optimal Pricing and network congestion: an attempt to modelize the Paris metro pricing

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Abstract

In this paper we show that the “Paris Metro Pricing” of Odlyzko (1997)can be solution f a maximization problem of the social welfare when the aversion of the consumers to the congestion is her/his private information. The solution obtained is rather robust insofar as there is no systematic divergence between the optimal function of congestion and that of first rank (in complete informations) and where the solution appears not very sensitive to the weighting used (represented by the opportunity cost of the public funds. Thus a private monopolist would implement a scheme giving results in term of congestion rates by types of users rather close to that which would choose a benevolent regulator.

The growth of the Internet was phenomenal and exceeded the most enthusiastic forecasts: the number of hosts, the number ofusers and the amount of the traffic doubled roughly every year since 1988. The price of this success was the increase in thecongestion on the whole of the networks of Internet, indeed "surfing" on the Internet is manifestly slow during the peak periods of traffic; some even estimate that 30% of traffic Internet concern the retransmission of lost packets! (see **??**Paxson (1997)).[1]

There are two major connected (linked) causes to this so important increase of the congestion on the networks of the Internet: The first is the

[1]The Internet is a whole of inter-connected "packet networks”, i.e an unspecified message (file, e-mail, pargraphe typed on a terminal) is a succession of information components a whole for the shipper and the recipient. The message is "cut out" of pieces called packets. The latter are sent independently the ones of the others on the networks of the Internet and can thus take various "roads". These packets are reassembled to form the original message when all the packages arrived at destination.

appearance of new increasingly band-width "greedy" applications.The second reason is obvioulsy the exponential increase of the net surfers. These two causes are connected because the more there are net surfers, the more those particular applications are used. This implies, obviously well, that the exponential increase of the request for band-width[2] is much more than proportional to the available band-width, i.e, the necessary capacity of the resources of network to treat (within a "suitable" time) the volume of information which circulate on the networks of the Internet.

This has an explanation: the Internet is privatized, indeed It does not exist at the present time any centralized authority which controls the Internet and the "managers of the networks" which are "profit maximizing clubs" concluded that increasing their capacity while increasing their price was not profitable. Thus, Internet, "the Networkof the networks" are largely marked by the presence of externalities of networks. In this work we concentrate only on the negative externalities of networks, i.e, the congestion: the value which an individual grants to a network of the Internet decreases with the number of individuals who use it.To control this phenomenon of congestion it is largely recognized that a tariffing of the Internet resources different from those into force currently is necessary. Indeed, the economists support that the broad heterogeneity in the uses of the Internet resources implies that an identical treatment of the packets circulating on the networks of the Internet (the current practice) is non-efficient and turn out to the "tragedy of the commons".

Many proposals based on usage pricing were made like those, recent, from Crémer and Hariton (1999). Full information is available on the Web site: (http://www.sims.berkeley.edu/resources/infoecon/) and in the collection of articles published by McKnight and Bailey. Additional references, summarized briefs, and criticisms are available (Clark (1996); Shenker (1995); Shenker, Clark, Estrin and Herzog (1996)). There is also interesting news proposals, such as the elegant PFP (Proportional Fair Pricing) of Gibbens and Kelly. Furthermore, varied techniques of "quality of service" are to be developed (for a general study and references, see Ferguson and Huston (1998)). There is also the "Smart Market" of Mackie-Mason and Varian (which inspired many authors) which differentiates the Net surfers according to their willingness to pay to send their packages of data on the networks of the Internet. It acts indeed of a second price bidding mechanism. However, to make it possible that packets with a high priority to completely block those of weaker priority

[2]In more technical terms, the processing capacity of the packets present on the networks of the Internet by the Internet resources, i.e, the CPU, hard disks... of the routers, servers of each network which represent the Internet.

violates the criterion of equity which appears important being with the eyes of the consumers.

The '"Paris Metro Pricing" of Odlysko (1997) is undoubtedly the most known tariffing of the Internet resources proposal. It into force applies to the Internet the tariffing concept of the Parisian Subway until the 80's. It acts of partition each network in several sub-networks (identical in term of band-width), each one of them treating all the "packets" equitably on the "best effort basis". These sub-networks would differ only in price paid to use them. It comes naturally that sub-networks whose prices are highest would attract less traffic and consequently would provide a better service, i.e, a higher flow.

Thus the application of this proposal would not make it possible to completely remove the congestion on the entirety of the networks but would allow some users, those whose the willingness to pay for sending their "packets of data" is the most important to suffer the least possible from the congestion, in other words to accelerate the routing of their "packets".

The PMP differs at the beginning from all these proposals by not maximizing some easily quantifiable objective function. It rather endeavours for its promoters to reach a maximum simplicity for the user and to take into account strong preferences of the users who were up to now difficult to introduce into formalized models.

We try here to formalize the concept of the "Paris Metro Pricing". An Internet service provider (ISP) in situation of monopoly offers to the net surfers the access to various sub-networks at different prices. The users, who are characterized by different degrees of aversion for the congestion, are distributed spontaneously between these sub-networks such that at the equilbrium the "more expensive" sub-networks are also the less congested ones. How the monopolist go does in an optimal way, in a situation where the degree of aversion for the congestion of each user is her/his private information, to assign to each type of user a part of the total capacity of which it lays out and a price of corresponding access? It is the object of this section which to try to answer this question in a precise way.

1 The model

The "Paris Metro Pricing" (PMP) differs at the beginning from every other proposition by not maximizing some objective function easily quantifiable. It tries rather according its promoters(developers) to reach a maximum simplicity for the user and account for the strong preferences of the users which have been as far a this difficult to introduce in formal models. We show here that the PMP can be the solution of a social welfare maximization problem when the degree of each consumer's aversion for congestion is his/her private information. The solution which we obtain is quite robust in so far as (i) there is no systematic divergence between the optimal congestion function we obtain and the first-best one (under complete information) and (ii) the solution appears quite unsensitive to the used level-headedness (represented by the opportunity cost of public funds). Thus a private monopoleur would make use of a scheme giving results in terms of congestion rates by types of users fairly close to the one which would be chosen by a kindly regulator under full information....

2 The model

We assume that there is a continuum of network users parametrized by a variable $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, with a strictly positive density $f(\theta)$ everywhere on this interval. The total number of users is normalized to 1. The surplus of a network user when he or she pays a price p for having access to this network and when the congestion rate is equal to y is

$$S(\theta, y, p) = u - \theta d(y) - p$$

where u is a strictly positive parameter, d is a continuously differentiable, strictly increasing and convex function . The rate of congestion y is defined as the ratio between the number of users of the subnetwork to which the type θ users connect and the capacity of this subnetwork. The value of θ measures the intensity of type-θ user's aversion to congestion. His/her value of θ is obviously a private information of each user.

In a "Paris Metro Pricing" scheme the network manager assigns to each type of users a fraction $K(\theta)$ of the entire network K and draws up a scale of differentiated access prices according to which users anticipate rationally the congestion rates $y(p)$ which follow from. This is an *implicit* relation,each user anticipating rationally the congestion rate which will be associated to each given access price. The manager assigns to each subnetwork a part of the total existing capacity K of the network

Each type θ of user chooses an access price and the congestion rate which solve the following program

$$\underset{p}{Max} \left[u - \theta d(y(p)) - p\right]$$

The first order condition of this problem is simply

$$-\theta d'(y(p))y'(p) - 1 = 0$$

or, equivalently, using the envelope theorem and noting $v(\theta) = \underset{p}{Max}\,[u - \theta d(y(p)) - p]$

$$v'(\theta) = -d(y(\theta)) \tag{1}$$

The second order condition of the user problem is $-\theta d^{'}(y(p))y"(p) - \theta d"(y(p))y'(p)^2 \leq 0$ which is equivalent to $p'(\theta) \geq 0$: the price paid by the users must increase at the equilibrium with their aversion to congestion. According to the first order condition above $y'(p) < 0$ which is equivalent to $y'(\theta) \leq 0$: the equilibrium rate of congestion must be an non-increasing function of the users aversion to congestion.

The network manager assigns a fraction $K(\theta)$ of the total capacity of the network to the type θ users and it must be checked at the equilibrium that $y(\theta) = \frac{f(\theta)}{K(\theta)}$ and, of course, that $\int_{\underline{\theta}}^{\overline{\theta}} K(\theta)d\theta = K$. Her/his objective is to maximize the social welfare. knowing that the opportunity cost of one dollar of public funds is $(1 + \lambda)$ due to the distortions associated to the taxes collect this social welfare is written as[3]

$$W = \int_{\underline{\theta}}^{\overline{\theta}} [u - \theta d(y(\theta)) - p(\theta) + (1 + \lambda)\,(p(\theta)]\, f(\theta)d\theta$$

2.1 The optimal second degree discrimination

Using the definition of $v(\theta)$ one can replace in W $p(\theta)$ by its value so as to obtain

$$W = \int_{\underline{\theta}}^{\overline{\theta}} [(1 + \lambda)(u - \theta d(y(\theta))) - \lambda v\,(\theta)]\, f(\theta)d\theta \tag{2}$$

So the problem of the regulator is to maximize W with respect to $y'(\theta)$ subject to the incentive constraints:

$$\begin{aligned} v'(\theta) &= -d(y(\theta)) \\ y'(\theta) &\leq 0 \end{aligned} \tag{3}$$

the participation constraint

$$v(\overline{\theta}) \geq 0$$

[3] $1 + \lambda$ can alternatively be considered as the ponderation granted to the profit in the objective function so that if $\lambda \to \infty$ we tend towards the case of a monopoly with lucrative goal.

and the "isoperimetric" constraint

$$\int_{\underline{\theta}}^{\overline{\theta}} K(\theta)d\theta = K \Leftrightarrow \int_{\underline{\theta}}^{\overline{\theta}} \frac{f(\theta)}{y(\theta)}d\theta = K. \tag{4}$$

To deal with this constraint we define a new variable $z(\theta) = \int_{\theta}^{\overline{\theta}} \frac{f(s)}{y(s)}ds$ which is so that $z'(\theta) = -\frac{f(\theta)}{y(\theta)}$, $z(\underline{\theta}) = K$ and $z(\overline{\theta}) = 0$.

We solve the problem using the Pontryagine maximum principle. To the state variables $v(\theta)$ and $y(\theta)$ we respectively associate $\delta(\theta)$ and $\eta(\theta)$. To the new variable $z(\theta)$ we associate $\mu(\theta)$. Let us now write the Hamiltonian of the problem as

$$H(\theta) = [(1+\lambda)(u - \theta d(y(\theta))) - \lambda v(\theta)] f(\theta) + \eta(\theta) y'(\theta) - \delta(\theta) d(y(\theta)) + \mu(\theta) z'(\theta)$$

which we maximize with respect to the control variable $y'(\theta)$ under the constraint $y'(\theta) \leq 0$. If we note $\gamma(\theta)$ the non-negative Lagrange multiplier associated to this constraint[4] we obtain the following necessary conditions of optimality:

$$\begin{aligned} &y'(\theta) = 0 \text{ if } \eta(\theta) \geq 0 \\ &y'(\theta) \in [-\infty, 0] \text{ if } \eta(\theta) = 0 \end{aligned} \tag{5}$$

$$\eta'(\theta) = -\frac{\partial H}{\partial y} = [(1+\lambda)\theta f(\theta) + \delta(\theta)]d'(y(\theta)) - \mu(\theta)\frac{f(\theta)}{y(\theta)^2} \tag{6}$$

$$\delta'(\theta) = -\frac{\partial H}{\partial v} = \lambda f(\theta) \tag{7}$$

$$\mu'(\theta) = -\frac{\partial H}{\partial z} = 0 \tag{8}$$

We deduce that $\mu(\theta) = \mu$ for every θ. Moreover from the condition (7) combined to the transversality condition $\delta(\underline{\theta}) v(\underline{\theta}) = 0$ we obtain

$$\delta(\theta) = \lambda F(\theta) \tag{9}$$

Henceforth we are going to focus on the *unconstrained solution* $y^*(\theta)$ obtained by discarding the incentive constraint $y'(\theta) \leq 0$ (or assuming that it is always satisfied). In this case $\eta(\theta) = 0 = \eta'(\theta)$ and $y^*(\theta)$ must then be such that (see equations (6) and (9):

[4] The Lagrangian is obtained by adding $-\gamma(\theta) y\prime(\theta)$ to the Hamiltonian.

$$y^*(\theta)^2 d'(y^*(\theta)) = \frac{\mu}{(1+\lambda)\theta + \lambda h(\theta)} \tag{10}$$

where $h(\theta) = \frac{F(\theta)}{f(\theta)}$ is the "hazard rate" which we will suppose classically a non-increasing function of θ.

If we note henceforth $g(y) = y^2 d'(y)$ the function g is strictly increasing[5]. Using the Inverse Function Theorem we then obtain

$$y^*(\theta) = g^{-1}\left(\frac{\mu}{(1+\lambda)\theta + \lambda h(\theta)}\right) \tag{11}$$

We note that, under the monotony hypothesis of the "hazard rate" done above, this solution is strictly decreasing in θ and verifies the second incentive constraint. We note also that $y^*(\theta)$ is increasing in μ and decreasing in λ.

Eventually the equilibrium value of μ is determined by the constraint (4):

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{f(\theta)}{g^{-1}\left(\frac{\mu}{(1+\lambda)\theta+\lambda h(\theta)}\right)} d\theta = K \tag{12}$$

This solution does exist and is unique since the left member is strictly decreasing in μ, tends towards $+\infty$ when $\mu \to 0$ and tends towards 0 as $\mu \to +\infty$. We show easily that μ is a decreasing function of K and an increasing function of λ.

This solution must be compared to the optimal solution under complete information which is simply given by the couple $(\widetilde{y}(\theta), \alpha)$ fulfilling the following conditions

$$\widetilde{y}(\theta) = g^{-1}\left(\frac{\alpha}{(1+\lambda)\theta}\right)$$

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{f(\theta)}{g^{-1}\left(\frac{\alpha}{(1+\lambda)\theta}\right)} d\theta = K \tag{13}$$

Indeed, under complete information, the regulator knows the type of each user, so their surplus will be equal to zero, in others terms

$$v(\theta) = 0, \nabla\theta$$

The problem of the regulator is then to maximize

[5] We supposed earlier that the function d is convex.

$$\widetilde{W} = \int_{\underline{\theta}}^{\overline{\theta}} \left[(1+\lambda)(u - \theta d(y(\theta)))\right] f(\theta) d\theta \tag{14}$$

subject to the constraint

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{f(\theta)}{y(\theta)} d\theta = K.$$

We build the Lagrangian:

$$L = \int_{\underline{\theta}}^{\overline{\theta}} [((1+\lambda)(u - \theta d(y(\theta))))f(\theta) - \alpha \frac{f(\theta)}{y(\theta)}] d\theta + \alpha K \tag{15}$$

where α is the multiplier of Lagrange non-negative associated to the constraint.

differentiating this expression with respect to $y(\theta)$ we obtain

$$\frac{\partial L}{\partial y(\theta)} = -(1+\lambda)\theta d'(\theta) f(\theta) + \alpha \frac{f(\theta)}{y(\theta)^2} = 0 \tag{16}$$

or equivalently

$$\widetilde{y}(\theta)^2 d'(\widetilde{y}(\theta)) = \frac{\alpha}{(1+\lambda)\theta}$$

We note as previously $g(y) = y^2 d'(y)$, and we obtain

$$\widetilde{y}(\theta) = g^{-1}\left(\frac{\alpha}{(1+\lambda)\theta}\right) \tag{17}$$

Proposition 1 *If θ is uniformly distributed on the interval $\left[0, \overline{\theta}\right]$*

(i) The optimal function of congestion rate is identical under complete and incomplete information: $y^*(\theta) = \widetilde{y}(\theta)$ for every $\theta \in \left[0, \overline{\theta}\right]$;

(ii) The optimal congestion rate $y^*(\theta)$ does not depend on the opportunity cost of public funds.

Proof: If μ is the unique solution of equation (12) thus, obviously, $\alpha = \frac{1+\lambda}{1+2\lambda}\mu$ is the unique solution of equation (13). Now straightforwardly $y^*(\theta) = \widetilde{y}(\theta)$ for every $\theta \in \left[0, \overline{\theta}\right]$. ■

The previous proposition establishes the identity between $y^*(\theta)$ and $\widetilde{y}(\theta)$ (and their common independence with respect to the opportunity cost λ of public funds) only in a particular case where the hazard rate is linear in θ. However there are noticeable differences in the general case between the results obtained in this model and standard results in the literature (see; for instance Mussa and Rosen (1978)). There are here no systematic distortions between the optimal congestion rates under complete and incomplete information such that, for instance, $y^*(\underline{\theta})$ would be always smaller than $\widetilde{y}(\underline{\theta})$. The intuition for this non-standard result lies obviously in the existence of a given capacity K and in the fact that equilibrium value of α and μ differ ($\mu > \alpha$). Moreover a variation of λ, the opportunity cost of public funds, has no systematic effect on the optimal congestion rate: actually an increase in λ results in an increase of μ...

Example: Let $d(y) = y$ and θ be distributed uniformly on the interval $[1, 2]$. Let $K = 100$ and $\lambda = 0.3$. In this case $y^*(\theta)$ and $\widetilde{y}(\theta)$ are represented below.

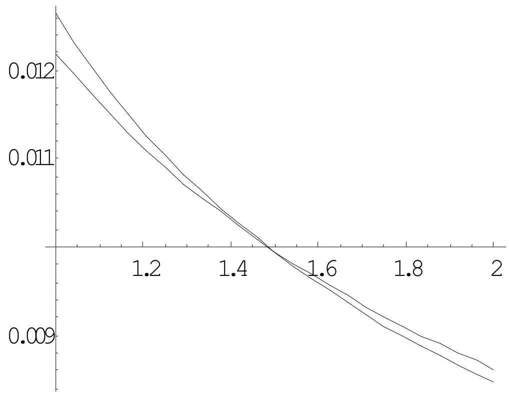

Figure 1:

In this example we see that $\widetilde{y}(\theta)$ is inferior (resp. superior) to $y^*(\theta)$ for low (resp. high) values of θ. The divergence is nevertheless limited enough to be neglected.

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