Estimating Structural Inflation Dynamics: A Reduced Form Solution for a Conundrum

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VERY PRELIMINARY

Abstract

In this paper we estimate a New Keynesian Phillips Curve (NKPC) based on a Vector Autoregressive (VAR) model. The strategy is to consider a VAR involving the inflation rate and the forcing variable(s), and exploits the rational expectations cross-equation restrictions, that the NKPC imposes on the VAR, to estimate the structural parameters. This methodolgy has the advantage to avoid the misspecifications of previous GMM and ML results. Moreover, in light of the potential instability in the dynamics of both processes, we also allow for time varying effects for all the parameters as well as in the mean and variance elements. We use the model to assemble evidence about the composition and evolution of the forward and backward looking components in the Euro area and United States inflation series.

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1 Introduction

Is the tamed inflation in industrialized countries an evidence of enough knowledge about inflation dynamics ? Given the increasing views challenging the insights of modern dynamic macroeconomics, the answer is certainly no. Since Lucas critique (1976), finding a relationship isolated from policy effect has been the holy grail of modern time economics. In the search of a process embedding a large variety of pricing environments, the "New Keynesian" paradigm and its tenants put the expectations formation process of private agents at the center of this quest, and gave rise to the nowadays workhorse of inflation dynamics, namely the New Keynesian Phillips Curve.

Theoretically derived under intertemporal micro-optimatisation, the extent to which the New Keynesian Phillips curve is able to replicate key dynamic features of empirical data, though, has been the subject of much debate, and grown the list of the conundrums still unresolved in applied economics. Now, as emphasized by Lucas and Sargent (1981), the question of whether a particular model is structural is an empirical, not theoretical, one. Two main approaches have been offered to assess the degree of forward lookingness in inflation, the “limited”information or single equation methodology, and the “full” information or system estimation. During the first years of this debate, it seemed like the estimation methods was clearly a decisive factor for the outcome of the empirical analysis. For instance, Galí and Gertler (1999), and Galí et al. (2001a, 2001b) found a dominant role for the forward looking component when using limited information methods in an “hybrid” version of the Phillips curve, which in turn, was argued to be a "good first approximation of inflation" in the US and Euro area. On the other hand, using full information methods, Fuhrer and Moore (1995) and Fuhrer (1997) present evidence on US inflation that seems to undermine the importance of forward-looking components as a relevant component of inflation dynamics.

However, it is well documented by now, that GMM estimates can be markedly biased in small samples and subject to “weak instruments” or “weak identification” issues (Stock et al., 2002). Recently, in order to cope with these deficiencies, Fuhrer and Olivei (2004) propose a GMM procedure that, instead of instrumenting by means of simple linear projections on the instruments set, uses projections that impose the dynamic constraints implied by the forward-looking relation. This “optimal instruments” procedure is argued to be similar to maximum likelihood estimation and indeed yield similar results as in Fuhrer and Moore (1995). Unlike Gali et al (2001a, 2001b), though, their results are based on the output gap as the driving variable, which is suggested to underestimate the empirical importance of the forward looking component (see Roberts, 1997 and Jensen, 2002).

Meanwhile, using a FIML approach, Ireland (2001) cannot reject the null hypothesis that inflation dynamics in the postwar U.S. are purely forward looking. The author's method consists of expressing the Euler equation and the VAR equations for the exogenous variables as a miniature dynamic general equilibrium (DGE) system. In doing so, this technique implicitly restricts the estimates such that all but one solution can be discarded on grounds of non-stationarity. The problem with this method is that imposing uniqueness may provide little economic sense, which can lead to severe misspecification in case the likelihood is maximized for a combination of parameters that implies multiple stable solutions.

Thus, as the research accumulates, it turns out that the analysis of the the degree of forward lookingness can be robuslty assessed less from the perspective of model specification and methodologies, than from the point of view of identification and testable restrictions.

Nevertheless, we argue in this paper that a system of equations is still the best framework for theoretical and empirical analyses about the stability and determination of the relative weight of forward and backward components in inflation dynamics. However, in light of the difficulties in identifying the structural parameters via a classical system approach, we follow a strand of the literature which recovers the values of these parameters using reduced form statistical models (see Cogley and Sbordone, 2004, Kurman, 2004). Specifically, we use the fact that the constrained coefficients of the rational expectation solution (RES) are higher-order polynomials in the unrestricted Euler equation parameters and VAR coefficients. The strategy is to consider a VAR involving the inflation rate and the forcing variable(s), and exploits the rational expectations cross-equation restrictions, that the NKPC imposes on the VAR, to estimate the structural parameters. We also argue that proceeding with system estimation under constraints is indeed necessary in some cases, since it enables to separately distinguish a role for lagged inflation arising from forecasting from the one arising from price-stickiness.

Besides, the identification task is also hardened because of the presence of the unit root often found in the inflation series. In such a case, two extreme specifications can give equivalent nonstationary dynamics even if the two equations are not equivalent. In this paper, we avoid this issue via the notion of persistence. Our strategy entails the modelisation of the mean as an I(1) exogenous inflation target to ensure that the deviations of inflation from such a mean are stationary. The masking effects of spurious high persistence for inference in the structural parameters is then avoided.

Moreover, some authors have argued that changes in the level of credibility of the central bank's commitment to attain their objective, should have an effect on the relative importance of forward-looking and backward-looking terms in inflation models (see Taylor, 1998, Sargent, 1999). Thus, we also allow for this possibility and describe the evolution of the law of motion for inflation by the use of time-varying parameters. As the structural model involves nonlinear cross-equation restrictions on the evolving parameters, we use the so-called Unscented Kalman Filter. This method is built on the principle that it is easier to approximate a probability distribution than an arbitrary nonlinear function (see Julier and Uhlmann, 1996). As such, this technique is superior to the Extended Kalman Filter since it does not require to linearize the measurements and evolution models using Taylor series expansions. The algorithm is coupled with the simulated annealing optimisation algorithm of Goffe (1996) for the optimisation of the fixed parameters, since it proved to be more robust than traditional numerical gradient algorithms.

The remainder of the work provides the relevant theory underlying this paper in section 2. Section 3 provides an explanation of the econometric methodology. Section 4 discusses the data used. Section 5 reports the empirical evidence. Finally, section 6 closes with a summary and some conclusions.

2 Theoretical Consideration

We first consider the hybrid Phillips curve equation defined as follows:

$$\pi_t = \gamma_f E_t(\pi_{t+1}) + \gamma_b \pi_{t-1} + \kappa mc_t + u_t \quad (1)$$

Where π_t denotes the inflation rate at time t, $E_t(\pi_{t+1})$ is the expectation conditional on time-t information of inflation at time t+1, mc_t is the real marginal cost (measured as deviation from steady state). The studies by Rotemberg (1982), Roberts (1995), Fuhrer and Moore (1995), Yun (1996), and Gali and Gertler (1999) are influential examples of the derivation of the forward looking Phillips curve. The addition of the backward looking component has been first motivated empirically, before being legitimised theoretically on the basis of several possible grounds. For example, Roberts (1997, 2001) and Ball (2000) assume that a fraction of agents use adaptive expectations, while Galí and Gertler (1999) assume that some firms have a non rational rule of thumb, which uses past inflation to set the optimal price. Without relaxing the assumptions of rational expectations but with frictions on price adjutment, the process may also be characterised as an hybrid Phillips curve (see Kozicki and Tinsley 2002).

Even though forming the basis for many empirical studies, the above specification is most of the time linearized around a constant, which represents the long-run anchor for inflation expectations (steady-state inflation rate) and is assumed to equal zero. The error term u_t is modelled as an i.i.d process or as a martingale difference sequence with respect to the available information set, i.e. u_t is assumed to be a contemporiously and serially uncorrelated white noise.

Different interpretations are provided in the literature for this term : a cost push shock, a measurement error characterizing prices and/or real marginal costs, a quantity reflecting differences in the information set of the economic agent and the econometrician, or more simply a term capturing deviations from the theory (Kurmann, 2004).

As the estimation by classical equation (for example OLS type of regression) has long been proved to be biased, it is now common practice to use either the Generalized Method of Moments (GMM) or the Maximum Likelihood (ML) approach. However, as pointed out by Nason and Smith (2004), identification may be easier in the system context than in the GMM. The lack of instruments or the problem of weak instruments is problematic for the estimation since they lead to GMM point estimates, hypothesis tests, and confidence intervals that are unreliable. Moreover, one should not forget that the existence and nature of a stationary solution remains a system property, implying that statements about the stationary properties of the rate of inflation hinges as importantly on the properties of the variables causing inflation.[1] As a matter of fact, the identification of the hybrid NKPC (nature of the solution, backward or forward) is indeed closely intertwinned with the nature of the driving variable process. To demonstrate this point, let's take a simple example where the Euler equation is augmented with a VAR(1) equation characterising the forcing variable process:

$$mc_t = \alpha_{mm} mc_{t-1} + \alpha_{m\pi} \pi_{t-1} + \varepsilon_{1t} \tag{2}$$

We shall note that we are agnostic about whether inflation Granger-cause marginal cost, as every econometrician should be when studying the behavior of two interacting variables.

Let's write now the reduced form solution for inflation as:

$$\pi_t = \alpha_{\pi\pi} \pi_{t-1} + \alpha_{\pi m} mc_{t-1} + \varepsilon_{2t}, \tag{3}$$

In this case, the coefficient γ_b on π_{t-1} reflects not only the structural parameter in equation (1), but also the forecasting rule for the forcing variable. For example, $\gamma_b = 0$ does not imply that $\alpha_{\pi\pi} = 0$.[2] Consequently, an investigator who incorrectly assumes that mc_t is strictly exogenous will deduce incorrect (i.e., biased) values of γ_f and γ_b , when performing unconstrained system

[1] Now we shall note that even if GMM methods can be seen as single estimation method, the requirements underlying its efficiency, such as the order and rank conditions, are intrinsically dependent on the properties of the forcing variable, through the choice of the instruments.

[2] This result is also based on Sargent (1987, chapter XI, part 24), who showed the relationship between strict exogeneity – in the classic terminology of Engle, Hendry and Richard (1983) - and Granger-causality.

estimation. Now, we know that the structural and reduced form parameters are linked each other by the following two equations:

$$\alpha_{\pi m} - \gamma_f * (\alpha_{\pi m} * \alpha_{mm} + \alpha_{\pi\pi} * \alpha_{\pi m}) = \kappa * \alpha_{mm} \tag{4}$$

$$\alpha_{\pi\pi} - \gamma_f * (\alpha_{\pi m} * \alpha_{m\pi} + \alpha_{\pi\pi} * \alpha_{\pi\pi}) - \gamma_b = \kappa * \alpha_{m\pi} \tag{5}$$

In most of the studies (see for instance Fuhrer and Moore, 1995, Jondeau and Le Bihan, 2003), the system is estimated by FIML, without imposing the necessary identification restriction, causing the parameters to be biased. Moreover, these same studies do not consider the properties of the system, which may also have drastic consequences on the estimation. To study the properties of the system, the latter can be cast in its standard second linear difference form following the early work of Blanchard and Kahn (1980):

$$E_t Y_{t+1} = A.Y_t + C.X_t, \tag{6}$$

where we have :

$$E_t Y_{t+1} = \begin{pmatrix} \pi_{t+1} \\ mc_{t+1} \\ \pi_t \\ mc_t \end{pmatrix}, \ A = \begin{pmatrix} -\gamma_F^{-1} & -\gamma_F^{-1}\kappa & -\gamma_F^{-1}\gamma_b & 0 \\ \alpha_{m\pi} & \alpha_{mm} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \tag{7}$$

$$C = I_4 \quad ; X = \begin{pmatrix} -\gamma_F^{-1} u_t \\ \varepsilon_{1t} \\ 0 \\ 0 \end{pmatrix}$$

It is well known that (7) has a unique stable rational expectations solution, if and only if the number of non-predetermined variables m in Y_t (one in our case, $E_t\pi_{t+1}$) equals the number of generalized eigenvalues h of (7) with modulus larger than one. If $h > m$, then there is no stable solution, whereas if all eigenvalues lie inside the unit circle ($h = 0$) there is an infinity of stable solutions. The generalized eigenvalues are given by the following polynomial equation:

$$\gamma_f \ \lambda^4 + (-1 - \gamma_f.\alpha_{mm}) \ \lambda^3 + (\gamma_b + \alpha_{mm} + \alpha_{m\pi}\kappa) \ \lambda^2 - (\alpha_{mm} \ \gamma_b) \ \lambda = 0 \tag{8}$$

By definition, we have one generalized eigenvalue such that $|\lambda_1| = 0 < 1$. The question of whether the system has a unique stable solution therefore boils down to whether the combination of the parameters γ_f, γ_b, κ, $\alpha_{m\pi}$, α_{mm} is such that $|\lambda_2| < 1$ and $|\lambda_3| < 1$, while at the same time and $|\lambda_4| > 1$. If this is indeed the case, then forward iteration allows to eliminate the unstable eigenvalues, and implies a unique and stable solution.

It is then evident, from (8), that the feedback parameters, $\alpha_{m\pi}$, α_{mm}, affect the roots properties of the system (see Fanelli, 2005). This shows then that in the presence of feedbacks from the inflation to the driving variable, it is not clear whether the NKPC can be reconciled either with a non-explosive inflation process or multiple stable solutions, without constraining the parameters opportunely.

2.1 Reduced Form and Cross Equation Restrictions

As pointed out by Sims (2002), algebrically, non-uniqueness or non-existence can arise because certain linear combination of Y_t are unrestricted by the matrix A.[3] Classical numerical estimation does not deal with this issue though. What lies behind the use of the ML is the restriction of the parameter space to regions where indeterminacy does not occur. As long as the combination of the parameters that maximizes the likelihood implies a unique stable solution, the inequality constraints do not bind and thus, imposing uniqueness has no influence on the ML estimation. However, the ML estimates may as well be located in a region of the parameter space with more than one stable solution. The inequality constraints imposed by the uniqueness condition bind and prevent the ML estimation from reaching its true maximum. Concretly, in our example, this means that there is not only one mapping from the structural parameters to the reduced form parameters. Genuine FIML estimation simply means constraining the estimation whereas constraining the equation as (4) and (5) is the solution.

To circumvent the multiplicity problem, Kurman (2004) provides a simple solution, exploiting the restrictions in (4) and (5), the author remarks that expressing the VAR coefficients for the forcing variable mc_t as a function of the structural parameters and the coefficients of the remaining VAR equations provides a unique mapping from the structural parameters and the VAR coefficients of the endogenous variables.

Hence, rewriting the cross-equation restrictions as explicit solutions to the VAR coefficients of the forcing variable altogether both solves the identification problem and circumvents the multiplicity problem. Moreover, as the direct estimation of the structural equations still remains

[3] As described by Sims (2000), the second source of uniqueness may arise because the model describes expectations of endogenous variables rather than the variables themselves.

a problematic issue, we exclude the structural New Keynesian Phillips curve from the system and instead carry out the estimation with the reduced from given in (3).

As long as the rational expectations solutions of the variables in the Euler equation have a state-space representation, their dynamics can be described by an infinite-order VAR process that does not contain any other variables. Moreover, using a VAR approximation instead of a more structural model, such as an Euler like-equation, to describe the dynamics of the forcing variable has the advantage that the estimation of this structural equation is not conditioned on other fundamental assumptions about the economy. Making inference on the structural parameters, or on the structural parameters of the forward counterpart, namely the reduced form, implies no loss of information if the parametric mapping between two representations is taken into explicit acute.We assume in the first place that the dynamics of the respective real marginal cost and inflation are well approximated by a bivariate VAR(1) process in the two variables.[4]The rational cross equation restrictions are then given by (4) and (5). The system has a single solution in the np coefficients of the VAR equations for inflation and the forcing variable of the hybrid NKPC and can expressed as:[5]

$$\left(\begin{array}{c} \pi_t \;=\; \alpha_{\pi\pi}\pi_{t-1} + \alpha_{\pi m} mc_t + \quad \varepsilon_{1t} \\ mc_t = \frac{\alpha_{\pi m} - \gamma_f * \alpha_{\pi\pi} * \alpha_{\pi m}}{\kappa + \gamma_f * \alpha_{\pi m}} mc_{t-1} + \frac{\alpha_{\pi\pi} - \gamma_f * \alpha_{\pi\pi}^2 - \gamma_b}{\kappa + \gamma_f * \alpha_{\pi m}} \pi_{t-1} + \; \varepsilon_{2t} \end{array} \right) \tag{9}$$

Proceeding with system estimation under constraints enables to separately distinguish a role for lagged in inflation arising from forecasting from the one arising from price-stickiness. Constraining the system as described above, i.e matching appropriately the reduced form solution with the structural form, will shed the light on the importance of backward and forward components[6].

[4]Previous studies report that despite the real marginal cost intrinsic persistence, there is no strong evidence of higher-order dynamics (see Nason and Smith, 2004 and Kurman, 2004) for the U.S. We argue then that this assumption is not too restrictive. However, we intend, in a second draft, to extend the analysis and involve multiple leads and lags of inflation in the NKPC (as in Cogley and Sbordone, 2004) which would lead to different cross equation restrictions.

[5]A non-structural VAR approach is used where the error terms of the model are not serially correlated. Moreover, we do not follow the New Keynesian Trinity Model (NKTM) by incorporating a monetary policy function. Recently Cho and Moreno, 2002, found the whole NKT model to be inconsistent with their data. The authors acknowledge that this might due to the fact that the Taylor rule does not describe accurately the way the Fed conducts monetary policy.

[6]Cogley and Sbordone (2004), Sbordone (2002 and 2003a) minimizes the distance between the model predicted path and the actual path of price level to select the parameters of the Phillips Curve. Rotemberg and Woodford (1997), Christiano, Eichenbaum and Evans (2001), and Boivin and Giannoni (2003) estimate general equilibrium models by minimizing the distance between the model-based impulse responses and the estimated VAR responses

In the next section, we elaborate on a set of issues that we believe to be relevant for the identification and estimation of the NKPC.

2.2 Model (Mis)Specification

Another issue arises from the presence of autocorrelation in the error term of (1) often found in empirical studies. Is the serial correlation an intrinsic feature of the New Keynesian Phillips Curve, or on the contrary, a symptom of more general model misspecification ? The overriding question is whether the New Keynesian Phillips curve equation when viewed as a statistical model, give rise to valid inference about the significance of the the forward and backward component. Using the Calvo-Rotemberg forward looking specification, the tenants of the NKPC tacitly assumes that serial correlation in the residuals is symptomatic of serial correlation in the true disturbances. In this set up, past inflation may appear to be important for determining current inflation, only because the shocks in the economy are positively correlated. They tend then to attribute the properties of strong autocorrelation to the error term in order to match the inertia found in tha data. However, econometrically speaking, this may not be commendable, since the underlying cause of the residual misspecification may be quite different, caused by omitted variables or an ill-founded functional form.[7] On the other hand, the tenants of the traditional Phillips curve argue that all of the underlying correlation is due to the backward looking component, leaving then no room for any substantial correlation in the residuals. However, it is highly difficult to get rid of the serial correlation in an hybrid Phillips curve since this correlation may stem from multiple reasons. The use of the VAR specification as modelled in (7) permits to avoid the deficiencies of the NKPC estimation, since the reduced form, by nature, can more easily deal with correlated residuals. Accordingly, we augment (3) and incorporate lagged differenced variables *a la* Dickey Fuller. Equation (3) becomes:

$$\pi_t = \alpha_{\pi\pi}\pi_{t-1} + \alpha_{\pi m} mc_{t-1} + \alpha_{\pi 1} \triangle \pi_{t-1} + ... + +\alpha_{\pi p} \triangle \pi_{t-p} + \varepsilon_{2t}, \tag{10}$$

Overall, this section echoes the voice of the previous part, and reinforces the view that, given the difficulties of the identification via direct estimation, it may be highly advisable to estimate a reduced form in a restrained fashion, embedding the structural parameters.

to monetary policy shock.

[7] In this example, we omit that the error in the variables may also be a source of misspecification.

2.3 Anchor Expectations

One last source that proved to influence the properties of the lag dynamics is the shifts in the long run anchor of agents' inflation expectations. As shown by Erceg and Levin (2003), learning about shifts in the policy target may be another source of persistence in the inflation process. Within the context of price stability, long-run inflation expectations should converge to the perceived inflation target of monetary policy, or the inflation target if known and credible.[8]

Besides, as recently expressed by the Inflation Persistence Network hosted by the ECB, inflation persistence refers to the tendency of inflation to converge slowly (or sluggishly) towards its long-run value following shocks of different nature. If one follows that definition, it is explicitly assumed that a good definition of the long run value is crucial in the assessment of inflation persistence (see also Marques, 2004, and Gadzinski, 2005).

We shall recall, though, that this perceived target or the long run value tend to incorporate small positive inflation rates[9]. Consequently, imposing the assumption that the long-run anchor for inflation expectations is zero either in a reduced form or the structural form is empirically unreasonable. The constant-zero assumption on the inflation expectations anchor is likely to lead to particularly misleading empirical results if the steady-state inflation rate changed within the sample. As a matter of fact, as shown by several authors (see Levin and Piger (2004), Gadzinski and Orlandi (2004), stationary series witnessing structural break(s) are often mistaken for I(1) processes, an empirical fact that exaggerates the degree of persistence in the series. Empirical evidence suggests that there have been shifts in the conditional mean of the inflation process. We follow then this strand of the literature and model the mean process in order to take into account its potential source of persistence. The way of accounting for shifts in the steady-state inflation is examined in the next section among the general set up of our empirical analysis.

3 Estimation Method

On the top of the questions raised above, come also to play the debate on time varying parameters. Has the persistence of inflation changed over time, as suggested by Cogley and Sargent (2004) and Gadzinski (2005) ? Has the weight on the forward looking component increased over time ?

[8] Under less optimistic but perhaps more realistic assumptions, or with non explicit inflation target, the perceived inflation value may depart from its long run value either way, which in turn may depend on the combination of the degree of transparency and credibility of the monetary authorities.

[9] No central banks in the world target a zero inflation rate. Moreover, the perceived inflation target is often found to overestimate the official target (see Kozicki and Tinsley 2002).

The time varying properties of the inflation process have been challenged a plethoria of times in the literature. While estimating a reduced form relationship, we ought to consider a time varying specification. Kalman filtering is used to describe the evolution of the law of motion for inflation as it enables to take on board all the points studied in the preceding sections. However, the equations in (9) involve nonlinear restrictions so that the Kalman filter needs to be augmented to deal with this nonlinearity. One of the straightforward extensions is the Extended Kalman Filter. The EKF is a minimum mean square error estimator based on the Taylor series expansion of the nonlinear function around the states estimates. However, because the EKF only uses the first order terms of the Taylor series expansion, it often introduces large errors in the estimates. This is especially evident when the models are highly nonlinear, the local linearity assumption breaks down and the effects of the higher order terms of the Taylor series expansion become significant. To deal with this issue, a new filter has been proposed: the Unscented Kalman Filter (UKF). Unlike the EKF, the UKF does not approximate the nonlinear process, but uses the true nonlinear models and rather approximates the distribution of the state random variable. This technique deterministically selects a set of sample points which completely capture the prior mean and covariance of the random variables accurately. This set of points is propagated through the nonlinear function, and only then the true posterior sample mean and covariance are calculated via the Kalman gain. Let's consider the following multidimensional normal state-space model:

$$
\begin{aligned}
Y_t &= h(\mu_t, \xi_t) = \mu_t + g(\xi_t).Y_{t-1} + \varepsilon_t \quad , \ \varepsilon_t \sim N(0, \sigma_\varepsilon^2) && (11)\\
\mu_t &= \mu_{t-1} + \varepsilon_{\mu,t} \quad , \ \varepsilon_{\mu,t} \sim i.i.d.N(0, \sigma_\mu^2) && (12)\\
\xi_t &= \xi_{t-1} + \varepsilon_{\xi,t} \quad , \ \varepsilon_{\xi,t} \sim i.i.d.N(0, \sigma_\xi^2) && (13)
\end{aligned}
$$

Where $Y_t = [\pi_t \ \ mc_t]$ and $h : \mathbb{R}^r \longrightarrow \mathbb{R}^2$ includes the linear and nonlinear functions of the parameters, $\alpha_{\pi\pi}$, $\alpha_{\pi m}$, γ_f, γ_b, κ defined in (9). All these time varying parameters as well as the time varying perceived inflation target μ_t can be represented as driftless random walks.

The first equation describing the inflation process in (9) is fully linear in its variables, whereas the equation describing excess demand is nonlinear in all the coefficients above. To deal with the nonlinear component of g, we use the UKF algorithm that updates the mean and the covariance to the posterior distribution of the states as follows:

1. Initialize with

$$
\begin{aligned}
\overline{x}_0 &= E[x_0] && (14)\\
P_0 &= E[(x_0-\overline{x}_0)((x_0-\overline{x}_0)^T] && (15)\\
\overline{x}_0^a &= E[x^a]=[x_0^T\ 0\ 0]^T && (16)
\end{aligned}
$$

$$
P_0 = E[(x_0-\overline{x}_0)((x_0-\overline{x}_0)^T] \ = \begin{bmatrix} P_0 & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & R \end{bmatrix} \tag{17}
$$

where $x_t = [\mu_t \ \ \xi_t]$.

2. For t $\in$ $\{1, ...N\}$,

a) Calculate sigma points

$$
\chi_{t-1}^a = [\overline{x}_{t-1}^a \quad \overline{x}_{t-1}^a \pm \sqrt{(n+\lambda)P_{t-1}^a}]
$$

b) Time update:

$$
\begin{aligned}
\chi_{t|t-1}^a &= \chi_{t-1}^a + \chi_{t-1}^v \\
\overline{x}_{t|t-1} &= \sum_{i=0}^{2n_a} W_i^{(m)} \chi_{i,t|t-1}^x \\
P_{t|t-1} &= \sum_{i=0}^{2n_a} W_i^{(m)} [\chi_{i,t|t-1}^x - \overline{x}_{t|t-1}][\chi_{i,t|t-1}^x - \overline{x}_{t|t-1}]^T \\
\gamma_{t|t-1} &= h(\chi_{t-1}^a, \chi_{t-1}^v) \\
\overline{y}_{t|t-1} &= \sum_{i=0}^{2n_a} W_i^{(m)} \gamma_{i,t|t-1}^x
\end{aligned}
$$

c) Measurement update equations:

$$
\begin{aligned}
P_{\overline{y}_t\overline{y}_t} &= \sum_{i=0}^{2n_a} W_i^{(c)}[\gamma^x_{i,t|t-1} - \overline{y}_{t|t-1}][\gamma^x_{i,t|t-1} - \overline{y}_{t|t-1}]^T \\
P_{\overline{x}_t\overline{y}_t} &= \sum_{i=0}^{2n_a} W_i^{(c)}[\chi^x_{i,t|t-1} - \overline{x}_{t|t-1}][\gamma^x_{i,t|t-1} - \overline{\gamma}_{t|t-1}]^T \\
K_t &= P_{\overline{x}_t\overline{y}_t} P_{\overline{y}_t\overline{y}_t} \\
\overline{x}_{t|t-1} &= \overline{x}_{t|t-1} + K_t(y - \overline{y}_{t|t-1}) \\
P_{t|t-1} &= P_{t|t-1} - K_t P_{\overline{y}_t\overline{y}_t} K_t
\end{aligned}
$$

where λ is a composite scaling parameter, $n_a = n_x + n_v + n_n$, R is the measurement noise variance covariance matrix, K is the Kalman gain and W_i the weights associated with the sigma points (see appendix 1 for further details). Note that no explicit calculation of the Jacobians or Hessians are necessary to implement the algorithm. It generates much accurate results than the EKF and in particular it generates much better estimates of the covariance of the states (since the EKF seems to underestimate this quantity).

The matrix, Q represents the variance-covariance matrix, which is set to be time invariant in the above specification. However, several authors including Sims (2001), Stock (2001) Bernanke and Mihov (1998), Kim and Nelson (1999) pointed to evidence that VAR innovation variances have changed over time, which might in turn exaggerate the time variation in ξ_t. There is much evidence to support a positive relation between the level and variance of inflation. As such, just like a mispecification of the level may affect the degree of persistence, so will a mispecification of the variance. A model with constant ξ and drifting Q would attribute a high inflation variance to an increase in innovation variances, while a model with drifting ξ and constant Q would attribute it to an increase in shock persistence. The evidence on inflation persistence without allowing time variation in the variance may be then subject to an artifact of model mispecification, as shown in Gadzinski (2005). To account for this, we follow Harvey, Ruiz and Sentena (1992) and include the error term in the state equation. The state space representation considered for the estimation is augmented as follows:

$$
\begin{bmatrix} \pi_t \\ mc_t \end{bmatrix} = \begin{bmatrix} 1 & \pi_{t-1} & 1 \\ 1 & mc_{t-1} & 1 \end{bmatrix} \begin{bmatrix} \mu_t \\ g_i(\xi_t) \\ \varepsilon_t \end{bmatrix} + \begin{bmatrix} \sum_{j=1}^{k-1} \alpha_i \bigtriangleup \pi_{t-j} \\ \sum_{j=1}^{l-1} \beta_i \bigtriangleup mc_{t-j} \end{bmatrix}, \tag{18}
$$

where we add to the system (18), the representation of the error terms defined by:

$$\begin{bmatrix} \varepsilon_{\pi,t} \\ \varepsilon_{mc,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{\varepsilon_{\pi,t}} & 0 \\ 0 & \sigma^2_{\varepsilon_{mc,t}} \end{bmatrix} \right), \tag{19}$$

where $\sigma^2_{\varepsilon,t} = [\sigma^2_{\varepsilon_{\pi,t}} \quad \sigma^2_{\varepsilon_{mc,t}}]$ is given by:

$$\sigma^2_{\varepsilon,t} = a_0 + a_1\sigma^2_{\varepsilon,t-1} + a_2\varepsilon^2_{t-1}. \tag{20}$$

In order to process with the Kalman filter, we need the ε^2_{t-1} term in order to calculate $\sigma^2_{\varepsilon,t}$. As in Harvey *et al*, the term is approximated by $E(\varepsilon^2_{t-1} \mid \psi_{t-1})$, where ψ_{t-1} is the information up to time $t-1$.

With

$$\varepsilon_{t-1} = E(\varepsilon_{t-1} \mid \psi_{t-1}) + (\varepsilon_{t-1} - E(\varepsilon_{t-1} \mid \psi_{t-1})), \tag{21}$$

we have:

$$E(\varepsilon^2_{t-1} \quad \mid \quad \psi_{t-1}) \tag{22}$$

$$= \quad E(\varepsilon_{t-1} \mid \quad \psi_{t-1})^2 + E((\varepsilon_{t-1} - E(\varepsilon_{t-1} \mid \psi_{t-1}))^2) \tag{23}$$

Where $E(\varepsilon_{t-1} \mid \psi_{t-1})$ is obtained from the last element of $\hat{\xi}_{t-1|t-1}$, and its mean squared error $E((\varepsilon_{t-1} - E(\varepsilon_{t-1} \mid \psi_{t-1}))^2$ is given by the last diagonal element of $P_{t-1|t-1}$.

4 Empirical Results

Figures 1 and 2 display the results for the HICP Euro area and CPI United States respectively. First, our results show that at any point in time the degree of forward lookingness is dominant both in the United States and Euro area. This echoes the results by Gali, Gertler, 1999 and Gali, Gertler and Lopez-Salido, 2001. As for the Euro area, the coefficient on the forward looking term remains stable at a value of 0.5 till it suddenly shoots up to the value of 1 in 1979 and then stabilises thereafter, implying the acceptation of the genuine version of the New Keynesian Phillips curve from this date on. The coefficient for the US increases slightly till 1975 Q4 and then peaks up to reach 0.8 in 1976 Q4. Then, the coefficient continues to vary over time but stay in a narrow band while accepting the pure New Keynesian Phillips curve only on a few rare occasions.

Unlike most of the studies, we did not impose the constraint $\gamma_f + \gamma_b + 1$. Consequently, the coefficient on the backward looking term cannot be induced from the values of γ_f . Moreover, contrary to the previous studies using a system approach, we allow the first lag of inflation to Granger-cause the marginal cost. Now, this implies that the coefficient γ_b on π_{t-1} reflects not only the structural parameter in equation (1), but also the forecasting rule for the forcing variable. This double signification may then explain the extra volatility and the negative values taken by this coefficient for the Euro area. The coefficient γ_b representing "intrinsic persistence" declines from the beginning and becomes non significant from 1979 onward. On the contrary, we note the relative stability of this coefficient for the United States, hoovering around zero after 1980. This casts some doubts about the relevance of the lagged inflation in forecasting the marginal cost for the US, in accordance with previous studies (see Nason and Smith, 2004).

Previous studies find a clear mapping between the monetary policy regime and the distribution of the persistence parameter (see Gaspar, Smets and Vestin, 2004). However, the fact that the rise in the degree of forward lookingness and the decline in persistence occured early in our sample casts some doubts about the sole dominance of the monetary policy in the determination of these coefficients.

Overall, reconciling the results of a time varying reduced form including time varying mean and variance, with a time invariant structural process is not possible according to our first results. Moreover, the bigger weight of the forward term seems warranted even when the expectations remain at a high level, which may in turn imply that not only the stance of the monetary authorities does indeed influence the degree of forward lookingness. Nevertheless, we should note that a stable monetary regime implies the stabilisation of the backward component to a low level.

Looking at the structural coefficient linking the marginal cost and inflation, we first note that the latter remains fairly stable over time, with some variability only at the end of the sample. The pattern of the coefficient is strinkingly similar between the US and Euro area whereas the latter witnesses some lower values along the sample. An interpretation of this evidence is that the inflation process in the EA is subject to a higher degree of rigidity, possibly due to a less competitive environment, more extensive price regulation or other formal or informal constraints on price setters. This interpration is indirectly confirmed by the analyses of the degree of product market regulation and competition conducted by the OECD, that clearly points to the existence of a less efficient price setting mechanism in most European countries, relative to the US.

For both countries, the coefficients starts declining at the end of the sample to reach its all time minimum around 2001. This decline is clearly visible in its reduced form counterpart, with a clear downward drift starting in the beginning of the nineties, which points toward a change in the inflation-output gap trade-off over time.

5 Conclusion

To be written

6 Appendix

The Unscented Kalman Filter proceeds as follows for the calculation of sigma points. A set of $2n_x + 1$ (n_x is the dimension of x) weighted samples or sigma points $\hat{S}_i = \{W_i, \chi_i\}$ are deterministically chosen so that they completely capture the true mean and covariance of the prior random variable x. A selection scheme that satisfies this requirement is:

$$
\begin{aligned}
\chi_0 &= \overline{x} \\
\chi_i &= \overline{x} \ + (\sqrt{(n_x + \lambda)P_x})_i && i = 1, .., n_x \\
\chi_i &= \overline{x} \ - (\sqrt{(n_x + \lambda)P_x})_i && i = n_x + 1, .., 2n_x \\
W_0^{(m)} &= \lambda/(n_x + \lambda) && \lambda = \alpha^2 (\ n_x + \tau) - n_x \\
W_0^{(c)} &= \lambda/(n_x + \lambda) + (1 - \alpha^2 + \beta) \\
W_i^{(m)} &= W_i^{(c)} = 1/\{2(n_x + \lambda)\} && i = 1, ..., 2n_x
\end{aligned}
$$

The parameter τ is a scaling parameter, its value is not critical, but to guarantee positive semidefiniteness of the covarance matrix, we choose $\tau \geq 0$. α controls the "size" of the sigma point distribution and should ideally be a small number to avoid sampling non local effects when the non linearities are strong. β is a non-negative weighting term which can be used to control the error in the kurtosis which affects the "heaviness" of the tails of the posterior distribution. For a gaussian prior, the optimal choice is $\beta = 2$.

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Euro area HICP Results

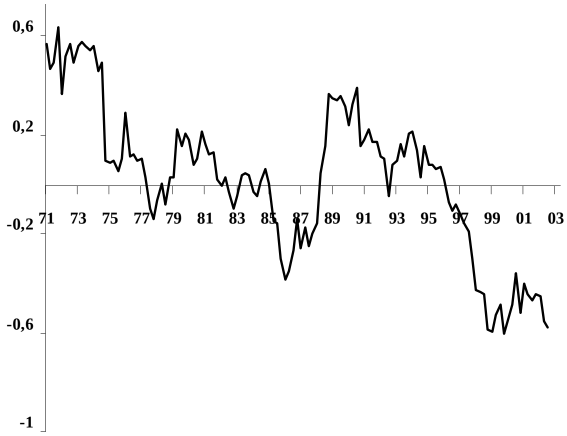

Figure 1: $\alpha_{\pi\pi}$

Figure 2: $\alpha_{\pi m}$

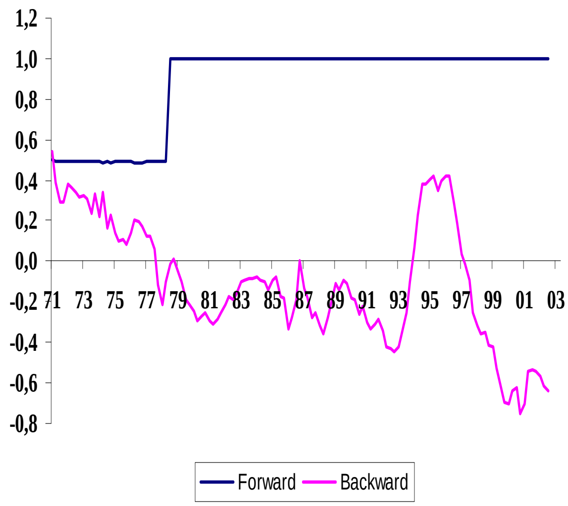

Figure 3: γ_f and γ_b

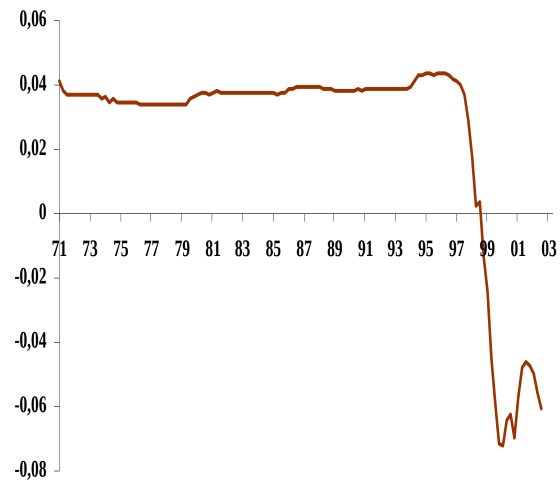

Figure 4: Slope κ

US CPI Results

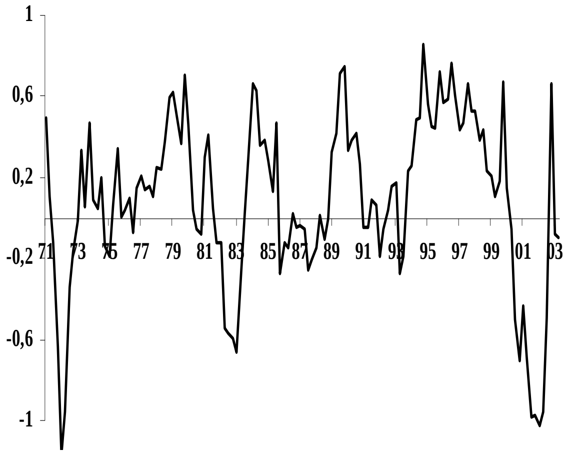

Figure 1: $\alpha_{\pi\pi}$

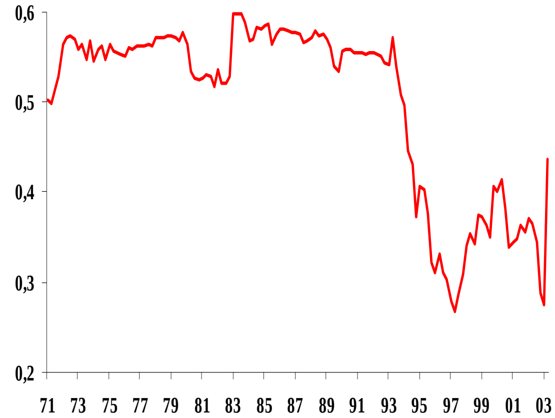

Figure 2: $\alpha_{\pi m}$

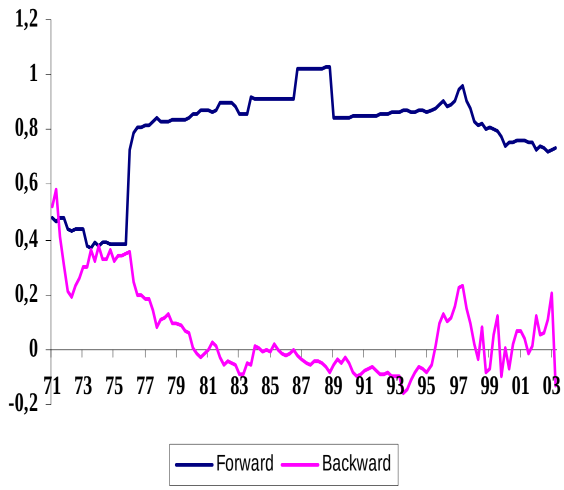

Figure 3: γ_f and γ_b

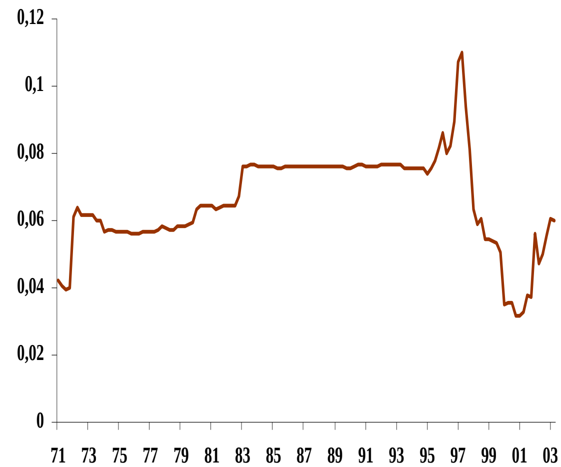

Figure 4: Slope κ