending on their current level of available funds, these individual will either save or dis-save to reach an equilibrium level of cash on hands, $x_{ss} \in [x_{ns}, \bar{x}]$ where they will stop saving.

To illustrate how the evolution in individual income, wealth and consumption are affected by the level of initial assets, I simulate sequences of these variables for two different levels of initial assets. The sequences are displayed in Figures 4 and 5.

Consider first the sequences displayed in Figure 4. The individual enters the labor market as wage worker with a relatively low level of initial assets, $a_0 = 0.5$. These initial conditions result in a realized level of cash on hands below the threshold, x_{ns}. Hence, rather than saving to become entrepreneur, this individual find it optimal to remain a wage worker unless an unanticipated sequence of positive income chocks is realized. After a couple of periods, the simulated sequence of a_t has decreased to the stationary

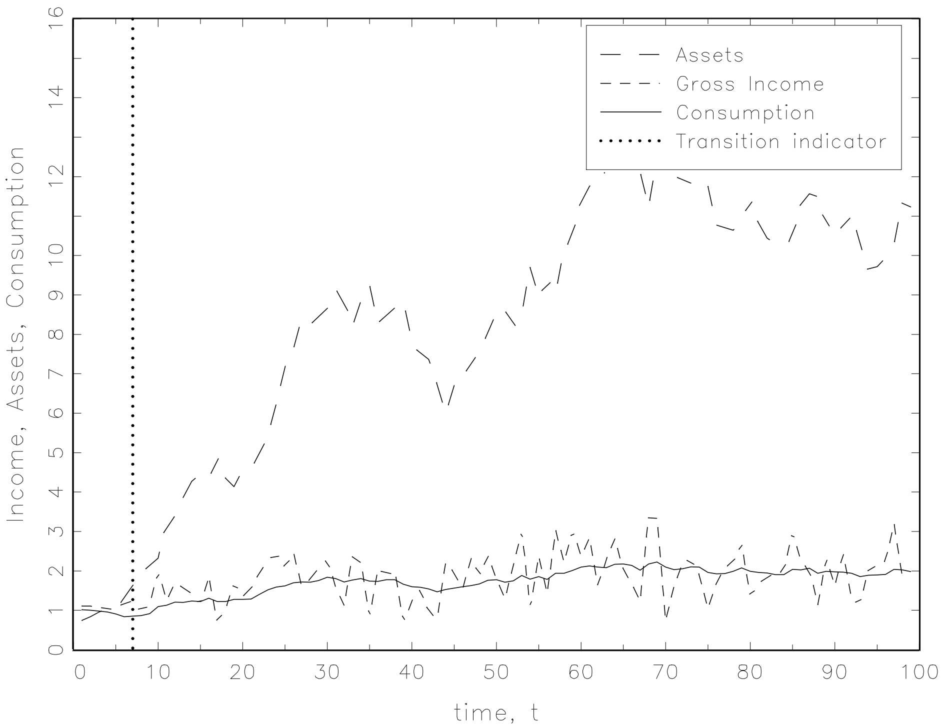
FIGURE 4. SIMULATIONS OF c_t, $y_t + r\alpha_t$ AND α_t, $a_0 = 0.5$



equilibrium, where precautionary savings are used as a buffer against negative income chocks. This individual behaves very much like the consumers in Deaton (1991). Hence, the following characteristics apply: First, consumption is notably smoother than income. Secondly, the downward spikes in consumption when liquid wealth stock-outs occur, are generally larger that the corresponding upward peaks. Consumption is therefore asymmetric, in the sense that mainly negative shocks are transmitted into consumption, whereas savings are used to smooth out positive shocks.

The displayed sequences in Figure 5 are associated with an individual entering the labor market as a wage worker with an intermediate level of initial wealth, $a_0 = 0.75$. Several things are worth noting. First, wage workers with an intermediate level of cash on hands, $x_t \in [x_{ns}, x_u]$ will have to save for several periods before entry to entrepreneurship is profitable. In fact, the wage worker associated with the simulated sequence in Figure 5 does not enter entrepreneurship until after seven periods of wage work. Hereafter,

FIGURE 5. SIMULATIONS OF c_t, $y_t + r\alpha_t$ AND α_t, $a_0 = 0.75$



the entrepreneur keeps saving until the entrepreneurial saving motive is dominated by impatience, i.e. the incentive to smooth consumption over the life-time.

Secondly, as the business becomes more profitable, consumption increases gradually over time to reach a higher equilibrium level.

Third, despite a very fluctuating entrepreneurial income, consumption is remarkably smoother for entrepreneurs than for wage-workers due to the higher stock of assets. This is due to the simplifying assumption that investments undertaken by the entrepreneur are fully reversible. Since invested assets are perfectly liquid, savings have a dual role: As working capital and to smooth out consumption.

This is opposed to Fafchamps and Pender (1997), where poor households fail to undertake a profitable investment that they could, in principle, self-finance because non-divisibility and irreversibility of the investment put it out of their reach. In the literature of investment under uncertainty, (see Dixit and Pendyck (1994)) it is emphasized that uncertainty works to decrease investments when investments are irreversible. I expect similar results would be found in this paper if entrepreneurial investments were (partly) irreversible. I.e. entrepreneurs would postpone some of the investments in the business and keep a buffer of liquid assets to smooth out consumption. In that sense, the combination of irreversibility and uncertainty introduces a value of waiting. A specific analysis of this phenomenon, would require an additional state variable (invested capital) and is therefore not pursued here.

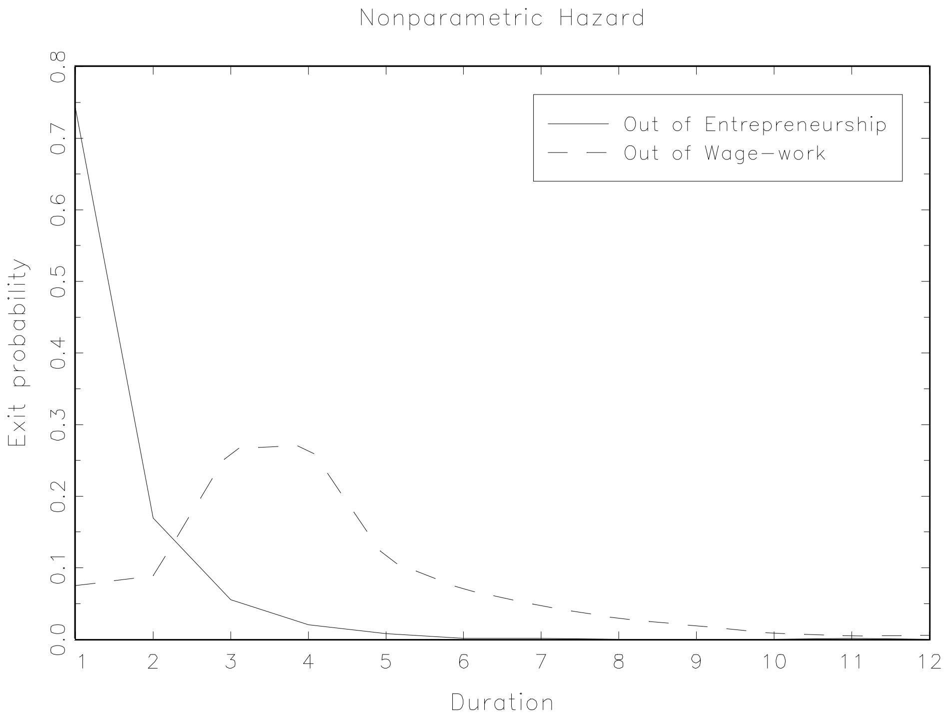
4.1.3. *Transition Patterns.* The model also generates well-defined transition patterns between entrepreneurship and ordinary wage work. To illustrate this aspect of the dynamics in the occupational choice, consider a population of individuals who enter the labor market as wage workers at an intermediate level of individual wealth, $a_0 = 0.75$.

Initially, these individuals will start saving to become entrepreneurs. Depending on the realized sequence of stochastic wages, some will fall into a 'poverty trap' and remain wage workers, others will save and enter entrepreneurship within a couple of periods. As soon as they become entrepreneurs, they accumulate capital to make the business more profitable and more resistant to negative shocks. Hence, individuals 'cycle' between the two occupational alternatives until they either fall into a poverty trap or accumulate enough assets to run a profitable business.

The implied transition behavior for these initially homogeneous agents can be analyzed by inspecting the occupational specific hazard functions displayed in Figure 6. The hazard functions are calculated as follows: The conditional probability that an individual exits a given initial state after a duration of τ periods, given τ periods of survival. Hence, the hazard function by construction will sum to one over τ.

Consider first the hazard out of entrepreneurship (the solid curve). Since, all individuals are initially wage workers saving to become entrepreneurs, the hazard function for entrepreneurs, is based on individuals that voluntarily entered entrepreneurship with a relatively low level of assets. Depending on the realized sequence of stochastic production shocks, some individuals will succeed in accumulating enough capital to resist negative shocks to production. In addition to the primary function as working capital,

FIGURE 6. SIMULATED HAZARD FUNCTIONS, $\alpha_0 = 0.75$



these assets serve as a buffer stock against the poverty trap. Therefore, the probability of exit to wage work will decrease with duration in entrepreneurship, i.e. the hazard function exhibits true negative duration dependence.

Now turn to the hazard function for wage workers (the dashed curve). Individuals who enter entrepreneurship will on average need 3 or 4 periods in wage employment to accumulate enough assets to start a business. If a wage worker receives a poor sequence of incomes for a longer period, available cash on hands falls below the threshold where it is no longer optimal to save to become an entrepreneur. Therefore, the exit probability decreases with duration. Note also that some individuals who re-entry wage work after a short duration in entrepreneurship, will relatively quickly re-exit to entrepreneurship[8]. Therefore an increased concentration around $\tau = 1$.

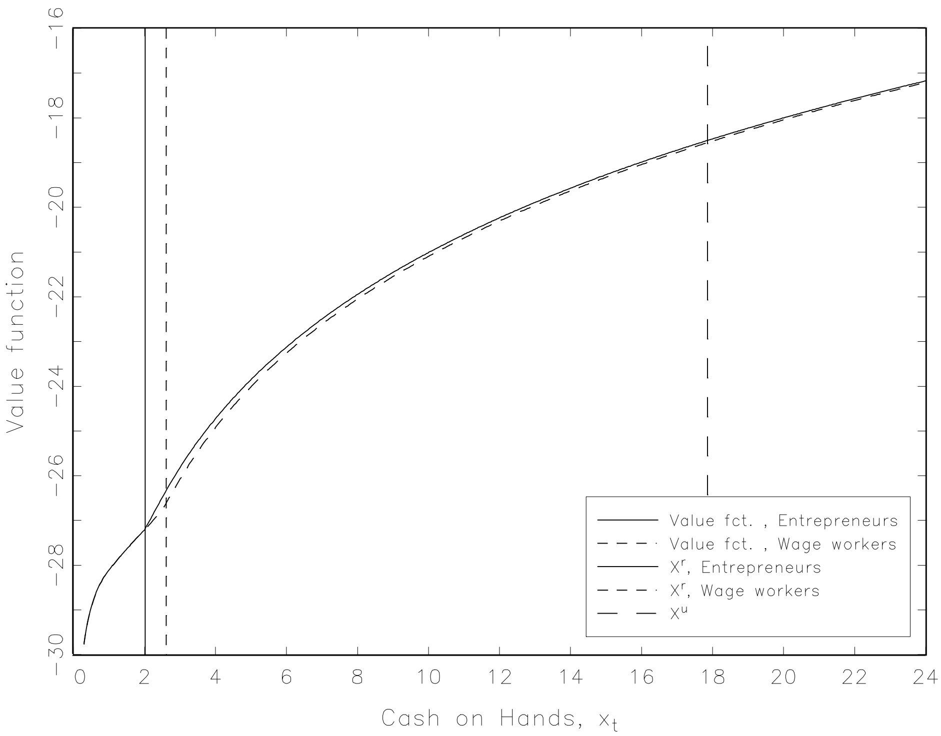
[8]Of course, this is not immediately evident from the hazard function. However, as closer inspection of the simulated sequence reveals this pattern

One important insight from the baseline scenario is that some individuals who expect to become entrepreneurs in the future, will save for several periods and accumulate a considerable amount of assets. As a back drop, note that precautionary savings models in Deaton (1991) and Caroll (1997) suggest that higher income uncertainty should lead to a higher level of precautionary savings. Previous authors (see e.g. Caroll and Samwick (1998)) has used this feature of the model to identify the level of precautionary savings from the cross sectional correlation between income risk measures and wealth holdings. As pointed out by Hurst, Lusardi, Kennickell, and Torrealba (2005), entrepreneurs may hold larger proportions of wealth for other reasons than precautionary savings. Therefore, entrepreneurs and wage workers have to be treated separately to identify the share of total liquid wealth which can be attributed to precautionary savings.

The results in this paper highlights the importance of conditioning on occupational status as entrepreneurial savings may constitute a significant share of total wealth for entrepreneurs. However, as we have seen *potential* entrepreneurs will start to save several periods prior to entry. Therefore, not only the current occupation is important for wealth accumulation also expected future occupations. Hence to mitigate a potential heterogeneity bias, we must appropriately control for expectations about future occupations too. Since these expectations are generally unobserved (and time varying), such conditioning is in general very difficult. Hence, to appropriately account for the composition of household savings, a full structural estimation of the present model can be a useful and perhaps necessary identification strategy.

4.2. **The Case with Business Start-up Costs.** The purpose of this section is to study how the presence of start up costs influence occupational choice and saving incentives. Figure 7 displays the numerical solution of the value function under the following model specification: Wage-workers deciding to become entrepreneurs incur an entry cost in the order of 20 pct, of expected wage income, i.e. $\phi^{entry} = 0.2$. The parameters of the model are otherwise identical to the baseline specification.

FIGURE 7. VALUE FUNCTION, $\phi^{entry} = 0.2$



To analyze the impact of entry cost, it is instructive to provide a few comparative remarks: First, in absence of transition costs (and human capital accumulation), wage workers and entrepreneurs with the same values of cash on hand, face the exact same future opportunities. Since, the state variable x_t, summarizes all information about the past that bears on current and future decisions, only the variable x_t is relevant for current and future saving decisions and occupational choices. Therefore, the value functions for wage-workers and entrepreneurs are identical equal in the absence of transition costs. Contrary to this, in the presence of start up costs, the value function is specific to the current occupation. In the following remark an important implication of this finding is formulated

Remark 1. *A sequence of occupational choices will not exhibit true state dependence unless transition costs exist. Hence, in the absence of transition costs, choosing a* *given occupation today, does not alter conditional choice probabilities in the future, i.e.* $P(i_{t+1}, x_{t+1}|x_t, i_t) = P(i_{t+1}, x_{t+1}|x_t)$

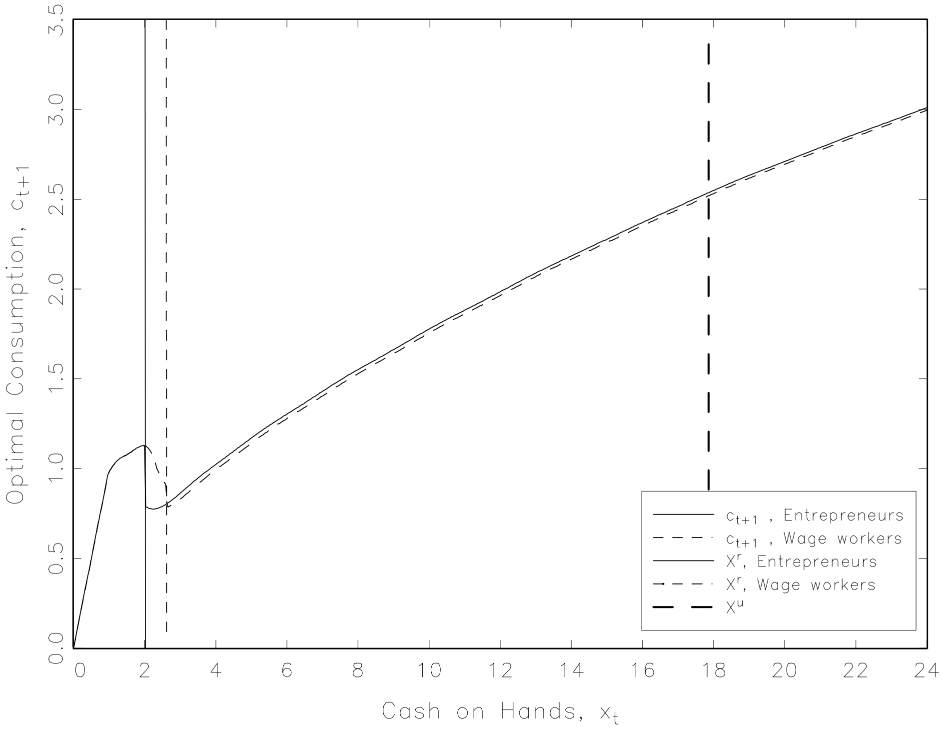
Not only is the reservation value of cash on hand, x_r higher for wage workers compared to entrepreneurs, $x_r^{we} > x_r^e$. The difference is larger than the entry cost, i.e. $x_r^{we} - x_r^e > \phi^{entry} = 0.2$. The explanation of this phenomenon is two-fold: In order to avoid losing invested entry costs, wage workers will postpone entry until they have accumulated enough assets to resist negative production chocks. Entrepreneurs, however, are willing to postpone exit to avoid paying the entry cost associated with potential re-entry to entrepreneurship. Consequently, being an entrepreneur represents an option value in terms of a wait-and-see option. If entry costs are large, the value of this option is important enough to make entrepreneurs willing to accept temporary income losses to keep their position and mitigate expected future entry payments. Therefore, compared to the case of no entry costs the reservation values x_r^e decreases. I summarize these findings, in the follow in remark

Remark 2. *In the presence of entry costs, we will se later entry and later exit. Due to the indivisibility and irreversibility of the entry costs, wage workers wish postpone investments in a business, whereas entrepreneurs are willing to cut consumption temporarily to keep their position as entrepreneurs. The combination of irreversibility, indivisibility and uncertainty introduces a value of waiting.*[9]

Figures 8 and 9 displays the policy function for consumption and the implied expected future gross income, $E(y_{t+1}) + ra_{t+1}$. Notice that the policy function for wage workers and entrepreneurs coincide for $x_t < x_r^e$. Regardless of the current occupation, the decision maker knows that he will be wage worker in the following period. Therefore, the same saving motives apply in both occupations and thus the division of cash on hand between savings and consumption is with equally identical. At $x_t = x_r^e$ consumption drops discretely for entrepreneurs. In fact, entrepreneurs are willing to reduce consumption

[9]This results is a common find in models of investment under uncertainty; see e.g., Dixit and Pendyck (1994), Fafchamps and Pender (1997), and Malchow-Møller and Thorsen (2005).

FIGURE 8. OPTIMAL CONSUMPTION, $\phi^{entry} = 0.2$

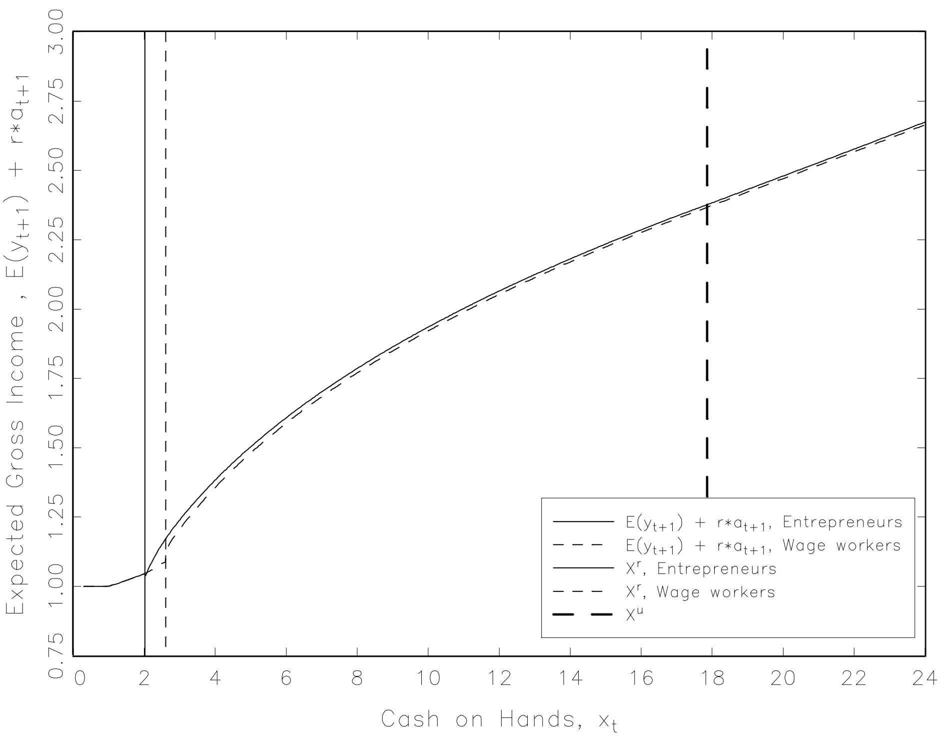


with 0.34 to maintain the business (corresponding to 34 pct of expected annual income as a wage worker or 70 pct more than the start-up cost)

The discontinuity in the policy function is due to the fact that the return to saving changes discretely at the switching point, x_r^e. If $x_t > x_r^e$ entrepreneurs know with certainty that they will be entrepreneurial the following period too. Therefore $dy_{t+1}/da_{t+1} = d\pi_{t+1}/da_{t+1} > 0$. If, on the other hand, $x_t < x_r^e$, they will exit entrepreneurship and become wage workers, $dy_{t+1}/da_{t+1} = 0$ for $x_t < x_r^e$.

Contrary to entrepreneurs, consumption decreases smoothly for wage workers right before the switching point x_r^{we}. As x_t approaches x_r^{we} the probability of future entry increases. Therefore, *expected* future returns to savings increase gradually. This is not the case for entrepreneurs with $x_t \in [x_r^e, x_r^{we}]$, since the entrepreneur knows *with certainty* that he will be an entrepreneur in the following period.

FIGURE 9. EXPECTED FUTURE GROSS INCOME, $\phi^{entry} = 0.2$



Again, it is evident from the consumption function for both entrepreneurs and wage workers, that saving incentives are very strong when $x_t > x_r^e$. In fact, the more liquidity constrained, the stronger the saving incentive. Note finally that entrepreneurs can consume more, since they have paid the entry cost already. Therefore, as x_t increases, the two policy functions converges due to increased ability to smooth out the entry cost over several periods.

To summarize how start up costs influence saving incentives and aspects of occupational choice, I formulate the following remark:

Remark 3. *Start up costs give an extra savings motive when credit constraints are binding: Wage workers who expect to enter entrepreneurship save to overcome entry-costs and the corresponding risk associated with entry. Entrepreneurs save to maintain their position as entrepreneurs to avoid potential costs associated with later re-entry.*

Figure 10. Simulated Hazard Function, $\phi = 0.2$ and $a_0 = 1.25$

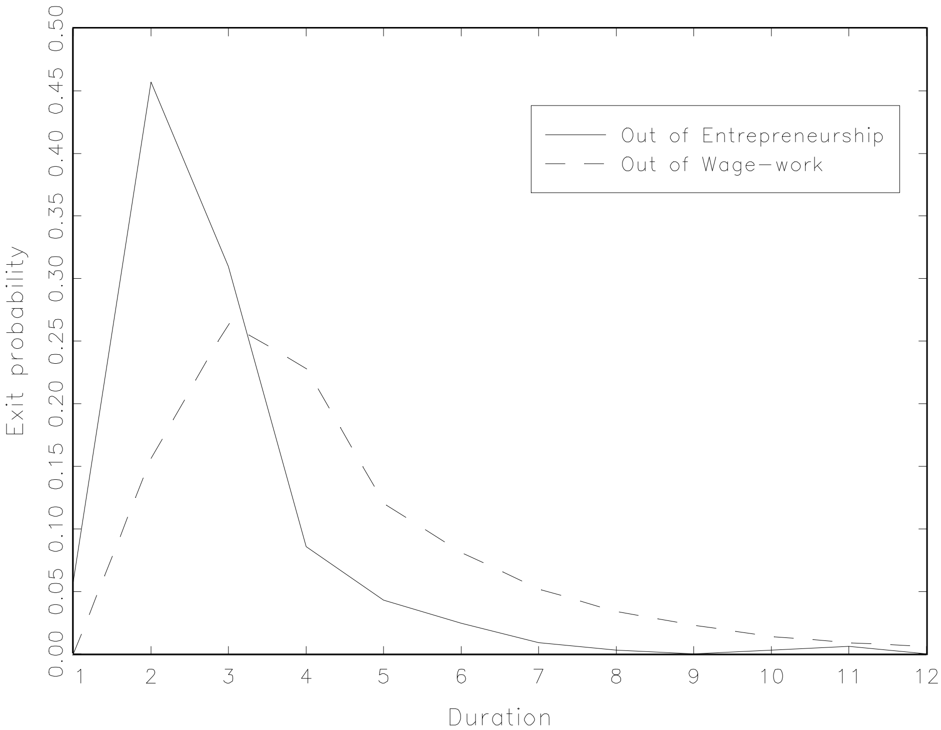


Figure 10 shows occupational specific hazard functions, for a population of individuals who enter the labor market as wage workers at an intermediate level of individual wealth, $a_0 = 1.25 \in [x_{ns}, x^e_{we}]$[10]. The presence of entry cost alters the transition patterns between entrepreneurship and ordinary wage work fundamentally: Due to start up costs, individuals will never enter entrepreneurship if there is a significant risk that they will not be able to maintain their business in the following periods. Therefore, the hazard initially increases with duration. This finding has two immediate implications for empirical analysis

Remark 4. *The transition pattern depends significantly on the type of the agent, whether he faces credit constraints or not, whether he faces transitions cost etc. This suggest that*

[10]The hazard is not directly comparable with the hazard from the previous section, since two different populations are considered. Due to increased reservation values for wage workers, x_r^{we}, none of the wage workers from the previous simulation would ever enter entrepreneurship. Therefore we consider a different population with a higher level of cash on hand.

estimation procedures in duration analysis should incorporate lots of heterogeneity. Not only in the intercept or scale of the hazard, but also the shape.

Remark 5. *Entrepreneurial hazard functions which are initially increasing followed by negative duration dependence are consistent with the presence of start-up costs or any other phenomenon, that generates an option value for the entrepreneur.*

Jørgensen (2005) provide a careful duration analysis of Danish start-ups. Using a large and comprehensive longitudinal firm database, he is able to identify all new start-up firms in 1994 and 1998 and the entrepreneur bind the firm. After carefully conditioning firm- and individual level characteristics, he find quite robust evidence that the hazard out of entrepreneurship, is initially increasing followed by a downward sloping hazard. As predicted by the model, these results are consistent with the existence of start-up cost.

5. Numerical Results - Entrepreneurial Human Capital Model

Until now we have we have treated entrepreneurial human ability as exogenously assigned to individuals and constant through time. In this section, however, I study how intertemporal incentives are altered when individuals accumulate entrepreneurial human capital. We shall consider two extreme cases: In the fist case, h_t is assumed to measure pure entrepreneurial experience. Each period, the entrepreneur gains one unit of entrepreneurial human capital while wage workers gain enough human capital to precisely offset the depreciation in human capital, when $h_t = 1$, i.e. $\Delta(e) = 1$ and $\Delta(we) = 1-\gamma$.[11] Since individuals accumulate entrepreneurial human capital only when they are entrepreneurs, entrepreneurial experience (or learning by doing) is treated as a behaviorally determined investment decision: Some individuals may find it optimal to incur a temporary income loss in exchange for increased future returns to their business.

[11]This is done for numerical convenience: Since the assumption ensures that h_t can never fall below 1, we can restrict attention to a compact subset of the states pace, $h_i \in [1, 1/(1-\gamma)]$.

Table 2. Baseline Parameter Values

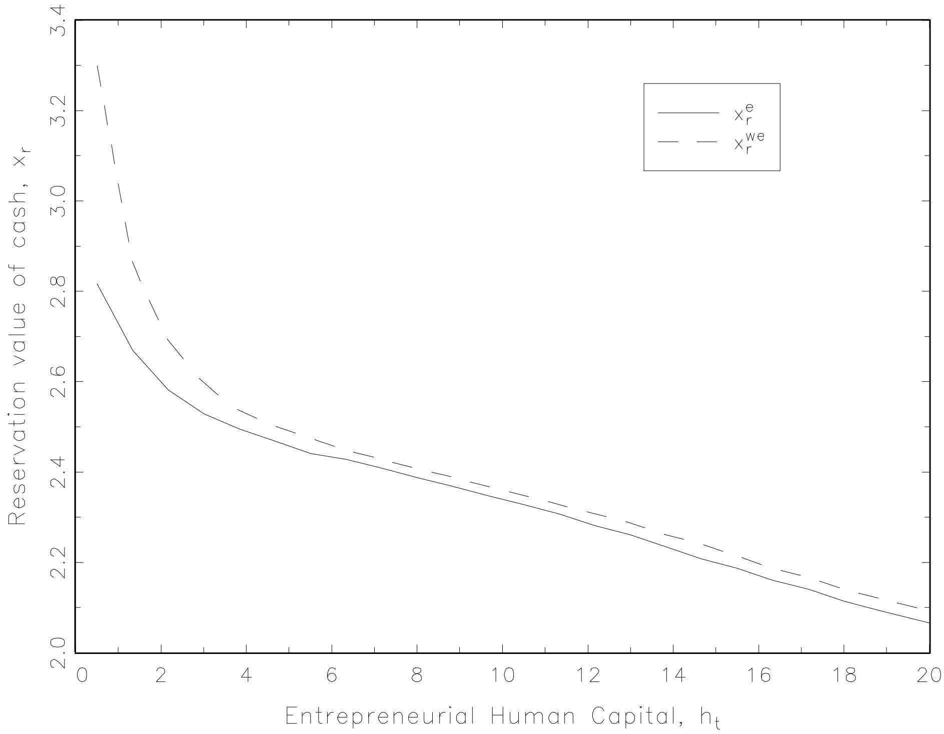
r	η	δ	θ	α^h	α^k	σ_ε^e	λ	w	σ_ε^{we}	γ	$\Delta(e)$	$\Delta(we)$	ϕ^{exit}	ϕ^{entry}
0.05	1.5	0.07	0.80	0.06	0.33	0.3	1	1	0.1	0.95	1 0	0 1	0	0

At the other extreme, h_t measures pure work experience such that productivity is assumed only to increase during wage work, i.e. $\Delta(we) = 1$ and $\Delta(e) = 1 - \gamma$. Knowledge spill-overs are thus assumed to be more important within firms than between firms. Admittedly, this assumptions is somewhat stylized, but it captures the idea that people learn more by working with and for other people.

When human capital accumulation does not differ across occupations, i.e. when $\Delta(we) = \Delta(e)$, human capital, h_t, is deterministic. Thereby, h_t is exogenous in the sense, that the occupational choice does not alter human capital accumulation. The implications of the model under this parametrization are not very different from the case without human capital accumulation, except that the hazard out of entrepreneurship exhibits a higher degree of negative duration dependence due to the trending productivity. When duration in entrepreneurship increases, productivity increases as well. But not due to duration in entrepreneurship, due to the course of time. Since the primary focus of this section is how saving and occupational choice is altered by human capital investment, I will not pursue the case of deterministic accumulation any further

The rest of the parameter values are chosen to be similar to the baseline parameters in the simple model with no transition costs, although with a few modifications. I let $\alpha^h = 0.06$ such that the expected increase in productivity is 6 pct. for a percentage increase in h_t. I let $\gamma = 0.95$, i.e. h_t depreciates with 5 pct. each period. I choose $\theta = 0.80$ such that the limit of $h_t^{a_h}$ approximately equals the baseline parameter value of θ in the simple model without human capital accumulation, i.e. $\theta^{\max} = \theta \left[max_i \left(\Delta(i)\right) / \left(1-\gamma\right)\right]^{\alpha_k} = 0.96$. The baseline parameter values used in the numerical simulations of the human capital model are listed in Table 5

FIGURE 11. RESERVATION VALUE OF CASH ON HAND, x_r



5.1. **Learning by doing.** Consider first the scenario where individuals accumulate human capital only while they are entrepreneurs. Since entrepreneurial earnings are monotonely increasing in human capital, h_t, the reservation value of cash on hand, x_r, is decreasing in h_t. This relationship between x_r and h_t is displayed in Figure 11 for wage workers and entrepreneurs respectively. First note that the reservation value of cash on hand depends not only on the level of human capital, but also on the current occupation. In general, wage workers have larger reservation values of cash, since human capital in the following period is lower. One can say that the exit from entrepreneurship is associated with an indirect cost due to the forgone opportunity to accumulate human capital. As a result the occupational choice is state dependent. Due to the diminishing return to human capital, the gab between the two reservation values $x_r^{we} - x_r^e$ diminishes with human capital.

Figure 12. Expected Future Gross Income - Wage Workers

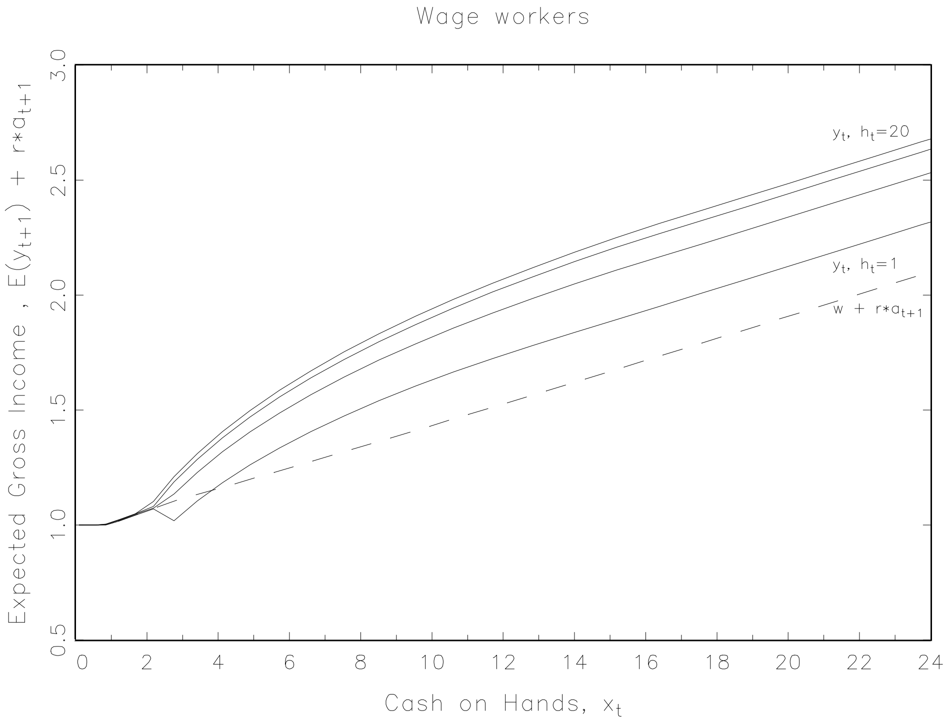
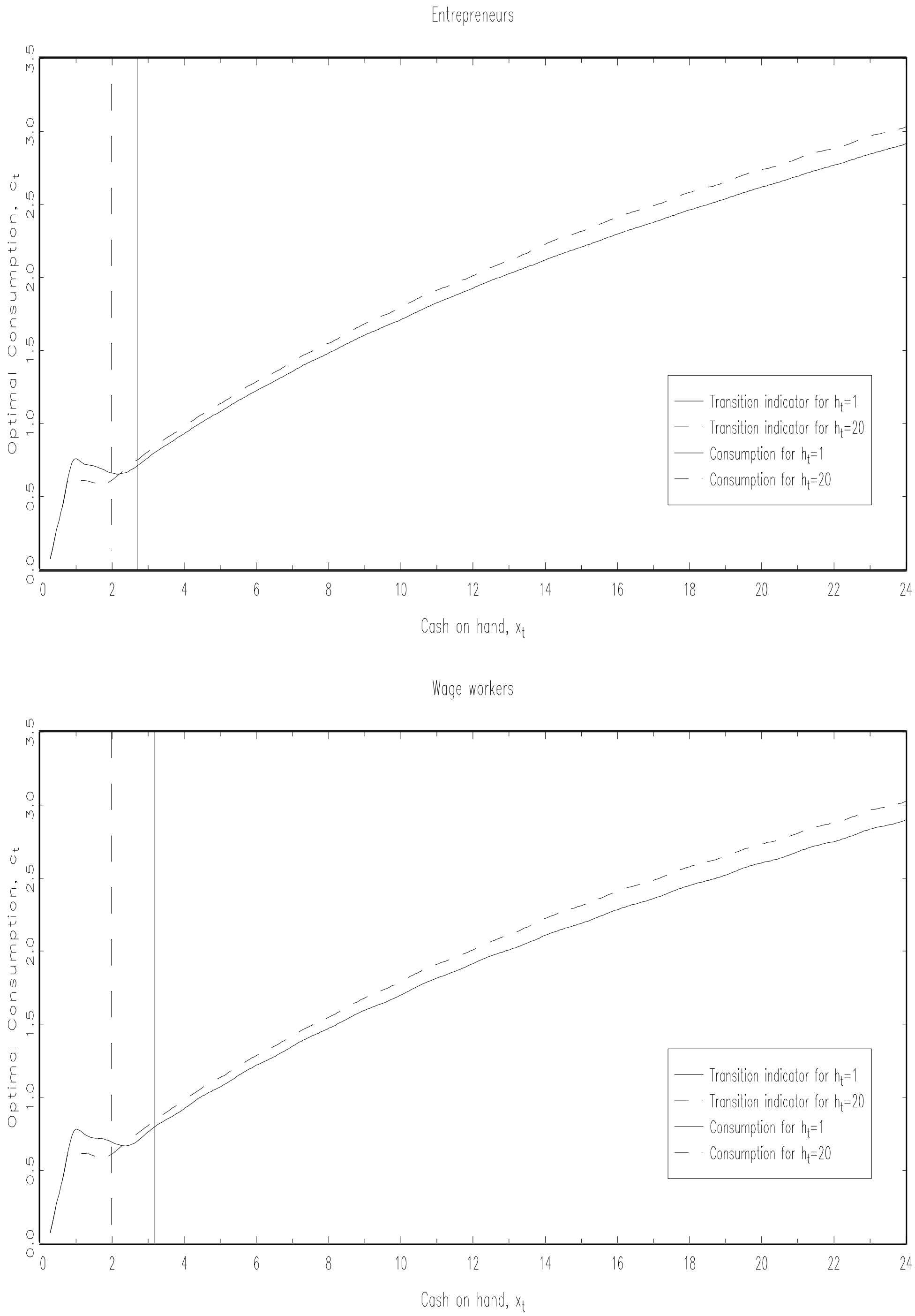


Figure 12 displays the relationship between wealth and expected future gross income for different values of human capital. Contrary to the case without human capital investments, individuals with a low level of human capital are willing to incur a temporary income loss, in exchange for an increase in expected future entrepreneurial earnings. Wage workers with a relatively low level of human capital and capital less than $x_r^{we}(h_t)$ earns the fixed wage, $w = 1$ plus interest of liquid assets holdings until he chooses to become entrepreneur at the switching point, $x_r^{we}(h_t)$.

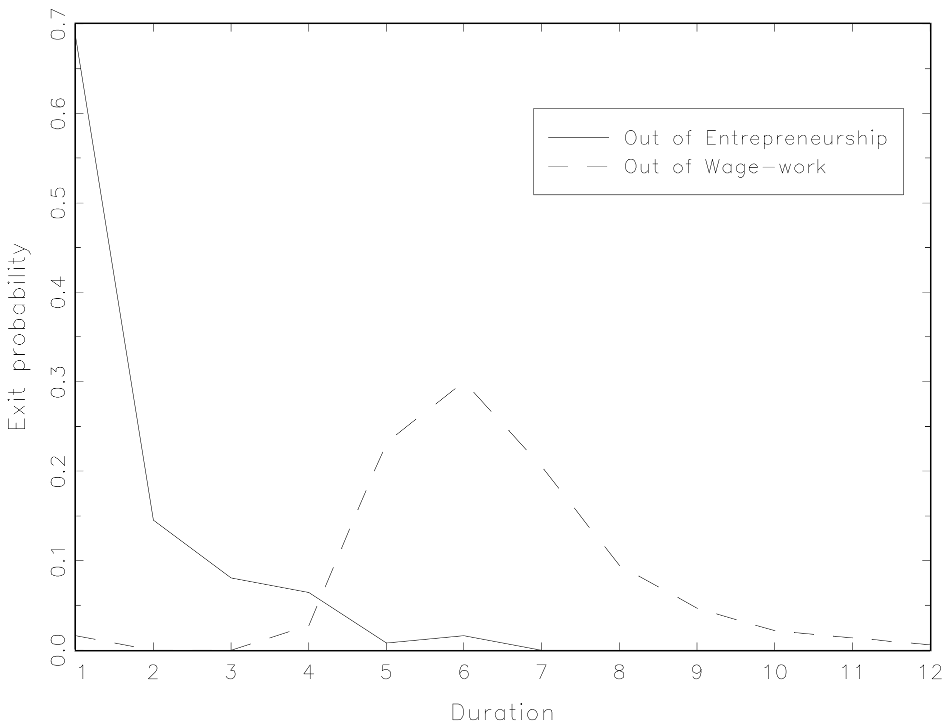
Figure 13 displays the policy function for consumption and occupational choice for entrepreneurs and wage workers respectively. The graphs are plotted for two different level of human capital, $h_t = 1$ and $h_t = 20$. It is evident from Figure 13 that individuals with a high level of human capital have much stronger saving incentives due to the higher return to investments. Indeed, for low levels of wealth, individuals with a high level of human capital will save more out of current wealth. In contrast, individuals with a low level of human capital have relatively low returns to investments in physical capital. As

Figure 13. Optimal Consumption



a consequence, individuals with relatively low levels of human capital have a additional incentive to entry: To gain more human capital.

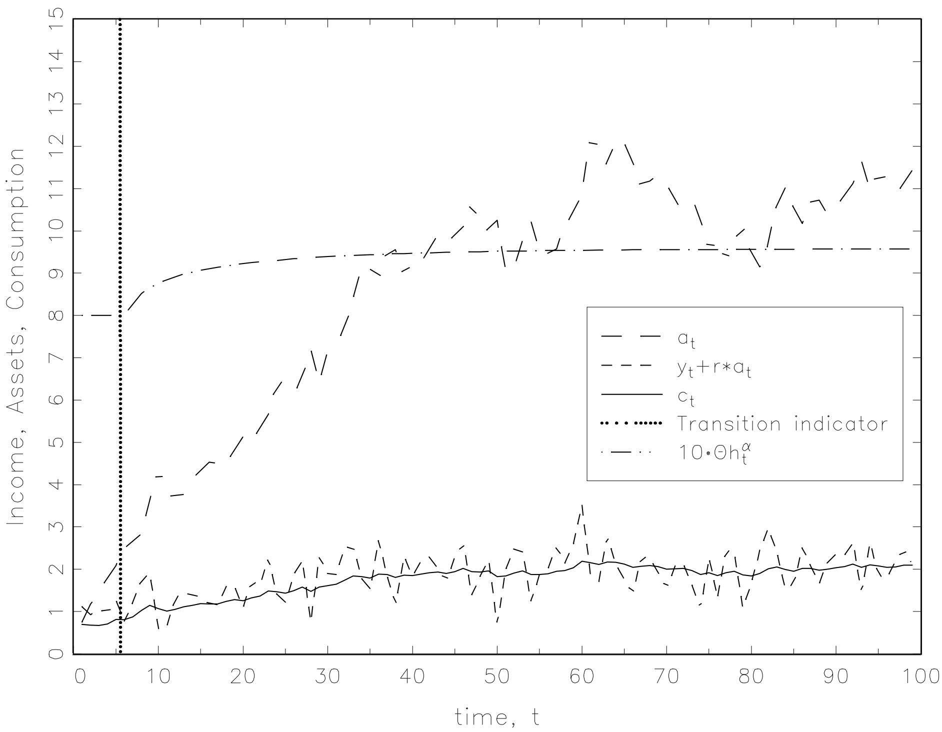
FIGURE 14. SIMULATED HAZARD FUNCTION, $a_0 = 0.75$



There is one important thing to note about the gab between the two reservation values $x_r^{we} - x_r^e$. The fact that individuals can only accumulate human capital in entrepreneurship creates a wedge between the two occupations: Not only is the reservation values of cash much higher for wage workers with no entrepreneurial experience, the saving incentives for constrained wage workers are weaker too. This combination reinforces significantly the impact of existing liquidity constraints on entry behavior .

In Figure 14 simulated hazard functions are displayed for entrepreneurs and wage workers respectively: The transition pattern is not qualitatively different from the case without human capital accumulation. However, the negative duration dependence for the hazard out of entrepreneurship is amplified by human capital accumulation in entrepreneurship: Since h_t increases with duration in entrepreneurship, the reservation value of cash, $x_r^e(h_t)$ will decrease with duration. As a consequence, the exit probability for entrepreneurs decreases rapidly with duration. A final thing to note is that wage workers

FIGURE 15. SIMULATED SEQUENCE OF INCOME, ASSETS AND CONSUMPTION, $\alpha_0 = 0.75$

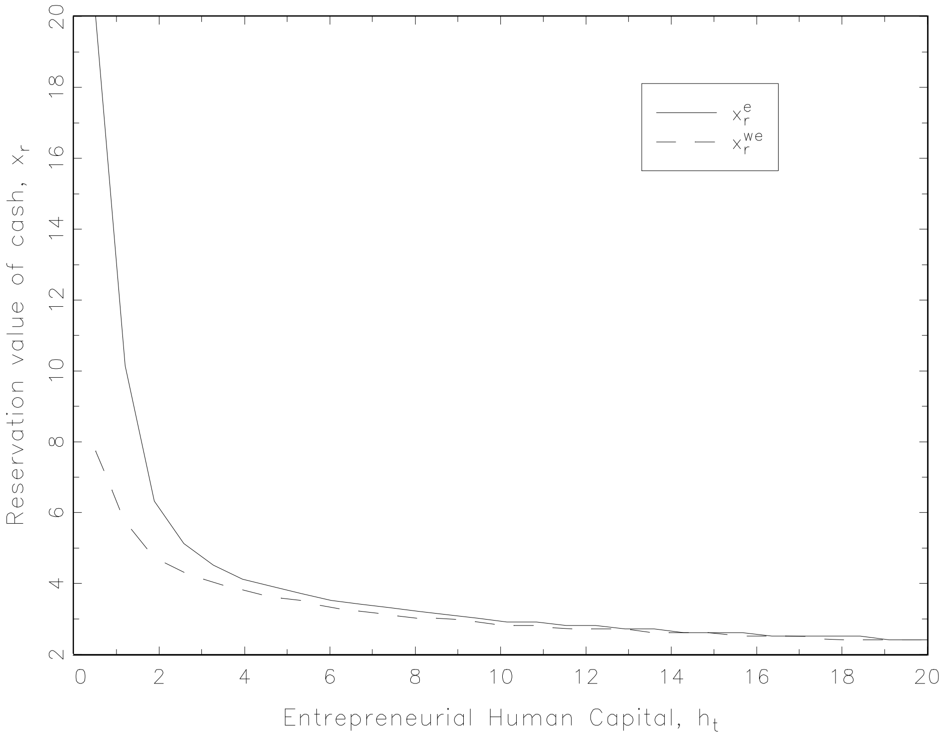


enter entrepreneurship later: On average, they will have to save for about 6-7 periods before entry.

Figure 15 graphs simulated sequences of assets, a_t, consumption, c_t, gross income, $y_t + ra_t$, and the implied value of entrepreneurial productivity multiplied by 10, $10\theta h_t^{\alpha_h}$.The simulation is done for an individual that enters the labor market with an initial level of assets of $a_0 = 0.75$.

5.2. **Human Capital Accumulation in Wage Work.** Now turn to the opposite extreme, where human capital accumulation primarily takes place during wage work. As seen in Figure 16, this accumulation scheme holds qualitatively different implications for the implied transition pattern between the two occupations. The first thing to note is that the dependence of the current occupation is reversed relative to the picture in Figure 11. Since individuals can enhance their future entrepreneurial productivity only

Figure 16. Reservation Value of Cash on Hand

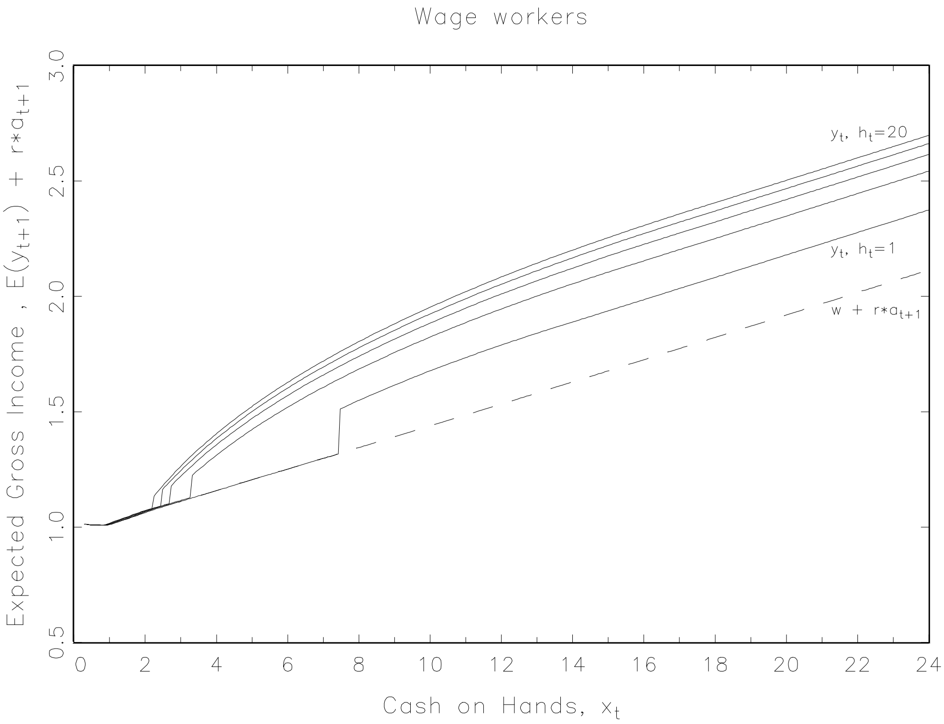


during wage work, human capital will be lower for entrepreneurs in the following period. As a result, entrepreneurs will have larger reservation values of cash on hand.

Figure 17 depicts expected future gross income for wage workers for different values of h_t (the solid lines) along with expected gross income if an individual (inoptimally) chooses to remain wage worker for all values of cash on hand (the dashed line). The picture is again reversed: Individuals are willing to *stay wage workers* although expected entrepreneurial earnings are higher. Hence, to balance the return to investments in human capital and physical capital, wage workers accept a temporary income loss in exchange for an increase in expected future entrepreneurial earnings. In fact, the expected income function jumps discretely in the switching point. The size of this jump represents the amount that individuals are willing to trade for an additional unit of human capital. Due to the decreasing returns to h_t, this amount decreases with h_t.

Figure 18 displays the policy functions for consumption and occupational choice for entrepreneurs and wage workers for two different levels of human capital, $h_t = 1$ and

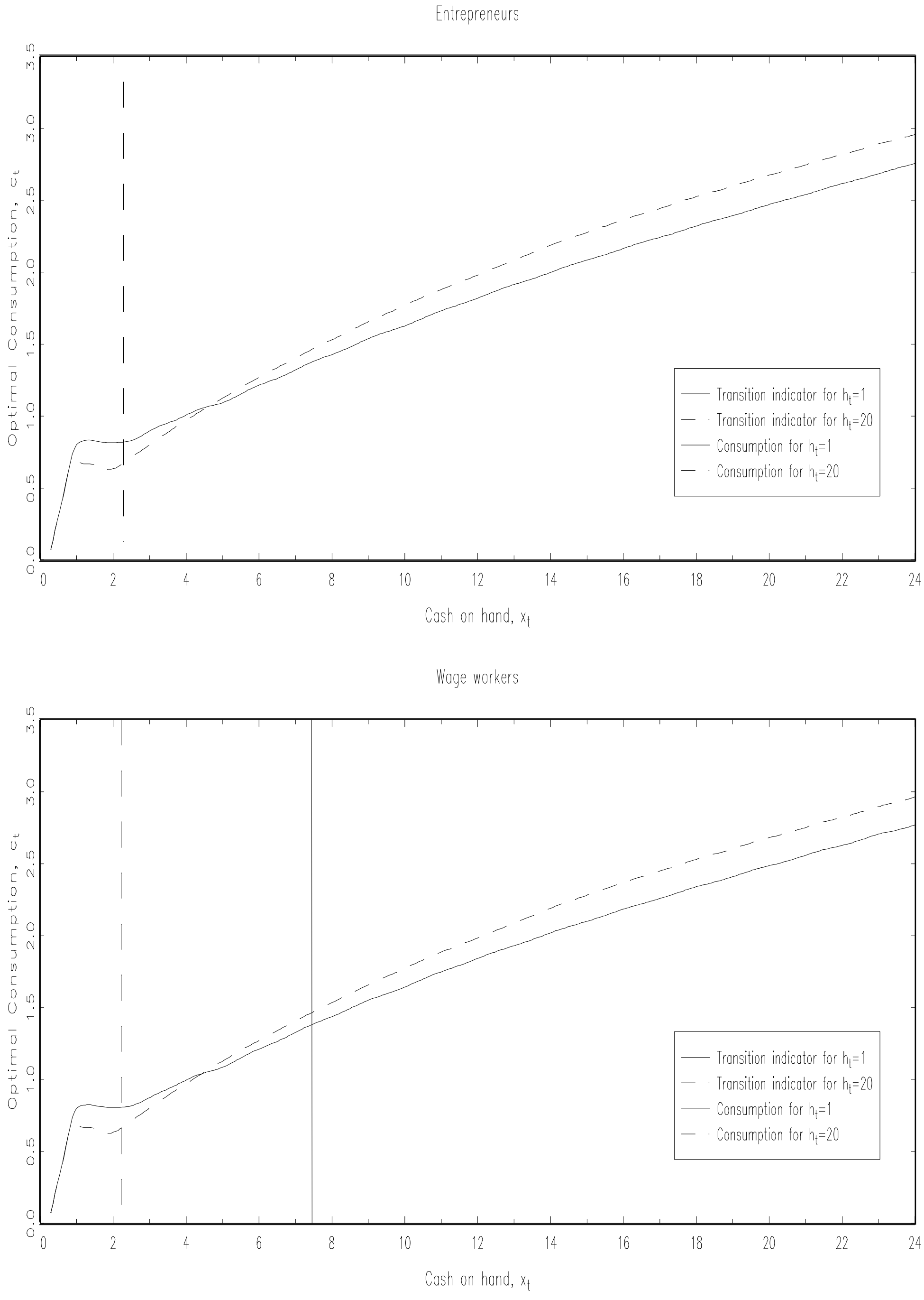
Figure 17. Expected Future Gross Income - Wage Workers



$h_t = 20$. The first thing to note is that entrepreneurs with $h_t = 1$, will switch to wage work regardless of their current level of wealth. Despite this transition to wage work, the entrepreneurial saving incentive remains since individuals with low levels of h_t expect entry (or re-entry) to entrepreneurship after a couple of periods in wage work.

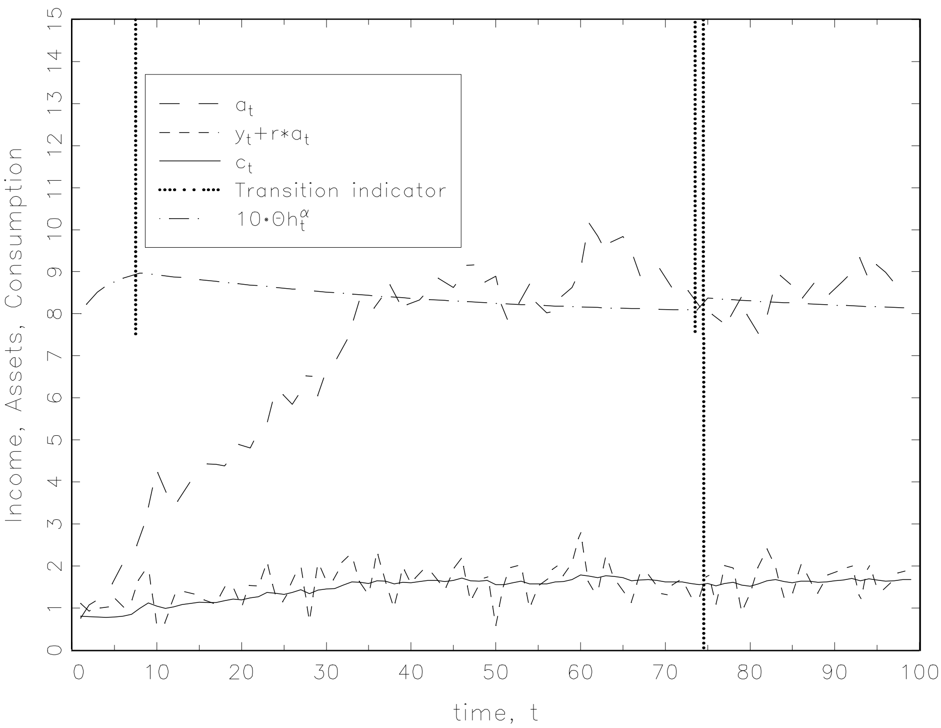
Figure 19 graphs simulated sequences of assets, a_t consumption, c_t gross income, $y_t + ra_t$, and the implied value of entrepreneurial productivity multiplied by 10, $10\theta h_t^{\alpha_h}$. The simulation is done for an individual that enters the labor market with an initial level of assets of $a_0 = 0.75$. Initially, the wage worker accumulates assets and human capital for a couple of periods. Hereafter, he switches to entrepreneurship to reap the benefits of his investments. At this point, the entrepreneur keeps saving to overcome binding liquidity constraints. However, since entrepreneurial human capital depreciates while access to financial assets simultaneously increases, the return to reinvestments in human capital will eventually become large enough to make the entrepreneur switch to wage work and thereby being able to accumulate more human capital. Note that the wage

FIGURE 18. OPTIMAL CONSUMPTION



workers re-entry entrepreneurship relatively fast (after only one period in wage work). Even more, the accumulated level of human capital is lower compared to the first entry.

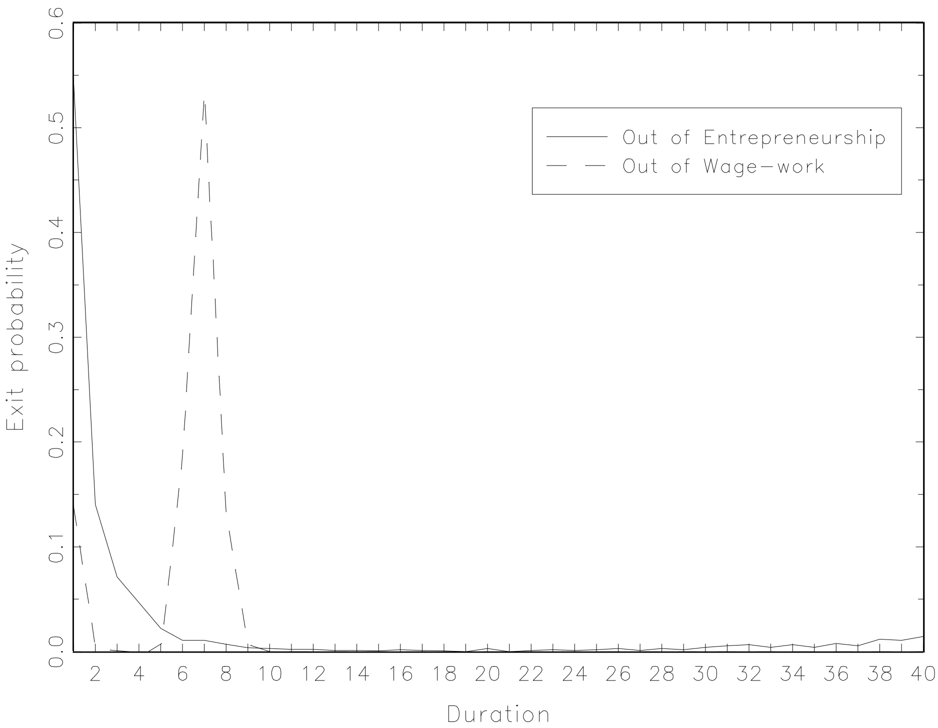
Figure 19. Simulated Sequence of Income, Assets and Consumption, $\alpha_0 = 0.75$



The reason is, that this agent faces a very large opportunity cost in terms of forgone entrepreneurial earnings, since accumulated assets are now much higher.

Figure 20 displays simulated hazard functions for a population that enters the labor market with an initial level of assets $a_0 = 0.75$. Consider first the hazard out of entrepreneurship. Again, we observe a decreasing hazard, although with one modification. After approximately 20 periods, the exit probability starts to increase again. At this point entrepreneurs have been exposed to human capital depreciation for several periods. Since the business is very capital intensive at this point, the return to human capital investments is high enough to induce a shift to wage work. From the hazard out of wage work, it is evident that these re-entries to wage work have a duration of only one period (the left peak). The hazard function has a second peak around 6-8 periods of wage work. The concentration here is due to the initial spell in wage work. Hence, on

FIGURE 20. SIMULATED HAZARD FUNCTION, $\alpha_0 = 0.75$



average, wage workers will need around 7 periods of wage work, before entrepreneurial activity is undertaken

5.2.1. *Credit Constraints and Human Capital Accumulation.* If entrepreneurial human capital is primarily accumulated in entrepreneurship, the importance of credit constraints is amplified significantly. Furthermore, since the importance of entrepreneurial human capital accumulation may differ substantially between industries and educational groups, the empirical question of whether human capital is primarily accumulated in wage work or entrepreneurship has important implications for the design of, e.g., public loan guaranties and business start-up schemes. To answer this important empirical question - a careful empirical analysis is required: Since entrepreneurial human capital is generally unobserved, a full estimation of the present structural model is needed to identify whether human capital spill-overs are most important within or between firms. I will try to address this issue in future work.

6. Conclusions and Directions for Future Research

To get a better understanding of the intertemporal incentives that underlie much of the behavior by entrepreneurs, I have developed an intertemporal model for saving, consumption, human capital accumulation and occupational choice in the presence of liquidity constraints, income uncertainty, and entry costs. I have done this by merging the set-up from the existing static models of entrepreneurship with the approach taken in the literature on intertemporal saving and consumption. Furthermore, I incorporate aspects from the literature on investment under uncertainty and the literature on human capital formation.

Using this model, I provide a theoretical foundation for analyzing a number of unresolved issues. I here summarize the six key findings: First, perhaps the most prominent feature of the model is that it generates a well-defined transition pattern characterized by continuous cycling in and out of entrepreneurship; a core phenomenon observed in the data.

Secondly, start up costs give an extra savings motive when credit constraints are binding: Wage workers who expect to enter entrepreneurship save to overcome entry-costs and the corresponding risk of losing the paid entry costs. Entrepreneurs save to maintain their position as entrepreneurs to avoid potential costs associated with later re-entry.

Third, in the presence of entry costs, wage workers wish to postpone investments in a business, whereas entrepreneurs are willing to cut consumption temporarily to keep their position as entrepreneurs. In that sense, the combination of entry costs and uncertainty introduces a value of waiting. Therefore, in the presence of entry costs, we will see later entry and later exit.

Fourth, in the absence of transition costs and human capital accumulation in wage work, hazard functions for entrepreneurs will generally exhibit true negative duration dependence. In the presence of entry costs, however, the probability of exit from entrepreneurship is initially increasing in elapsed duration and is then followed by a downward sloping hazard; a prediction which is consistent with what we observe in the Danish register data on firm start-ups. Hence, the model provides a plausible explanation of observed exit behavior for Danish entrepreneurs.

Fifth, if entrepreneurial human capital is primarily accumulated in entrepreneurship, the importance of credit constraints is amplified significantly, whereas the opposite is the case if individuals primarily acquire entrepreneurial skills in paid employment.

Therefore, whether human capital is primarily accumulated in wage work or entrepreneurship has important implications for the design of policies that help entrepreneurs to circumvent binding credit constraints. To answer this important question, a full estimation of the present structural model can help to identify whether human capital spill-overs are most important within or between firms. I will try to address this issue in future work.

Sixth, the entrepreneurial saving motive is affected by the tightness of credit constraints in a non-monotone way: This result has important implications for an understanding of the effects of changes in credit policy, e.g. government loan guaranties. While such a policy would induce relatively productive individuals to increase their savings, less productive individuals will decrease their savings. This kind of policy will increase wealth inequality in the economy and could be associated with increased probability of default. On the other hand, relaxing the credit constraints increase the probability of entry relatively more for productive individuals - due to the increased willingness to use savings to overcome credit constraints. However, to appropriately account for these effects, a full structural estimation is needed.

The insight of the model also has important implications for future research: First, analysis of precautionary savings using the cross sectional correlation between income uncertainty and wealth must take into account that households may hold large proportions of wealth which is related to expectations about the decision to become entrepreneurs in the future. Hence, to mitigate a potential heterogeneity bias it is not sufficient to control for the current occupation, we must appropriately control for expectations about future occupations too. Since these expectations are generally unobserved (and time varying), such conditioning is in general very difficult. Therefore, to appropriately account for the composition of household savings, a full structural estimation of the present model can be a useful and perhaps necessary identification strategy.

Secondly, since transition patterns depend significantly on the type of the agent, i.e. whether he faces credit constraints or not, whether he faces transitions cost etc, estimation procedures in duration analysis should incorporate lots of heterogeneity; not only in the intercept or scale of the hazard, but also in the shape.

To fully understand the importance of credit constraints, entry costs and entrepreneurial risks for the importance of different saving motives it is necessary to estimate the distribution of entrepreneurial ability in the population, the significance of different human capital accumulation schemes, returns to factors of production, preference parameters, etc. Hence, a full structural estimation of the model is needed. This is a very interesting project that is subject to ongoing research.

However, a full estimation of the present model on micro data is a non-trivial task. For each evaluation of the likelihood function or the moments used for identification, we will have to solve a complex dynamic programming problem. Therefore, algorithms used to solve to the model must be developed further to make the estimation feasible.

7. Appendix - Proof of Propositions

Proof of Proposition 1. Consider an individual who chooses to become entrepreneur in the following period but is constrained by capital, i.e. $i_{t+1} = e$ and $k_{t+1} = \lambda a_{t+1}$. For a marginal increase in future assets, this individual can expect to increase entrepreneurial earnings with the following amount

$$\Xi \equiv E\left[\frac{dy_{t+1}\left(i_{t+1}, h_{t+1}, a_{t+1}, \varepsilon_{t+1}\left(i_{t+1}\right)\right)}{da_{t+1}}\right] = \alpha_k \theta a_{t+1}^{\alpha_k - 1} h_{t+1}^{\alpha_h} \lambda^{\alpha_k} - r\lambda > 0$$

Note that, Ξ is concave in λ since $\alpha_k \in]0,1[$ and positive if the individual is credit constrained. For unconstrained entrepreneurs $\Xi = 0$.

Furthermore, Ξ is increasing in λ if $\theta h_{t+1}^{\alpha_h} \alpha_k^2 a_{t+1}^{\alpha_k - 1} \lambda^{\alpha_k - 1} > r$. For a given value of r, a_{t+1} and λ, this condition hold for large enough $\theta h_{t+1}^{\alpha_h}$.Hence, relatively productive individuals with a relatively low asset holdings, will increase their savings when λ increases.

On the other hand, Ξ is decreasing in λ if $\theta h_{t+1}^{\alpha_h} \alpha_k^2 a_{t+1}^{\alpha_k - 1} \lambda^{\alpha_k - 1} < r$. For a given value of r, $\theta h_{t+1}^{\alpha_h}$ and a_{t+1}, this condition holds for large enough λ.Hence, when credit constraints become less binding, the expected return to increased savings approaches zero. □

Proof of Proposition 2. Since $h_{t+1}(h_t)$ is monotone increasing with slope $h'_{t+1}(h_t) = \gamma < 1$, then $h_{t+1}(h_t)$ will cross the 45 degree line from above in a unique fixed point $\tilde{h}$ Thereby for any $h_{high} \geq \tilde{h} < \infty$ we must have that $h_{t+1}(h_{high}) \leq h_{high}$ (since $h_{t+1}(h_{high})$ is below the 45 degree line for all $h_{high} \geq \tilde{h} < \infty$) □

Proof of Proposition 3. We wish to show that there exists a $a^* < \infty$ such that if $a_t \leq a^*$ then $a_{t+1} \leq a^*$. Hence it is sufficient to prove that exists a $a^* < \infty$ such that $a_t \geq a_{t+1}$ holds for all $a_t \geq a^*$. That is,

$$\begin{aligned} a_t &\geq a_{t+1} = (1+r)\left(a_t + y_t - c_t - \phi\left(s_t, s\right)\right) \Rightarrow \\ c_t &\geq \frac{r}{1+r} a_t + y_t - \phi\left(s_t, s_{t+1}\right) \end{aligned} \tag{7.1}$$

should hold for all $a_t \geq a^*$.

Since y_t is bounded from above by $y^{\max}$ and $\phi(s_t, s_{t+1})$ is bounded from below by zero, a sufficient condition for the inequality (7.1) can be formulated as

$$c \geq \frac{r}{1+r} a + y^{\max}$$

where

$$y^{\max} = \max \left\{ \begin{array}{c} w, \\ \theta \left(\lambda a_t\right)^{\alpha_k} \left(h^{high}\right)^{\alpha_h} \varepsilon^{\max} - r\lambda a_t, \\ \theta \left[\frac{\theta \alpha_k}{r} \left(h^{high}\right)^{\alpha_h}\right]^{\frac{\alpha_k}{1-\alpha_k}} \left(h^{high}\right)^{\alpha_h} \varepsilon^{\max} - r \left(\frac{\alpha_k \theta}{r} \left(h^{high}\right)^{\alpha_h}\right)^{1/(1-\alpha_k)} \end{array} \right\}$$

is a finite since $\alpha_k < 1$ and $r > 0$

Consider for a moment the corresponding deterministic model, where individuals are endowed with initial assets A_t. In this case, optimal consumption can be expressed as (see Caroll (1997))

$$c_t^{det} = \left(1 - \left(\frac{1}{1+\delta}\right)^{\frac{1}{\eta}} \left(\frac{1}{1+r}\right)^{1-\frac{1}{\eta}}\right) \left(a_t + \sum_{i=t+1}^{\infty} (1+r)^{t-i} \left(y\left(i_i, h_{i,} a_i\right) - \phi\left(i_i, i_{i+1}\right)\right)\right)$$

Since income is bounded from below by zero and $\phi\left(s_t, s_{t+1}\right)$ is bounded from above by ϕ Consumption in the stochastic model can newer fall below

$$\tilde{c}_t^{det} = \left(1 - \left(\frac{1}{1+\delta}\right)^{\frac{1}{\eta}} \left(\frac{1}{1+r}\right)^{1-\frac{1}{\eta}}\right) \left(a_t - \frac{1}{r}\phi\right)$$

Therefore we must have that $c > \tilde{c}^{det}$ and we can therefore formulate a sufficient condition for (7.1) given by

$$\left(\left(\frac{1+r}{1+\delta}\right)^{\frac{1}{\eta}} \frac{1}{1+r} - 1\right) \frac{1}{r}\phi > \frac{1}{1+r} \left(1 - \left(\frac{1+r}{1+\delta}\right)^{\frac{1}{\eta}}\right) a_t + y^{\max}$$

Since the left hand side is constant and the right hand side is increasing in a_t there must exist some a^* such that for $a_t \geq a^*$, the inequality is a true statement. Or equally true, there must exist some $a^* < \infty$ such that such that $a_t \geq a_{t+1}$ holds for all $a_t \geq a^*$. This completes the proof $\square$

References

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109(3), 659–684.

Ben-Porath, Y. (1967): "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy*, 75(4), 352–365.

Blanchflower, D. G., and A. J. Oswald (1998): "What Makes an Entrepreneur?," *Journal of Labor Economics*, 16, 26–60.

Blinder, A. S., and Y. Weiss (1976): "Human Capital and Labor Supply: A Synthesis," *Journal of Political Economy*, 84(3), 449–472.

Brock, W. A., and D. S. Evans (1986): *The Economics of Small Business: Their Role and Regulation in the US Economy.* Holmes and Meier, New York.

Browning, M., and A. Lusardi (1996): "Household Saving: Micro Theories and Micro Facts," *Journal of Economic Literature*, 34(4), 1797–1855.

Buera, F. J. (2003): "A Dynamic Model of Entrepreneurship with Borrowing Constraints," Chicago University, mimeo.

Cagetti, M., and M. DeNardi (2002): "Entrepreneurship, Frictions and Wealth," Federal Reserve Bank of Minneapolis Working paper no. 620.

Caroll, C., and A. Samwick (1998): "How Important is Precautionary Savings?," *Review of Economics and Statistics*, 80(3), 41–71.

Caroll, C. D. (1997): "Buffer-Stock Savings and the Life-Cycle/Permanent Income Hypothesis," *Quarterly Journal of Economics*, 112(1), 1–55.

——— (2004): "Liquidity Constraints and Precautionary Savings," Mimeo.

Caroll, C. D., and M. S. Kimball (1996): "On the Concavity of the Consumption Function," *Econometrica*, 64(4), 981–992.

Cramer, J. S., and C. M. V. Praag (2001): "The Roots of Entrepreneurship and Labour Demand: Individual Ability and Low Risk Aversion," *Economia 2001, pp. 45-62.*

Deaton, A. (1991): "Savings and Liquidity Constraints," *Econometrica*, 59, 1121–1248.

Denardo, E. V. (1967): "Contraction Mappings in the Theory Underlying Dynamic Programming," *SIAM Review*, 9(2), 165–177.

Dixit, A., and R. S. Pendyck (1994): *Investment under Uncertainty.* Princeton University Press, Princeton, New Jersey.

Dunn, T., and D. Holtz-Eakin (2000): "Financial Capital, Human Capital and the Transition to Self-Employment: Evidence from Intergenerational Links," *Journal of Labour Economics 18 pp. 282-305 (2000).*

Evans, D. S., and B. Jovanovic (1989): "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints," *Journal of Political Economy*, 97, 808–827.

Fafchamps, M., and J. Pender (1997): "Precautionary Savings, Credit Constraints, and Irreversible Investments: Theory and Evidence from Semiarid India," *Journal of Business and Economic Statistics*, 15(2), 180–194.

Flavin, M., and S. Nakagawa (2005): "A Model of Housing in the Presence of Adjustment Costs: A Structural Interpretation of Habit Persistence," Unpublished Manuscript, University of California, San Diego.

Fonseca, R., P. Lopez-Garzia, and C. A. Pissarides (2001): "Entrepreneurship, Start-Up Costs and Employment," *European Economic Reveiew*, 45, 692–705.

Friedman, M. (1957): *A Theory of the Consumption Function.* . Princeton University Press.

Gentry, W. M., and R. G. Hubbard (2000): "Entrepreneurship and Household Saving," mimeo.

Hamilton, B. H. (2000): "Does Entrepreneurhip Pay? An Empirical Analysis of the Returns to Self-Employment," *Journal of Political Economy*, 108(3), 604–631.

Harrison, G. W., M. I. Lau, and E. E. Rutstrom (2004): "Estimating Risk Attitudes in Denmark: A Field Experiment," *CEBR Discussion Paper*, 2004-10.

Harrison, G. W., M. I. Lau, and M. B. Williams (2002): "Estimating Individual Discount Rates in Denmark: A Field Experiment," *American Economic Review*, 92(5), 1606–1617.

Holmes, T. J., and J. A. Schmitz, Jr (1990): "A Theory of Entreprebeurship and Its Application to the Study of Business Transfers," *Journal of Political Economy*, 98, 265–294.

Holtz-Eakin, D., D. Joulfaian, and H. S. Rosen (1994): "Entrepreneurial Decisions and Liquidity Constraints," *The RAND Journal of Economics*, 25, 334–347.

Hurst, E., and A. Lusardi (2004): "Liquidity Constraints, Household Wealth, and Entrepreneurship," *Journal of Political Economy*, 112(2), 319–347.

Hurst, E., A. Lusardi, A. Kennickell, and F. Torrealba (2005): "Precautionary Savings and Entrepreneurship," unpublished manuscript, University of Chicago.

Jørgensen, R. (2005): "A Duration Analysis of Danish Start-Ups," *CEBR Student Paper*, 2005-08.

Keane, M. P., and K. I. Wolpin (1997): "The Career Decisions of Young Men," *Journal of Political Economy*, 105(3), 473–522.

Kihlstrom, R. E., and J.-J. Laffont (1979): "A Generel Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk Aversion," *The Journal of Political Economy*, 87(4), 719–748.

Kihlstrom, R. E., and J. J. Laffont (1983): "Taxation and Risk Taking in General Equilibrium Models with Free Entry," *Journal of Public Economics*, 21, 159–181.

Kimball, M. S. (1990): "Precautionary Saving in the Small and in the Large," *Econometrica*, 58(1), 53–73.

Knight, F. (1921): *Risk, Uncertainty and Profit*. The University of Chicago Press, Chicago/London, 1971 edn.

Malchow-Møller, N., J. R. Markusen, and J. R. Skaksen (2005): "Wage Compression, Learning, and Self-Employment," *CEBR Discussion Paper*, 2005-02.

Malchow-Møller, N., and B. J. Thorsen (2005): "Repeated Real Options: Optimal Investments and a Good Rule of Thumb," *Journal of Economic Dynamics and Control*, 29, 1025–1041.

Quadrini, V. (2000): "Entrepreneurship, Saving and Social Mobility," *Review of Economic Dynamics*, 3, 1–40.

Quadrini, V., and J.-V. Rios-Rull (1997): "Understanding the U.S. Distribution of Wealth," *Federal Reserve Bank of Minneapolis Quarterly Review*, 21(2), 22–36.

Rust, J. (1987): "Optimal Replacement of GMC Bus Engines: An Emprirical Model of Harold Zurcher," *Econometrica*, 55(5), 999–1033.

——— (1996): "Numerical Dynamic Programming in Economics," Chapter 14 in H. Amman, D. Kendrick and J. Rust (eds.) Handbook of Computational Economics.

Stokey, N. L., and R. E. Lucas (1989): *Recursive Methods in Economic Dynamics*. Harvard University Press, Cambridge.

Tarajima, Y. (2004): "Education, Self-Employment and Wealth Inequality," mimeo, University of Pennsylvania.

Centre for Economic and Business Research (CEBR)

E-mail address: `bsc@cebr.dk`

URL: `www.cebr.dk/bsc`