Dynamic Duopoly Competition with Switching Costs and Network Externalities*

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January 2004

Abstract

This paper analyzes competition in a two-period differentiated-products duopoly in the presence of both switching costs and network effects. We show that they have opposite implications on the demand side, specially in the first period. While the former reduces demand elasticities, the latter increases them. We derive the symmetric subgame perfect equilibrium outcome of the two period competition. Increases in marginal network benefits imply lower prices in both periods while the effects of switching costs are ambiguous. However, when network effects are strong, and switching costs are moderate, prices in both periods may be lower than those in a market without network effects and switching costs.

Keywords: dynamic duopoly, monotone comparative statics, network effects, switching costs.

JEL Classification: C73, D21, D43, L13, L21.

*We would like to thank Volkswagen Stiftung for the generous financial support which made this research possible. All errors are ours.

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1 Introduction

A product exhibits *network effects* when its value increases in the number of its users. On the other hand, switching costs arise when consumers face frictions to change the brand they consume either due to relationship specific investments or contractual obligations. We would like to refrain from starting this paper with mentioning telecommunication services or computer operating systems as prime examples of products exhibiting network effects and/or switching costs. Examples are plenty, and are presented in an illustrative manner elsewhere as in Shapiro and Varian (1998), Katz and Shapiro (1994), Klemperer (1995) and Farrell and Klemperer (2002). Interesting dynamic issues arising in industries with such characteristics have attracted economists' attention not only due to the intellectual challenges, but also due to increasing public policy debates over the operation of these industries. Thus, there is a large body of literature on network effects and switching costs which arose mainly in the last two decades. In fact, the recent survey by Farrell and Klemperer (2002) (FK hereafter) contains 35 pages of references suggesting a mature and saturated knowledge base.

We will follow FK in summarizing the main results of literature, and begin by noting that both network effects and switching costs could potentially extend the consumer choice problem dynamically. Current choices of consumers affect their future consumption leading to state dependent demands. Thus, expectations of consumers on future pricing policies and future size of sales of a firm play a key role in determining outcomes. In certain cases, historical accidents may determine long run behavior of a given industry. Firms face incentives inducing them to adopt "*bargains-then-ripoffs* type pricing policies. That is, early on firms compete fiercely to lock-in consumers, in order to exploit them in the future when switching costs are present, and in order to increase the willingness to pay of future generations in case of network effects. Locking into an inferior standard, excessive private incentives for incompatibility, distorted incentives for entry are common features of models studying such industries. In most models, consumers do not switch between brands in equilibrium. A message FK delivers is the similarity of outcomes in models with network effects and models with switching costs. For a full review of the literature, we refer the reader to FK and the references therein.

Surprisingly, however, there is no model which studies industries where both network effects and switching costs are present. Taking the risk of stating the obvious, we would like to note that network effects have *no* dynamic consequences when switching costs are nil. In such a case, it is completely optimal for consumers to be myopic, as they can switch between brands as they please every period. In a typical dynamic network effects model[1], consumers are assumed to purchase only once, usually as soon as they arrive to the market, and stay with their choice forever even though the net present value of buying an alternative might become positive at a future date.

To the best of our knowledge, all existing dynamic models of network effects presume lock-in, that is sufficiently high switching costs preventing consumers from switching between brands. Hence, it is curious whether parallels drawn between the results of models with switching costs and those with network effects are due to genuine similarity between switching costs and network effects, or just an artifact of presumption of lock-in in network effects models. Our goal in this paper is to attempt to identify the consequences of network effects and switching costs both on consumer behavior and strategies of firms when they co-exist in a meaningful way.

We adopt a very stylized model of preferences which allows consumers to switch between brands in equilibrium. We build on the model of Klemperer (1987) by simply appending a network benefit term to the valuation of products.[2] We will consider a simple two-period price setting model of competition between two firms which are horizontally differentiated à la Hotelling. Only some of the consumers survive to the next period. Those that leave the market are replaced by new consumers. Furthermore, some consumers receive a taste shock which changes their location on the unit interval, thus they might wish to switch the brand they buy. We assume that switching costs are sufficiently low that at least some of these consumers will

[1] See, for example, Farrell and Saloner (1986), Katz and Shapiro (1992).

[2] When network effects vanish, the model boils down to that of Klemperer (1987).

be able to change the brand they purchase. The rest of the consumers are rigid, that is, their preferences remain as in period one as well as they have very high costs hindering any desire to switch in the second period.[3]

Consumers form rational expectations of not only current network sizes but also of future network sizes and prices. Given prices consumers are able to compute fractions of current and future consumers buying from each firm correctly. For rational expectations demands to be well-behaved, that is to avoid situations where firms can corner the market, we assume that the marginal network benefits are sufficiently low.

We derive a subgame perfect equilibrium where firms share the market in both periods, in the second period some of the consumers with changing preferences switch the brands, while the rigid consumers purchase again from the same firm they shopped in the first period. This behavior could be supported in equilibrium only for certain parameters constellations. We derive sufficient conditions on the parameters, which simply states that switching costs must be sufficiently low to induce switching in the second period as well as the network effects in order to avoid tipping towards one product in each period, and the size of the population of rigid consumers must be sufficiently small.

The rational expectations demand we derive for each period exhibit interesting properties. First period demands become more price sensitive with higher marginal network benefits. In contrast, however, increasing switching costs reduce price sensitivity of first period demands. Thus, in the first period switching costs and network effects operate in completely opposite directions. In the second period, both switching costs and network effects imply a positive shift in demand for a firm which carries over a market share more than one half. However, the latter effect is present only when there are switching costs. That is, in the absence of switching costs, network effects have no dynamic consequences. The second period demands become more price

[3] We keep the rigid segment in the model in order to preserve the parallels with Klemperer (1987). Alternatively, one could view our model as one with a distribution of switching costs in the population.

sensitive when marginal network benefits increase, while switching costs have no impact on the price sensitivity.

Second period equilibrium prices increase with the customer base of a firm carried over from the first period. However, the subgame perfect equilibrium outcome of the two period competition is symmetric. Thus, in both periods firms share the market equally. Second period prices increase in the share of rigid consumers, decrease with marginal network effects and are not affected by the switching costs.

First period prices unambiguously reduced by higher marginal network benefits. However, switching costs may have different effects on the equilibrium prices. We explore how switching costs and network effects impact the equilibrium prices in the first period by means of Monotone Comparative Statics. This allows us to uncover the mechanisms that change equilibrium prices in response to a change in one of these features.

First period equilibrium prices turn out to be a quadratic-convex function of the switching costs faced by flexible consumers. Thus, for certain parameter constellations, increasing switching costs reduce first period prices. This occurs when marginal second period profits respond to a change in the switching costs more than first period marginal profits. We show that this could occur when switching costs are low, in particular, when there are no rigid consumers, there is no impact of a change in switching costs on marginal first period profits around zero, while second period marginal profits decrease in switching costs. Thus introducing slightly higher switching costs decrease first period prices.

In section 2, we present the model. We derive the equilibrium and discuss its properties in section 3. Section 4 concludes.

2 The Model

As in Klemperer (1987), we analyze competition between two firms over two periods. The consumer population have peculiar characteristics. For example, they value the number of consumers purchasing a brand, they incur switching costs if they wish to change their brand choice, and some of them have changing tastes. The model is very stylized and builds on Klemperer (1987) by simply appending a network benefit component to the utilities of consumers. In fact, it is equivalent to that of Klemperer (1987) when network effects vanish. Let us begin by describing features of consumer utilities which remain the same over the two periods. We assume that consumers have a reservation price,[4] denoted by v, that is sufficiently high so that all consumers on the market buy as soon as they arrive. The reservation price is the same for both products and all consumers in any period. Furthermore, each consumer has an affinity towards one of the brands in each period which could be due to effects of a typical consumer's social circle or exposure to different marketing mixes. We capture this affinity by means of standard Hotelling horizontal differentiation model.

Thus, we assume firms a and b are located at opposite ends of the unit interval, that is $L_a = 0$ and $L_b = 1$ where L_i denotes the location of firm i. Consumers are assumed to be uniformly distributed between the two firms, and their location characterizes their ideal product. However, due to the fact that they will have to consume one of the available brands, they incur a disutility proportional to the distance to the product considered. Formally, if a consumer located at $x \in [0, 1]$ makes her purchase from i, she incurs a utility reduction equal to $t \mid x - L_i \mid$, where t measures the magnitude of this reduction, or with standard terminology unit "transport" costs.

Moreover, consumers derive a network benefit proportional to the number of other consumers purchasing a given product. That is, if a product is bought by N consumers, then each of these consumers derive a benefit equal to kN, where k measures the magnitude of network effects. In particular, we will require the magnitude of the network effects to be less than that of the transport costs—in particular, $k < 2t/3$, in order to avoid situations where one firm corners the market. Hence, in each period, both firms will have positive sales.

The consumer population evolve in different ways from period one to two. Particularly, only

[4]We could also refer to this term as the stand alone value, as customary in the network effects literature.

a fraction, $1-\nu$, of the first period consumers survive to the next period, and those who leave the market are replaced by new unattached consumers. A second group of mass μ, receive a taste shock, and thus, are relocated along the unit interval. This taste shock could be interpreted as a change in a consumer's affinity due to changes in her social circle, as well as exposure to a different marketing mix. We assume the taste shock to be independent of first period tastes; admittedly a rather strong assumption. Hence, some consumers may find a product different than what they have bought in the first period more attractive. However, to change the brand they consume, they will have to incur a switching cost, s, which we assume to be sufficiently small so that some consumers switch in equilibrium. The rest of the consumer population, with a mass of $1-\mu-\nu$, is rigid in their tastes and face much higher switching costs, s_r.[5] Therefore, they continue to purchase from the firm which they bought in the first period. For expositional ease, we refer to unattached second period consumers[6] as *new* (n), the group with changing preferences and low switching costs as *flexible* (f), and those with high switching costs and constant tastes as *rigid* (r) consumers.

In summary, the net first period utility of a consumer located at $x \in [0,1]$ can be written as

$$U_1^i(x, p_1^i, N_1^i) = v + kN_1^i - \mid x - L_i \mid t - p_1^i, \qquad i = \{a, b\}$$

where N_τ^i and p_τ^i represent the expected network size and the retail price of firm i at period τ. While the second period utility, which also is a function of their type and first period choice, is given by

$$U_{2h}^{i|j}(x, p_2^i, N_2^i) = v + kN_2^i - \mid x - L_i \mid t - p_2^i - I(h = f, i \neq j)s - I(h = r, i \neq j)s_r,$$

where $i \in \{a, b\}$,$j \in \{0, a, b\}$, $h \in \{n, f, r\}$ and $I(h = g, i \neq j)$ is the indicator function which

[5] Notice that even if they have changing preferences, sufficiently high switching costs would prevent them from switching.

[6] One could include this group to those with changing preferences, but assume that this group incurs zero switching costs. Thus, if we allow everybody to change their references, we arrive at a model with a distribution of switching costs; namely, none, moderate, and high.

equals one whenever $h = g$ and $i \neq j$, and zero otherwise where $g \in \{f, r\}$.

Consumers choose that brand which maximizes their utility. This problem is relatively easier in the second period, since consumers will learn their types, and given this information and their expectations on the contemporary network sizes, they will select the brand which provides them with the highest net benefit. On the other hand, the first period choice is significantly more involved. First, consumers are uncertain about which group they will belong to in the second period. Moreover, given that their choice this period constrains their behavior due to potential switching costs, they need to have beliefs about future. Given prices in the first period, they need to form expectations about current and future network sizes and future prices. We will adopt Rational Expectations (RE) as the mechanism for expectation formation. Therefore, a typical consumer will select the brand that maximizes the discounted sum of lifetime utilities,

$$U^j(x, p_1^j, N_1^j) = U_1^j(x, p_1^j, N_1^j) + \delta E\big[U_{2h}^{i|j}(\chi, p_2^i, N_2^i) \mid j, p_1^a, p_1^b\big],$$

where δ is the discount factor and $E[\cdot \mid \cdot]$ is the conditional expectations operator. Expectations are taken over the distribution of types, and distribution of potential second period tastes. Notice that the cumulative expected utilities depend only on the first period observables. Consumers compute N_1^j,N_2^j and p_2^j rationally. That is, N_τ^j is a demand function conditional on prices in τ, and delivers the realized network sizes for all relevant prices for $\tau = 1, 2$. In the first period, consumers solve the firms' problem in the future and anticipate second period equilibrium prices and therefore network sizes exactly.

Firms select prices in order to maximize their discounted cumulative profits. For simplicity we assume that firms have the same discount factor as consumers, δ. We assume away fixed costs, and normalize marginal costs zero which is quite an innocent assumption given the linearity of consumer utilities. We presume that firms cannot differentiate among old locked-in and new consumers, and thus, restrict firms strategies to nondiscriminatory, linear prices.

3 The Two-Period Game

Given the preferences we have introduced, we will look for a subgame perfect equilibrium in prices. However, we have certain ex ante restrictions on the nature of this equilibrium. We presume that the market is covered and shared in each period which in turn requires conditions on v and k. Furthermore, we assume some of the flexible consumers are able to switch in equilibrium in both directions which imposes an upper bound on s. On the other hand, the switching costs faced by the rigid types needs to be sufficiently high so that they continue buying the same brand in the second period. Furthermore, these main features occur for each level of market share firms carry over from the first period. In the following, we first derive the equilibrium outcome assuming that the conditions which make such outcomes possible are met, and then provide restrictions on the parameters to indeed satisfy these conditions.

3.1 The Second Period

We start solving the two-period game by finding the second period equilibrium prices. We first need to derive second period demand functions in order to construct the profits. Due to switching costs, the second period choices of consumers which remain in the market depend on their first period choices, therefore we need to find the demands from each consumer group. Let us first consider the new unattached consumers who are distributed uniformly along the unit interval with mass ν. This group simply compares the utilities from product a and b, and select the brand which provides them the highest net benefit. Thus, finding the indifferent consumer is sufficient to identify the demand faced by firms from this group of consumers. Formally, let $d_2^{a|0}$ denote this location, then it must satisfy $U_{2n}^{a|0}(d_2^{a|0}, p_2^a, N_2^a) = U_{2n}^{b|0}(1-d_2^{a|0}, p_2^b, 1-N_2^a)$ where p_2^i and N_2^i are the second period price and expected network size of firm i respectively. Notice due to the fact that consumers are uniformly distributed, $d_2^{a|0}$ also is equal to the fraction of new consumers buying from firm a. Solving this equation yields,

$$d_2^{a|0} = \frac{1}{2} + \frac{k}{2t}[2N_2^a - 1] + \frac{1}{2t}[p_2^b - p_2^a], \tag{1}$$

and $d_2^{b|0} = 1 - d_2^{a|0}$.

The flexible group of consumers, which has a mass of μ, evaluate each product anew as they now are placed at a different point along the unit interval. For example, one consumer who was at 0 in period one, could very well be located at 1 in period 2. Therefore, who they have bought from in the first period has a crucial impact on their second period choice. Let us first consider those who have bought from a in the first period. Identifying the demands of this group of consumers is once again equivalent to finding the indifferent consumer with one difference. Even though firm 2 announces a retail price of p_2^b, consumers face $p_2^b + s$ when they consider buying from b. Recall our prevailing assumption that switching costs, s, are sufficiently low that some will prefer switching to firm b. Let us denote the fraction of consumers from this group which prefer firm a by $d_2^{a|a}$. Then, it must satisfy $U_{2f}^{a|a}(d_2^{a|a}, p_2^a, N_2^a) = U_{2f}^{b|a}(1 - d_2^{a|a}, p_2^b, 1 - N_2^a)$, and is given by

$$d_2^{a|a} = \frac{1}{2} + \frac{k}{2t}[2N_2^a - 1] + \frac{1}{2t}[p_2^b + s - p_2^a]. \tag{2}$$

The fraction of consumers who has bought a in the first period, but prefers b in the second period is simply $d_2^{b|a} = 1 - d_2^{a|a}$. Applying similar arguments, the fraction of flexible consumers who have purchased from b and switches to a, $d_2^{a|b}$, solves $U_{2f}^{a|b}(d_2^{b|a}, p_2^a, N_2^a) = U_{2f}^{b|b}(1 - d_2^{a|b}, p_2^b, 1 - N_2^a)$, and is given by

$$d_2^{a|b} = \frac{1}{2} + \frac{k}{2t}[2N_2^a - 1] + \frac{1}{2t}[p_2^b - p_2^a - s]. \tag{3}$$

Likewise, the fraction of consumers who remain loyal to b is $d_2^{b|b} = 1 - d_2^{a|b}$. Notice that these consumers perceive firm a's price as $p_2^a + s$.

Finally, the fraction of consumers with unchanged preferences $(1 - \nu - \mu)$ will choose in the second period exactly the same brand as before, since their switching cost s_r is assumed to be sufficiently high.

Therefore, the total second period demands for technology a and b may be expressed as:

$$d_2^i = \mu[d_2^{i|a} N_1^a + d_2^{i|b} N_1^b] + (1-\mu-\nu)N_1^i + \nu d_2^{i|0}, \qquad i = \{a, b\}. \tag{4}$$

Rational expectations about the network sizes imply $N_2^a = d_2^a$ and $N_2^b = 1 - N_2^a = 1 - d_2^a = d_2^b$. Let

$$\alpha = \frac{s\mu + t(1-\mu-\nu)}{2(t - k\mu - k\nu)}$$

and

$$\beta = \frac{\mu + \nu}{2(t - k\mu - k\nu)}.$$

Solving (4) with imposing the rational expectations restrictions yields

$$d_2^a = \frac{1}{2} + \beta(p_2^b - p_2^a) + \alpha[2N_1^a - 1], \tag{5}$$

and $d_2^b = 1 - d_2^a$. For each firm to face downward sloping demand curves, $t - k\mu - k\nu > 0$ must hold; that is, the network benefits must be small relatively to the transportation costs or the share of rigid consumers must be relatively high.

It is easy to verify that whenever $t - k\mu - k\nu > 0$

$$\frac{\partial \alpha}{\partial s} = \frac{\mu}{2(t - k\mu - k\nu)} > 0,$$

$$\frac{\partial \beta}{\partial s} = 0,$$

$$\frac{\partial \alpha}{\partial k} = \frac{(s\mu + t(1-\mu-\nu))(\mu+\nu)}{2(t - k\mu - k\nu)^2} = 2\alpha\beta > 0,$$

and

$$\frac{\partial \beta}{\partial k} = \frac{(\mu+\nu)^2}{2(t - k\mu - k\nu)^2} = 2\beta^2 > 0.$$

Thus, a close inspection of (5) suggests that, the second period demand of a firm with a user base larger than one half shifts outward, while the demand of the other firm contracts when switching costs increase. However, the price responsiveness of the demand is not affected by the same change which is an artifact of our two period model. Since there is no future, neither the new consumers nor the flexible ones need to worry about low current prices implying high ones in the future and vice versa.

Similarly, a slight increase in k not only shifts the demand of the firm with a higher user base outward, but also makes the demands more sensitive to price differentials. The price effect is due to rational expectations; i.e., consumers observing a price differential expect the demand of the lower price firm to increase both due to an increase in utility via price directly and via the network benefits indirectly. The upward shift in demand however occurs only when coupled with high switching costs. When $s = 0$ and $\nu + \mu = 1$, that is when there are no switching costs and no rigid consumers, the network effects only have an impact via price sensitivities.

It is exactly this point which have not received much attention in the literature. The similarities between results obtained in models with switching costs and in models with network effects arise due to the outward shift of the demand when consumers are locked-in. But what locks consumers in is not the network effects, it is the switching costs which are usually assumed to be very high that no one switches. When we derive first period demands below, we would demonstrate similar patterns in the first period demands also.

Given that the fixed and marginal costs are normalized to zero, the second period profit functions of the firms are simply their revenues and given by $\Pi_2^a = p_2^a d_2^a$ and $\Pi_2^b = p_2^b d_2^b$. And due to the linearity of demands, the profit functions are concave in strategies of each firm.[7] Thus, first order conditions(FOCs) describe a candidate Nash equilibrium with prices given by

$$\begin{aligned} p_2^i &= \frac{1}{2\beta} + \frac{\alpha}{3\beta}[2N_1^i - 1] \\ &= \frac{t}{\nu + \mu} - k + \frac{1}{3}\frac{(s\,\mu + t(1 - \nu - \mu))\left(2\,{N_1}^i - 1\right)}{\nu + \mu}, \qquad i = \{a, b\}. \end{aligned} \tag{6}$$

However, note that we have imposed certain behavioral assumption on the demand side when we derived rational expectations demands. These behavioral assumptions translate to constraints which firms should take into account when formulating their best responses. Namely,

[7] Notice that $\beta \geq 0$.

we require some consumers to switch brands in the second period which is only possible when $0 < d_2^{a|b} \leq d_2^{a|a} < 1$. If this condition holds, then $0 < d_2^{a|0} < 1$, since $d_2^{a|b} \leq d_2^{a|0} \leq d_2^{a|a}$. Observe that both $d_2^{a|a}$ and $d_2^{a|b}$ will be functions of N_1^i in equilibrium, therefore these restrictions should hold for every possible value of N_1^i, namely $0 \leq N_1^i \leq 1$. This is necessary, since, in the first period, firms would foresee the equilibrium in the second period. By restricting our attention to cases where the postulated behavior occurs for each N_1^i, we avoid situations where second period profit functions become non-differentiable for certain strategies that might arise in the first period.

Furthermore, as $\mu+\nu \to 0$, firms place a higher weight on revenues from the rigid consumers, and since these consumers face very high switching costs, each firm will find it most profitable to exploit this segment to the fullest extent. Namely, they might select their prices in order to drive the net surplus of the marginal rigid consumer to zero, which in turn might drive their demand from new and flexible consumers to zero. In fact, such a strategy may be profitable for any value of $\mu + \nu$ when v is sufficiently large. Remember also that we have assumed v to be sufficiently large that all consumers buy for reasonable ranges of prices. Even though we do not explicitly derive the necessary conditions, we assume that v is sufficiently large to induce all consumers to participate, while it is sufficiently small that neither firm finds it optimal to just serve the rigid consumers for all $0 \leq N_1^i \leq 1$ and all reasonable prices of the other firm.

Let

$$\mathcal{P} = \left\{ \mu + \nu \in \left[\frac{2}{5}, 1\right], k \in \left[0, k_{max}\right], s \in \left[0, s_{max}\right] \right\}$$

where

$$s_{max} = t - \frac{t(1-\nu)(1-k\beta)}{t\beta + (2\mu+\nu)(1-k\beta)},$$

and

$$k_{max} = \frac{2}{3}t.$$

At the candidate equilibrium prices, the type of behavior we have postulated, i.e. new consumers buy from both firms, some flexible consumers switch to the other brand, and rigid ones stick to their first period choice, is realized if parameters belong to $\mathcal{P}$.

Lemma 1 *A sufficient condition for the second period prices given in (6) to constitute a Nash equilibrium is* $(\mu + \nu, k, s) \in \mathcal{P}$*. At these prices, the equilibrium demands are*

$$d_2^i = \frac{1}{2} + \frac{1}{3}\alpha(2N_1^i - 1) \tag{7}$$

and the equilibrium profits are

$$\Pi_2^i(N_1^i) = \frac{1}{36\beta}[3 + 2\alpha(2N_1^i - 1)]^2 \tag{8}$$

where $i = \{a, b\}$*.*

Proof. See Appendix.

We would like to emphasize once again that the restrictions defining $\mathcal{P}$ are only sufficient but not necessary to induce the postulated behavior. Thus, the equilibrium prices are valid for a larger set of parameters. The second period equilibrium prices have a few interesting properties. First observe that, whenever $k < k_{max}$, $t - k\mu - k\nu > 0$, thus both α and β are positive and demands are downward sloping. Therefore, a firm carrying over a market share that is larger than one half from the first period is able to charge a higher price. Moreover, the second period profits are increasing in the first period customer base which makes lock-in valuable. This would give both firms incentives compete more fiercely in the first period.

Both second period equilibrium prices and profits increase in switching costs when a firm has a customer base, N_1^i, that is larger than one half. Thus, an increase in the switching costs benefit the dominant firm in the first period more. On the other hand, an increase in the network effects, k, decrease both the price and profit of a firm which carries over more than half of the first period consumers to the second period. Thus, switching costs and network effects are forces which act in completely opposite directions in equilibrium in the second period.

If the firms carry over a market share that is closed to one half from the first period and $\mu + \nu \to 1$, then the second period equilibrium prices could well fall below t, the price which would have prevailed in the absence of switching costs and network effects. Thus, the presence of network effects might lead even second period to be a fiercely competitive period.

3.2 The First Period

Consumers face a much more complicated task in decision making in the first period. They need to evaluate a stream of benefits for two periods for each product in order to select one. Consumers do not know their type initially, thus they need to figure out their second period actions conditional on first period choices. With probability ν, they will leave the market in which case they receive no benefits, while with probability $1-\mu-\nu$, they will be rigid and will buy the same brand as in the first period. They will belong to the group of flexible consumers, i.e. will be redistributed along the unit interval, with probability μ. Therefore, they will switch to the other brand with some probability whichever brand they buy in the first period. For example, if they are considering the second period benefit conditional on having bought brand a in the first period, they will know that they will switch to brand b if their new location along the unit interval is larger than $d_2^{a|a}$ and remain with brand a otherwise. Formally, their expected benefit will be

$$EU^a = \int_0^{d_2^{a|a}} U_{2f}^{a|a}(\chi, p_2^a, N_2^a)d\chi + \int_{d^{a|a}}^1 U_{2f}^{b|a}(\chi, p_2^b, N_2^b)d\chi.$$

Similarly, the expected benefit of a flexible consumer conditional on buying brand b in the first period can be written as

$$EU^b = \int_0^{d_2^{a|b}} U_{2f}^{a|b}(\chi, p_2^a, N_2^a)d\chi + \int_{d^{a|b}}^1 U_{2f}^{b|b}(\chi, p_2^b, N_2^b)d\chi.$$

In doing these complicated calculations, we assume that consumers rationally infer next period prices (p_2^a, p_2^b), next period network sizes (N_2^a, N_2^b) and critical values determining whether they switch or not, $(d_2^{a|a}, d_2^{a|b})$. Observe that each of these quantities in equilibrium turns out to be a function of first period customer base of each brand, and thus, first period prices. Therefore, given first period prices, rational consumers should be able to compute first period demands which in turn determine second period equilibrium prices, network sizes, and critical values.

Therefore, in the first period, a rational consumer chooses that brand which delivers highest lifetime utility, that is they compare

$$U^a(x, p_1^a, N_1^a) = [r - p_1^a - tx + kN_1^a] + \delta\Big[\mu EU^a + (1-\mu-\nu)\big[r - p_2^a - tx + kN_2^a\big]\Big]$$

and

$$U^b(x, p_1^b, N_1^b) = [r - p_1^b - t(1-x) + kN_1^b] + \delta\Big[\mu EU^b + (1-\mu-\nu)\big[r - p_2^b - t(1-x) + kN_2^b\big]\Big].$$

Computing the difference yields

$$\begin{aligned} U^a(x, p_1^a, N_1^a) - U^b(x, p_1^b, N_1^b) &= p_1^b - p_1^a + (t - k + 2kN_1^a - 2tx) \\ &\quad + \frac{\delta s\mu}{t}\big(k(2N_2^a - 1) + p_2^b - p_2^a\big) \\ &\quad + \delta(1-\mu-\nu)\big(p_2^b - p_2^a + (t - k + 2kN_2^a - 2tx)\big), \end{aligned} \tag{9}$$

where we have used $N_1^b = 1 - N_1^a$ and $N_2^b = 1 - N_2^a$. Observe that the right hand side of (9) is decreasing in x, the distance from brand a. Thus, if there is a consumer indifferent between brands, all those consumers to the left will purchase brand a and to right will buy brand b.

The demands faced by firms, therefore, can be identified by finding the location of the indifferent consumer in the first period. Let d_1^a denote this location then it solves

$$U^a(d_1^a, p_1^a, N_1^a) - U^b(d_1^a, p_1^b, N_1^b) = 0. \tag{10}$$

Rational expectations on first period network sizes require $d_1^a = N_1^a = 1 - N_1^b = 1 - d_1^b$. Imposing this condition as well as substituting second period prices from (6), second period rational expectations network sizes from (7), we solve (10) for the first period demands yielding

$$d_1^i = \frac{1}{2} + \frac{p_1^j - p_1^i}{\gamma}, \qquad i \in \{a, b\} \quad \text{and} \quad j \neq i, \tag{11}$$

where

$$\gamma = 2(t-k) + 2t\delta(1-\mu-\nu) + \frac{4}{3}\frac{\alpha\delta(s\mu + t(1-\mu+\nu))(1-k\beta)}{t\beta}.$$

Whenever $k \leq k_{max}$, we have $t - k > 0$ and $1 - k\beta > 0$, implying $\gamma > 0$, therefore first period demands are downward sloping.[8]

Switching costs and network effects have their impact on the first period demands through γ. A brief inspection reveals that

$$\frac{\partial \gamma}{\partial k} = -2 - \frac{8\delta\alpha^2}{3} < 0,$$

and

$$\frac{\partial \gamma}{\partial s} = \frac{8\delta\mu\alpha(1 - k\beta)}{3t\beta} > 0.$$

That is, the first period demand becomes more sensitive to prices with an increase in network effects, while they become less price sensitive with an increase in switching costs. The latter is due to the fact that, rational consumers forecast a larger price for the firm which carries over a larger customer base to the second period—the lower priced firm in the first period. Hence, consumers do not easily buy in to initial price cuts. On the other hand, a lower price in the first period implies a larger group of consumers who would be "locked-in" in the second period implying a larger network benefit. A lower price in the first period allows consumers to coordinate on one firm not only in the first period but also in the second period. The difference between switching costs and network effects are starker in the first period; they are demand side forces in completely opposite directions.

The cumulative profit functions of the firms are

$$\Pi^a = p_1^a d_1^a + \delta \Pi_2^a(d_1^a)$$

and

$$\Pi^b = p_1^b d_1^b + \delta \Pi_2^b(d_1^b),$$

where (Π_2^a, Π_2^b) are the second period equilibrium profits given in (8). For both these profit functions to be concave,

$$4\delta\alpha^2 < 9\gamma\beta$$

[8]We would like to note that when $k = 0$, γ reduces to y introduced in Klemperer (1987) pp. 148.

must hold. We show in the appendix that when we restrict our attention to parameters in $\mathcal{P}$, this indeed is true. Hence solving the FOCs yields symmetric first period candidate equilibrium prices given by

$$p_1^a = p_1^b = \frac{1}{2}\gamma - \frac{2\delta\alpha}{3\beta}. \tag{12}$$

At these prices the profits of the firms are equal and given by

$$\Pi^i = \frac{\gamma}{4} + \frac{\delta}{4\beta} - \frac{\delta\alpha}{3\beta}, \tag{13}$$

which we show, in the appendix, to be non-negative when $(\mu + \nu, k, s) \in \mathcal{P}$.

Proposition 1 *A sufficient condition for the prices given in (12) to constitute a Nash equilibrium in the first period is* $\{\mu + \nu, k, s)\} \in \mathcal{P}$. *The equilibrium first period demands turn out to be*

$$d_1^a = d_1^b = \frac{1}{2}. \tag{14}$$

Given the first period customer bases, the second period equilibrium prices are also symmetric and given by

$$p_2^a = p_2^b = \frac{t}{\mu + \nu} - k \tag{15}$$

while the equilibrium demands in the second period also turn out be

$$d_2^a = d_2^b = \frac{1}{2}. \tag{16}$$

Proof. See appendix.

The second period equilibrium prices[9] are always positive, while the first period prices could be negative. One likely configuration where first period prices can be negative occurs when $s \to s_{max}$, $k \to k_{max}$, $\delta \to 1$, $\mu + \nu \to 1$ and $\mu > 1/2$. The first period equilibrium prices are quadratic convex in s—a feature we explore further when $\mu + \nu \to 1$ below. But it is important

[9]Note once again that we only provide sufficient conditions in proposition 1, and the equilibrium is valid for a larger set of parameters.

to note that the quadratic convex nature of equilibrium prices implies that for some parameter constellations, increasing the switching cost slightly may lead to a decrease in first period prices. This is particularly interesting since we have previously shown that increasing the switching costs reduces demand elasticities in the first period. However, it also increases the value of carrying over a larger user base to the second period.

On the other hand, both first and second period prices decrease in the marginal network benefits, k.[10] Therefore, when there are network effects, markets with switching costs may not be as anticompetitive. In fact, the higher the network effects the lower the prices, and there may be cases where prices fall below those in a market without rigid consumers, network effects and switching costs. That is, any worry policy makers might have concerning high prices due to switching costs may not be necessary in the presence of strong network effects.

Let p_τ^{sk} be the equilibrium prices in the presence of both switching costs and network effects, p_τ^k be the equilibrium price when only network effects exist, p_τ^s be the equilibrium price with just switching costs and p_τ be the equilibrium price without switching costs and network effects in period τ. It is easy to verify that $p_2 = t/(\mu+\nu)$ and $p_1 = t(1+\delta(1-\mu-\nu)/3)$. Observe that as $\mu+\nu \to 1$, both p_1 and p_2 approach t, the price that would have prevailed in a static standard Hotelling model. In the following proposition, we summarize relationships of these prices both in the first and second period.

Proposition 2 (Price Orders)

1. *The second period equilibrium prices can be ordered in two ways:*
   - *Low network benefits:* $p_2^s > p_2^{sk} > p_2 > p_2^k$
   - *High network benefits:* $p_2^s \geq p_2 > p_2^{sk} > p_2^k$,
2. *The first period equilibrium prices must fulfill two conditions:*

[10] First period price are decreasing in k, since γ is decreasing in k, while α/β is constant in k.

- $p_1^s > p_1^{sk}$ *and* $p_1 > p_1^k$

Notice the indeterminacy of first period price rankings. As we have noted above, first period prices could fall below marginal cost—zero, in our model—for certain parameter constellations. The clear message, however, is that the presence of network effects reduce prices in both periods. In the next subsection, we investigate the incentives of firms in setting their prices in the first period, and try to uncover the mechanisms through which switching costs and network effects shape the first period equilibrium.

3.3 The Monotone Comparative Statics

In this subsection, we explore further the impact of switching costs and network effects on first period prices. We employ monotone comparative statics (MCS) to uncover the mechanisms both these features affect the incentives of the firms.[11] The MCS allow us to separate the impact of switching costs and network benefits on the equilibrium prices into first and second period effects.[12] In the arguments below we maintain the assumption that $\{\mu + \nu, k, s\} \in \mathcal{P}$.

The derivative of the best response function of firm a, $R(p_1^b, s, k)$ with respect to switching costs can be written as[13]

$$\frac{\partial}{\partial s} R(p_1^b, s, k) = -\frac{\frac{\partial^2}{\partial s \partial p_1^a} \Pi^a(R(p_1^b, s, k), p_1^b, s, k)}{\frac{\partial^2}{\partial {p_1^a}^2} \Pi^a(R(p_1^b, s, k), p_1^b, s, k)}.$$

The denominator is negative because the profits are concave in p_1^a. Thus, the sign of the left hand side depends on the sign of the numerator, which is determined by a few non-zero partial derivatives and given by

$$\frac{\partial^2 \Pi^a}{\partial s \partial p_1^a} = p_1^a \frac{\partial^2 d_1^a}{\partial s \partial p_1^a} + \delta p_2^a \left[\left(\frac{\partial d_2^a}{\partial p_2^b} \frac{\partial p_2^b}{\partial d_1^a} + \frac{\partial d_2^a}{\partial d_1^a} \right) \frac{\partial^2 d_1^a}{\partial s \partial p_1^a} + \left(\frac{\partial d_2^a}{\partial p_2^b} \frac{\partial^2 p_2^b}{\partial s \partial d_1^a} + \frac{\partial^2 d_2^a}{\partial s \partial d_1^a} \right) \frac{\partial d_1^a}{\partial p_1^a} \right] \tag{17}$$

[11] In doing so we follow Vives (1999),pp.34-39.

[12] Below, we will only present the results; see appendix for their derivation.

[13] Similar arguments apply for the best response function of firm b due to symmetry.

In writing (17), we suppressed arguments of $d_1^a = d_1^a(p_1^a, p_1^b, s, k)$, $p_2^b = p_2^b(d_1^a(p_1^a, p_1^b, s, k), s, k)$, and $d_2^a = d_2^a(p_2^a(d_1^a(p_1^a, p_1^b, s, k), s, k), p_2^b(d_1^a(p_1^a, p_1^b, s, k), s, k), d_1^a(p_1^a, p_1^b, s, k), s, k)$. When we evaluate (17) at the first period equilibrium prices, we obtain the direction of change in the best response function of firm a in the first period due to a change in s around the equilibrium. After substituting the expressions for partial derivatives and equilibrium prices, we get

$$\begin{aligned}\frac{\partial^2 \Pi^a}{\partial s \partial p_1^a} &= \underbrace{\frac{3\gamma\beta - 4\alpha\delta}{6\beta}\frac{8\delta\mu\alpha(1-k\beta)}{3t\gamma^2\beta}}_{+/-} \\ &\quad + \delta\frac{1}{2\beta}\Bigg[\underbrace{\beta\left(-\frac{2\alpha}{3\beta}\right)\frac{8\delta\mu\alpha(1-k\beta)}{3t\gamma^2\beta}}_{-} + \underbrace{2\alpha\frac{8\delta\mu\alpha(1-k\beta)}{3t\gamma^2\beta}}_{+} \\ &\quad + \underbrace{\beta\left(-\frac{2\mu}{3(\mu+\nu)}\right)\left(-\frac{1}{\gamma}\right)}_{+} + \underbrace{\frac{\mu}{t-\mu k-\nu k}\left(-\frac{1}{\gamma}\right)}_{-}\Bigg],\end{aligned} \tag{18}$$

which after simplifications yields

$$\frac{\partial^2 \Pi^a}{\partial p_1^a \partial s} = \frac{2\delta\,\mu}{3\gamma t\beta}\left(2\,\alpha\,(1-k\,\beta) - \frac{t\,\beta}{\nu+\mu}\right). \tag{19}$$

All derivatives in (17) have definite signs except the first period effect, which depends on the sign of first period prices. Switching costs affect second period marginal profits through several different channels, however overall effect is ambiguous.[14] When first period prices are positive, marginal first period profits are increasing in the switching costs. This is due to the increase in γ, which in turn reduces first period demand elasticity. The reduction in the first period demand elasticity also impact second period incentives, first, through a negative indirect effect of the second period price of firm b, p_2^b, on firm a's second period demand, d_2^a (2nd term in (17)); and second, through a positive direct effect of the first period demand, d_1^a, on the second period demand (3rd term in (17)). The overall contribution of these three effects on the incentives of firm a is positive yielding incentives to increase first period price, irrespective of the sign of the first period equilibrium prices.

[14] All the arguments below are based on the sign of partial derivatives given in (18), however they are followed best referring to equation (17).

The fourth term in (17) results from the decrease in the responsiveness of second period price of firm b, p_2^b, to the first period demand, d_1^a, due to an increase in switching costs. When the first period price, p_1^a, increases, the first period demand, d_1^a, falls, which encourages firm b to raise its second period price, p_2^b, and therefore the second period demand, d_2^a, and profits increase. When switching costs grow in magnitude, the increase in firm b's price is more, leading to incentives for firm a to increase first period prices. The fifth term is due to a positive change in the responsiveness of the second period demand, d_2^a, to the first period demand when switching costs increase. An increase in the first period price p_1^a decreases the first period demand d_1^a, which decreases the second period demand d_2^a and the profits. When switching costs rise, the second period demand decreases more implying a negative impact on profits which leads to an incentive to decrease first period prices. It is straightforward to show that these two effects combined have a negative sign.

The overall direction of the movement of the best response function of firm a is ambiguous. However, notice that the term in parenthesis in (19) is linearly increasing in s.[15] Thus, if the right hand side of (17) is positive at $s = 0$, it is positive for all s. Otherwise, best response function of firm a shifts downwards around the equilibrium for small s, implying lower equilibrium prices with a slightly higher s.

A similar analysis can be performed for network benefits. The change in the marginal profits due to a change in marginal network benefits, k, is induced through three non-zero partial derivatives

$$\frac{\partial^2 \Pi^a}{\partial k \partial p_1^a} = p_1^a \frac{\partial^2 d_1^a}{\partial k \partial p_1^a} + \delta p_2^a \left[\frac{\partial d_2^a}{\partial p_2^b} \frac{\partial p_2^b}{\partial d_1^a} + \frac{\partial d_2^a}{\partial d_1^a} \right] \frac{\partial^2 d_1^a}{\partial k \partial p_1^a}, \tag{20}$$

[15] This is due to the fact that α is linearly increasing in s.

after the substituting the expressions for partial derivatives and equilibrium prices we obtain

$$\begin{aligned}\frac{\partial^2\Pi^a}{\partial k\partial p_1^a} &= \underbrace{\frac{3\gamma\beta-4\alpha\delta}{6\beta}\Big(-\frac{6+8\delta\alpha^2}{3\gamma^2}\Big)}_{+/-}+\delta\frac{1}{2\beta}\Big[\underbrace{\beta\Big(-\frac{2\alpha}{3\beta}\Big)\Big(-\frac{6+8\delta\alpha^2}{3\gamma^2}\Big)}_{+}+\underbrace{2\alpha\Big(-\frac{6+8\delta\alpha^2}{3\gamma^2}\Big)}_{-}\Big] \\ &= -\frac{3+4\delta\alpha^2}{3\gamma}<0. \qquad (21)\end{aligned}$$

Apart from the first period term, once again the effects of an increase in network benefits have well determined signs. The overall effect is unambiguously negative, implying that best response function of firm a shifts down around the equilibrium. Since firm b's reaction function moves also downward, equilibrium is obtained at lower prices when network effects increase.

In contrast to the switching costs, higher network benefits result in a more elastic demand in the first period for positive prices. And, the impact of increasing network effects on the cumulative profits only operate through this increase in first period demand elasticity as can be seen in (20). The second period effects go through two channels, first a negative indirect effect of firm b's second period price, p_2^b, on firm a's second period demand, d_2^a, and second, through a direct positive impact of the first period demand of firm a on the second period demand.

In summmary, the MCS we present in (18) and (21) not only allow us to disentangle the first and second period effects of changes in switching costs and network effects on pricing incentives of the firms in the first period, but also to identify the channels which these changes affect incentives.

3.4 The Case without Rigid Consumers

A few of the results we have presented in the previous subsection become sharper when we consider the case without rigid consumers, i.e., $\mu+\nu\to 1$. This simply assumes that every consumer could potentially switch, and firms as well as consumers are all aware of this possibility. A brief inspection of (12), suggest that second period prices decrease, however, the effects in the first period prices is not immediately seen as decreasing the size of the rigid consumers negatively impact second period profits and could reduce the first period incentives to lock customers in. Nevertheless, if we keep μ constant and let $\rho = \mu + \nu$, i.e., replace rigid consumers with new unattached ones, it is possible to show that

$$\frac{\partial p_1^a}{\partial \rho} = -\frac{1}{3}\,\frac{\left(2\,s\,\mu\,(t-k\,\rho)^2\,(s\,\mu+t)+(s\,\mu\,k-t\,(t-k))^2\,\rho^2\right)\delta}{\rho^2 t\,(t-k\,\rho)^2} < 0,$$

thus also first period prices decrease.

Next, we investigate the effect of changes in switching costs faced by flexible consumers and network effects when $\mu + \nu \to 1$. Substituting $\mu = 1 - \nu$ in (17), after simplifications, yields

$$\frac{\partial^2 \Pi^a}{\partial s \partial p_1^a} = \frac{\delta(1-\nu)[s(1-\nu)(2t-3k) - t(t-k)]}{\delta(1-\nu)^2 s^2(2t-3k) + 3t(t-k)^2} \tag{22}$$

The denominator is always positive when $k < k_{max}$. Hence, the sign of this expression depends on the numerator, which is positive for $s > \frac{t(t-k)}{(1-\nu)(2t-3k)}$ and negative otherwise. More importantly, when $s = 0$, a slight increase in switching costs shift both best response functions downwards leading to lower first period equilibrium prices. Thus, when switching costs are sufficiently low, the incentives to exploit locked-in consumers in the second period dominates and first period competition is fiercer.

In particular, consider the case when switching costs are zero, which implies first period equilibrium prices of $p_1^i = t - k$. After substitution of $\mu = 1 - \nu$ and $s = 0$ in (18), one can show that

$$\left.\frac{\partial^2 \Pi^a}{\partial s \partial p_1^a}\right|_{s=0,\mu=1-\nu} = 0 + \delta(t-k)\left[0 + 0 + \frac{1-\nu}{6(t-k)^2} - \frac{1-\nu}{2(t-k)^2}\right] = -\frac{\delta(1-\nu)}{3(t-k)}.$$

Clearly the decrease in equilibrium prices is due to the impact of small increase in switching costs on future profits, since the first period effect is identically zero. A similar result can be found in Doganoglu (2004) where it is shown in a fully dynamic setup that small switching costs might lead to lower prices in steady state compared to no switching costs.

4 Conclusions

We have analyzed the two-period model of duopolistic competition with switching costs and network effects, built on the model of Klemperer (1987). Hopefully, this paper can be considered as a first step to analyze models where switching costs and network effects coexist as in most real life situations.

We have shown that switching costs and network effects are forces in opposite directions early on. That is, consumer would like to be part of a network which would be large in the future. But firms with large user bases are able to sustain high prices in the presence of switching costs, thus reducing their attractiveness for consumers early on. The clear signs of these phenomena can be exemplified in our first period demands, which becomes more(less) price sensitive with an increase in marginal network benefits (switching costs). When there are no switching costs, the size of a network does not play an important dynamic role, as can be seen in the second period demands we have derived. Network effects increase price sensitivity of consumers when expectations are rationally formed, since in this case a price decrease not only has a direct positive impact on the utility of a marginal customer, but also an indirect effect through the network benefits.

Subgame perfect(SP) equilibrium prices in both tend to be lower as network effects increase. In both periods, this is due to increased price elasticity of demands faced by firms. The effects of switching costs, on the other hand, on the equilibrium outcomes are less clear cut. In the second period, the switching costs faced by flexible consumers tend to increase the price charged by a firm which carries over an installed base larger than one half. However, in the SP equilibrium of the whole game, switching costs have no effect. Nevertheless, the second period on the SP equilibrium prices increase with an increase in the size of the population of rigid consumers.

The effects of switching costs on the first period equilibrium prices is ambiguous. Even though the firms face demands with a lower price elasticity when switching costs increase, they also face a more profitable second period when they are able lock in more than half of the first period consumers. While the former effect leads to incentives to increase prices, the latter encourages firms to reduce their prices. Hence, the equilibrium is attained when these opposing incentives are balanced.

However, there is no telling ex-ante where this balancing would occur. Thus our first period prices could be lower than those in a market without network effects and switching costs. We show that the first period prices are quadratic-convex functions of the level of switching costs, therefore for certain parameter values increasing switching costs may reduce equilibrium prices. We can see this clearly in our example without rigid consumers, where around zero, increasing switching costs have no impact on marginal first period profits while it reduces second period profits, leading to lower equilibrium prices.

This point is very important to note, as the common message in the literature studying fully dynamic models is that switching costs unambiguously increase steady state prices. A common assumption shared by all the fully dynamic models is that switching costs are sufficiently high that consumers do not expect to switch, and in equilibrium there is no switching. However, it is not obvious whether this would be the case for small switching costs as our model in this paper suggests. Essentially, one would expect that in a fully dynamic model, firms would be in a situation much like the first period, but at the same time have some installed bases. Thus in a steady state, we expect the prices to inherit the same U-shape as we have derived in this paper. In a related model, Doganoglu (2004) shows that in fact for small switching costs steady state prices may below those in a market without switching costs.

Naturally, there are a few directions in which one could extend this paper. We think that a fully dynamic analysis is warranted, which we are in the process of developing. A more interesting issue is related to the market size. In this model, the total demand is completely inelastic, that is all consumers buy no matter what. However, both network effects and switching costs are likely to have effects on the participation incentives of the marginal consumer. A thorough welfare analysis could only be conducted, when these incentives are taken into account.

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5 Appendix

Proof of Lemma 1

We have imposed certain behavioral assumption on the demand side when we derived rational expectations demands in the second period. Namely, market is covered, both firms make positive sales to all the consumer types, and some flexible customers of each firm find it better to switch to the other brand. Switching in both directions occur, for each $N_1^a \in [0,1]$, only when

$$0 < d_2^{a|b} \leq d_2^{a|a} < 1,$$

which in turn implies $0 < d_2^{a|0} < 1$, since $d_2^{a|b} \leq d_2^{a|0} \leq d_2^{a|a}$.

When we evaluate $d_2^{a|a}$ given in (2), at the second period rational expectations demands and equilibrium prices, we obtain

$$1 - d_2^{a|a} = \frac{2}{3}\,\frac{\alpha\,(1-k\,\beta)\,N_1^a}{t\,\beta} + \frac{1}{2}\,\frac{t-s}{t} - \frac{1}{3}\,\frac{\alpha\,(1-k\,\beta)}{t\,\beta}. \tag{23}$$

Similarly, when we evaluate $d_2^{a|b}$ given in (3),

$$d_2^{a|b} = -\frac{2}{3}\,\frac{\alpha\,(1-k\,\beta)\,N_1^a}{t\,\beta} + \frac{1}{2}\,\frac{t-s}{t} + \frac{1}{3}\,\frac{\alpha\,(1-k\,\beta)}{t\,\beta} \tag{24}$$

For switching to occur in between both brands, we need the right hand sides of both (23) and (24) to be positive for all $N_1^a \in [0,1]$. Observe that $1 - d_2^{a|a}$ increases in N_1^a when $k \leq k_{max}$, therefore we need to evaluate the right hand side of (23) at $N_1^a = 0$. On the other hand, $d_2^{a|b}$ decreases in N_1^a requiring us to evaluate the right hand side of (24) at $N_1^a = 1$. It is easy to verify that both (23) and (24), evaluated at $N_1^a = 0$ and $N_1^a = 1$ respectively lead to the same condition, namely,

$$\frac{1}{2}\,\frac{t-s}{t} - \frac{1}{3}\,\frac{\alpha\,(1-k\,\beta)}{t\,\beta} > 0,$$

which yields, after substituting for α and simplifying, an upper bound on s given by

$$s \leq t - \frac{t(1-\nu)(1-k\beta)}{t\beta + (2\mu+\nu)(1-k\beta)} \equiv s_{max}.$$

It is easy to see that $s_{max} \leq t$, whenever $1 - k\beta > 0$ which holds when $k \leq 2t/3 \equiv k_{max}$. However, in order to guarantee that there are some positive switching costs, we need $s_{max} > 0$. Observe that

$$\frac{\partial}{\partial k} s_{max} = \frac{3}{4}\,\frac{t(1-\nu)(\mu+\nu)^2}{(t\beta + (2\mu+\nu)(1-k\beta))^2(t-k\mu-k\nu)^2} > 0,$$

therefore the smallest value of s_{max} occurs when $k = 0$, and given by

$$s_{max}\big|_{k=0} = \frac{t}{5\mu+3\nu}\big[5(\mu+\nu) - 2\big],$$

and is positive whenever $\mu + \nu > 2/5$. Therefore whenever $(\mu + \nu, s, k) \in \mathcal{P}$, some first period consumers of firm a switch to firm b, while some customer of b switch to a and the new customers buy from both firms in equilibrium for all $N_1^a \in [0, 1]$. By assumption, we have that v, the reservation price of consumers, is sufficiently large that all consumer buy, while it is sufficiently small that neither firm prefers to serve just the rigid consumers. Therefore, prices given in (6) constitute a Nash equilibrium in the second period.

■

Proof of Proposition 1

We solve the FOCs of each firm simultaneously implied by the cumulative profit functions to obtain the candidate first period equilibrium prices given in (12). To show that these prices indeed constitute a Nash equilibrium in the first period, we need to prove then when $(\mu+\nu, k, s) \in \mathcal{P}$, γ is nonnegative, cumulative profit functions are concave in the price of each firm's own price, and equilibrium profits are nonnegative.

Concavity of profit functions:

Concavity of profit function requires

$$0 < \frac{4}{9}\frac{\delta\alpha^2}{\beta} < \gamma,$$

therefore, whenever profit functions are concave $\gamma > 0$. Rewriting the above condition, we require

$$H = \gamma - \frac{4}{9}\frac{\delta\alpha^2}{\beta} > 0,$$

which yields

$$H = 2\,t - 2\,k + 2\,\delta\,t\,\left(1 - \nu - \mu\right) + \frac{2}{9}\,\frac{\left(s\,\mu + t - t\,\nu - t\,\mu\right)^2\delta\,\left(-9\,k\,\nu + 5\,t - 9\,k\,\mu\right)}{t\,\left(\nu + \mu\right)\left(t - k\,\mu - k\,\nu\right)},$$

when we substitute the expressions for α and β. It is straightforward to verify that

$$\frac{\partial H}{\partial k} = -2 - \frac{8}{9}\,\frac{\left(s\,\mu + t - t\,\nu - t\,\mu\right)^2\delta}{\left(t - k\,\mu - k\,\nu\right)^2} < 0.$$

Thus, it is sufficient to show that $H > 0$ when $k = k_{max}$. Evaluating, H at $k = k_{max}$ yields

$$\begin{aligned} H \;\; = \;\; & \frac{2}{3}\,t + 2\,\delta\,t\,\left(1 - \nu - \mu\right) - \frac{2}{3}\,\frac{\delta\,\left(s\,\mu + t\,\left(1 - \nu - \mu\right)\right)^2}{t\,\left(3 - 2\,\mu - 2\,\nu\right)} \\ & + 4\,\frac{\delta\,\left(s\,\mu + t\,\left(1 - \nu - \mu\right)\right)^2\left(1 - \nu - \mu\right)}{t\,\left(\nu + \mu\right)\left(3 - 2\,\mu - 2\,\nu\right)} \end{aligned} \tag{25}$$

Notice that only the third term is negative. Furthermore, $s\mu + t(1-\mu-\nu) < t$, thus the third term becomes even more negative when we replace $s\mu + t(1-\mu-\nu)$ with t. Hence, it is easy to verify that

$$\begin{aligned} H &\geq \frac{2}{3}t - \frac{2}{3}\frac{\delta\, t}{(3-2\,\mu-2\,\nu)} \\ &= \frac{2t}{3}\big[1 - \frac{\delta}{3-2(\mu+\nu)}\big] \geq 0, \end{aligned}$$

since $\delta \leq 1$ and $3-2(\mu+\nu) \geq 1$. Therefore, the profit function are concave in firm's own prices. Moreover, $\gamma > 0$. □

Equilibrium profits are non-negative:

The equilibrium profit from both periods is given (13) and this expression is once again decreasing in k. Thus, it is sufficient to verify that profits are non-negative when $k = k_{max}$. Substituting the expressions for γ, α, β and $k = k_{max}$ in (13) yields

$$\begin{aligned} G &= \frac{t}{6} + \frac{1}{2}\delta\, t\;(1-\nu-\mu) + \frac{\delta\;(s\,\mu + t\;(1-\nu-\mu))^2\,(1-\nu-\mu)}{t\;(\nu+\mu)\,(3-2\,\mu-2\,\nu)} \\ &\quad -\frac{1}{3}\frac{\delta\;(s\,\mu+t\;(1-\nu-\mu))}{\nu+\mu} + \frac{1}{6}\frac{\delta t(3-2\mu-2\nu)}{\mu+\nu}. \end{aligned}$$

Notice that only the fourth term is negative. It is easy to verify that

$$\begin{aligned} G &\geq \frac{t}{6} - \frac{1}{3}\frac{\delta s\mu}{\nu+\mu} + \frac{1}{6}\frac{\delta t}{\mu+\nu} \\ &\geq \frac{t}{6}\big[1 - \frac{\delta\mu}{\mu+\nu}\big] + \frac{t}{6}\frac{\delta(1-\mu)}{\mu+\nu} \geq 0, \end{aligned}$$

where the first inequality follows from simply ignoring second and third terms in G, second inequality from $s \leq t$ and the last from the fact that $\delta < 1$, $s \leq t$, $\mu \leq 1$ and $\mu/(\mu+\nu) \leq 1$. Therefore, in equilibrium firms obtain nonnegative profits, even if they charge below cost prices in the first period. □

Each firm's profit function is concave in its own price, and equilibrium profits are nonnegative when $(\mu+\nu, k, s) \in \mathcal{P}$, hence solution of FOCs indeed describe an equilibrium.

■

Monotone comparative statics

A convenient method to disentangle effects of certain variables on firms incentives is to use Monotone Comparative Statics (MCS). Essentially, MCS tells us how the best response function of a firm will shift if one of the model parameters changes.

Consider two-period profit of firm a for a fixed p_1^a and p_1^b:

$$\Pi^a(p_1^a, p_1^b, s, k) \quad = \quad p_1^a d_1^a(p_1^a, p_1^b, s, k) + \delta \Pi_2^a(d_1^a(p_1^a, p_1^b, s, k), s, k),$$

where

$$\Pi_2^a(d_1^a(p_1^a, p_1^b, s, k), s, k) \quad = \quad p_2^a(d_1^a(p_1^a, p_1^b, s, k), s, k) d_2^a,$$

and

$$d_2^a = d_2^a(p_2^a(d_1^a(p_1^a, p_1^b, s, k), s, k), p_2^b(d_1^a(p_1^a, p_1^b, s, k), s, k), d_1^a(p_1^a, p_1^b, s, k), s, k).$$

We will suppress the dependency of d_1^a, p_2^a, p_2^b and d_2^a on their arguments below for ease of exposition.

Let $R(p_1^b, s, k)$ denote the best response price of firm a when firm b charges p_1^b. Then, the FOC implies

$$\frac{\partial}{\partial p_1^a} \Pi^a(R(p_1^b, s, k), p_1^b, s, k) = 0,$$

and therefore

$$\frac{\partial}{\partial s} R(p_1^b, s, k) = -\frac{\frac{\partial^2}{\partial s \partial p_1^a} \Pi^a(R(p_1^b, s, k), p_1^b, s, k)}{\frac{\partial^2}{\partial {p_1^a}^2} \Pi^a(R(p_1^b, s, k), p_1^b, s, k)}.$$

A similar expression also applies for changes in marginal network benefits, k. Given that profit function of firm a is concave in p_1^a, it is easy to see that

$$\text{sign}\left[\frac{\partial}{\partial s} R(p_1^b, s, k)\right] = \text{sign}\left[\frac{\partial^2}{\partial s \partial p_1^a} \Pi^a(R(p_1^b, s, k), p_1^b, s, k)\right].$$

We can, therefore, find out how the best response of a firm will change with respect to a change in switching costs, and more importantly identify through which channels this change takes place, by looking at the derivative of the first order condition with respect to the switching costs holding first period prices at p_1^b and firm a's best response to it as constant. A particularly appealing choice is setting p_1^b to the first period equilibrium price of firm b, in which case firm a's best response is simply also to charge the equilibrium price. Thus, MCS around the equilibrium prices will allow us characterize the equilibrium incentives of firms in a transparent manner.

The FOC of firm a in the first period is

$$\frac{\partial^2 \Pi^a}{\partial p_1^a} = d_1^a + p_1^a \frac{\partial d_1^a}{\partial p_1^a} + \delta \left[\left(d_2^a + p_2^a \frac{\partial d_2^a}{\partial p_2^a} \right) \frac{\partial p_2^a}{\partial d_1^a} \frac{\partial d_1^a}{\partial p_1^a} + p_2^a \frac{\partial d_2^a}{\partial p_2^b} \frac{\partial p_2^b}{\partial d_1^a} \frac{\partial d_1^a}{\partial p_1^a} + p_2^a \frac{\partial d_2^a}{\partial d_1^a} \frac{\partial d_1^a}{\partial p_1^a} \right].$$

Observe that

$$d_2^a + p_2^a \frac{\partial d_2^a}{\partial p_2^a} = 0,$$

since it is the FOC in the second period, and it is identically zero because in a subgame perfect equilibrium firm's take equilibrium payoffs from future given at their equilibrium level. Therefore, the derivative of the FOC with respect to switching costs, holding first period prices constant, amounts to

$$
\begin{aligned}
\frac{\partial^2 \Pi^a}{\partial s \partial p_1^a} &= \frac{\partial d_1^a}{\partial s} + p_1^a \frac{\partial^2 d_1^a}{\partial s \partial p_1^a} \\
&+\delta \Bigg[\left(\frac{\partial p_2^a}{\partial d_1^a}\frac{\partial d_1^a}{\partial s} + \frac{\partial p_2^a}{\partial s} \right) \frac{\partial d_2^a}{\partial p_2^b}\frac{\partial p_2^b}{\partial d_1^a}\frac{\partial d_1^a}{\partial p_1^a} \\
&+p_2^a \left(\frac{\partial^2 d_2^a}{\partial s \partial p_2^b}\frac{\partial p_2^b}{\partial d_1^a}\frac{\partial d_1^a}{\partial p_1^a} + \frac{\partial d_2^a}{\partial p_2^b}\frac{\partial^2 p_2^b}{\partial s \partial d_1^a}\frac{\partial d_1^a}{\partial p_1^a} + \frac{\partial d_2^a}{\partial p_2^b}\frac{\partial p_2^b}{\partial d_1^a}\frac{\partial^2 d_1^a}{\partial s \partial p_1^a} \right) \\
&+ \quad \left(\frac{\partial p_2^a}{\partial d_1^a}\frac{\partial d_1^a}{\partial s} + \frac{\partial p_2^a}{\partial s} \right) \frac{\partial d_2^a}{\partial d_1^a}\frac{\partial d_1^a}{\partial p_1^a} \\
&+p_2^a \left(\frac{\partial^2 d_2^a}{\partial s \partial d_1^a}\frac{\partial d_1^a}{\partial p_1^a} + \frac{\partial d_2^a}{\partial d_1^a}\frac{\partial^2 d_1^a}{\partial s \partial p_1^a} \right) \Bigg].
\end{aligned}
$$

Let us first list a few partial derivatives which will also be useful in our analysis with respect to network effects. It is straightforward to confirm that

$$
\begin{aligned}
&\frac{\partial d_1^a}{\partial p_1^a} = -\frac{1}{\gamma}, \qquad \frac{\partial d_2^a}{\partial d_1^a} = 2\alpha, \\
&\frac{\partial d_2^a}{\partial p_2^b} = \beta, \qquad \frac{\partial p_2^b}{\partial d_1^a} = -\frac{2\alpha}{3\beta}, \qquad \frac{\partial p_2^a}{\partial d_1^a} = \frac{2\alpha}{3\beta}.
\end{aligned}
$$

The partial derivatives of involving d_1^a with respect to switching costs are

$$
\frac{\partial d_1^a}{\partial s} = -(p_1^a - p_1^b)\frac{1}{\gamma^2}\frac{\partial \gamma}{\partial s} = 0, \qquad \frac{\partial^2 d_1^a}{\partial s \partial p_1^a} = \frac{1}{\gamma^2}\frac{\partial \gamma}{\partial s} = \frac{8\delta\mu\alpha(1-k\beta)}{3t\gamma^2\beta},
$$

and those involving d_2^a are

$$
\frac{\partial^2 d_2^a}{\partial s \partial d_1^a} = \frac{\mu}{t - \mu k - \nu k}, \qquad \frac{\partial^2 d_2^a}{\partial s \partial p_2^b} = 0.
$$

We have only one term involving p_2^a given by

$$
\frac{\partial p_2^a}{\partial s} = \frac{2(p_1^b - p_1^a)}{3\gamma\beta}\frac{\partial \alpha}{\partial s} = 0,
$$

and another involving p_2^b which is given by

$$
\frac{\partial^2 p_2^b}{\partial s \partial d_1^a} = -\frac{2\mu}{3(\mu+\nu)}.
$$

After eliminating the elements equal to zero we arrive at

$$\frac{\partial^2 \Pi^a}{\partial s \partial p_1^a} = p_1^a \frac{\partial^2 d_1^a}{\partial p_1^a \partial s} + \delta p_2^a \Bigg[\Bigg(\frac{\partial d_2^a}{\partial p_2^b}\frac{\partial p_2^b}{\partial d_1^a} + \frac{\partial d_2^a}{\partial d_1^a}\Bigg)\frac{\partial^2 d_1^a}{\partial s \partial p_1^a} + \Bigg(\frac{\partial d_2^a}{\partial p_2^b}\frac{\partial^2 p_2^b}{\partial s \partial d_1^a} + \frac{\partial^2 d_2^a}{\partial s \partial d_1^a}\Bigg)\frac{\partial d_1^a}{\partial p_1^a}\Bigg]$$

and after substituting the partial derivatives and equilibrium price we obtain

$$\begin{aligned}\frac{\partial^2 \Pi^a}{\partial s \partial p_1^a} &= \underbrace{\frac{3\gamma\beta - 4\alpha\delta}{6\beta}\frac{8\delta\mu\alpha(1-k\beta)}{3t\gamma^2\beta}}_{+/-} \\ &+\delta\frac{1}{2\beta}\Bigg[\underbrace{\beta\Bigg(-\frac{2\alpha}{3\beta}\Bigg)\frac{8\delta\mu\alpha(1-k\beta)}{3t\gamma^2\beta}}_{-} + \underbrace{2\alpha\frac{8\delta\mu\alpha(1-k\beta)}{3t\gamma^2\beta}}_{+} \\ &\underbrace{\beta\Bigg(-\frac{2\mu}{3(\mu+\nu)}\Bigg)\Bigg(-\frac{1}{\gamma}\Bigg)}_{+} + \underbrace{\frac{\mu}{t-\mu k-\nu k}\Bigg(-\frac{1}{\gamma}\Bigg)}_{-}\Bigg]\end{aligned}$$

A similar analysis might be conducted for network benefits. The derivative of the first order condition with respect to marginal network benefits, k, yields

$$\begin{aligned}\frac{\partial^2 \Pi^a}{\partial k \partial p_1^a} &= \frac{\partial d_1^a}{\partial k} + p_1^a\frac{\partial^2 d_1^a}{\partial k \partial p_1^a} \\ &+\delta\Bigg[\Bigg(\frac{\partial p_2^a}{\partial d_1^a}\frac{\partial d_1^a}{\partial k} + \frac{\partial p_2^a}{\partial k}\Bigg)\frac{\partial d_2^a}{\partial p_2^b}\frac{\partial p_2^b}{\partial d_1^a}\frac{\partial d_1^a}{\partial p_1^a} \\ &+p_2^a\Bigg(\frac{\partial^2 d_2^a}{\partial k \partial p_2^b}\frac{\partial p_2^b}{\partial d_1^a}\frac{\partial d_1^a}{\partial p_1^a} + \frac{\partial d_2^a}{\partial p_2^b}\frac{\partial^2 p_2^b}{\partial k \partial d_1^a}\frac{\partial d_1^a}{\partial p_1^a} + \frac{\partial d_2^a}{\partial p_2^b}\frac{\partial p_2^b}{\partial d_1^a}\frac{\partial^2 d_1^a}{\partial k \partial p_1^a}\Bigg) \\ &+ \quad \Bigg(\frac{\partial p_2^a}{\partial d_1^a}\frac{d_1^a}{\partial k} + \frac{\partial p_2^a}{\partial k}\Bigg)\frac{\partial d_2^a}{\partial d_1^a}\frac{\partial d_1^a}{\partial p_1^a} \\ &+p_2^a\Bigg(\frac{\partial^2 d_2^a}{\partial k \partial d_1^a}\frac{\partial d_1^a}{\partial p_1^a} + \frac{\partial d_2^a}{\partial d_1^a}\frac{\partial^2 d_1^a}{\partial k \partial p_1^a}\Bigg)\Bigg].\end{aligned}$$

The partial derivatives with respect to marginal network benefits involving d_1^a are

$$\frac{\partial d_1^a}{\partial k} = -(p_1^a - p_1^b)\frac{1}{\gamma^2}\frac{\partial \gamma}{\partial k} = 0, \qquad \frac{\partial^2 d_1^a}{\partial k \partial p_1^a} = \frac{1}{\gamma^2}\frac{\partial \gamma}{\partial k} = -\frac{6+8\delta\alpha^2}{3\gamma^2},$$

those involving d_2^a are

$$\frac{\partial^2 d_2^a}{\partial k \partial d_1^a} = 4\alpha\beta, \qquad \frac{\partial^2 d_2^a}{\partial k \partial p_2^b} = 2\beta^2.$$

Once again, we have only one term involving p_2^a given by

$$\frac{\partial p_2^a}{\partial k} = \frac{2d_1^a - 1}{3\beta}\frac{\partial \alpha}{\partial k} - \frac{3 + 2\alpha(2d_1^a - 1)}{6\beta^2}\frac{\partial \beta}{\partial k} + \frac{2\alpha}{3\beta}\frac{\partial d_1^a}{\partial k} = -1,$$

and another involving p_2^b which is given by

$$\frac{\partial^2 p_2^b}{\partial k \partial d_1^a} = 0.$$

Thus, after eliminating elements equal to zero results in

$$\begin{aligned}
\frac{\partial^2 \Pi^a}{\partial k \partial p_1^a} &= p_1^a \frac{\partial^2 d_1^a}{\partial k \partial p_1^a} \\
&+ \delta p_2^a \Bigg[\frac{1}{p_2^a} \frac{\partial p_2^a}{\partial k} \frac{\partial d_2^a}{\partial p_2^b} \frac{\partial p_2^b}{\partial d_1^a} \frac{\partial d_1^a}{\partial p_1^a} + \frac{\partial^2 d_2^a}{\partial k \partial p_2^b} \frac{\partial p_2^b}{\partial d_1^a} \frac{\partial d_1^a}{\partial p_1^a} + \frac{\partial d_2^a}{\partial p_2^b} \frac{\partial p_2^b}{\partial d_1^a} \frac{\partial^2 d_1^a}{\partial k \partial p_1^a} \\
&+ \quad \frac{1}{p_2^a} \frac{\partial p_2^a}{\partial k} \frac{\partial d_2^a}{\partial d_1^a} \frac{\partial d_1^a}{\partial p_1^a} + \frac{\partial^2 d_2^a}{\partial k \partial d_1^a} \frac{\partial d_1^a}{\partial p_1^a} + \frac{\partial d_2^a}{\partial d_1^a} \frac{\partial^2 d_1^a}{\partial \partial p_1^a} \Bigg].
\end{aligned} \tag{26}$$

Notice that

$$-\frac{1}{p_2^a} \frac{\partial p_2^a}{\partial k} \frac{\partial d_2^a}{\partial p_2^b} = \frac{\partial^2 d_2^a}{\partial k \partial p_2^b} = 2\beta^2,$$

as well as

$$-\frac{1}{p_2^a} \frac{\partial p_2^a}{\partial k} \frac{\partial d_2^a}{\partial d_1^a} = \frac{\partial^2 d_2^a}{\partial k \partial d_1^a} = 4\alpha\beta,$$

thus, second and third, fifth and sixth terms cancel each other in (26), leaving only

$$\frac{\partial^2 \Pi^a}{\partial k \partial p_1^a} = p_1^a \frac{\partial^2 d_1^a}{\partial k \partial p_1^a} + \delta p_2^a \left[\frac{\partial d_2^a}{\partial p_2^b} \frac{\partial p_2^b}{\partial d_1^a} + \frac{\partial d_2^a}{\partial d_1^a} \right] \frac{\partial^2 d_1^a}{\partial k \partial p_1^a}.$$

After substituting the partial derivatives and equilibrium prices we obtain

$$\begin{aligned}
\frac{\partial^2 \Pi^a}{\partial p_1^a \partial k} &= \underbrace{\frac{3\gamma\beta - 4\alpha\delta}{6\beta} \left(-\frac{6 + 8\delta\alpha^2}{3\gamma^2} \right)}_{+/-} \\
&+ \delta \frac{1}{2\beta} \Bigg[\underbrace{\beta \left(-\frac{2\alpha}{3\beta} \right) \left(-\frac{6 + 8\delta\alpha^2}{3\gamma^2} \right)}_{+} + \underbrace{2\alpha \left(-\frac{6 + 8\delta\alpha^2}{3\gamma^2} \right)}_{-} \Bigg]
\end{aligned} \tag{27}$$