Can Ambiguity in Electoral Competition be Explained by Projection Effects in Voters' Perceptions?
!!!Preliminary!!!

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Abstract

Studies in political science and psychology suggest that voters' perceptions of political positions depend on their personal views of the candidates. A voter who likes/dislikes a candidate will perceive his position as closer to/further from his own than it really is (*projection*). Clearly these effects should be most pronounced when candidate positions are ambiguous. Thus a generally well liked candidate will have an incentive to take an ambiguous position. In this paper we construct a simple model to see under which conditions this incentive survives in the strategic setting of electoral competition.

1   Introduction

Ambiguous statements are prevalent in electoral competition (!!!examples to come!!!). Assuming politicians are rational that means that at least some politicians have incentives to be ambiguous and that these incentives survive the strategic setting of an election. The question then is where these incentives come from. In this paper we formulate a model that gives one possible answer to that question. The model is a standard Downsian model extended by the possibility of taking ambiguous positions and behavioral assumptions on voter perception inspired by political psychology.

A number of theoretical models on ambiguity in electoral competition exists in the literature. The seminal paper is Shepsle (1972). He extends the standard Downsian model by forcing one of the candidates (the challenger) to take a lottery position. The voters are expected utility maximizers. The main result is that if a majority of voters are risk loving on an interval containing the median, then the challenger can beat an incumbent at the median by taking a lottery position with mean equal to the median. However both existence and non-existence of a winning (lottery) position for the challenger can occur.

Page (1976) is critical of Shepsle's theory of ambiguity. He notes that the prediction of ambiguity is not very strong because the challenger may not have a winning strategy. Also he questions whether (a majority of) voters are really risk loving. And he argues that lottery positions are not a good way of modelling ambiguous political positions because candidates do not express their positions in ways that can easily be perceived as objective probability distributions. Rather candidates speak very vaguely about their positions or do not mention them at all.

Page also presents his own theory of political ambiguity, *emphasis allocation theory*. He considers a multidimensional space of policy and valence dimensions. Candidates choose which dimensions (issues) to emphasize and choose positions in these dimensions. They are vague/ambiguous on issues they do not put any emphasis on. Voters evaluate a candidate by summing the utilities of the candidate's positions on the issues, weigthed by the candidate's emphasis on each issue. In an example it is shown that this leads to emphasis on consensus issues and ambiguity on issues of conflict, no matter what the risk preferences of the voters are.

Later models of ambiguity include McKelvey (1980), Glazer (1990), Alesina and Cukierman (1990), Aragones and Neeman (2000), Aragones and Postlewaite (2002) and Meirowitz (2005). Among the explanations of ambiguity are uncertainty about candidate and median voter preferences (Glazer (1990)) and sufficiently strong candidate preference for freedom in office (Aragones and Neeman (2000)). None of the explanations offered are similar to the one we will suggest in this paper.

As mentioned earlier our model is based on behavioral assumptions about voter perception of ambiguous positions. We will use the rest of the introduction to present these assumptions and their background in psychology in a non-formal way.

We take the view that an ambiguous policy position should not be modelled by an objective probability distribution on the policy space. This is in line with Page (1976) where it is argued that ambiguous candidates do not express their stands in a way that is properly characterized in terms of probability distributions/lotteries. Instead ambiguous candidates give vaguely conflicting hints about their stands and only precise statements about what they do not stand for (or they do not give any impression of their positions at all) (p. 744-5). We claim that these observations

support modelling an ambiguous strategy as a subset of the policy space, which will be the approach in this paper.

When ambiguous positions are represented by sets instead of probability distributions it makes the voter evaluation/ranking of positions less obvious because the application of expected utility is not straightforward. For the voters to use expected utility they need to somehow associate probability distributions with the ambiguous strategies they have to rank in order to decide which candidate to vote for. This approach allows for subjectivity in voter perception, which is a central aspect of the model.

Thus we need a theory for how the perception of each voter is formed, i.e. how he associates a probability distribution with a subset of policies representing an ambiguous position. For that we get inspiration from psychology, more specifically from *Balance Theory*, which is a theory claiming that the attitudes and beliefs of a person (voter) will be formed or changed to achieve "cognitive balance" (see Granberg (1993), p. 75-83). Suppose we consider a person (voter) P who has attitudes towards another person (political candidate) O and an issue X (e.g. more or less redistribution) and a belief about O's attitude towards X. These are denoted P-O, P-X and O-X. There is cognitive balance if P agrees with and likes O or disagrees with and dislikes O. For example, if P likes O, favors more redistribution and believes that O has the same opinion, then there is cognitive balance and the set of attitudes and belief is stable. If P likes O, favors more redistribution and thinks that O favors less redistribution then there is cognitive imbalance and the theory predicts that either P-O, P-X or X-O will change. These two situations can be illustrated by the following diagrams.
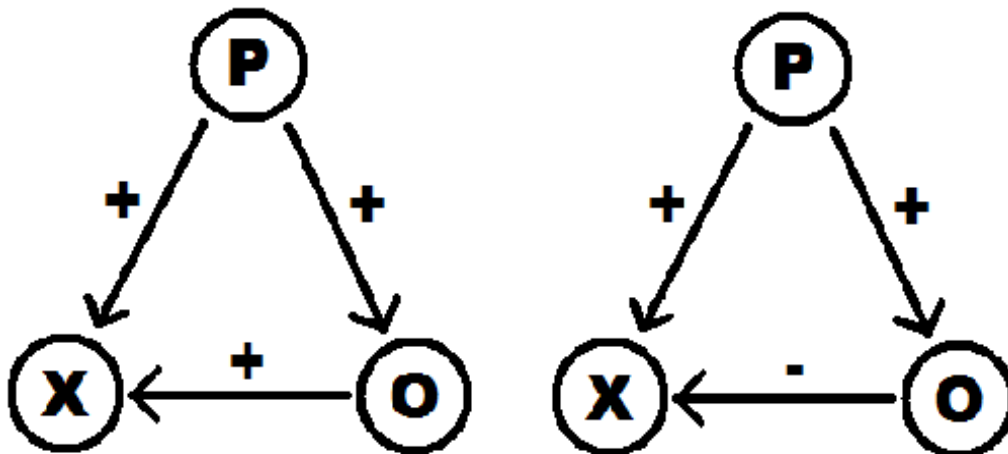
A balanced and an imbalanced set of attitudes and beliefs.

Here we will assume that before the campaign voters have formed a non-policy related attitude toward each candidate (P-O) and that they have preferences over

the policy space (P-X). We also assume that neither of these attitudes will change during the campaign. Then, by Balance Theory, each voter's perception of the positions of the candidates (O-X) should depend on his attitudes towards the candidates and his policy preferences. Roughly speaking a voter will, in his mind, "pull" a well liked candidate towards his own preferred position and "push" a disliked candidate away from that position. This phenomenon is called *projection* or, if we want to distinguish between pulling and pushing, *assimilation* and *contrast*. Originally these terms come from *Social Judgment Theory*, but the idea that projection of some communicated message depends on the receivers attitude towards the communicator comes from Balance Theory (see Granberg (1993), p. 83-88).

Granberg (1993) cites a lot of empirical studies on projection effects in elections. Generally they support Balance Theory in the sense that voters' attitudes and beliefs are balanced although it seems to be more important to agree with a favored candidate than to disagree with a non-favored candidate. But it is hard empirically to identify how balance is achieved. Is balance reached by projection (adjusting O-X), rational selection (adjusting P-O) or persuasion (adjusting P-X)? According to Granberg there is not yet a satisfactory empirical answer to that question, but some work on separating projection from rational selection and persuasion suggests that projection is really happening (and so is some combination of rational selection and persuasion).

A more recent empirical study is Merrill, Grofman & Adams (2001) where data from elections in Norway, France and the US are used. They also find results that can be interpreted as assimilation and contrast effects. There seems to be more assimilation than contrast. However they show that rational selection (choosing the closest candidate) together with random variation in voters' placement of candidates and/or variable voter-perception of the scale can generate results that looks like the empirical findings that are consistent with assimilation. But rational selection does not generate results similar to the empirical findings consistent with contrast. Thus there may not really be less contrast than assimilation, since part of what looks like assimilation could be rational selection (together with random variation/scale perception effects). And in some cases (US elections) there might even be more contrast than assimilation. But, even taking the rational selection caveat into consideration, "...some real assimilation is still likely to be present." (footnote 8, p. 219).

If there were no limits to projection then the positions of the candidates would become irrelevant and only non-policy related attitudes would matter for the outcome of an election. So we assume that there can be no "counter factual" perception bias in the following sense. A certain position (a single point in the policy space) is perceived correctly by all voters. And each voter will associate an ambiguous position with some probability distribution that does not put any probability mass on policies outside the subset given by the ambiguous position. In that way

we insure that voting is still policy dependent to a large extend. But projection can matter if at least one of the candidates chooses to be ambiguous. This approach to explaining political ambiguity is briefly considered in the following quote from Page (1976): "A different theoretical approach might recognize that perceptions vary, and seek incentives for ambiguity in the fact that it permits citizens to project or selectively perceive that candidates stand for whatever they want them to." (p. 748, footnote 38). Also, Social Judgment Theory claims that ambiguous messages leaves more room for projection (Granberg (1993), p. 83-84).

This was a non-formal presentation of the non-standard assumptions in the model and their background. It should already now be clear that when voters are risk averse only a generally well liked candidate can have an incentive to be ambiguous. This incentive of course depends on the strength of the assimilation effect. We will use our formal model to analyze under which conditions assimilation can infact lead to political ambiguity, even when voters are risk averse. By assuming that voters are risk averse we make sure that assimilation is really the force that drives our results.

2   The Model

Our starting point is a standard one-dimensional spatial model with two candidates. As mentioned above we will add to that model the possibility for candidates to announce ambiguous policy positions and introduce projection effects in voter perception of such positions. In the following we will describe the model in details.

2.1   The Candidates

Before the election the two candidates announce policy positions. Each candidate can announce either a certain position or an ambiguous position. A certain position is simply represented by a point in the policy space $\mathbb{R}$. An ambiguous position is represented by a compact interval of non-zero length. We will assume that the maximum length of an interval representing an ambiguous position is 2 (an innocent normalization). Thus the strategy space for both candidates can be written as

$$S = \{[A - a, A + a] \mid A \in \mathbb{R},\ 0 \leq a \leq 1\}. \tag{1}$$

Announced positions are credible in the sense that the candidate who wins the election must enact a policy in his announced interval.

The candidates care only about winning or loosing, they have no policy preferences. Thus their preference relation over the outcome of the election is given by

(we assume that a draw is resolved by a fair coin toss)

$$win \succ draw \succ loose. \tag{2}$$

The outcome of the election is, of course, decided by how the voters evaluate the policy positions of the candidates. That will be described in the following subsection. All the information about the voters is known by the candidates.

2.2 The Voters

There is a continuum of voters and each of them has a preferred point in the policy space $\mathbb{R}$. The distribution of their preferred points is given by a density function $v : \mathbb{R} \to \mathbb{R}$. We make the rather innocent assumptions that v is continuous and that the support of v is an interval. Without loss of generality we assume that the median voter is located at $x_0 = 0$. Thus we have

$$\int_{-\infty}^{0} v(x)dx = \int_{0}^{\infty} v(x)dx = \frac{1}{2}. \tag{3}$$

Each voter has a utility function over policies. Let the utility function of the median voter be $u_0 : \mathbb{R} \to \mathbb{R}$. Then the utility function u_{x_0} of a voter with preferred point at $x_0 \in \mathbb{R}$ is defined by

$$u_{x_0}(x) = u_0(x - x_0) \quad \text{for all} \quad x \in \mathbb{R}. \tag{4}$$

We assume that u_0 is symmetric around 0 and C^2 with

$$
\begin{aligned}
u_0'(x) &\lesseqgtr 0 \quad \text{for all} \quad x \gtreqless 0, \\
u_0''(x) &< 0 \quad \text{for all} \quad x \in \mathbb{R}.
\end{aligned} \tag{5}
$$

Thus all voters are strictly risk averse.

We are now ready to describe how voters decide on which candidate to vote for. If both candidates announce a certain position then each voter simply votes for the candidate announcing the position that gives the highest utility. If at least one of the candidates announces an ambiguous position then it is less obvious how the voter should decide on who to cast his vote for. We would like to use expected utility, but that is not straightforward since an ambiguous position is represented by an interval of policies rather than a probability distribution over policies. Thus, for a voter to use expected utility to evaluate an ambiguous position he has to somehow associate a probability distribution with the interval representing the position. The distribution represents the voter's *perception* of the ambiguous position. How voters perceive ambiguous positions is really the central part of our model and we will use the rest of this section to describe it.

   The main idea is that a voter's perception of an ambiguous position depends on whether he has a positive or negative (non-policy related) attitude towards the candidate announcing it and where his preferred policy is placed relative to the interval representing the position. If the voter likes the candidate then he will put most of the probability mass on the points in the interval that are closest to his preferred policy (*assimilation*) and if the voter dislikes the candidate then he will do the opposite (*contrast*). We will make the assumption that all voters have a positive attitude towards candidate 1 and a non-positive (i.e. neutral or negative) attitude towards candidate 2. It is clearly not realistic that all voters have identical attitudes toward the candidates, but it is the obvious starting point and an interesting benchmark case.

   Now we are ready to describe our model of voter perception of ambiguous positions in details. First consider a voter with preferred point $x_0 \geq 1$ and suppose that candidate 1 announces the ambiguous position $[-1, 1]$ (thus we are modelling assimilation in a case with maximal ambiguity around the median). Then the probability distribution that the voter associates with the ambiguous position is given by a density function $f : [-1, 1] \rightarrow \mathbb{R}$ that is weakly increasing and non-constant, i.e.

$$x \leq y \;\Rightarrow\; f(x) \leq f(y) \quad \text{and} \quad f(-1) < f(1). \tag{6}$$

We will also assume that f is continuous at the end-points -1 and 1. Note that this implies that the cumulative distribution function F strictly first order stochastically dominates that of the uniform distribution on $[-1, 1]$.
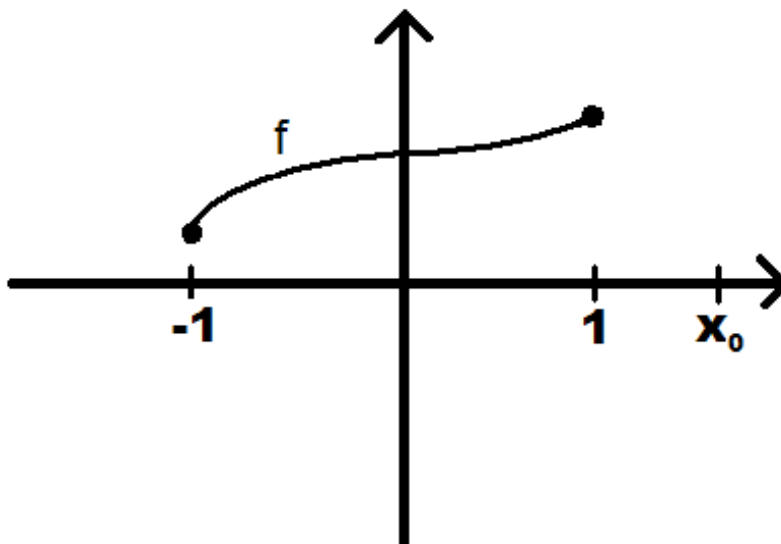
An example of the function f.

Next, consider the case where, for some $0 < a < 1$, the voter has preferred point $x_0 \geq a$ and the candidate chooses the ambiguous position $[-a, a]$. Then a simple way to model the assimilation is to assume that the voters perception is given by the density function $f_a : [-a, a] \to \mathbb{R}$ defined by

$$f_a(x) = \frac{1}{a} f(\frac{x}{a}) \quad \text{for all} \quad x \in [-a, a]. \tag{7}$$

By modelling assimilation of $[-a, a]$ this way we assume that the strength of the assimilation effect does not decrease with the level of ambiguity in a sense that will be explained in the section on extensions of the model. In that section we will present a more general way of modelling assimilation for $a < 1$ that allows for decreasing assimilation. But for now we will stick to the simple model so that we do not bury the idea of the paper in technical details.

Until now we have only considered ambiguity around the median. Suppose that candidate 1 announces an interval $[A - a, A + a]$ with $A \neq 0$. The perception of a voter with $x_0 \geq A + a$ will then be represented by the translated density function $f_a^A : [A - a, A + a] \to \mathbb{R}$ defined by

$$f_a^A(x) = f_a(x - A) \quad \text{for all} \quad x \in [-a + A, a + A]. \tag{8}$$

Thus we have modelled the assimilation of any ambiguous position by voters to the right of this position. The modelling of assimilation for voters to the left of an ambiguous position follows by symmetry (reflect the density function in the midpoint of the interval).

Suppose now that candidate 2 announces an ambiguous position. Then the perception of the voters having a negative attitude towards him will display a contrast effect. Having modelled assimilation for voters outside the interval, it is easy to model contrast for these voters. We simply define the contrast-perception of a voter to the right (left) of the interval as the assimilation-perception of a voter to the left (right) of the interval. The perception of the voters having a neutral attitude towards the candidate is given by the uniform distribution on the interval. So neutral voters all have the same (unbiased) perception of the ambiguous strategy.

It is difficult, for a general f, to make a tractable model of assimilation and contrast for voters with preferred points in the interior of an ambiguous position. So we will leave the modelling of the perceptions of "interior voters" to a special case where f has a very simple functional form. That special case will be treated in detail in the next section.

3 Results

In this section we will answer two questions. The first one is whether candidate 1 can beat any position of candidate 2. Or, to put it differently, if candidate 1 will always win the sequential election game where candidate 2 moves first. The second (and harder) question is whether candidate 1 has a winning strategy when announcements are simultaneous. To answer that we will restrict ourselves to a special case of the model where the perceptions of interior voters can fairly easily be modelled.

3.1 Candidate 2 moves first

First note that candidate 1 can always win the sequential moves game if he can win when candidate 2 announces the median (follows easily from the facts that all voters are risk averse and that the median beats any other certain position). Since voters are locally risk neutral and the assimilation effect does not decrease with the level of ambiguity it is not surprising that candidate 1 can beat the median by announcing $[-a, a]$ for an a sufficiently close to zero. The formal statement and a proof is given below.

Theorem 3.1 *Suppose candidate 2 moves first and announces the median. Then there exists some $a' > 0$ such that for any $0 < a \le a'$, candidate 1 wins the election by announcing $[-a, a]$.*

Proof. For each $0 < a \le b \le 1$ define

$$B_b = \{x_0 \in \mathbb{R} \setminus \{0\} \mid |\frac{u''_{x_0}(0)}{u'_{x_0}(0)}| < \frac{2E(f)}{b}\}$$

and

$$B_{b,a}^{\max} = \{x_0 \in \mathbb{R} \setminus [-a, a] \mid \max_{-a \le y \le a} -u''_{x_0}(y) < \frac{2E(f)}{b}|u'_{x_0}(0)|\}.$$

Then the theorem is proved by proving the following three claims.

- Claim 1: There exists a $b' > 0$ such that a strict majority of voters have preferred points in $B_{b'}$.

- Claim 2: There exists an $a' > 0$ such that a strict majority of voters have preferred points in $B_{b',a}^{\max}$ for any $0 < a \le a'$.

- Claim 3: Suppose candidate 1 announces $[-a, a]$ for some $0 < a \le a'$ and candidate 2 announces 0 (the median). Then any voter with a preferred point in $B_{b',a}^{\max}$ (a strict majority by claim 2) will vote for candidate 1.

Proof of claim 1: $|\frac{u''_{x_0}(0)}{u'_{x_0}(0)}| = |\frac{u''_0(-x_0)}{u'_0(-x_0)}|$ is a continuous function (of x_0) on $\mathbb{R}\backslash\{0\}$. Therefore it is bounded on compact subsets of $\mathbb{R}\backslash\{0\}$. And thus it follows that for any $n \in \mathbb{N}$ there exists a $b > 0$ such that

$$\{x_0 \in \mathbb{R} \mid \frac{1}{n} \leq |x_0| \leq n\} \subseteq B_b.$$

Since there exists an N such that a strict majority has preferred points in $\{x_0 \in \mathbb{R} \mid \frac{1}{N} \leq |x_0| \leq N\}$ the claim follows immediately.

Proof of claim 2: Using the continuity of u'' it is easily seen that

$$\bigcup_{0<a\leq b'} B^{\max}_{b',a} = B_{b'}.$$

Thus it follows, from claim 1, that there exists a strict majority of voters with preferred points in $\cup_{0<a\leq b'} B^{\max}_{b',a}$. And then the claim follows easily from the fact that $a_1 \leq a_2 \Rightarrow B^{\max}_{b',a_2} \subseteq B^{\max}_{b',a_1}$.

Proof of claim 3: Let $0 < a \leq a'$ and consider a voter with preferred point $x_0 \in B^{\max}_{b',a}$. If the voter votes for candidate 1, then so does a voter with preferred point $-x_0$. Therefore it suffices to look at the case $x_0 > 0$. The voter's utility of the position of candidate 1 is

$$\int_{-a}^{a} u_{x_0}(x) f_a(x) dx.$$

Thus we have to show that

$$\int_{-a}^{a} u_{x_0}(x) f_a(x) dx > u_{x_0}(0).$$

By Taylors Theorem we get that for all $x \in \mathbb{R}$ there exists a ξ between 0 and x such that

$$u_{x_0}(x) = u_{x_0}(0) + u'_{x_0}(0)x + \frac{u''_{x_0}(\xi)}{2}x^2.$$

By the definition of $B^{\max}_{b',a}$ it then follows that for all $x \in [-a, a]$,

$$u_{x_0}(x) \geq u_{x_0}(0) + u'_{x_0}(0)x - u'_{x_0}(0)\frac{E(f)}{b'}x^2.$$

Using that inequality we get

$$\int_{-a}^{a} u_{x_0}(x) f_a(x) dx$$

$$\geq \int_{-a}^{a} (u_{x_0}(0) + u'_{x_0}(0)x - u'_{x_0}(0)\frac{E(f)}{b'}x^2) f_a(x) dx$$

$$= u_{x_0}(0) + u'_{x_0}(0)(\int_{-a}^{a} x f_a(x) dx - \frac{E(f)}{b'} \int_{-a}^{a} x^2 f_a(x) dx)$$

$$> u_{x_0}(0) + u'_{x_0}(0)(\int_{-a}^{a} x f_a(x) dx - \frac{E(f)}{b'} a^2)$$

$$\geq u_{x_0}(0) + u'_{x_0}(0)(\int_{-a}^{a} x f_a(x) dx - E(f)a)$$

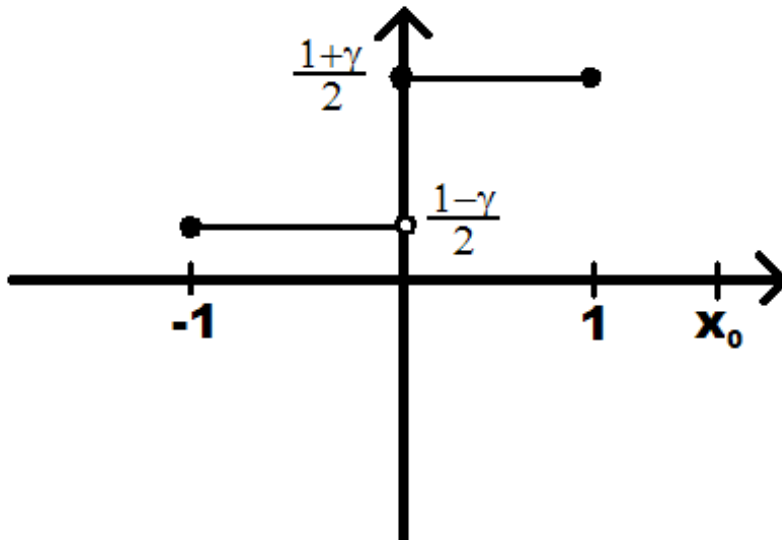$$= u_{x_0}(0) + u'_{x_0}(0)a(\int_{-1}^{1} x f_a(x) dx - E(f))$$

$$= u_{x_0}(0).$$

□

3.2   Simultaneous moves

As mentioned above we will only consider a special case of the model. Let f have the following simple form

$$f(x) = \begin{cases} \frac{1-\gamma}{2} & \text{if} \quad x \in [-1, 0) \\ \frac{1+\gamma}{2} & \text{if} \quad x \in [0, 1] \end{cases} , \tag{9}$$

where $0 < \gamma \leq 1$ is a parameter.

The special form of f.

As described earlier specifying f defines assimilation (and contrast) of any ambiguous strategy for "exterior voters", i.e. voters with a preferred points outside the interval. For example, if candidate 1 announces $[-a, a]$ the perception of a voter with $x_0 \geq a$ is given by

$$f_a(x) = \begin{cases} \frac{1-\gamma}{2a} & \text{if} \quad x \in [-a, 0) \\ \frac{1+\gamma}{2a} & \text{if} \quad x \in [0, a] \end{cases} \quad . \tag{10}$$

With this simple type of projection for exterior voters there is a fairly straightforward way to extend it to interior voters. Suppose candidate 1 announces $[-1, 1]$. Then the perception of a voter with $0 \leq x_0 \leq 1$ is given by the density f^{x_0} defined by

$$f^{x_0} = f \quad \text{if} \quad x_0 \in [\frac{1}{2}, 1] \quad \text{and}$$

$$f^{x_0}(x) = \begin{cases} \frac{1-\gamma}{2} & \text{if} \quad x \notin [x_0 - \frac{1}{2}, x_0 + \frac{1}{2}] \\ \frac{1+\gamma}{2} & \text{if} \quad x \in [x_0 - \frac{1}{2}, x_0 + \frac{1}{2}] \end{cases} \quad \text{if} \quad x_0 \in [0, \frac{1}{2}]. \tag{11}$$
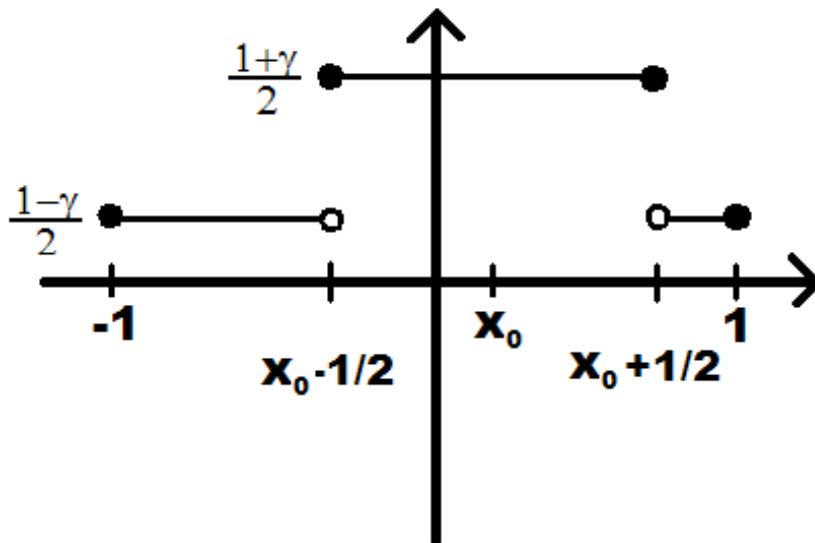
f^{x_0} for an $x_0 \in [0, \frac{1}{2}]$.

This is infact all we need to model assimilation (and contrast) for interior voters, because the extension to all interior voters and all intervals is completely analogous to the one we did for exterior voters in the general case. For example, if candidate 1 announces $[-a, a]$ the perception of a voter with $0 \le x_0 \le \frac{a}{2}$ is given by

$$f_a^{x_0}(x) = \begin{cases} \frac{1-\gamma}{2a} & \text{if} \quad x \notin [x_0 - \frac{a}{2}, x_0 + \frac{a}{2}] \\ \frac{1+\gamma}{2a} & \text{if} \quad x \in [x_0 - \frac{a}{2}, x_0 + \frac{a}{2}] \end{cases} . \tag{12}$$

The final assumption of this special case is that voter utility is quadratic, i.e. that $u_0(x) = -x^2$.

This special case enables us to answer the question of whether candidate 1 has a winning strategy or not, i.e. if he can announce a position (which obviously must be ambiguous) such that he will win the election no matter what candidate 2 does.

It turns out that the answer depends on γ. There exists a $0 < \gamma^* < 1$ such that for $\gamma \le \gamma^*$ the answer is negative and for $\gamma > \gamma^*$ the answer is positive. The result is formulated in the following theorem.

Theorem 3.2 *Consider the special case of the model described above and let $\gamma^* = \frac{1}{2}(\sqrt{\frac{19}{3}} - 1)$ (≈ 0.758). Then the following two statements hold.*

1. *Suppose $0 \le \gamma \le \gamma^*$. Then candidate 1 does not have a winning strategy. More specifically, if candidate 1 announces the ambiguous position $[A-a, A+$*

a] then candidate 2 can either win (if $\gamma < \gamma^*$) or get a draw (if $\gamma = \gamma^*$) by announcing the certain position x^* given by

$$x^* = \begin{cases} A + \frac{\gamma a}{2} & \text{if} \quad A \le 0 \\ A - \frac{\gamma a}{2} & \text{if} \quad A > 0 \end{cases} . \tag{13}$$

(And if candidate 1 announces a certain position then candidate 2 can win or get a draw by announcing the median.)

2. *Suppose $\gamma^* < \gamma \le 1$. Then there exists an $a' > 0$ such that, for any $0 < a \le a'$, $[-a, a]$ is a winning strategy for candidate 1.*

Proof. 1): We will only do the proof for $A = 0$. The extension to $A \ne 0$ is straightforward.

First let $\gamma < \gamma^*$, $0 < a \le 1$ and $x^* = \frac{\gamma a}{2}$. Candidate 2 beats $[-a, a]$ by announcing x^* if, for some $\varepsilon > 0$,

$$U_{x_0}([-a, a]) < u_{x_0}(x^*) \quad \text{for all} \quad x_0 > -\varepsilon, \tag{14}$$

where $U_{x_0}([-a, a])$ denotes the utility of $[-a, a]$ (announced by candidate 1) for a voter with preferred point x_0.

First consider voters with $x_0 \ge \frac{a}{2}$. Then we have

$$\begin{aligned} U_{x_0}([-a, a]) &= \frac{1 - \gamma}{2a} \int_{-a}^{0} -(x - x_0)^2 dx + \frac{1 + \gamma}{2a} \int_{0}^{a} -(x - x_0)^2 dx \\ &= \dots = -x_0^2 + \gamma a x_0 - \frac{a^2}{3} \end{aligned} \tag{15}$$

and

$$u_{x_0}(x^*) = -(x^* - x_0)^2 = -(\frac{\gamma a}{2} - x_0)^2 = -x_0^2 + \gamma a x_0 - \frac{\gamma^2 a^2}{4}. \tag{16}$$

Obviously, $\frac{\gamma^2 a^2}{4} < \frac{a^2}{3}$, so these voters will vote for candidate 2.

Then consider voters with $-\frac{a}{2} < x_0 < \frac{a}{2}$. In this case we have

$$\begin{aligned} U_{x_0}([-a, a]) &= \frac{1 - \gamma}{2a} \int_{-a}^{x_0 - \frac{a}{2}} -(x - x_0)^2 dx \\ &\quad + \frac{1 + \gamma}{2a} \int_{x_0 - \frac{a}{2}}^{x_0 + \frac{a}{2}} -(x - x_0)^2 dx + \frac{1 - \gamma}{2a} \int_{x_0 + \frac{a}{2}}^{a} -(x - x_0)^2 dx \\ &= \dots = -(1 - \gamma)x_0^2 - a^2(\frac{1}{3} - \frac{\gamma}{4}) \end{aligned} \tag{17}$$

and the same expression for $u_{x_0}(x^*)$ as before. Thus we just have to check whether there exists an $\varepsilon > 0$ such that

$$\gamma x_0^2 - \gamma a x_0 + a^2(\frac{\gamma^2}{4} + \frac{\gamma}{4} - \frac{1}{3}) < 0 \quad \text{for all} \quad -\varepsilon < x_0 < \frac{a}{2}. \tag{18}$$

The inequality holds for all x_0 between the roots of the polynomial on the left hand side. These are

$$x_0^{\pm} = \frac{a}{2} \pm \frac{a}{2}\sqrt{\frac{4}{3\gamma} - \gamma}. \tag{19}$$

So we just have to show that $\frac{4}{3\gamma} - \gamma > 1$. It is easily checked that this inequality is satisfied for all $0 \le \gamma < \gamma^*$.

For $\gamma = \gamma^*$ it follows easily from what we did above that

$$U_{x_0}([-a, a]) < u_{x_0}(x^*) \quad \text{for all} \quad x_0 > 0. \tag{20}$$

Thus, if candidate 1 announces $[-a, a]$, then candidate 2 can always get a draw by announcing x^*.

2): Let $\gamma > \gamma^*$ and define γ' by

$$\gamma' = \frac{\gamma}{4(\gamma - \sqrt{\frac{4-3\gamma}{3}})}. \tag{21}$$

Note that since $\gamma > \gamma^*$, γ' is well-defined and positive. Then pick an $a' > 0$ such that there are strictly less than 50% of voters in the interval $[-a'\gamma', a'\gamma']$, i.e. such that

$$\int_{-a'\gamma'}^{a'\gamma'} v(x)dx < \frac{1}{2}. \tag{22}$$

We will show that this a' "does the job". So let $0 < a \le a'$ and $x^* \in \mathbb{R}$ and assume that candidate 1 announces $[-a, a]$ and candidate 2 announces x^*. We have to show that a strict majority of voters will vote for candidate 1. We will only do that for $x^* \ge 0$. The proof for $x^* < 0$ is completely analogous.

First consider the case $x^* > \frac{a}{2}\sqrt{\frac{4-3\gamma}{3}}$. It is easily seen that

$$\begin{aligned} u_{x_0}(x^*) &= -x_0^2 + 2x^*x_0 - x^{*2} \quad \text{and} \\ U_{x_0}([-a, a]) &= \begin{cases} -x_0^2 - \gamma a x_0 - \frac{a^2}{3} & \text{if} \quad x_0 \le -\frac{a}{2} \\ -(1-\gamma)x_0^2 - a^2(\frac{1}{3} - \frac{\gamma}{4}) & \text{if} \quad -\frac{a}{2} < x_0 < \frac{a}{2} \end{cases} \end{aligned} \tag{23}$$

Using these expressions it is straightforward to check that

$$U_{x_0}([-a,a]) > u_{x_0}(x^*) \quad \text{for all} \quad x_0 \leq 0. \tag{24}$$

Thus a strict majority will vote for candidate 1 (note that both sides of the inequality are continuous in x_0 so the inequality also holds for positive values of x_0 sufficiently close to 0).

Finally consider the remaining case, i.e. $0 \leq x^* \leq \frac{a}{2}\sqrt{\frac{4-3\gamma}{3}}$. We claim that for these x^*,

$$\{ x_0 \in \mathbb{R} \mid U_{x_0}([-a,a]) \leq u_{x_0}(x^*) \} \subseteq [-a'\gamma', a'\gamma'] , \tag{25}$$

which means that a strict majority of voters will vote for candidate 1. Thus we have to show that

$$|x_0| > a'\gamma' \quad \Rightarrow \quad U_{x_0}([-a,a]) > u_{x_0}(x^*). \tag{26}$$

Since $x^* \geq 0$ it suffices to show that

$$x_0 > a'\gamma' \quad \Rightarrow \quad U_{x_0}([-a,a]) > u_{x_0}(x^*). \tag{27}$$

And since $a'\gamma' > a\gamma' > \frac{a}{2} > \frac{a}{2}\gamma > \frac{a}{2}\sqrt{\frac{4-3\gamma}{3}}$ that holds true if

$$x_0 > a'\gamma' \quad \Rightarrow \quad U_{x_0}([-a,a]) > u_{x_0}(\frac{a}{2}\sqrt{\frac{4-3\gamma}{3}}). \tag{28}$$

By straightforward calculations it is seen that the inequality

$$U_{x_0}([-a,a]) > u_{x_0}(\frac{a}{2}\sqrt{\frac{4-3\gamma}{3}}). \tag{29}$$

is satisfied if $x_0 > a\gamma'$. Thus we are done. $\square$

4 Extensions of the Model

We will first generalize our model of assimilation. Let $(f_a)_{a \in (0,1]}$ be a family of functions such that each f_a is a density function on $[-a,a]$. Assume that each f_a is weakly increasing, non-constant and continuous at the end points. For $0 < a \leq 1$ define

$$E_a = \frac{E(f_a)}{aE(f_1)}.$$

E_a is a first order measure of the strength of the assimilation effect. If $E_a < 1$ then the assimilation is weaker for a than for the maximal level of ambiguity (1) in the sense that the mean of the perception has decreased proportionally faster than the level of ambiguity. We will assume that E_a weakly decreases as a decreases (so that assimilation does not increase as the level of ambiguity decreases).

It is important how strong the assimilation effect is as $a \to 0$. Therefore define

$$E_0 = \lim_{a \to 0} E_a.$$

If $E_0 = 0$ then the first order assimilation effect disappears as $a \to 0$. Otherwise it survives to some extend as the level of ambiguity goes to zero. Note that for the model used in the previous sections ($f_a(x) = \frac{1}{a} f(\frac{x}{a})$) we have $E_a = 1$ for all $0 < a \leq 1$ and therefore $E_0 = 1$. Thus it represents a case where the first order assimilation effect does not decrease at all as the level of ambiguity decreases. While that does make things nice and simple it is not an obviously true or innocent assumption. So it makes sense to do a robustness check by using the more general model.

The following theorem gives sufficient conditions for candidate 1 to be able to beat any strategy of candidate 2 in the generalized model (of course it still suffices to check if candidate 1 can win when candidate 2 announces the median).

Theorem 4.1 *Suppose candidate 2 moves first and announces the median. Then the following statements hold.*

1. *Suppose $E_0 > 0$. Then there exists an $a' > 0$ such that for any $0 < a \leq a'$, candidate 1 wins the election by announcing $[-a, a]$.*

2. *Suppose $E_0 = 0$. Also suppose that there exists a $b > 0$ such that a strict majority have preferred points in the set*

$$B_b = \{x_0 \in \mathbb{R} \setminus \{0\} \mid |\frac{u''_{x_0}(0)}{u'_{x_0}(0)}| < \frac{2E(f_1)}{b} E_b\}.$$

Then there exists an $a' > 0$ such that for any $0 < a \leq a'$, candidate 1 wins the election by announcing $[-a, a]$.

Proof. !!!With some straightforward changes the proof from the simple case also works here!!! $\square$

We see that as long as some of the first order assimilation effect is preserved as $a \to 0$ candidate 1 can beat any position of candidate 2. When $E_0 = 0$ we only have a sufficient condition that depends on the electorate, i.e. the distribution of

preferred points and the voters' absolute risk aversion at the median. Note however that if $\frac{E_b}{b}$ goes to infinity as b goes to zero then we still have that candidate 1 can beat any position of candidate 2.

The general model of assimilation presented above is to general for us to answer the question of whether candidate 1 has a winning strategy in the simultaneous move game. But we can generalize the result presented earlier to a situation where assimilation decreases with the level of ambiguity. Given some f we can get a model allowing for decreasing assimilation by defining

$$f_a(x) = h(a)\frac{1}{a}f(\frac{x}{a}) + (1 - h(a))\frac{1}{2a},$$

where $h : (0, 1] \to [0, 1]$ is continuous, weakly increasing and satisfies $h(1) = 1$. If we let $h(a) = 1$ for all $a \in (0, 1]$ then we get the simple model. If h is strictly increasing then we have decreasing assimilation as the level of ambiguity (a) decreases. Because in that case

$$E(f_a) = \int_{-a}^{a} x(h(a)\frac{1}{a}f(\frac{x}{a}) + (1 - h(a))\frac{1}{2a})dx = h(a)E(f_a) = h(a)aE(f).$$

So we have that, for all $0 < a \leq 1$,

$$E_a = h(a)$$

and

$$E_0 = \lim_{a \to 0} h(a).$$

If f is the piecewise constant function considered earlier, then we have

$$f_a(x) = \left\{ \begin{array}{l} h(a)\frac{1-\gamma}{2a} + (1 - h(a))\frac{1}{2a} \text{ for } -a \leq x < 0 \\ h(a)\frac{1+\gamma}{2a} + (1 - h(a))\frac{1}{2a} \text{ for } 0 \leq x \leq a \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \frac{1-\gamma h(a)}{2a} \text{ for } -a \leq x < 0 \\ \frac{1+\gamma h(a)}{2a} \text{ for } 0 \leq x \leq a \end{array} \right\}.$$

The definition of the density functions representing the perceptions of interior voters is then straightforward. With this more general model of assimilation for $a < 1$ we have the following sufficient condition for candidate 1 to have a winning strategy in the simultaneous move game. All other assumptions are the same as they were earlier (quadratic utility).

Theorem 4.2 *Suppose $\gamma E_0 > \gamma^*$. Then there exists an $a' > 0$ such that for any $0 < a \leq a'$, $[-a, a]$ is a winning strategy for candidate 1.*

Proof. !!!With some straightforward changes the proof of statement 2 from the simple case also works here!!! □

Obviously it is still true that if $\gamma \leq \gamma^*$ then candidate 1 does not have a winning strategy (we have made the assimilation effect and thus the advantage of candidate 1 weaker). So the only remaining question is what happens when $\gamma > \gamma^*$ and $\gamma E_0 \leq \gamma^*$. !!!More to come!!!

5   Discussion

We have seen that assimilation in voters' perceptions of ambiguous policy positions can lead to ambiguity in electoral competition. Ambiguity can be a winning strategy for a generally well liked candidate, even when announcements are simultaneous. However we must remember that this result relies on a strong assimilation effect and the unrealistic assumption that one of the candidates is well liked by all voters and that the other candidate is not well liked by any voters. So maybe assimilation by itself is not enough to explain political ambiguity. But even if that is the case it could still be an important contributing factor, for example if it is combined with the assumption that candidates have a preference for freedom in office (see Aragones and Neeman (2000)).

If assimilation is a significant part of the explanation of ambiguity in electoral competition we would expect to see candidates that are generally well liked on a personal level to be more ambiguous. It would be interesting to see some empirical or experimental work on that (although we recognize that there are lots of problems in doing such work). It would also be interesting to see some empirical or experimental work on the relationship between ambiguity and assimilation/contrast among voters.

There are also possibilities for further theoretical research. As mentioned above one possibility is to include candidate preferences for freedom in office into the model. That might give predictions of ambiguity with more realistic assumptions on the assimilation part of the model. Other possibilities are to look at candidates with policy preferences or candidates that are uncertain about the distribution of voters' preferred points. If candidates have policy preferences then there will obviously be a lot of informational issues, so such a model will be substantially more complicated that the one presented in this paper. Candidate uncertainty about the distribution of voters may be easier to include in the model.

6 References

1. Alesina, A. & Cukierman, A. (1990). The Politics of Ambiguity, *The Quarterly Journal of Economics* 105: 829-850.

2. Aragonès, E. & Neeman, Z. (2000). Strategic Ambiguity in Electoral Competition, *Journal of Theoretical Politics* 12(2): 183-204.

3. Aragonès, E. & Postlewaite, A. (2002). Ambiguity in Election Games, *Review of Economic Design* 7: 233-255.

4. Fishburn, P. C. (1972). Lotteries and Social Choice, *Journal of Economic Theory* 5: 189-207.

5. Glazer, A. (1990). The Strategy of Candidate Ambiguity, *The American Political Science Review* 84: 237-241.

6. Granberg, D. (1993). Political Perception. In Iyengar & McGuire, *Explorations in Political Psychology* (p. 70-112). Duke University Press.

7. McKelvey, R. D. (1980). Ambiguity in Spatial Models of Policy Formation, *Public Choice* 35: 385-402.

8. Meirowitz, A. (2005). Informational Party Primaries and Strategic Ambiguity, *Journal of Theoretical Politics* 17(1): 107-136.

9. Merrill, III, S., Grofman, B. & Adams, J. (2001). Assimilation and contrast effects in voter projections of party locations: Evidence from Norway, France and the USA, *European Journal of Political Research* 40: 199-221.

10. Page, B. (1976). The Theory of Political Ambiguity, *The American Political Science Review* 70: 742-752.

11. Shepsle, K. (1972). The Strategy of Ambiguity: Uncertainty and Electoral Competition, *The American Political Science Review* 66: 555-568.

12. Sherrod, D. R. (1971-1972). Selective Perception of Political Candidates, *The Public Opinion Quarterly* 35(4): 554-562.