# State of technology, innovation and finance

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Sep 2004[1]
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**Abstract**

This paper investigates the link between the state of technology and type of financial contract, in order to understand if innovating firms have an advantage or disadvantage of backwardness. The state of technology is relevant because entrepreneurs and investors have different incentives if a sector is at the frontier (so that it can only innovate) or inside the frontier (so it can also imitate). Different incentives have important consequences on the type of contract for financing innovation.

The first finding of this paper is that for a sector at the frontier there externalities if innovation is financed with equity, but not with debt. In fact, with debt there is adverse selection, so that only unskilled firms are financed, while with equity there is a pooling equilibrium such that both are financed.

Second, when these results are related to the distance to the frontier parameter, then it emerges that equity is preferred by sectors which are either very distant from the frontier and by those which are at the frontier, while debt is chosen by sectors at intermediate distance. This use of financial contracts has a direct influence on the growth rate of a sector. In fact, while the growth rate is decreasing as a sector gets closer to the frontier, in the intermediate region of debt it suffers an even stronger fall, due to non participation of H-types to the patent race.

## 1 Introduction

In recent years there has been a wide consensus on the central role that innovation plays for growth in modern economies. However, despite this general statement, not all countries seem to innovate at the same rate, and still huge differences remain across countries. Moreover, innovation is very diffused in countries where not only productivity is high, but also financial markets are well developed. For this reason much of the

[1] Acknowledgements: I am very grateful to my supervisor L. Bottazzi, and to J. Ventura F. and R. Ranciere for very useful comments. I am also grateful to J. Shapiro, R. Marimon, A. Caggese and G. Gancia and all participants to the Innovation Workshop Seminar and GRASS Serminar at UPF for helpful discussion. All errors are mine. Most of the proofs are in the appendix section, some others are available on request: francesco.bripi@uni-bocconi.it

theoretical and empirical literature has argued that institutions play an important role on growth and in particular on the links between financial development and growth.

On the other hand, it is evident that the fact that innovation and productivity growth do not depend only on domestic factors (like institutions and/or financial development), but also on how much technology is diffused across countries, in the sense that world's best technology may spread from leader countries to technology laggards. In this respect, the question is: do countries benefit from being backward or from being at the frontier? While some literature has focused on productivity differences or financial development, there is no consensus about the so called "advantage of backwardness", in the sense that there is an advantage if "*the further a country falls behind the world's technology leaders the easier it is for that country to progress technologically simply by implementing new technologies that have been discovered elsewhere.*"[2] If we accept the view that laggard countries have an advantage of backwardness, then we should see long run convergence in the growth rate to frontier countries; and the reverse (divergence) if we allow for a disadvantage of backwardness. However, the empirical evidence seems quite mixed: while on one hand there are some countries catching up very quickly to the richest ones, on the other hand, some very rich countries seem to grow at different rates.[3]

One way to understand this puzzle is not to consider the economy as a whole, but sector by sector. In this way, one may better disentangle the effects of the distance to the world frontier by considering each sector separately. Then, the aim of this paper is to understand if countries, for each sector, have some advantages in being at the frontier of research or not, taking the financial development and other factors as given. Moreover, taking the financial development as given means that only two basic financial contracts are considered: equity and debt. Debt is considered because it is the most natural and diffused financing contract, while equities because they are also very diffused and also because they play a considerable role in the financing of innovation business projects (Black and Gilson, 1998; Kaplan and Stromberg, 2003). This allows to better link the state of technology to the type of contracts used to finance innovation activities and it is necessary to understand how the incentives to innovation are affected by the state of technology only and not also to financial development. Being at the frontier or inside of it simply implies different technology choices available as it will be explained below. This allows to specify how simply the distance to the frontier determines advantages or disadvantages of economic agents with respect to producers which are already at the frontier. Moreover, understanding this point is relevant to undertake the correct policies, in presence or absence of other factors, that favour innovation and growth.

The state of the technology is represented by the distance to the frontier of research. For each productive sector, a country may be $at$ the frontier or $inside$ of it. In the second case, quality improvements can be obtained by undertaking innovation projects, such that they go through the existing frontier, or by imitation projects which improve the quality much less, and do not reach the existing frontier. If the sector is already at the frontier, then quality improvements can only be obtained by innovation, since

[2] Aghion Howitt, Foulkes (2003), page 1. the original idea of advantage of backwardness is due to Gensckenkon (1962).

[3] See for contrasting views Barro, Sala-i-Martin (2004), chap. 8 and Howitt (2000).

there is no possibility of imitation. So, being at the frontier or inside implies a different technological choice, which also alters the structure of incentives for the entrepreneurs. It will be shown that even if leader and backward sectors have the same financial system and technology can diffuse across countries without restrictions, some sectors will face a lower growth rate than their optimal level when they are close but still behind the world frontier. This result is quite similar to other studies that have focused on the problems of middle income countries, even if they refer to the economy at macro level (Aghion, Bacchetta, Banerjee, 2003).

In this sense this paper is related to different important strands of literature. On one hand, it is related to the literature of financial development and growth. De la Fuente and Marin (1996) develop a model in which financial intermediaries emerge endogenously to reduce uncertainty inefficiencies due to unobservable effort by entrepreneurs. Cooley and Smith (1998) instead focus on the role of financial markets in promoting specialization, learning by doing and so growth. Differently from them, this paper does not try to understand how financial markets emerge, but how firms perform differently when they are more or less close to the technology frontier, taking the same choice of financial contracts (debt/equity) available at any state of technology. Boyd and Smith (1998) focus also on the endogenous evolution of financial markets as an economy grows, and similarly to here they consider the substitute role of debt and equity in financing investments.

It is also related to the literature linking the agency problems in accessing external finance. In this sense, Bernanke and Gertler (1989) and Aghion *et al. (1999)* have assumed that the access to external finance is limited by a multiple of the entrepreneur's own wealth. Differently from them, here no such a mechanism is assumed, as for innovation activites usually entrepreneurs have no collateral at all. Moreover, with particular regard to innovation, Guiso (1998) has argued that banks ration credit to innovative firms and rather prefer to finance traditional activities, because they know more about their businesses than about high tech firms: while it is widely accepted that for innovation projects specialized financial investors play a specific the role, in this paper financial frictions are simply due to different skills of entrepreneurs which is their own private information.

Third, the paper is also related to a recent contribution by Aghion, Acemoglu, Zilibotti (2003). That paper links the distance to the technology frontier on the type of investment managers choose, highlighting the risk of poverty traps. It does so using a specific assumption on the form of the TFP change of a sector, which depends on the relative skills of managers to imitation or innovation projects. Analogously, in this paper there is a growth path which emerges endogenously along the distance to the frontier parameter; however, differently from them, here the shape of a sector TFP is not determined exogenously, but it is simply stated in the most general possible way.

Finally, due to the presence of asymmetric information in financial contracts, this paper is also related to two basic papers on this argument: Stiglitz and Weiss (1981) and De Meza and Webb (1987).

The paper is structured as follows: in section 2, I model a sector firm *inside* the frontier with debt financing; in section 3, I model the same sector *at* the frontier with debt. Then, in section 4 there is a sector firm *inside* the frontier with equity financing and in section 5 the model with equity is extended to innovation in sectors *at* the frontier. Then, section 6 shows the distance to the frontier is associated with each form of financing (zero values) and in section 7 contains the consequences of these choices on the growth rates and some policy recommendations. Section 8 concludes.

# 2 Debt inside the frontier

Consider a closed economy where there are three kinds of agents: entrepreneurs, specialized workers and non-specialized workers. Entrepreneurs lead individual firms for the production of intermediate or final goods. Specialized workers are researchers working in the intermediate sector firms, while non-specialized workers are employed in the final good production process. In the economy there are $V$ intermediate productive sectors $(0, 1, 2, .....v, ...., V)$. These inputs are used to produce a unique final good $(Y)$ according to the following function:

$$Y_t = \frac{1}{\alpha} \sum_{v=1}^{V} A(v)_t^{1-\alpha} k(v)_t^{\alpha} \tag{1}$$

where $A(v)_t$ is the TFP of firms operating in sector $v$ (it also represents the quality level of that good), $k(v)_t$ is the total amount of input $v$ used in the final good production. Non specialized labour has been normalized to 1 for simplicity. The final good is produced competitively, while the intermediate goods are produced by a monopolist, but in the R&D process each firm is in competition with any other to patent first in the race.

At the beginning of period $t$, each intermediate good $v$ is produced taking the sector TFP of the previous period $[A(v)_{t-1}]$ as given. Any intermediate good may be produced either at the same quality level of the previous year or at a higher level; in this last case it will be created either by imitation or by innovation. If a better quality is introduced in the market by a firm, then this will grab all the market. This means that innovations are drastic: if an innovation is successfully implemented, the old technology immediately becomes obsolete. So, if at least one entrepreneur tries to improve the quality, there is no argument for the others no to do it as well. So, all entrepreneurs willing to sell the $v$ good will try to improve the quality.

In order to improve the quality of an intermediate good, an entrepreneur of a sector inside the frontier may either do an imitation or an innovation project. Since the improvement in quality is to be done within a period (see below for the timing), imitation consists in copying simply from the best world leader (the frontier) as much as possible within a period. It is obvious that very probably within a period it is quite hard to reach the frontier simply by copying the world best technology.

Otherwise, an entrepreneur inside the frontier may do innovation that consists in trying to improve the home quality by copying more quickly than what imitation can do in one period of time. This innovation process, if successful, will bring about the sector's TFP at the frontier; therefore, innovation moves the frontier forward.[4] It could

[4] Throughout the paper I will always refer to the world frontier as the "frontier." When referring to a

be argued that if the other country is very much inside the frontier, then it is very hard to reach it and to go beyond it with a single project. Some literature has put forth this point arguing about the disadvantage of backwardness (Papageorgiou, 2002). However, another part of the literature has also argued about the advantage of backwardness (Gerschenkron, 1962).[5] While the point is debated among economists, we take the view that there is no restriction or other impediment to adopt foreign technology. There are two reasons for this choice. First this allows to compare innovation projects in sectors at the frontier and inside the frontier more easily. Second, by proceeding in this way, we are able to derive the dis/advantage of backwardness endogenously.

The improvement in quality is fixed. In the case of imitation, this is just of a factor $\gamma$ ($\gamma > 1$) with respect to the previous period quality index $[A(v)_{t-1}]$. In this case the quality does not reach the frontier, and the good can be sold in the home market with monopoly profits (I am implicitly assuming that there is no international trade in intermediates[6]). In the case of innovation, the improvement in quality is of a factor $\gamma$ with respect to the previous period frontier level $[\bar{A}(v)_{t-1}]$.[7]

Moreover, while the timing for the final goods is irrelevant, it matters for the intermediate goods. In fact, since all entrepreneurs in sector $v$ are willing to improve the quality, they will try to do it either by imitation or by innovation. This implies that for each period, there are two subperiods; in the first subperiod, innovation and imitation are undertaken, while in the second subperiod, the intermediate product is produced and sold. During the first subperiod, the first firm (either doing imitation or innovation) succeeding in the new product gets a patent which lasts for one period only, and which allows to sell the intermediate as a monopolist. This means that in the choice of which kind of project to undertake, if imitation or innovation, either ways are good to win the race and get the monopoly patent. However, notice that if both an innovation and an imitation projects are successful for the same intermediate good, the entrepreneur producing the good of imitation quality will sell nothing, because the innovation good is a far better quality one, and the entrepreneur owning its patent will grab all the market. In this case, the value of the imitation patent is zero.

In the second subperiod, the intermediate good is produced and sold. At the beginning of the following period, the game starts again with the previous level of knowledge being a publicly available to all.

Obviously, this timing has the advantage of being simple, but at the cost of imposing some restrictions. First, I am not considering that discoveries may come at different times than just one period (an innovator might get the discovery just soon after an imitator has got the patent); in fact, the timing of the periods here is at regular intervals,[8] but it does not correspond to the actual timing of new products being brought to the market, which may not be regular. Moreover, while the monopoly right comes naturally from taking a higher quality product into the market, it is not obvious that it has to last

country frontier this is explicitly mentioned.

5 For a similar approach see Howitt (2000), and Aghion et al. (2002).

6 For a similar approach see Howitt (2000).

7 In the figure, and just in this figure, the notation of $A_t$ is switched to bold: $\mathbf{A}_t$.

8 The timing here is at regular intervals, a neccessary condition for differential calculus to work.

for one period only. Actually, it should last as long as another better quality product is introduced, but also this assumption has been done for keeping the model tractable.[9]

The quality of intermediate products is represented by the sector TFP, $(A(v)_t)$, and it evolves through time. If at time $t$ an innovative project succeeds, the "quality" improvement is of an amount $\gamma$ (with $\gamma > 1$): $\gamma \bar{A}_{t-1}(v)$ ; while if it fails, the "quality" improvement of that innovation is null.

In formulas:

$$A_t(v) = \begin{cases} \gamma \bar{A}_{t-1}(v) & \text{if innovation succeeds} \\ 0 & \text{if innovation fails} \end{cases} \tag{2}$$

However, in the case of failure of all innovators, if imitation in the same sector $v$ is undertaken by another entrepreneur, there is a quality improvement in the TFP of a smaller amount $\gamma$: $\gamma A_{t-1}(v)$ . Consequently, the overall improvement of TFP in each intermediate sector $v$ in each period $t$ is

$$A_t(v) = \begin{cases} \gamma \bar{A}_{t-1}(v) & \text{if innovation succeeds} \\ \gamma A_{t-1}(v) & \text{if imitation succeeds} \end{cases} \tag{3}$$

If we consider the number of entrepreneurs to be sufficiently large in each sector, then it is possible to consider (3) as the process underlying the evolution of TFP in each intermediate sector. Moreover, each intermediate product $v$ is produced by a profit maximizer entrepreneur, which may either be an innovator or an imitator. As shown above, if she imitates she will reach a lower quality improvement than what a successful innovator can do.

As an assumption, consider that imitation does not require any specific skill, while innovation requires a certain degree of skill or "intelligence." With respect to this, entrepreneurs are assumed to be of two types: those with low entrepreneurial skills (L-types), who correspondingly have a lower probability of success, and those with high entrepreneurial skills (H-types), who correspondingly have a higher probability of success. The difference in skills will be explained more in detail below.

Finally, for each sector there is a finite amount of entrepreneurs $x$, where an amount $H$ are H-types and another amount $L$ are L-types: $x = H + L$.

## 2.1 Imitation

An intermediate sector firm doing imitation will improve the sector TFP according to the following law of motion of TFP:[10]

$$A_t = \begin{cases} \gamma A_{t-1} & \text{with prob. } N_t^{\beta} \left(1 - \frac{p}{x-s}\right)^{x-s} \\ 0 & \text{otherwise} \end{cases}$$

that is to say that imitation will bring about an increase in TFP as big of a $\gamma$ factor with a probability of success for imitation that depends on the amount of funds invested

[9] We are assuming implicitly that giving the patent for at least one period allows one to extract some rents for a period that are sufficient to repay for the R&D investment.

[10] Since now on, we drop the $(v)$ notation for the sector TFP, where it creates no confusion.

in R&D ($N_t^{\beta}$, where $0 < \beta < 1$)[11] and iff all firms undertaking innovation ($x - s$) fail (with probability $\left(1 - \frac{p}{x-s}\right)^{x-s}$): consider that a successful innovation will displace imitation, and so every entrepreneur will try to undertake innovation.[12]

If all costs are financed by external finance (debt) at the gross interest rate $R_t$, expected total profits of imitation ($E\left(\Pi_M\right)_t$) are

$$E\left(\Pi_M\right)_t = N_t^{\beta}\left(1 - \frac{p}{x-s}\right)^{x-s}\left[\pi(M) - R_t N_t\right] \tag{4}$$

That is to say that with a probability $N_t^{\beta}\left(1 - \frac{p}{x-s}\right)^{x-s}$, the entrepreneur is successful at imitation; in this case she will enjoy profits from imitation $\pi_t(M)$, and will be able to repay its debt, for the amount of funds borrowed $N_t$ ($N_t$ is the number of specialized workers employed and also the amount of funds borrowed [13]) at the interest rate $R_t$.

Moreover, (4) can be changed (see Appendix A) in the following way:

$$E\left(\Pi_M\right)_t = N_t^{\beta}\left(1 - \frac{p}{x-s}\right)^{x-s}\left[\delta(\chi) - R_t n_M\right] A_t \tag{5}$$

where $\pi_t(M) = \delta(\chi)A_t(v)$ and $n_M$ represents the efficiency of specialized workers in the R&D process, that is derived taking into account the so called "fishing out" effect: innovating becomes more and more difficult as the quality of the good is higher. Therefore, the relation between $n_M$ and $N_t$ is:[14]

$$n_M = \frac{N_t}{A_t(v)} \tag{6}$$

where $A_t(v)$ is the target level of quality that the entrepreneur hopes to reach if she is successful with her imitation in sector $v$. Notice that as this target level of quality increases (an increase in $A_t$), the lower is the efficiency of each unit of funds ($n_M$), or it is more difficult is to improve the quality.

Finally, notice that imitation is independent of the type of entrepreneur (H or L); instead, the type matters for innovation.

11 For a similar approach on the construction of the probability of success, see Aghion, Howitt (2004).

12 $p$ is the probability of success of innovation of an entrepreneur independently from the other competitors in the R&D race. Taking competitors into account, the probability becomes $\frac{p}{x}$, while the joint probability that $x - s$ (s are imitators) innovators fail is $\left(\frac{p}{x-s}\right)^{x-s}$.

13 Notice that the salary of workers has been normalized to 1 for simplicity.

14 Aghion and Howitt (2003) use a similar approach; the difference is that they compute the fishing out effect with respect to the frontier level of technology: $n_t = \frac{N_t}{A_t(v)}$, explaining this with the advanmtage of backwardness (Gensckenkon, 1962); however, differently from them, Papageorgiou (2002) has argued that the distance to the frontier may also have a drawback: disadvantage of backwardness. This last approach has been adopted here.

## 2.2 Innovation inside the frontier

Focusing on innovation, again if the innovating entrepreneur is successful, she will then be a monopolist in this sector at least for one period; on the other hand, in case of failure, she gains nothing and defaults on her debt.[15] She will then maximize the expected profits of innovation ($E(\Pi^i)_t$)

$$E(\Pi^i)_t = N_t^{\beta} \frac{p_i}{x} [\pi(I) - R_t N_t] \qquad \text{for } i = H, L \tag{7}$$

Similarly to imitation, expected profits from innovation are: a) financed externally by an investor at the interest rate $R_t$; b) increasing in the amount of production profits $\pi_t(I)$; c) dependent on the probability of success $N_t^{\beta} \frac{p_i}{x}$, which depends positively on the amount of R&D investment ($N_t^{\beta}$) an intrinsic probability of success ($p_i$) and on the actual number of entrepreneurs participating in the patent race for innovation ($x$).[16] Differently, from imitation, innovation depends on: d) $p_i$, the probability of success depends on the type: it is lower for L-types than for H-types: $0 < p_L < p_H < 1$.[17]

The two types of entrepreneurs differ in the "entrepreneurship" skill. I call it *entrepreneurship* because it represents any kind of skills that makes innovation easier for some entrepreneurs and less easy for others.[18] For example, this could be due to organizational or deeper knowledge of the sector of some entrepreneurs with respect to others; obviously, it does not refer to the skills of specialized workers who are homogeneous with regard to this characteristic.

Moreover, we can take into account the fishing out effect also for innovation. In this case, the target quality is respect to the "would be" quality level, which in this case is the frontier quality level ($\bar{A}_t$):

$$n_I = \frac{N_t}{\bar{A}_t} \tag{8}$$

Due to free entry, each of the two types entrepreneurs will choose to undertake innovation if expected profits are higher. So, considering that production profits are $\delta(\chi)\bar{A}_t$ (See Appendix A for details), and taking into account (8), expected profits (7) of L-type entrepreneurs become:

$$E\left(\Pi^L\right)_t = N_t^{\beta} \frac{p_L}{x} [\delta(\chi) - R_t n_I] \bar{A}_t \tag{9}$$

and similarly for H-types, expected profits are:

$$E\left(\Pi^H\right)_t = N_t^{\beta} \frac{p_H}{x} [\delta(\chi) - R_t n_I] \bar{A}_t \tag{10}$$

[15] A period sufficient for the next innovation or imitation to be introduced. For simplicity, the reader could think that the period is at least long an year.

[16] For a similar approach on probabilities of success, see Zeira (2003).

[17] The probability of success here is modelled by assuming that implicitly, and in a quite realistic fashion, that there cannot be two winners at the same time. For a similar approach, see Zeira (2003).

[18] For a review of the literature on the concept of entrepreneurship see Audretsch (2002).

## 2.3 Entrepreneurs indifference curves

Notice, first, that each entrepreneur would like to maximize expected profits. However, due to competition and free entry in the R&D entry, they raise their costs ($N_t$ and $R_t$) squeezing expected profits as much as possible. To determine how much expected profits are squeezed, notice that since entrepreneurs are risk neutral, they will prefer to undertake innovation inasmuch as expected profits from this activity are higher than those of the alternative activity (imitation). This is why we are interested to derive the indifference curves of both types of entrepreneurs. Then, the participation constraint to innovation for both types is given by: $E\left(\Pi^i\right) \geq E\left(\Pi_M\right)$ for $\quad i = H, L$.

In particular, for L-types we have:

$$N_t^{\beta} \frac{p_L}{x} \left[\delta(\chi) - R_t n_I\right] \bar{A}_t \geq N_t^{\beta} \left(1 - \frac{\overset{\frown}{p}}{x-1}\right)^{x-1} \left[\delta(\chi) - R_t n_M\right] A_t \tag{11}$$

where $\overset{\frown}{\Omega} = \left(1 - \frac{\overset{\frown}{p}}{x-1}\right)^{x-1}$ [19]. Solving (11) requires imposing that in equilibrium, because of competition in the innovation market, profits are brought down until the marginal entrepreneur is indifferent between the two activities. Then, we can derive the following the entrepreneurs indifference curves of L-types (see Appendix B):

$$N_L^E : N_t = \frac{\delta(\chi)\bar{A}_{t-1}}{R_t} \left[\frac{1 - \frac{x}{p_L}\overset{\frown}{\Omega} a_t}{1 - \frac{x}{p_L}\overset{\frown}{\Omega}}\right] \tag{12}$$

where $a_t = \frac{A_t}{\bar{A}_t}$ is an inverse index of distance to the frontier. Consider that since entrepreneurs in the R&D race are competitive and there is free entry, their expected profits would be driven to zero. This does not happen because of the participation constraint in (11), so expected profits fall until they reach those of imitation. At this level there is a locus of combinations of $N_t$ and $R_t$ which describes L-types indifference curve.

In a similar way, the participation constraint of H-types is:

$$N_t^{\beta} \tfrac{p_L}{x} \left[\delta(\chi) - R_t n_I\right] \bar{A}_t \geq N_t^{\beta} \left(1 - \tfrac{\overset{\frown}{p}}{x-1}\right)^{x-1} \left[\delta(\chi) - R_t n_M\right] A_t$$

from which we derive H-types indifference curve:

$$N_H^E : N_t = \frac{\delta(\chi)\bar{A}_{t-1}}{R_t} \left[\frac{1 - \frac{x}{p_H}\overset{\frown}{\Omega} a_t}{1 - \frac{x}{p_H}\overset{\frown}{\Omega}}\right] \tag{13}$$

The two curves are shown in figure 1.

Notice that both curves represent the highest amount of funds that H and L-type firms are willing to employ in order to undertake innovation for any interest rate $R_t$.

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[19] Notice that $\overset{\frown}{p} = \frac{Hp_H + *p_L}{H+L}$, that is the average probability.

The position of the two curves reveals an important observation. Consider the case that both high and low skills entrepreneurs are charged the same interest rate (say $R$). Then, since L-types are less efficient (lower probability of success: $p_L < p_H$), for this interest rate they will need to employ a higher amount of funds in order to undertake innovation, instead of imitation. Indeed, when comparing the two indifference curves as in figure 1, one can check that for every level of funds demanded by both types ($N_t$)

the indifference curve of L-types will always lie above that of H-types.[20]

Finally, notice that (check with the derivation in Appendix B) for each type, the innovation area is the area south-west of the respective indifference curves. The meaning of this is that since firms compete each other in order to be the first to patent, they raise their costs ($R_t$ and $N_t$) as much as possible. At the indifference curve ( and beyond this curve), the costs are so high that it is more convenient (in expected values) to undertake the inferior project (imitation).

### 2.4 Investors zero profit curves

In this part I will show the behaviour of the investors when they have to finance an imitation or an innovation project.

Assume that both types of entrepreneurs have no initial wealth at all, and that there is no collateral on which the lender can secure the loan. Excluding collateral for innovation activities is not a strong assumption inasmuch as innovative firms mostly rely on the human capital of its researchers (specialized workers in this framework).[21] Therefore, all total costs have to be financed with external finance and without collateral.

Moreover, consider the following assumption on investors: *investors are perfectly competitive, there is free entry and they are risk neutral*.

Since we are mainly interested to innovation, I will mainly focus on this case. Consider that the investment is in the amount of $N_t$ at the gross interest rate $R_t$. Moreover, the investor knows that the entrepreneur (of types $i$) will repay the debt for the full amount at the agreed interest only if the project is successful, which happens with probability $p_i$, otherwise she will get nothing. Therefore, the expected return $E[R_f^i]_t$ of the investor ($f$) is

$$E[R_f^i]_t = \begin{cases} N_t & \text{with prob. } N_t^{\beta} \frac{p_i}{x} \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } i = H, L$$

while the expected profit $E[\Pi_f^i]_t$ is

$$E[\Pi_f^i]_t = N_t^{\beta} \frac{p_i}{x} R_t N_t - N_t \qquad \text{for } i = H, L^{22} \tag{14}$$

[20] Notice that the derivative of $N^d$ with respect to $p_i$ is negative: $\frac{\partial N_i^d}{\partial p_i} < 0$ for $a_t < 1$.

[21] For the role of collateral, see Besanko and Thakor (1987) for a model of credit where collateral is a sortice device to avoid problems asymmetric information. Stiglitz and Weiss (1981) have an opposing view on the role of collateral.

[22] With regard to imitation we can proceed in a similar way. Then, expected profits of an investor financing an imitation project ($E[\Pi_f^M]_t$) are:

$E[\Pi_f^M]_t = q\,(1 + r_t)\, N_t - N_t$

An investor can either finance innovation and imitation. Due to competition and free entry among investors, when financing innovation, her expected profits will be driven to zero:

$$E\left(\Pi_f^i\right)_t = N_t^{\beta}\frac{p_i}{x}R_tN_t - N_t \ = 0 \qquad \text{for } i = H, L \tag{15}$$

This implies immediately that the zero profit curve of the investor financing an innovation project to each type is:

$$N_L^Z = \left(\frac{x}{p_L}\frac{1}{R_t}\right)^{\frac{1}{\beta}} \tag{16}$$

if she is a L-type, while

$$N_H^Z = \left(\frac{x}{p_H}\frac{1}{R_t}\right)^{\frac{1}{\beta}} \tag{17}$$

if she is an H-type.

Notice that both curves are downward sloping with the amount of funds. The reason for this is that the probability of success is increasing in the amount of funds, so that more funds are invested, the lower the risk of failure and so also the lower the interest rate charged can be.[23]

Moreover, notice that the areas where financing for innovation takes place is north-east with respect to the each zero profit curve. This is due to the fact that on one hand $R_t$ are sources of income for the investor, and on the other $N_t$ is increasing the probability of repayment to the investor, so that expected profits increase in both the variables.

## 2.5 Equilibrium inside the frontier with debt

From the indifference curves of each type (12 and 13) and from the investors zero profit curves (16 and 17), we can derive the equilibrium amount of funds and interest rate of both types:

$$N_{d,L}^* = \frac{x}{p_L}\left[\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}}\left[\frac{1-\frac{x}{p_L}\overset{\frown}{\Omega}a_t}{1-\frac{x}{p_L}\overset{\frown}{\Omega}}\right]^{\frac{1}{1-\beta}} \tag{18}$$

$$N_{d,H}^* = \frac{x}{p_H}\left[\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}}\left[\frac{1-\frac{x}{p_H}\overset{\frown}{\Omega}a_t}{1-\frac{x}{p_H}\overset{\frown}{\Omega}}\right]^{\frac{1}{1-\beta}} \tag{19}$$

$$R_{d,L}^* = \left(\frac{x}{p_L}\right)^{\frac{1}{\beta}}\left[\frac{1}{\delta\left(\chi\right)\bar{A}_t}\right]^{\frac{\beta}{1-\beta}}\left[\frac{1-\frac{x}{p_L}\overset{\frown}{\Omega}}{1-\frac{x}{p_L}\overset{\frown}{\Omega}a_t}\right]^{\frac{\beta}{1-\beta}}$$

[23] As a suggestion for the reader, notice that if the probability were not dependent on $N_t$, then the supply curve would be flat.

$$R_{d,H}^{*}=\left(\frac{x}{p_H}\right)^{\frac{1}{\beta}}\left[\frac{1}{\delta\left(\chi\right)\bar{A}_t}\right]^{\frac{\beta}{1-\beta}}\left[\frac{1-\frac{x}{p_H}\widehat{\Omega}}{1-\frac{x}{p_H}\widehat{\Omega}a_t}\right]^{\frac{\beta}{1-\beta}}$$

Notice that both $N_{d,L}^{*}$ and $N_{d,H}^{*}$ depend positively on $\bar{A}_t$. That is to say that as technology improves, more and more funds are necessary to finance R&D innovative projects. This has no direct implications on the model, as I astray from general equilibrium implications.[24]

Therefore, if we had symmetric information, the two equilibria would be in points A and B as shown in figure 2.

## 2.6 Asymmetric information equilibrium

Until now, the model has been built assuming that types were known. However, this is not the case in reality. Therefore, it is important to see how the model works under asymmetric information, that is when entrepreneurs know their type and this is their private information, while investors do not know the true type of entrepreneurs.

Since the skills matter only for innovation, the effects of asymmetric information will be shown only for innovation. We have seen that since $N_L^E$ is always higher than $N_H^E$, L-types, being worse at the business, have to pay a higher interest rate for any amount of funds. It follows that when the type is private information, there is an incentive for L-types to demand the same amount of H-types so that they may pay a lower interest rate. To see this consider for example that if H-types demand an amount of funds $N_H$ (see figure 3), then L-types can demand do the same amount. By doing this, L-types can claim to be H-types as well. This implies that since the investor doesn't know the real type of each entrepreneur, she will offer a pooling contract at point D, with an average interest rate of those of the two types $[R_D]$. In our case, this average rate is given by the average of the two zero profit curves $N_H^Z$ and $N_L^Z$. So, if there is a total of $x$ entrepreneurs, the average interest rate at $N_t = N_0$ is a $E(R_t^*|_{N_t=N_H})$ given by:[25]

$$E(R_t|_{N_t=N_H}) = \left(LR_L^* + HR_H^*\right)/x \tag{20}$$

Obviously, this rate is lower than any rate that L-types would be charged if their type were known, so they would accept this rate, because not only they could still undertake innovation, but would gain the interest rate difference $R_C - R_D$. On the other hand, since $R_D$ is higher than the equilibrium interest rate of H-types, these would be paying a rate higher than what they actually would pay if their type were known as well. This implies that H-types are out of the innovation market, while L-types remain.

Then, notice that offering the pooling contract offered (at point D) is not an equilibrium contract, neither for the investor nor for the entrepreneur. In fact, since only L-types remain in the market, their type would be revealed to the investors. It follows

[24] Notice that this feature is common to all the equilibrium amount of funds: inside/at the frontier, with debt/equities.

[25] For simplicity, the indifference curves are assumed to be linear in the graphs, where this does not create confusion.

that in equilibrium the probability of success changes accordingly; indeed, only $L$ entrepreneurs will undertake innovation and the rest $H$ will undertake imitation. Since both the investor and the L-type entrepreneurs know this, the equilibrium amount of funds for innovation projects is given by:

$$N_{d,L}^{*\prime} = \frac{L}{p_L}\left[\delta(\chi)\bar{A}_t\right]^{\frac{1}{1-\beta}}\left[\frac{1-\frac{L}{p_L}\Omega_L a_t}{1-\frac{L}{p_L}\Omega_L}\right]^{\frac{1}{1-\beta}} \tag{21}$$

at the interest rate:

$$R_{d,L}^{*\prime} = \left(\frac{L}{p_L}\right)^{\frac{1}{\beta}}\left[\frac{1}{\delta(\chi)\bar{A}_t}\right]^{\frac{\beta}{1-\beta}}\left[\frac{1-\frac{L}{p_L}\Omega_L}{1-\frac{L}{p_L}\Omega_L a_t}\right]^{\frac{\beta}{1-\beta}} \tag{22}$$

It can be shown that L-types, once offered the contract $[N_L^{*\prime}, r_L^{*\prime}]$, increase their expected profits with respect to the case of $[N_L^*, r_L^*]$ (see Appendix B). Therefore, by hiding their type, L-types are better off, even if the equilibrium is revealing.

So, we have one important adverse selection result for a sector inside the frontier:

**Proposition 1** *If an intermediate sector $v$ is inside the frontier, there is a separating equilibrium for innovation, with H-types undertaking imitation and L-types undertaking innovation. This equilibrium is revealing for L-types, and it is at the point $[N_L^{*\prime}, r_L^{*\prime}]$.*

Notice that this result is quite standard in the literature of financing with asymmetric information, as in the paper of Stiglitz and Weiss (1981). In fact, even if their work is a credit model for a generic investment project, while here the project to be financed is an innovation project [which depends on the state of technology in absolute $\left(\bar{A}_t\right)$ and relative terms $(a_t)$], the result is the same, because the mechanism at work is the same: adverse selection due to asymmetric information.

# 3 Debt at the frontier

At the frontier there is not imitation, so the only way for an entrepreneur to increase the quality of a product is to undertake innovation. This implies that since there is no imitation, there is no participation constraint either for investors and for entrepreneurs. Then, in order to derive the equilibrium for both types, we need first derive the indifference curve, and then the zero profit one, in this different economic environment. Finally, consider that the derivation of the results are very similar to those of a sector inside the frontier.

## 3.1 Entrepreneurs indifference curves

To derive the entrepreneurs indifference curves, consider first that expected profits of L-types are:[26]

[26] $\bar{A}_t(v)$ represents the $v$ sector TFP at the frontier.

$$E\left(\bar{\Pi}^{L}\right)_{t} = N_t^{\beta}\frac{p_L}{x}\left[\delta(\chi) - R_t n_I\right]\bar{A}_t \tag{23}$$

and those of H-types are:

$$E\left(\bar{\Pi}^{H}\right)_{t} = N_t^{\beta}\frac{p_H}{x}\left[\delta(\chi) - R_t n_I\right]\bar{A}_t \tag{24}$$

Because of competition among entrepreneurs, their expected profits would be driven to the minimum possible amount: each entrepreneur will undertake innovation if the expected profits from this business are higher than those of an alternative activity, which in this case it is not imitation, but simply a riskless bond which transfers wealth to the future, at the net interest rate $r$. This financial technology gives a net return of $rN_t^{\beta}$. So, the participation constraint for an L-type becomes:

$$N_t^{\beta}\tfrac{p_L}{x}\left[\delta(\chi) - R_t n_I\right]\bar{A}_t \geq rN_t^{\beta}$$

From the previous condition we can derive the indifference curve of L-types:

$$\bar{N}_L^{E} : N_t = \frac{1}{R_t}\left[\delta(\chi)\bar{A}_t - r\frac{x}{p_L}\right] \tag{25}$$

while from a similar condition, we have the indifference curve of H-types:

$$\bar{N}_H^{E} : N_t = \frac{1}{R_t}\left[\delta(\chi)\bar{A}_t - r\frac{x}{p_H}\right] \tag{26}$$

Differently from the curves derived for a sector inside the frontier, in this case the indifference curves have an inverse order: $\bar{N}_H^{E}$is higher than $\bar{N}_L^{E}$. The reason of this inverted order can be explained with the following example. Consider an L-type entrepreneur being inside the frontier willing to undertake innovation. Then, starting from a generic point (F in figure 4), she can increase costs as long as this leaves her indifferent between innovation and imitation. Supposing that she just increases $N_t$ along the rate $R_0$, then she will have to increase them more than H-types in order to reach the indifference, because she is less skilled than H-types, so she reaches point J. She wants to avoid to end in the imitation area, because it is too risky, as a successful innovator would make her investment become worthless! Instead, if she were at the frontier, she could just increase the funds to invest until $N_K$ (and not until a generic point J'), where she is again on the indifference curve, but now the alternative business is a bond which will surely give her a return in the next period, independently from what other entrepreneurs do.

## 3.2 Investors zero profit curves and symmetric equilibrium

In order to derive the curves of investors, consider that the expected profit of an investor at the frontier financing entrepreneur is

$$E[\bar{\Pi}_f^{i}]_t = N_t^{\beta}\frac{p_i}{x}R_t N_t - N_t \qquad \text{for } i = H, L$$

imposing the zero profit condition we get (as in the case of a sector inside the frontier) for L and H-types respectively, we get the zero profit curves for funds of both types are exactly similar to those of a sector inside the frontier:

$$\bar{N}_L^Z : N_t = \left( \frac{1}{R_t} \frac{x}{p_L} \right)^{\frac{1}{\beta}} \tag{27}$$

$$\bar{N}_H^Z : N_t = \left( \frac{1}{R_t} \frac{x}{p_H} \right)^{\frac{1}{\beta}} \tag{28}$$

In turn, the symmetric information equilibrium values of the amount of funds and interest rates for both types (see Appendix D) are:

$$\bar{N}_{d,L}^* = \left( \frac{x}{p_L} \right)^{\frac{1}{\beta^2(\beta-1)}} \left[ \delta(\chi) \bar{A}_t - \left( \frac{p_L}{x} \right)^{\frac{1-\beta}{\beta}} \right]^{\frac{1}{1-\beta}} \tag{29}$$

$$\bar{R}_{d,L}^* = \left( \frac{x}{p_L} \right)^{\frac{1}{1-\beta}} \left[ \delta(\chi) \bar{A}_t - r \left( \frac{p_L}{x} \right)^{\frac{1-\beta}{\beta}} \right]^{\frac{\beta}{\beta-1}} \tag{30}$$

$$\bar{N}_{d,H}^* = \left( \frac{x}{p_H} \right)^{\frac{1}{\beta^2(\beta-1)}} \left[ \delta(\chi) \bar{A}_t - \left( \frac{p_H}{x} \right)^{\frac{1-\beta}{\beta}} \right]^{\frac{1}{1-\beta}} \tag{31}$$

$$\bar{R}_{d,H}^* = \left( \frac{x}{p_H} \right)^{\frac{1}{1-\beta}} \left[ \delta(\chi) \bar{A}_t - r \left( \frac{p_H}{x} \right)^{\frac{1-\beta}{\beta}} \right]^{\frac{\beta}{\beta-1}} \tag{32}$$

with $\bar{N}_{d,H}^* > \bar{N}_{d,L}^*$ and $\bar{R}_{d,H}^* > \bar{R}_{d,L}^*$.

Notice that differently from a sector inside the frontier, the interest rates of H-types is lower, while the amount of funds is higher. They are shown in figure 5.

### 3.3 Asymmetric information equilibrium

As for the previous section, it is important to see how the model works under asymmetric information, where entrepreneurs know their type and this is their private information, while investors do not.

Notice that the respective equilibrium values (of symmetric information) for H and L-types are at points A and B. Is there an incentive for one of the two groups to hide its type? The answer is yes. In fact, consider what happens if an H-type entrepreneur were to hide her type, and consequently demand the same amount of L-types ($\bar{N}_L$). The investor would be offering a pooling contract, at an average interest rate at point D. At point D, L-types would be charged a higher interest rate than at their point A, and so would not be undertaking innovation. H-types, in turn, would be charged a lower interest rate than they should be, therefore gaining from hiding their type the difference in the interest rate $R_C - R_D$. Notice, then, that the point D pooling contract is not an equilibrium contract, because only H-types would be undertaking innovation, and so recognized for their type. For L-types, the contract at point D implies an interest rate too high to undertake innovation, so they opt for imitation. This implies that at the frontier there is a positive selection, as stated in the proposition below:

**Proposition 2** *If an intermediate sector $v$ is at the frontier, there is a separating equilibrium for innovation, with H-types undertaking innovation and L-types undertaking imitation. This equilibrium is revealing for L-types, and it is at the point* $\left[\bar{N}_{d,L}^{*\prime}, \bar{R}_{d,H}^{*\prime}\right]$.

**Proposition 3** *Asymmetric information (in innovation) has different consequences in a sector inside the frontier and in one at the frontier. In the sector inside the frontier, it drives H-types out of the innovation market and they undertake imitation, while innovation is undertaken by L-types. In a sector at the frontier, it is the reverse: H-types undertake innovation while L-types undertake imitation.*

# 4 Equities inside the frontier

In the previous section we have found that the debt contract always creates separating equilibria, excluding one or the other type according that it is *inside* or *at* the frontier. Since this is not a satisfactory result, especially in the case inside the frontier (because of adverse selection), it may be interesting to see what happens under an a different contract. In particular, we will try to see if there is any sensible improvement with an equity contract.

If innovation projects are financed with equities, it means that the entrepreneur will have to give up part of firm ownership to the investor. In turn, the investor will get a share of profits in exchange of the investment and an insider control over the firm. In the following subsection we will focus on equities without any form of monitoring in order to show that this is sufficient to remove the inefficiencies that we have with debt.

## 4.1 Entrepreneurs indifference curves

Consider an entrepreneur undertaking imitation financed with equities by an external investor. Let $\theta$ be the share of innovation profits of the entrepreneur for imitation. Then, adapting (4), her expected profits from imitation are

$$E\left(\Pi_{eq}^{M}\right)_{t} = \theta N_t^{\beta}\left(1-\frac{p}{x-s}\right)^{x-s}\pi(M) = \theta N_t^{\beta}\left(1-\frac{p}{x-s}\right)^{x-s}\delta\left(\chi\right)A_t$$

while the expected profits from an innovation project financed with equities are:

$$E(\Pi_{eq}^{i})_t = \eta_i N_t^{\beta}\frac{p_i}{x}\pi(I) = \eta_i N_t^{\beta}\frac{p_i}{x}\delta\left(\chi\right)\bar{A}_t \qquad \text{for } i = H, L$$

where $\eta_i$ is the share of profits of the entrepreneur (it depends on the type: $i$).

In order to derive the indifference curve in this case, consider that an entrepreneur is willing to undertake innovation if the expected profits are higher; so, for example, for L-types, we have $E(\Pi_{eq}^{i})_t \geq E\left(\Pi_{eq}^{M}\right)_t$. Imposing this constraint implies:

$$\eta_L N_t^{\beta}\tfrac{p_L}{x}\delta\left(\chi\right)\bar{A}_t \geq \theta N_t^{\beta}\breve{\Omega}\delta\left(\chi\right)A_t$$

from which we derive the following:

$$\eta_L^E = \frac{x}{p_L}\overset{\frown}{\Omega}\theta a_t \tag{33}$$

which is the minimum share of profits desired by the entrepreneur in order for him to undertake innovation. Similarly, for H-types we have:

$$\eta_H^E = \frac{x}{p_H}\overset{\frown}{\Omega}\theta a_t \tag{34}$$

Notice that both of the indifference curves represent the minimum share of profits desired by the entrepreneurs such that she undertakes innovation. Moreover, it is: $\eta_L^E > \eta_H^E$, that is that since they are flat (not dependent on $N_t$), for any amount of funds L-types, being worse at the business (innovation) would accept to undertake it only for a higher share of profits than what H-types are willing to do.

## 4.2 Investors zero profit curves and symmetric equilibrium

On the supply side, the investor financing innovation gets the following expected profits (financing an L-type):

$$E(\Pi_{eq,f}^L)_t = (1-\eta_L)\, N_t^\beta \frac{p_L}{x}\pi(I) - N_t = (1-\eta_L)\, N_t^\beta \frac{p_L}{x}\delta\left(\chi\right)\bar{A}_t - N_t$$

Imposing the free entry condition, we have the following zero profit curve (see Appendix C):

$$N_{L,eq}^Z : N_t = \left[\frac{p_L}{x}\left(1-\eta\right)\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}} \tag{35}$$

Similarly, for H-types, we can derive:

$$N_{H,eq}^Z : N_t = \left[\frac{p_H}{x}\left(1-\eta\right)\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}}$$

Notice that both the zero profit curves are downward sloping (see figure 6). This is due to the fact that the more funds the investor puts to finance the innovation project, the higher her reward has to be (share of profits: $1-\eta$), and the lower the reward for the entrepreneur. Moreover, notice that the areas where financing for innovation takes place is south-west with respect to the each zero profit curve. The reason is that as the share of profits to the investor increases (lower $\eta_t$), the more funds the investor is willing to invest into the project.

From the indifference and the zero profit curves derived above we have the following equilibrium values (see Appendix C):

$$N_{L,eq}^* = \left[\frac{p_L}{x}\left(1-\frac{x}{p_L}\overset{\frown}{\Omega}\theta a_t\right)\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}} \tag{36}$$

$$\eta_L^* = \frac{x}{p_L}\overset{\frown}{\Omega}\theta a_t$$

for L-types and

$$N_{H,eq}^{*}=\left[\frac{p_H}{x}\left(1-\frac{x}{p_H}\overset{\frown}{\Omega}\theta a_t\right)\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}} \tag{37}$$

$$\eta_H^{*}=\frac{x}{p_H}\overset{\frown}{\Omega}\theta a_t$$

with $\eta_L^{*}>\eta_H^{*}$, and $N_{H,eq}^{*}>N_{L,eq}^{*}$, as shown in the graph below

## 4.3 Pooling equilibrium

Consider now the separating equilibrium in points A and B. Is this equilibrium optimal? The answer is no. In fact, following figure 6, consider, for example, the case that the investor were to offer a pooling contract in point D because she is unable to discern the true type, where both types demand for $N_{L,eq}^{*}$, and also have the same share ($\eta_D$). Then, L-types would be better off because for the same amount of funds they gain an additional share of profits ($\eta_D-\eta_A$) and H-types would benefit even more because their share of profits increases by the difference ($\eta_D-\eta_B$). Therefore, the pooling contract in D is a Pareto improvement with respect to the symmetric information separating equilibrium (points A and B). Finally, notice that the pooling contract in D is an equilibrium contract such that H-types subsidize (for the difference $\eta_D-\eta_A$) L-types; however, despite this subsidy H-types are still better off than if they were to accept the separating contract in B.

Therefore, we can derive the following result:

**Proposition 4** *In a sector inside the frontier, if innovation is financed with equities, there is a pooling equilibrium such that H-types subsidize L-types. This equilibrium creates no rationing for no type and clears the market.*

# 5 Equities at the frontier

## 5.1 Entrepreneurs indifference curves

In the sector at the frontier, expected profits of the entrepreneurs are the following

$$E(\overline{\Pi}_{eq}^{i})_t=\overline{\eta}_i N_t^{\beta}\frac{p_i}{x}\overset{\frown}{\Omega}\pi(I)=\overline{\eta}_i N_t^{\beta}\frac{p_i}{x}\overset{\frown}{\Omega}\delta\left(\chi\right)\bar{A}_t \qquad \text{for } i=H,L$$

where $\overline{\eta}_i$is the share of profits of the entrepreneur innovating at the frontier. At the frontier, since there is no imitation, competition drives expected profits to the lower bound determined by the net return on the riskless asset. Therefore, the innovation participation constraint is given by $E(\overline{\Pi}_{eq}^{i})_t\geq rN_t^{\beta}$ and from this, the indifference curve of the entrepreneur (for each type) is derived:

$$\overline{\eta}_L^{E}=\frac{rx}{p_L\overset{\frown}{\Omega}\delta\left(\chi\right)\bar{A}_t} \tag{38}$$

$$\overline{\eta}_H^E = \frac{rx}{p_H \overset{\frown}{\Omega} \delta\left(\chi\right) \bar{A}_t} \tag{39}$$

Notice that $\overline{\eta}_L^E$ and $\overline{\eta}_H^E$ are independent of the amount of funds invested, and that $\overline{\eta}_L^d > \overline{\eta}_H^E$.

## 5.2 Investors zero profit curves and symmetric equilibrium

In order to derive the zero profit curve of the investors, we need to consider the expected profits:

$$E(\bar{\Pi}_{eq,f}^i)_t = (1-\overline{\eta}_i)_i\, N_t^\beta \frac{p_i}{x} \overset{\frown}{\Omega} \pi(I) - N_t = (1-\overline{\eta}_i)_i\, N_t^\beta \frac{p_i}{x} \overset{\frown}{\Omega} \delta\left(\chi\right) \bar{A}_t - N_t \quad \text{for } i = H, L$$

Also here, imposing the free entry condition gives (derived in a way similar to the case of equities inside the frontier, in Appendix C):

$$\bar{N}_{L,eq}^Z = \left[\left(1-\overline{\eta}_L\right) \frac{p_L}{x} \overset{\frown}{\Omega} \delta\left(\chi\right) \bar{A}_t\right]^{\frac{1}{1-\beta}}$$

$$\bar{N}_{H,eq}^Z = \left[\left(1-\overline{\eta}_L\right) \frac{p_H}{x} \overset{\frown}{\Omega} \delta\left(\chi\right) \bar{A}_t\right]^{\frac{1}{1-\beta}}$$

In a symmetric information equilibrium the amount of funds would be (see Appendix C for an analogous case):

$$\bar{N}_{L,eq}^* = \left[\frac{p_L \overset{\frown}{\Omega} \delta\left(\chi\right) \bar{A}_t - rx}{x}\right]^{\frac{1}{1-\beta}} \tag{40}$$

$$\bar{N}_{H,eq}^* = \left[\frac{p_H \overset{\frown}{\Omega} \delta\left(\chi\right) \bar{A}_t - rx}{x}\right]^{\frac{1}{1-\beta}} \tag{41}$$

and the equilibrium shares are just given by (38) and (39). The graph for the asymmetric information case is the same of the case inside the frontier (figure 6).

## 5.3 Pooling equilibrium

Consider now the separating equilibrium in points A and B as reported in figure 7.

As in the case inside the frontier, the problem is if that this contract is not optimal. In fact, again consider, for example, that the investor were to offer a pooling contract in point D (because she is unable to discern the true type), where both types demand for $\bar{N}_{L,eq}^*$, and also have the same share ($\overline{\eta}_D$). Then, L-types would be better-off because for the same amount of funds they gain an additional share of profits ($\overline{\eta}_D - \overline{\eta}_A$), while also H-types are better-off because they also increase their share of profits ($\overline{\eta}_D - \overline{\eta}_B$). Therefore, the pooling contract in D is a Pareto improvement with respect to the symmetric information separating equilibrium (points A and B), and it is also an equilibrium contract with asymmetric information.

**Proposition 5** *In a sector at the frontier, if innovation is financed with equities, there is a pooling equilibrium such that H-types subsidize L-types. This equilibrium creates no rationing for no type and clears the market.*

And more in general, summing up all the results obtained insofar:

**Proposition 6** *If an innovation project is undertaken when the sector is inside the frontier, then there is adverse selection if it is financed with debt, while there is a pooling contract if it is financed with equities. When the sector is at the frontier, then there is positive selection if it is financed with debt, while there is a pooling contract if it is financed with equities.*

# 6 Zero Values

Until now we have found the equilibrium contracts for each situation (inside/at the frontier) and type of contract (debt/equities) careless of when these contracts are actually implemented. It turns out that not all contracts are always available, and that the choice crucially depends on the distance to the frontier index $a_t$.

To understand this point, in this section I derive the zero values, that is the values of the index of distance to the frontier such that the equilibrium amount of funds either with debt and with equities (for sectors inside the frontier) are non negative. This allows to determine which contract will be used according to the initial state of technology (distance to the frontier).

Consider the case of debt first.

a) The equilibrium amount for funds of L-types (18) is given by the following condition: $N_{d,L}^{*\prime} \geq 0 \implies \frac{L}{p_L}\left[\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}}\left[\frac{1-\frac{L}{p_L}\Omega_L a_t}{1-\frac{L}{p_L}\Omega_L}\right]^{\frac{1}{1-\beta}} \geq 0$. This relation only requires that $a_t \geq a_{d,L} = \frac{p_L}{\widehat{\Omega}x}$.

b) Similarly, for H-types, the condition (given by 19) is $N_{d,H}^{*\prime} \geq 0$ requires that $a_t \geq a_{d,H} = \frac{p_H}{\widehat{\Omega}x}$.

Notice that from points a) and b) we have that debt is allowed as long as $a_t > a_{d,i}$ (for $i = H, L$), which implies that if $a_t$ is below one or both the thresholds, the debt contract will not be applied inasmuch being the sector too much far from the frontier. That is, the production profit of the alternative activity (imitation) $[\delta\left(\chi\right)A_t]$ is so low with respect to one of innovation $[\delta\left(\chi\right)\bar{A}_t]$, that as entrepreneurs raise costs (from the origin to north east: see figure 1) just a little, the inferior technology becomes immediately more convenient (in expected values).[27] In other terms, as we get closer to the frontier ($A_t$ increases) production profits increase as well, and this makes imitation relatively more convenient for any pair of costs $(N_t, R_t)$; so it is necessary to increase them more (outward shift of $N_i^E$) to reach the indifference locus of points.

[27] In other terms, the entrepreneurs indifference curve is squeezed to the origin: see figure 1.

For equities, in a similar way, we can derive the threshold values for the equilibrium amount of funds.

c) For L-types (from 36) is positive for $a_t \leq a_{eq,L} = \frac{p_L}{\widehat{\Omega} x \theta}$.

d) For H-types (from 37) it is positive for $a_t \leq a_{eq,H} = \frac{p_H}{\widehat{\Omega} x \theta}$.

Notice in this case that both the threshold values are determined by the conflicting interests of the two shareholders: entrepreneur and investor. In fact, since the entrepreneur's share ($\eta_i^E = \frac{x}{p_i} \widehat{\Omega} \theta a_t$) is increasing with the distance to the frontier index $a_t$, as the sector gets too close to the frontier, the investor's share is squeezed until it reaches zero at $a_{eq,i}$ (for $i = H, L$). This is why there is an upper limit for both $N^*_{eq,L}$ and $N^*_{eq,H}$ .

Putting these results together, we have the following ranking:

$$0 < a_{d,L} < a_{eq,L} < a_{d,H} < a_{eq,H} < 1$$

as shown in figure 8.

From figure 8 it is evident that there are 6 regions, 5 *inside* the frontier and the sixth is the region *at* the frontier. In order to better understand this ranking, following figure 9, it is possible to see that for example for $a_t < a_{eq,L}$, L-types entrepreneurs may demand funds with equities, or that for $a_t < a_{d,H}$ H-type entrepreneurs may demand funds with a debt contract. Finally, notice that for $a_t = 1$ either debt and equities are available choices for H and L-types (not sown in figure 9).

Given the regions outlined above, we are now able to determine which contracts will be used according to the distance to the frontier (see figures 9 and 10). In order to understand this, consider that in region I, we have $a_t < a_{d,L}, a_{d,H}$, that is the distance to the frontier is too high ($a_t$ too small) for financing with debt of both types, but also $a_t < a_{eq,L}, a_{eq,H}$ that means that it is low enough to allow financing with equities of both types. So, in region I innovation is financed with equities and, as such there is a pooling equilibrium as stated in proposition 4. In region II, the distance to the frontier is still sufficiently low for financing with equities ($a_t < a_{eq,L}, a_{eq,H}$), but it is also sufficient for financing with debt of L-types only ($a_{d,L} < a_t < a_{d,H}$). Then, L-types may choose debt or equities, while H-types can only choose equities. This implies that both types will be financed in region II.[28] Similarly, the reasoning can be done for regions III, IV, V and VI. The results are summarized in figure 10.

Notice that all of the regions are with both types undertaking innovation, except for region 5, where only L-types are financed. So, we can summarize the results as shown in figure 11.

In all the regions where both types have access to finance in equilibrium, they use equities: this is in regions I to IV (inside) and in region VI (at the frontier); in region V, only one type (low skilled) has access to finance and it is in debt form. The threshold inside the frontier is at $a_t = a_{eq,H}$. Dividing the distance span as in figure 11 is a convenient way to derive the growth rates of productivity as it will be done below.

[28] H-types have no choice but equities, while L-types can choose. So, if they choose debt they are recognized as L-types and not rationed, while if they choose equities they are not rationed as well (see proposition 6).

Moreover, this distinction is crucial to understand why the productivity growth rate is lower for a sector close to the frontier (region V), as it will be clear in the next section.

Finally, notice that from figures 10 and 11 we have financing with equities for both types. Recall that at the frontier, only H-types have access in equilibrium to debt finance at the frontier (proposition 2), while both types have access to equity finance (proposition 5). So, while L-types may only get finance through equities, H-types may choose between equity and debt and in any case both types would be financed.

# 7 Growth Rates

From the findings of the previous section, we can derive the growth path of sector along all of the regions spanning on the distance to the frontier parameter $a_t$. First of all, notice that given the form of the final output production function, for a generic sector, the productivity growth rate ($g$) is the same of the output growth rate,[29] where $g$ is given by

$$g = \frac{\partial \ln Y}{\partial A} \cong \frac{E\left(A_t\right) - A_{t-1}}{A_{t-1}} = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

and $E\left(A_t\right)$ is the expected value of TFP in the period $t$:

$$E\left(A_t\right) = \begin{cases} \gamma \bar{A}_{t-1} & \text{if Innovation succeeds with prob. } \breve{p} N_t^{\beta} \\ \gamma A_{t-1} & \text{if Imitation succeeds with prob. } \left(1-\breve{p}\right) N_t^{\beta} \end{cases}$$

or in other terms

$$g = \frac{E(A_t) - A_{t-1}}{A_{t-1}} = \gamma N_t^{\beta} \left[ \frac{\breve{p} \bar{A}_{t-1}}{A_{t-1}} + \frac{\left(1-\breve{p}\right) A_{t-1}}{A_{t-1}} \right] - 1 \text{ or}$$

$$g = \gamma N_t^{\beta} \left[ \frac{\breve{p}}{a_{t-1}} + \left(1 - \breve{p}\right) \right] - 1 \tag{42}$$

Notice that $g$ is decreasing in the distance to the frontier ($g_a < 0$), that is as the sector is closer to the frontier index the lower the expected growth rate. This result depends on assuming that innovation is possible also at early stages of development. Moreover, consider that the growth rate varies according to $N_t^{\beta}$, which is the equilibrium amount of funds invested, which varies according to the contract that is used in equilibrium. So, instead of deriving the productivity growth rates for each region (from I to VI), it will be sufficient to consider the one in regions I-IV, the one in region

[29] Notice that from (1), in log form we have that:
g$=\frac{\partial \ln Y}{\partial A} = \alpha \frac{\frac{\partial \ln k}{\partial A}}{k} \dot{A} + (1-\alpha) \frac{\dot{A}}{A}$
where $\frac{\partial \ln k}{\partial A} = \chi^{\frac{1}{\alpha-1}}$ and $k = \chi^{\frac{1}{\alpha-1}} A$. Therefore,
$\frac{\partial \ln Y}{\partial \ln A} = \alpha \frac{\chi^{\frac{1}{\alpha-1}} \dot{A}}{\chi^{\frac{1}{\alpha-1}} A} + (1-\alpha) \frac{\dot{A}}{A} \Longrightarrow$
$\Longrightarrow g = \frac{\partial \ln Y}{\partial \ln A} = \frac{\dot{A}}{A}$.

V and the one in region VI. It follows that the productivity growth rate for the first four regions ($g_1$) is

$$g_1 = \gamma N_{eq,L}^{*\beta} \left[ \frac{\breve{p}}{a_{t-1}} + \left(1 - \breve{p}\right) \right] - 1 \tag{43}$$

where $N_t = N_{eq,L}^*$ due to private information (see proposition 4), while the one for region V is

$$g_2 = \gamma N_{d,L}^{*\beta} \left[ \frac{\breve{p}}{a_{t-1}} + \left(1 - \breve{p}\right) \right] - 1 \tag{44}$$

where $N_t = N_{d,L}^*$ due to private information (see proposition 1) and the one for region VI is[30]

$$g_3 = \bar{N}_{eq,L}^{*\beta} \left[ \breve{p}\,(\gamma - 1) + 1 \right] - 1 \tag{45}$$

where $N_t = \bar{N}_{eq,L}^{*}$ due to private information (see proposition 6).[31]

In order to have a better understanding of how these growth rates vary, consider that $g_1$ and $g_2$ differ just for the amount of funds invested in equilibrium, and since $N_{eq,L}^* > N_{d,L}^*$ (see proof in Appendix E), therefore we have that $g_1 > g_2$. Moreover, it can be shown (see Appendix E for details) that for realistic values of $\gamma$ and $p_L$, it is $g_3 > g_2$. So, the growth path can seen as in figure 12.

This figure tells us that the growth rate is ever decreasing with the distance to the frontier, as for $g_1$. As mentioned above, this result is due to the assumption that there is no obstacle for a very backward sector to innovate, that is to reach the frontier in only one period. Moreover, after the threshold level at $a_{eq,H}$, the growth rate ($g_2$) is still decreasing, but it has downward jump due to the fact that in this region (region V), only debt can be used, and since the use of debt inside the frontier causes adverse selection (no participation of H-types) and in equilibrium the amount of funds invested is lower (remember that $N_{eq,L}^* > N_{d,L}^*$), the growth rate falls down. However, this path changes again at the frontier. In fact, at this point, due to the absence of the alternative of imitation, the structure of incentives changes, and the growth rate ($g_3$) increases again.

To have an idea of the magnitude of this loss of growth, I have run a simulation with the following parameter values: $p_H = 0,7$, $p_L = 0,25$, $\bar{A}_t = 100$, $\gamma = 1,1$,

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30 To derive the growth rate at the frontier ($g_3$), consider that

$$E\left(\bar{A}_t\right) = \begin{cases} \gamma \bar{A}_{t-1} & \text{if Inn. succeeds with prob.} \breve{p} N_t^{\beta} \\ \bar{A}_{t-1} & \text{if Innovation fails with prob. } \left(1 - \breve{p}\right) N_t^{\beta} \end{cases}$$

so

$$g_3 = \frac{E(A_t) - A_{t-1}}{A_{t-1}} = N_{eq,L}^{*\beta} \left[ \frac{\gamma \bar{A}_{t-1} \breve{p}}{\bar{A}_{t-1}} + \left(1 - \breve{p}\right) \frac{\bar{A}_{t-1}}{\bar{A}_{t-1}} \right] - 1$$

or also:

$$g_3 = N_{eq,L}^{*\beta} \left[ \gamma \breve{p} + \left(1 - \breve{p}\right) \right] - 1$$

from which the value above is derived.

31 Note that at the frontier L-types are rationed if they use debt, but not with equities: so they surely choose equities. On the other hand, H-types are never rationed, neither with debt nor with equities, so they are free to choose both contracts. The indeterminacy of H-types can be solved considering that if they choose equities they are (in equilibrium) above the indifference curve ($\eta_{eq,H}^{E}$): this makes them better-off than if they were to choose debt (where in equilibrium they would be on the indifference curve).

$x = 50$, $H = 25$, $L = 25$, $\Omega = 0,61893$. With these values, the loss in the rate of growth occurs at the distance coefficient $a_t = 0,51$ (that is at half of the distance) and $g$ falls from 6,22% to 3,64%, that is the growth rate falls by 41%!

### 7.1 Policy implications

Despite the aim of this paper is not to provide and test policy recommendations, the model exposed insofar suggests some ideas on how public authorities may intervene in order to promote innovation and growth. In this sense, consider that the main negative implication of the model is the drop of the growth rate ($g_2$) in the intermediate region where financing is with debt. Therefore, public authorities might intervene in favour of sectors which are in this intermediate region, in two ways.

The first way is to consider R&D subsidizing policy aimed to H-types, so as to include them in the R&D race; if also H-types are participants of the R&D race, then the growth rate would be higher, even if descending as the sector gets closer to the frontier. Therefore, public support of R&D would have a rationale for sectors inside the frontier (intermediate regions), while it would not for a sector at the frontier.

However, there is also another way public authorities might use and that would probably be less costly for the government. To show how it works, consider that the threshold where the intermediate region starts is at $a_{eq,H} = \frac{p_H}{\overset{\backsim}{\Omega} x\theta}$. Now, while $p_H$, $x$ and all components of $\overset{\backsim}{\Omega}$ are determined, recall that $\theta$ is the parameter for the share of profits of the entrepreneur undertaking imitation inside the frontier (with equities). This parameter has been kept exogenous for simplicity. Moreover, consider that if it would be decreased, then $a_{eq,H}$ would increase,[32] and as a result the intermediate debt region (V) would be shorter and might eventually shrink. This means that if for a given sector $v$, public authorities could incentive entrepreneurs and investors in a way so as to reduce the share of profits of entrepreneurs in imitation, then the low growth region would be shorter, and sector $v$ would fall in the initial region of equities inside the frontier (regions I to IV).

## 8 Conclusions

In this paper, I have tried to understand if countries have an advantage or disadvantage of backwardness, taking the financial system as given. In order to answer to this question I have focused on the link between the state of technology and type of contract for financing an innovation project. This is to say, studying the incentives for either the investor and the entrepreneur when a sector is at the frontier so that it can only innovate) or inside the frontier (so it can also imitate).

The first finding of this paper is that for a sector at the frontier there externalities if innovation is financed with equities, but not with debt. In fact, if innovation is financed with debt in a sector inside the frontier, there is always an incentive for low skilled entrepreneurs to hide their type; this creates adverse selection so that highly skilled are driven out of the innovation race. Instead, if innovation is financed with equities, then

[32] in terms of figures 11 and 12, $a_{eq,H}$ would move to the right.

there are always positive externalities for either a sector inside and for a sector at the frontier, because there is a pooling equilibrium such that both are financed.

Second, when these results are related to the distance to the frontier parameter, then it emerges that equities are preferred by sectors which are either very distant and at the frontier, while debt is chosen by middle distant sectors. The consequence of this use of financial contracts is in the growth rates. In fact, while this is decreasing as a sector gets closer to the frontier, in the intermediate region of debt it drops down even more. This is due to adverse selection which implies non participation of H-types to the patent race.

Then, the contribution of this paper is to highlight at which stages of development there is a risk of low growth. Therefore, it turns out that policy measures intended to stimulate innovation should be undertaken for sectors which are in this region, that is for sectors which are close but still behind the technology frontier. While some policy measures are suggested, I leave to future research the study of which of these are most appropriate.

References


[1] Acemoglu D., Aghion P., Zilibotti F., ”Distance to frontier, Selection, and economic growth”, NBER WP, no. 9066, Jul 2002.

[2] Aghion P., Bacchetta P., Banerjee A., "Financial Development and the instability of Open Economies", mimeo, 2003

[3] Aghion, P., Howitt, P:, ”A model of growth through creative destruction”, Econometrica, vol. 60, 1992.

[4] Aghion, P., Howitt, P., ”Growth with Quality improving Innovations: an integrated framework”, chap. 2 in Handbook of Economic Growth, 2004, forthcoming.

[5] Akerlof, M., ”Lemons market: quality uncertainty and the market mechanism" Quarterly Journal of Economics, 1970.

[6] Audretsch, D.B., ”Entrepreneurship: a survey of the Literature”, mimeo prepared for the European Commission, Enterprise Directorate General, Jul 2002.

[7] Barro, R.J. and Sala-i-Martin, X., Economic growth, MIT Press, 2004.

[8] Bernanke, B. and Gertler, M., ”Agency Costs, Net Worth and business fluctuations”, American Economic Review, vol. 79, 1989, pp. 14-31.

[9] Besanko, D. and Thakor, A., ”Collateral and Rationing: Sorting Equilibria in Monopolistic and Competitive Credit Markets”, in International Economic Review, vol. 28, no. 3, 1987, pp.671-89.

[10] Black, B. and Gilson, R., ”Venture capital and the structure of Capital markets: banks versus stock markets”, Journal of Financial Economics, vol. 47, 1998, pp. 243-277.

[11] Boyd, J.H., and Smith, B.D., ”The evolution of debt and equity markets in economic development” in Economic Theory, vol. 12, 1998, pp. 519-560.

[12] Cooley, T.F., and Smith, B.D., ”Financial markets, Specialization and learning by doing”, Research in Economics, vol. 52, 1998, pp. 333-361.

[13] De la Fuente, A. and Marin, J.M., ”Innovation, Bank monitoring and endogenous financial development”, Journal of Monetary Economics, vol. 38, 1996, pp. 269-301.

[14] De Meza, D. and Webb, D., ”Too much investment: a problem of asymmetric information” Quarterly Journal of Economics, vol. 102, no. 2, pp.281-292.

[15] Evans, D.S., Jovanovic, B., ”An estimated model of entrepreneurial choice under Liquidity Constraint”, Journal of Political Economy, vol. 97, no. 4, Aug 1989.

[16] Fazzari, S.M., Hubbard, R.G., Petersen, B.C., ”Financing Constraints and Corporate Investment”, Brooking Papers on economic activity, no. 3, 1988.

[17] Gerschenkron, A., Economic Backwardness in Historical Perspective, Harvard University Press, Cambridge MA, 1962.

[18] Gentry, W., Hubbard, G., ”Entrepreneurship and saving”, mimeo, Columbia Business, School, 2001.

[19] Gompers P., Lerner, J., ”The Venture Capital Cycle”, MIT Press, 1999.

[20] Guiso, L., ”High tech Firms and credit rationing”, CEPR DP, no. 1696, Oct 1997.

[21] Howitt, P., Endogenous Growth and Cross-Country Income Differences, American Economic Review, vol. 90, no. 4, 2000.

[22] Kaplan, S. and Stromberg, P., ”Financial Contracting Theory Meets the Real World: An Empirical Analysis of Venture Capital Contracts”, in Review of Economic Studies, no.2, vol. 70, Apr 2003.

[23] Ozerturk, S., ”Risk Sharing, Risk Shifting and Optimality of Convertible Debt in Venture Capital”, mimeo, 2002.

[24] Papageorgiou, C., ”Technology adoption, Human capital and Growth theory”, in

Review of Development Economics, vol. 6 no. 3, 2002.

[25] Stiglitz, J.E., Weiss, A., ”Credit Rationing in Markets with Imperfect Information”, American Economic Review, vol. 71, no. 3, Jun 1981.

[26] Zeira, J., ”Innovation, Patent races and Endogenous growth”, CEPR DP, no. 3974, 2003.

# Appendix A. Production Profits

In order to better understand the expression form of production profits $\pi_t(v)$, consider that in order to produce $x(v)$ units of the intermediate $v$, it is necessary to employ one unit of the final good; so production profits are:

$$\pi_t(v) = p_t(v)k_t(v) - k_t(v) = [p_t(v) - 1]k_t(v)$$

where $p_t(v)$ is the price of the intermediate good. Now, since the final good is produced competitively, the price of the intermediate good can be set to a constant ($\chi$) that is equal to the marginal product with respect to that input. Then, in the case of innovation the price is:

$$p_t = \chi = \frac{\vartheta Y_t}{\vartheta k(v)_t} = \bar{A}_t(v)^{1-\alpha} k(v)_t^{\alpha-1} \tag{A-1}$$

while in the case of imitation it is:

$$p_t = \chi = \frac{\vartheta Y_t}{\vartheta k(v)_t} = A_t(v)^{1-\alpha} k(v)_t^{\alpha-1} \tag{A-2}$$

So (following only the innovation case for simplicity), the optimal demand for the input is:

$$k(v)_t = \chi^{\frac{1}{\alpha-1}} \bar{A}_t(v) \tag{A-3}$$

and production profits are:

$$\pi_t(v) = (\chi - 1)\, \chi^{\frac{1}{\alpha-1}} \bar{A}_t(v)$$

or if we set $\delta(\chi) = (\chi - 1)\, \chi^{\frac{1}{\alpha-1}}$, they become:

$$\pi_t(I) = \delta(\chi)\bar{A}_t(v) \tag{A-4}$$

where the TFP here is $\bar{A}_t(v) = \gamma A_{t-1}(v)$ if innovation is successful; in the case of imitation they become:

$$\pi_t(M) = \delta(\chi)\bar{A}_t(v)$$

where the TFP here is $A_t(v) = \gamma A_{t-1}(v)$.

## Appendix B. Debt inside the frontier

In this section, I derive the entrepreneurs indifference curve and the equilibrium with symmetric and asymmetric information. The derivations of L-types only are shown.

### Entrepreneurs indifference curves

I derive the indifference curve of L-types. From the participation constraint: $E\left(\Pi^i\right) \geq E\left(\Pi_M\right) \quad i = H, L$ one can derive the indifference curve of entrepreneurs willing to undertake innovation. In fact, imposing that the constraint is binding and written in full detail (see (11)), we have:

$$N_t^{\beta}\frac{p_L}{x}\left[\delta(\chi) - R_t n_I\right]\bar{A}_t = N_t^{\beta}\left(1 - \frac{\breve{p}}{x}\right)^{x-1}\left[\delta(\chi) - R_t n_M\right]A_t$$

where $\breve{p}$ is the average probability. Set $\breve{\Omega} = \left(1 - \frac{\breve{p}}{x}\right)^{x-1}$, delete $N_t^{\beta}$ and divide by $\bar{A}_t$on both sides; then we get $\delta(\chi) - R_t n_I = \frac{x}{p_L}\breve{\Omega}\left[\delta(\chi)a_t - R_t n_M\right]$

where $a_t = \frac{A_t}{\bar{A}_t}$ is the inverse index of distance to the frontier. Notice that $0 < a_t < 1$.

By (8) and (6), replace $N_t$ on both sides:

$\delta(\chi) - R_t\frac{N_t}{\bar{A}_t} = \frac{x}{p_L}\breve{\Omega}\delta(\chi)a_t - \frac{x}{p_L}\breve{\Omega}R_t\frac{N_t}{\bar{A}_t}$

From this expression we get

$$N_L^E : N_t = \frac{\delta(\chi)\bar{A}_t}{(1 + r_t)}\left[\frac{\left(1 - \frac{x}{p_L}\breve{\Omega}a_t\right)}{\left(1 - \frac{x}{p_L}\breve{\Omega}\right)}\right]$$

which is our L-types indifference curve.

### Equilibrium with symmetric information

To derive the equilibrium amount of funds of L-types, we just need equalize entrepreneurs indifference curve with investors zero profit curve. For L-types (H-types are derived in a similar way) we have: $N_L^E = N_L^Z$, that is:

$$\frac{\delta(\chi)\bar{A}_t}{(1+r_t)}\left[\frac{\left(1-\frac{x}{p_L}\breve{\Omega}a_t\right)}{\left(1-\frac{x}{p_L}\breve{\Omega}a_t\right)}\right] = \left(\frac{x}{p_L}\frac{1}{R_t}\right)^{\frac{1}{\beta}}$$

which regrouping for $R_t$ gives:

$$R_t^{\frac{\beta-1}{\beta}} = \left(\frac{x}{p_L}\right)^{\frac{1}{\beta}}\delta\left(\chi\right)\bar{A}_t\left[\frac{\left(1-\frac{x}{p_L}\breve{\Omega}a_t\right)}{\left(1-\frac{x}{p_L}\breve{\Omega}a_t\right)}\right]$$

or finally:

$$R_L^* = \left(\frac{x}{p_L}\right)^{\frac{1}{\beta-1}}\left[\delta\left(\chi\right)\bar{A}_t\right]^{\frac{\beta}{\beta-1}}\left[\frac{\left(1 - \frac{x}{p_L}\breve{\Omega}a_t\right)}{\left(1 - \frac{x}{p_L}\breve{\Omega}\right)}\right]^{\frac{\beta}{\beta-1}} \qquad \text{(B-1)}$$

which is the equilibrium interest rate for L-types in the case of symmetric information.

To find the equilibrium amount of funds, insert (B-1) into the zero profit curve and get:

$$N_L = \left(\frac{x}{p_L}\right)^{\frac{1}{\beta}} \left\{ \left(\frac{x}{p_L}\right)^{\frac{1}{\beta-1}} \left[\delta\left(\chi\right)\bar{A}_t\right]^{\frac{\beta}{\beta-1}} \left[\frac{\left(1-\frac{x}{p_L}\widehat{\Omega}a_t\right)}{\left(1-\frac{x}{p_L}\widehat{\Omega}a_t\right)}\right]^{\frac{\beta}{\beta-1}} \right\}^{\frac{1}{\beta}}$$

or

$$N_L^* = \frac{x}{p_L}\left[\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{\beta-1}} \left[\frac{\left(1-\frac{x}{p_L}\widehat{\Omega}a_t\right)}{\left(1-\frac{x}{p_L}\widehat{\Omega}\right)}\right]^{\frac{1}{1-\beta}} \tag{B-2}$$

which is the amount of funds in equilibrium of symmetric information for L-types.

### Equilibrium with asymmetric information

...to be computed....

$$N_L^{*\prime} = \frac{L}{p_L}\left[\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{\beta-1}} \left[\frac{\left(1-\frac{L}{p_L}\Omega_L a_t\right)}{\left(1-\frac{L}{p_L}\Omega_L\right)}\right]^{\frac{1}{1-\beta}}$$

## Appendix C. Equities inside the frontier

In this part, I derive only the investor indifference curve of L-types only by simply imposing the zero profit condition and then the equilibrium values.

### Investors zero profit curves

Impose the zero profit condition on L-types expected profits:

$$E(\Pi^L_{eq,f})_t = \left(1-\eta_L\right)N_t^{\beta}\tfrac{p_L}{x}\delta\left(\chi\right)\bar{A}_t - N_t = 0$$

Setting everything on one side, we have:

$$N_t^{\beta-1}\tfrac{p_L}{x}\left(1-\eta_L\right)\delta\left(\chi\right)\bar{A}_t = 1$$

or also:

$$N^Z_{L,eq} = \left[\frac{p_L}{x}\left(1-\eta_L\right)\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}} \tag{C-1}$$

which is the zero profit curve.

### Equilibrium with symmetric information

To derive the equilibrium values of L-types, consider that for the share of equities the value is given by (33) for L-types, while for the equilibrium amount of funds, consider to plug (33) into the zero profit curve (C-1):

$$N^*_{L,eq} = \left[\frac{p_L}{x}\left(1-\frac{x}{p_L}\widehat{\Omega}\theta a_t\right)\delta\left(\chi\right)\bar{A}_t\right]^{\frac{1}{1-\beta}}$$

# Appendix D. Debt at the frontier

In this section the equilibrium values of the amount of funds and interest rates are found for innovation projects financed with debt in a sector which is at the frontier.

First, equalize the entrepreneurs indifference curve with the investors zero profit curve $\bar{N}_L^E = \bar{N}_L^Z$(for L-types only):

$$\frac{1}{R_t}\left[\delta(\chi)\bar{A}_t - r\frac{x}{p_L}\right] = \left(\frac{1}{R_t}\frac{x}{p_L}\right)^{\frac{1}{\beta}}$$

then regroup in terms of $R_t$:

$$R_t^{\frac{\beta-1}{\beta}} = \left(\frac{x}{p_L}\right)^{-\frac{1}{\beta}}\delta(\chi)\bar{A}_t - r\left(\frac{x}{p_L}\right)^{\frac{\beta}{\beta-1}}$$

or also:

$$\bar{R}_L^* = \left(\frac{x}{p_L}\right)^{\frac{1}{1-\beta}}\left[\delta(\chi)\bar{A}_t - r\left(\frac{p_L}{x}\right)^{\frac{1-\beta}{\beta}}\right]^{\frac{\beta}{\beta-1}} \tag{D-1}$$

In order to find the equilibrium amount of funds, insert (D-1) into the zero profit curve (27):

$$\bar{N}_L = \left(\frac{1}{R_t}\frac{x}{p_L}\right)^{\frac{1}{\beta}} = \left(\frac{x}{p_L}\right)^{\frac{1}{\beta}}\left\{\left(\frac{x}{p_L}\right)^{\frac{1}{1-\beta}}\left[\delta(\chi)\bar{A}_t - r\left(\frac{p_L}{x}\right)^{\frac{1-\beta}{\beta}}\right]^{\frac{\beta}{1-\beta}}\right\}^{\frac{1}{\beta}}$$

rearranging this gives:

$$\bar{N}_L^* = \left(\frac{x}{p_L}\right)^{\frac{1}{\beta^2(\beta-1)}}\left[\delta(\chi)\bar{A}_t - r\left(\frac{p_L}{x}\right)^{\frac{1-\beta}{\beta}}\right]^{\frac{1}{1-\beta}} \tag{D-2}$$

# Appendix E. Growth Rates

In this first part I show that $g_1 > g_2$.

Notice that since $g_1$ and $g_2$ are

$$g_1 = \gamma N_{eq,L}^{\beta}\left[\frac{\breve{p}}{a_{t-1}} + \left(1-\breve{p}\right)\right] - 1 \quad \text{and} \quad g_2 = \gamma N_{d,L}^{*\beta}\left[\frac{\breve{p}}{a_{t-1}} + \left(1-\breve{p}\right)\right] - 1$$

it is sufficient to show that $N_{eq,L}^* > N_{d,L}^*$ . So, taking the ratio between the two (we just need it to be >1!), we have:

$$\frac{N_{eq,L}^*}{N_{d,L}^*} = \frac{\left[\delta(\chi)\bar{A}_t\right]^{\frac{1}{1-\beta}}\left[\frac{p_L}{x}\left(1-\frac{x}{p_L}\breve{\Omega}\theta a_t\right)\right]^{\frac{1}{1-\beta}}}{\frac{x}{p_L}\left[\delta(\chi)\bar{A}_t\right]^{\frac{1}{1-\beta}}\left[\frac{1-\frac{x}{p_L}\breve{\Omega}a_t}{1-\frac{x}{p_L}\breve{\Omega}}\right]^{\frac{1}{1-\beta}}}$$

or also

$$\frac{N_{eq,L}^*}{N_{d,L}^*} = \left(\frac{p_L}{x}\right)^{\frac{2-\beta}{1-\beta}}\left(\frac{\frac{p_L - x\breve{\Omega}\theta a_t}{p_L}}{\frac{p_L - x\breve{\Omega}\theta a_t}{p_L}}\frac{p_L - x\breve{\Omega}}{p_L}\right)^{\frac{1}{1-\beta}} \Longrightarrow \left(\frac{p_L}{x}\right)^{\frac{2-\beta}{1-\beta}}\left(\frac{p_L - x\breve{\Omega}}{p_L}\right)^{\frac{1}{1-\beta}}$$

which is $> 1$ for $x$ not too large![33]

[33] by a simulation, with $p_L = 0,3$, it takes that x<98,26, which far sufficient for our being realistic.

In the second part we will show that $g_3 > g_2$.

First, notice that since $\bar{N}^*_{eq,L} < N^*_{d,L}$ , then we can write $\bar{N}^*_{eq,L} = (1-f)\, N^*_{d,L}$, where $0 < f < 1$. Define the difference between the growth rates as

$$dg_{3,2} = g_3 - g_2 \Longrightarrow dg_{3,2} =$$
$$N^{*\beta}_{d,L}\left\{(1-f)^{\beta}\left(\gamma\breve{p} - \breve{p} + 1\right) - \gamma\left[p_L\left(1 + \frac{1}{a_{t-1}}\right) - 1\right]\right\}$$

Define $J = (1-f)^{\beta}\left[\breve{p}\,(\gamma - 1) + 1\right]$; notice that $J > 0$. In order to show that $g_3 > g_2$, we simply need to show that $dg_{3,2} > 0$. So we have $dg_{3,2} = N^{*\beta}_{d,L}\left\{J + \gamma\left[1 - p_L\left(1 + \frac{1}{a_{t-1}}\right)\right]\right\} > 0 \Longrightarrow J + \gamma\left[1 - p_L\left(1 + \frac{1}{a_{t-1}}\right)\right] > 0$ or also

$\gamma\,(1 - p_L) + J > \frac{\gamma}{a_{t-1}} p_L$, which is true for

$a_t > \frac{\gamma p_L}{\gamma(1-p_L)+J} = a^*$. Therefore, we have that $g_3 > g_2$ for $a_t > a^*$, which is true for reasonable value of parameters. In fact, from a simulation with reasonable values:[$f = 0,7; \gamma = 1,05; p_H = 0,7; \beta = 0,3$], it turns out that for $p_L \leq 0,55$ , $a^* < 0,4848$. So, the restriction that $a_t > a^*$ is not binding inasmuch as for the same parameters, it turns out that $a_{eq,L} = 0,51$[34] (recall that the intermediate region starts from $a_{eq,L}$)!

[34] See section 7.