Analysing Monetary Policy with an Estimated DSGE Model*

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September 2004

Abstract

Keywords:
JEL Classifications:

VERY PRELIMINARY:
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EDGE Jamboree
Dublin, September 2004

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†The activities of EPRU (Economic Policy Research Unit) are financed by a grant from The Danish National Research Foundation.

1 Introduction

The purpose of this paper is two-fold; first, we seek to establish a suitable framework for analysis of monetary policy in a small open economy which is firmly founded in economic theory as well as empirically relevant; secondly, we use the suggested framework to analyse the structural dynamics of a small open economy with microfounded nominal rigidities with particular emphasis on the monetary transaction mechanism. In particular, we apply our model to Danish data and consider the dynamics of this economy under a fixed exchange rate regime.

We follow the current trend in this literature and consider the cashless limiting economy; that is, we consider an economy where money-based transactions are sufficiently unimportant for the utility of real consumption to be safely ignored. In this case, monetary policy is used to determine the short-term interest rate, which we consider to be the empirically relevant case. Woodford (2003) argues convincingly in favour of this approach.

The particular model we use is based on the small open DSGE economy constructed by Kollmann (2001, 2002). Thus, we have imperfect competition in the labour market as well as the market for intermediate goods. In both markets, suppliers update their prices infrequently and thus have to consider future market conditions in order to set their prices optimally. While Kollmann calibrated his model, we perform a Bayesian estimation of the model by combining the likelihood function with prior distributions for the structural parameters. Following Smets and Wouters (2003) we postulate that all stochastic volatility in the model is of structural nature. This necessitates that we extend the number of structural shocks from those included in Kollmann's model.

This paper is strongly inspired by the work of Smets and Wouters (2003) who in turn draw heavily on the model constructed by Christiano, Eichenbaum, and Evans (2001) (henceforth the CEE model). There are three major differences between our analysis and that of Smets and Wouters. Firstly, we consider a small open model instead of a closed economy. This expansion is of critical importance to us, since we seek a framework which can address the relative merits of a fixed exchange rate regime and an independent monetary policy. Secondly, we have chosen to keep our theoretical model fairly close to that of Kollmann (2001, 2002), thus ignoring some of the frictions which were included in the CEE model in order to capture the empirically observed inertia in the data. Thirdly, whereas Smets and Wouters (2003) had to make the dubious assumption that monetary policy in the current euro area was properly described by a generalised Taylor rule throughout the period 1980-1999, we can make the more appealing assumption that the Danish monetary policy consisted of an imperfect peg on the euro (and the D-mark before 1999).

2 The Model

In this section we build a fully micro-founded DSGE model with staggered setting of wages and prices. The model is fairly rich in variables and parameters which we summarise in Appendix A.

2.1 Final Goods

Domestic final goods are produced from Dixit-Stiglitz aggregates of a continuum of tradable intermediate goods. These are produced domestically and abroad;

$$Q_t^i = \left[\int_0^1 q^i(s)^{\frac{1}{1+\nu}}\, ds\right]^{1+\nu_t}, \; i = d, m.$$

Z_t is the production of final goods using the Cobb Douglas technology

$$Z_t = \left(\frac{Q_t^d}{\alpha^d}\right)^{\alpha^d} \left(\frac{Q_t^m}{\alpha^m}\right)^{\alpha^m}, \; \alpha^d + \alpha^m = 1. \tag{2.1}$$

Domestic firms face the problem of minimizing the cost of producing Z_t units of the final good;

$$\min_{Q_t^d, Q_t^m} P_t^d Q_t^d + P_t^m Q_t^m \tag{2.2}$$

$$\text{s.t.} \qquad \left(\frac{Q_t^d}{\alpha^d}\right)^{\alpha^d} \left(\frac{Q_t^m}{\alpha^m}\right)^{\alpha^m} = Z_t, \tag{2.3}$$

where individual intermediate-goods prices are $p_t^i(s)$ and appropriate CES price indices are given as

$$P_t^i \equiv \left[\int_0^1 p_t^i(s)^{1-\nu_t}\, ds\right]^{1/(1-\nu_t)}, \; i = d, m. \tag{2.4}$$

The associated Lagrangian is

$$\begin{aligned} \mathcal{L} &= P_t^d Q_t^d + P_t^m Q_t^m \\ &\quad -\lambda_t \left[\left(\frac{Q_t^d}{\alpha^d}\right)^{\alpha^d} \left(\frac{Q_t^f}{1-\alpha^d}\right)^{1-\alpha^d} - Z_t\right], \end{aligned}$$

and the first order condition with respect to Q_t^i gives us

$$P_t^i = \lambda_t \alpha^i \frac{Z_t}{Q_t^i} \Longrightarrow \frac{Q_t^i}{\alpha^i} = \lambda_t \frac{Z_t}{P_t^i} \tag{2.5}$$

Inserting equation (2.5) in the budget constraint (2.2) yields

$$\left(P_t^d\right)^{\alpha^d} \left(P_t^m\right)^{1-\alpha^d} = \lambda_t. \tag{2.6}$$

The appropriate aggregate price index P_t is the cost of producing one unit of the final good ($Z_t = 1$). Thus, we use equation (2.5) again to obtain

$$\begin{aligned} &\left(\frac{Q_t^d}{\alpha^d}\right)^{\alpha^d} \left(\frac{Q_t^{EU,d}}{1-\alpha^d}\right)^{1-\alpha^d} = 1 \Rightarrow \\ P_t &\equiv P_t^d Q_t^d + P_t^m Q_t^m \\ &= \lambda_t \left(\alpha^d + \alpha^m\right) = \lambda_t \\ &= \left(P_t^d\right)^{\alpha^d} \left(P_t^m\right)^{\alpha^m} \end{aligned} \tag{2.7}$$

Note that P_t is thus the marginal cost of the final-goods producing firm. With perfect competition in the final-goods market, the price of one unit is thus P_t.

Profit-maximizing demands are found by combining (2.5) with (2.7);

$$Q_t^i = \alpha^i \lambda_t \frac{Z_t}{P_t^i} = \alpha^i \frac{P_t Z_t}{P_t^i}, \; i = d, m. \tag{2.8}$$

2.2 Intermediate Goods

Intermediate-goods producers have access to Cobb-Douglas technology

$$y_t(s) = \theta_t K_t(s)^{\psi} L_t(s)^{1-\psi}, \ 0 < \psi < 1,$$

and operate in a monopolistic competitive market, where each producer sets the price of her variety, taking other prices as given and supplying whatever amount is demanded at the price set. Cost minimization implies the following first-order conditions

$$\left.\begin{array}{r}(1-\psi)\, y_t(s)/L_t(s) = W_t \\ \psi y_t(s)/K_t(s) = R_t,\end{array}\right\} \Rightarrow \frac{L_t(s)}{K_t(s)} = \frac{1-\psi}{\psi}\frac{R_t}{W_t},$$

where R_t is the rental price of capital and W_t is the wage rate. Thus,

$$\begin{aligned}
L_t(s) &= \frac{1-\psi}{\psi}\frac{R_t}{W_t}K_t(s) \Rightarrow && (2.9)\\
y_t(s) &= \theta_t K(s)_t^{\psi}\left(\frac{1-\psi}{\psi}\frac{R_t}{W_t}K_t(s)\right)^{1-\psi} = \theta_t K_t(s)\left(\frac{1-\psi}{\psi}\frac{R_t}{W_t}\right)^{1-\psi} \Rightarrow \\
K_t(s) &= \frac{1}{\theta_t}\left(\frac{1-\psi}{\psi}\frac{R_t}{W_t}\right)^{-(1-\psi)} y_t(s), && (2.10)\\
L_t(s) &= \frac{1}{\theta_t}\left(\frac{\psi}{1-\psi}\frac{W_t}{R_t}\right)^{-\psi} y_t(s).
\end{aligned}$$

Hence, the firm's total and marginal costs are

$$\begin{aligned}
TC(y_t(s)) &= W_t L_t + R_t K_t \\
&= \frac{1}{\theta_t}\left[W_t\left(\frac{\psi}{1-\psi}\frac{W_t}{R_t}\right)^{-\psi} + R_t\left(\frac{1-\psi}{\psi}\frac{R_t}{W_t}\right)^{-(1-\psi)}\right] y_t(s) \\
&= \frac{1}{\theta_t}\left[W_t^{1-\psi}R_t^{\psi}\left(\frac{\psi}{1-\psi}\right)^{-\psi} + W_t^{1-\psi}R_t^{\psi}\left(\frac{\psi}{1-\psi}\right)^{1-\psi}\right] y_t(s) \\
&= \frac{1}{\theta_t}W_t^{1-\psi}R_t^{\psi}\left[\left(\frac{\psi}{1-\psi}\right)^{-\psi} + \left(\frac{\psi}{1-\psi}\right)^{1-\psi}\right] y_t(s) \\
&= \frac{1}{\theta_t}W_t^{1-\psi}R_t^{\psi}\left[\left(\frac{\psi}{1-\psi}\right)^{-\psi}\frac{1}{1-\psi}\right] y_t(s) \\
&= \frac{1}{\theta_t}W_t^{1-\psi}R_t^{\psi}\psi^{-\psi}(1-\psi)^{-(1-\psi)} y_t(s) \Rightarrow \\
MC_t &= \frac{1}{\theta_t}W_t^{1-\psi}R_t^{\psi}\psi^{-\psi}(1-\psi)^{-(1-\psi)}. && (2.11)
\end{aligned}$$

Following Calvo (1983), we assume that the firm only reoptimizes its prices in any given period with probability $1-d$. Producers sell their good variety to both domestic and foreign final-goods producers; $y_t(s) = q_t^d(s) + q_t^m(s)$ and can price discriminate between the two markets. As is well-known from the Dixit-Stiglitz models, final-good producers demand individiual varieties of intermediaries as follows

$$q_t^i(s) = \left(\frac{p_t^i(s)}{P_t^i}\right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^i, \ i = d, m.$$

Firm profits are thus

$$\begin{aligned}\pi^{dx}\left(p_t^d(s), p_t^x(s)\right) &= \left(p_t^d(s) - MC_t\right) q_t^d(s) + \left(e_t p_t^x(s) - MC_t\right) q_t^x(s) \\ &= \left(p_t^d(s) - MC_t\right)\left(\frac{p_t^d(s)}{P_t^d}\right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^d \\ &\quad + \left(e_t p_t^x(s) - MC_t\right)\left(\frac{p_t^x(s)}{P_t^x}\right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^x\end{aligned} \tag{2.12}$$

Here we assumed Dixit-Stiglitz demands from foreign final-goods producers;

$$\begin{aligned} q_t^x(s) &= \left(\frac{p_t^x(s)}{P_t^x}\right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^x, \\ Q_t^x &= \left(\frac{P_t^x}{P_t^*}\right)^{-\eta} Y_t^*,\ \eta > 0, \end{aligned} \tag{2.13}$$

where the foreign aggregates P_t^*, Y_t^* are exogenous.

Likewise, foreign exporters generate the following profits in the domestic market;

$$\pi^m\left(p_t^m(s)\right) = \left(p_t^m(s) - e_t P_t^*\right)\left(\frac{p_t^m(s)}{P_t^m}\right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^m$$

Hence, a domestic firm reoptimizing its domestic price faces the following problem:

$$\begin{aligned} p_{t,t}^d &= \arg\max_{\omega} \sum_{\tau=0}^{\infty} d^\tau E_t\left[\rho_{t,t+\tau}\pi^{dx}\left(\omega, p_t^x(s)\right)\right], \\ \rho_{t,t+\tau} &\equiv \beta^\tau\left(U_{C,t+\tau}/U_{C,t}\right)\left(P_t/P_{t+\tau}\right) \end{aligned} \tag{2.14}$$

where $\rho_{t,t+\tau}$ appropriately discounts profits at time $t+\tau$, and d^τ is the probability that the current pricing decision is still in effect in period $\tau+\tau$. Substituting from the profit expression (2.12) yields

$$\begin{aligned} &\sum_{\tau=0}^{\infty} d^\tau E_t\left[\rho_{t,t+\tau}\left(p_{t,t}^d - MC_{t+\tau}\right)\left(\frac{p_{t,t}^d}{P_{t+\tau}^d}\right)^{-\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} Q_{t+\tau}^d\right] \\ = &\sum_{\tau=0}^{\infty} d^\tau E_t\left[\left(p_{t,t}^d - MC_{t+\tau}\right)\left(p_{t,t}^d\right)^{-\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} \rho_{t,t+\tau}\left(P_{t+\tau}^d\right)^{\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} Q_{t+\tau}^d\right], \end{aligned}$$

resulting in the following first-order condition;

$$\begin{aligned} &\sum_{\tau=0}^{\infty} d^\tau E_t\left[\left(-\frac{1}{\nu_{t+\tau}}\left(p_{t,t}^d\right)^{-\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} + \frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}\left(p_{t,t}^d\right)^{-\frac{1+2\nu_{t+\tau}}{\nu_{t+\tau}}} MC_{t+\tau}\right)\rho_{t,t+\tau}\left(P_{t+\tau}^d\right)^{\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} Q_{t+\tau}^d\right] = 0 \Rightarrow \\ &\sum_{\tau=0}^{\infty} d^\tau E_t\left[\left(p_{t,t}^d - (1+\nu_{t+\tau}) MC_{t+\tau}\right)\rho_{t,t+\tau}\left(\frac{p_{t,t}^d}{P_{t+\tau}^d}\right)^{-\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} \frac{Q_{t+\tau}^d}{p_{t,t}^d \nu_{t+\tau}}\right] = 0. \end{aligned} \tag{2.15}$$

Analogously, the optimal price for sales to foreign final-goods producers is determined from the following condition;

$$\sum_{\tau=0}^{\infty} d^\tau E_t\left[\left(e_{t+\tau}p_{t,t}^x - (1+\nu_{t+\tau}) MC_{t+\tau}\right)\rho_{t,t+\tau}\left(\frac{p_{t,t}^x}{P_{t+\tau}^x}\right)^{-\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} \frac{Q_{t+\tau}^x}{p_{t,t}^x \nu_{t+\tau}}\right] = 0. \tag{2.16}$$

Import firms are owned by risk-neutral foreigners who discount future profits at the foreign nominal interest rate $R_{t,t+\tau} \equiv \Pi_{s=t}^{t+\tau} (1+i_s^*)^{-1}$. Thus, when they reoptimize, they set their prices in order to maximize discounted future profits measured in foreign units;

$$\begin{aligned}
p_{t,t}^m &= \arg\max_{\omega} \sum_{\tau=0}^{\infty} d^\tau E_t \left[R_{t,t+\tau} \pi^m(\omega) / e_{t+\tau} \right] \\
&= \sum_{\tau=0}^{\infty} d^\tau E_t \left[R_{t,t+\tau} \pi^m(\omega) / e_{t+\tau} \right] \\
&= \sum_{\tau=0}^{\infty} d^\tau E_t \left[R_{t,t+\tau} \left(p_{t,t}^m - e_{t+\tau} P_{t+\tau}^* \right) \left(\frac{p_{t,t}^m}{P_{t+\tau}^m} \right)^{-\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} Q_{t+\tau}^m / e_{t+\tau} \right] \\
&= \sum_{\tau=0}^{\infty} d^\tau E_t \left[R_{t,t+\tau} \left(p_{t,t}^m / e_{t+\tau} - P_{t+\tau}^* \right) \left(p_{t,t}^m \right)^{-\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} \left(P_{t+\tau}^m \right)^{\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} Q_{t+\tau}^m \right],
\end{aligned}$$

with first-order condition

$$\sum_{\tau=0}^{\infty} d^\tau E_t \left[\left(-\frac{1}{\nu_{t+\tau}} p_{t,t}^{m\,-\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} / e_{t+\tau} + \frac{1+\nu_{t+\tau}}{\nu_{t+\tau}} \left(p_{t,t}^m \right)^{-\frac{1+2\nu_{t+\tau}}{\nu_{t+\tau}}} P_{t+\tau}^* \right) \left(P_{t+\tau}^m \right)^{\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} Q_{t+\tau}^m \right] = 0 \Rightarrow$$

$$\sum_{\tau=0}^{\infty} d^\tau E_t \left[\left(p_{t,t}^m(s) / e_{t+\tau} - (1+\nu_{t+\tau}) P_{t+\tau}^* \right) \left(\frac{p_{t,t}^m}{P_{t+\tau}^m} \right)^{-\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} \frac{Q_{t+\tau}^m}{p_{t,t}^m \nu_{t+\tau}} \right] = 0. \tag{2.17}$$

Considering aggregate Dixit-Stiglitz prices of the intermediate goods (equation (2.4)), we apply to the law of large numbers and the fact that the fraction d of firms that reoptimize is completely random to find that

$$\begin{aligned}
\left(P_t^i \right)^{-\frac{1}{\nu_t}} &= \int_0^1 p_t^i(s)^{-\frac{1}{\nu_t}} ds = d \int_0^1 \left(p_{t-1}^i(s) \right)^{-\frac{1}{\nu_t}} ds + (1-d) \left(p_{t,t}^i \right)^{-\frac{1}{\nu_t}} \\
&= d \left(P_{t-1}^i \right)^{-\frac{1}{\nu_t}} + (1-d) \left(p_{t,t}^i \right)^{-\frac{1}{\nu_t}}, \; i = d, m, x.
\end{aligned} \tag{2.18}$$

2.3 Households

A representative household is characterized by the following preferences with external habit formation in consumption:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^*, L_t) \right], \; U(C_t^*, L_t) = \left[u(C_t^*) - v(L_t) \right], \tag{2.19}$$

where ζ_t^b represents a shock to the discount rate and ζ_t^L represents a shock to the labour supply. We define

$$C_t^* = C_t - h\bar{C}_{t-1} \tag{2.20}$$

where $\bar{C}_t$ is the average consumption level, which is considered exogenous to the representative household.

As in Kollmann (2001), we assume that the representative household supplies a continuum of labour service varieties, i.e., $l_t(j)$, $j \in [0,1]$.[1] These labour services enter as a Dixit-Stiglitz aggregate in the

[1] This should mathematically be identical to the case where individual households each supply one variety of labor services and completely diversify individual income uncertainty in a security market.

intermediate-goods firm production; thus, letting $l_t(s,j)$ be the amount of labour service j utilized by firm s we find that firm s uses the following amount of labour services;

$$L_t(s) = \left[\int_0^1 l_t(s,j)^{\frac{1}{1+\gamma}}\, dj\right]^{1+\gamma_t}, \ \gamma_t > 1, \tag{2.21}$$

and total labour is $L_t = \int_0^1 L_t(s)\, ds$.

In addition to consumption, the representative household can invest in domestic and foreign one-period bonds as well as in domestic capital. Capital K_t earns rental rate R_t and accumulates as follows;

$$K_{t+1} = K_t(1-\delta) + I_t - \phi(K_{t+1}, K_t), \ 0 < \delta < 1, \tag{2.22}$$

where $\phi(K_{t+1}, K_t)$ is an adjustment cost. Domestic bonds A_t earns net interest i_t, while the interest i_t^f accruing to foreign bonds B_t held by domestic agents deviates from the foreign interest level i_t^* as follows;

$$\left(1 + i_t^f\right) = \Omega_t^j (1 + i_t^*), \tag{2.23}$$

$$\Omega_t = \upsilon_t \exp\left\{-\lambda \frac{e_t B_{t+1}}{P_t \Xi}\right\}, \ \Xi = \frac{eP^x Q^x}{P} \tag{2.24}$$

where Ξ is the steady-state value of export in units of the domestic final good. Thus, the interest on foreign bonds is growing in the foreign debt level which ensures the existence of a unique equilibrium, cf. Schmitt-Grohe and Uribe (2003), while υ_t is a stochastic shock which can also be interpreted as a UIP shock.

The household owns domestic firms and thus earns profit from the intermediate-goods firms and rental rates (R_t) on the capital in addition to wage income from its variety of labour services. The budget constraint is thus

$$\begin{aligned} &A_{t+1} + e_t B_{t+1} + P_t(C_t + I_t) = \\ &A_t(1 + i_{t-1}) + e_t B_t\left(1 + i_{t-1}^f\right) + R_t K_t + \int_0^1 \pi_t^{dx}(s)\, ds + \int_0^1 \int_0^1 w_t(j)\, l_t(s,j)\, dj ds. \end{aligned} \tag{2.25}$$

Wage setting is staggered a la Calvo (1983). That is, in each period the household only optimize the wage of labour type j (styled $w_t(j)$) with probability $1 - D$. The household takes the average wage rate $W_t = \left[\int_0^1 w_t(j)^{-\frac{1}{1+\gamma_t}}\, dj\right]^{-(1+\gamma_t)}$ as given when it chooses the optimal wage $w_{t,t}$, and will meet any demand for the given type of labour;

$$l_t(j) = \int_0^1 l(s,j)\, ds. \tag{2.26}$$

The household thus faces the following problem

$$\max_{\{C_t, A_{t+1}, B_{t+1}, K_{t+1} w_{t,t}\}_{t=0}^{\infty}} E_0\left[\sum_{t=0}^{\infty} \beta^t U(C_t^*, L_t)\right] \tag{2.27}$$

s.t. (2.20)-(2.26).

Solving (2.25) with respect to C_t yields

$$\begin{aligned}
C_t &= \frac{1}{P_t}\left\{A_t\left(1+i_{t-1}\right)+e_t^{EU}B_t^{EU}\left(1+i_{t-1}^{EU}\right)+e_t^{ROW}B_t^{ROW}\left(1+i_{t-1}^{ROW}\right)\right. \\
&\quad \left.+R_tK_t-A_{t+1}-e_tB_{t+1}\right\} \\
&\quad -I_t+\frac{1}{P_t}\left[\int_0^1 \pi_t^{dx}\left(s\right)ds+\int_0^1\int_0^1 w_t\left(j\right)l_t\left(s,j\right)djds\right] \iff \\
C_t^* &= \frac{1}{P_t}\left[A_t\left(1+i_{t-1}\right)+e_tB_t\left(1+i_{t-1}^f\right)+R_tK_t-A_{t+1}-e_tB_{t+1}\right] \\
&\quad -\left[K_{t+1}-K_t\left(1-\delta\right)+\phi\left(K_{t+1},K_t\right)\right] \\
&\quad +\frac{1}{P_t}\left[\int_0^1 \pi_t^{dx}\left(s\right)ds+\int_0^1\int_0^1 w_t\left(j\right)l_t\left(s,j\right)djds\right]-h\bar{C}_{t-1},
\end{aligned}$$

Thus, an interior solution to the household's problem (2.27) yields the following first-order conditions: Capital,

$$\begin{aligned}
\beta^tU_{C,t}\left[-1-\phi_{1,t}\right]+\beta^{t+1}E_t\left[U_{C,t+1}\left(\frac{R_{t+1}}{P_{t+1}}+\left(1-\delta\right)-\phi_{2,t+1}\right)\right] &= 0 \iff \\
\beta^{t+1}E_t\left[U_{C,t+1}\left(\frac{R_{t+1}}{P_{t+1}}+\left(1-\delta\right)-\phi_{2,t+1}\right)\right] &= \beta^tU_{C^*,t}\left[1+\phi_{1,t}\right] \iff \\
\beta\frac{E_t\left[U_{C,t+1}\left(\frac{R_{t+1}}{P_{t+1}}+\left(1-\delta\right)-\phi_{2,t+1}\right)\right]}{U_{C,t}\left[1+\phi_{1,t}\right]} &= 1 \iff \\
E_t\left[\rho_{t,t+1}\frac{P_{t+1}}{P_t}\frac{\left(\frac{R_{t+1}}{P_{t+1}}+\left(1-\delta\right)-\phi_{2,t+1}\right)}{\left[1+\phi_{1,t}\right]}\right] &= 1. \qquad (2.28)
\end{aligned}$$

Domestic bonds,

$$\begin{aligned}
\beta^tU_{C,t}\left[-\frac{1}{P_t}\right]+\beta^{t+1}E_t\left[U_{C,t+1}\left(\frac{1+i_t}{P_{t+1}}\right)\right] &= 0 \iff \\
\left(1+i_t\right)\beta E_t\left[\frac{U_{C,t+1}}{U_{C,t}}\frac{P_t}{P_{t+1}}\right] &= 1 \iff \\
\left(1+i_t\right)E_t\left[\rho_{t,t+1}\right] &= 1. \qquad (2.29)
\end{aligned}$$

Foreign bonds,

$$\begin{aligned}
\beta^t U_{C^*,t}\left(-\frac{e_t}{P_t}\right)+\beta^{t+1}E_t\left[U_{C^*,t+1}\left(\frac{e_{t+1}\left(1+i_t^f\right)}{P_{t+1}}\right)\right] &= 0 \Longleftrightarrow \\
\beta E_t\left[U_{C^*,t+1}\left(\frac{e_{t+1}\left(1+i_t^f\right)}{P_{t+1}}\right)\right] &= U_{C^*,t}\frac{e_t}{P_t} \Longleftrightarrow \\
\left(1+i_t^f\right)\beta E_t\left[\frac{U_{C^*,t+1}}{U_{C^*,t}}\frac{e_{t+1}}{e_t}\frac{P_t}{P_{t+1}}\right] &= 1 \Longleftrightarrow \\
\left(1+i_t^f\right)E_t\left[\rho_{t,t+1}\frac{e_{t+1}}{e_t}\right] &= 1, \qquad (2.30)
\end{aligned}$$

where $\rho_{t,t+k}$ is defined in equation (2.14) above.

Since the household meets the demand for labour at its chosen wage level, we find the following relations;

$$\begin{aligned}
l_\tau(s,j) &= \left(\frac{w_{t,t}}{W_\tau}\right)^{-\frac{1+\gamma_t}{\gamma_t}}\frac{1-\psi}{\psi}\frac{R_\tau}{W_\tau}K_\tau(s) \Rightarrow \\
\frac{dl_t(s,j)}{dw_t(j)} &= -\frac{1+\gamma_t}{\gamma_t}w_{t,t}^{-\frac{1+2\gamma_t}{\gamma_t}}\frac{1-\psi}{\psi}R_tK_t(s)W_t^{\frac{1}{\gamma_t}} \Rightarrow \\
\int\frac{dl_t(s,j)}{dw_t(j)}ds &= -\frac{1+\gamma_t}{\gamma_t}w_{t,t}^{-\frac{1+2\gamma_t}{\gamma_t}}\chi_t, \\
\text{and}\int\frac{d\left(l_t(s)w_t(j)\right)}{dw_t(j)}ds &= -\frac{1}{\gamma_t}w_{t,t}^{-\frac{1+\gamma_t}{\gamma_t}}\chi_t, \\
\text{where } \chi_t &\equiv \frac{1-\psi}{\psi}R_t\int K_t(s)\,dsW_t^{\frac{1}{\gamma_t}} = \frac{1-\psi}{\psi}R_tK_tW_t^{\frac{1}{\gamma_t}}. \qquad (2.31)
\end{aligned}$$

Thus, we finally obtain the following first-order condition with respect to the wage rate;

$$\begin{aligned}
&\sum_{\tau=0}^{\infty}(\beta D)^\tau E_t\left[-\frac{U_{C,t+\tau}}{P_{t+\tau}}\frac{1}{\gamma_{t+\tau}}w_{t,t}^{-\frac{1+\gamma_{t+\tau}}{\gamma_{t+\tau}}}\chi_{t+\tau}+U_{L,t+\tau}\frac{1+\gamma_{t+\tau}}{\gamma_{t+\tau}}w_{t,t}^{-\frac{1+2\gamma_{t+\tau}}{\gamma_{t+\tau}}}\chi_{t+\tau}\right]=0 \Rightarrow \\
&\sum_{\tau=0}^{\infty}(\beta D)^\tau\frac{\chi_{t+\tau}}{\gamma_{t+\tau}}w_{t,t}^{-\frac{1+2\gamma_{t+\tau}}{\gamma_{t+\tau}}}E_t\left[\frac{U_{C,t+\tau}}{P_{t+\tau}}w_{t,t}-\left(1+\gamma_{t+\tau}\right)U_{L,t+\tau}\right]=0. \qquad (2.32)
\end{aligned}$$

Analogously to equation (2.18), the aggregate wage level is determined as

$$W_t=\left[D\left(W_{t-1}\right)^{-\frac{1}{\gamma_t}}+(1-D)\left(w_{t,t}\right)^{-\frac{1}{\gamma_t}}\right]^{-\gamma_t}. \qquad (2.33)$$

2.4 Specifying Utility Functions and Capital Adjustment Costs

We now make particular assumptions for the functional forms for the felicity function, the disutility of labour, and the capital adjustment costs. Thus, utility is specified as follows;

$$\begin{aligned}
u(C) &= \frac{C^{1-\sigma_C}}{1-\sigma_C},\ v(L) = \frac{L^{1+\sigma_L}}{1+\sigma_L} \Rightarrow \\
U(C_t^*, L_t) &= \zeta_t^b \left[\frac{\left(C_t - h\bar{C}_{t-1}\right)_t^{1-\sigma_C}}{1-\sigma_C} - \zeta_t^L \frac{L_t^{1+\sigma_L}}{1+\sigma_L} \right] \Rightarrow \\
U_{C,t} &\equiv \frac{\partial U(C_t^*, L_t)}{\partial C_t} = \zeta_t^b \left(C_t - h\bar{C}_{t-1}\right)^{-\sigma_C} \\
&= \zeta_t^b \left(C_t - h\bar{C}_{t-1}\right)^{-\sigma_C}, \\
\text{and } U_{L,t} &\equiv \frac{\partial U(C_t^*, L_t)}{\partial L_t} = \zeta_t^b \zeta_t^L L_t^{\sigma_L},
\end{aligned}$$

where we have imposed the condition $\bar{C}_t = C_t$.

Capital costs are assumed to be quadratic;

$$\begin{aligned}
\phi(K', K) &= \frac{\Phi}{2} \frac{(K' - K)^2}{K} \Rightarrow \\
\phi_{1,t} &\equiv \frac{\partial \phi(K_{t+1}, K_t)}{K_{t+1}} = \Phi \frac{K_{t+1} - K_t}{K_t}, \\
\phi_{2,t} &\equiv \frac{\partial \phi(K_{t+1}, K_t)}{K_{t+1}} = \frac{\Phi}{2} \left(1 - \left(\frac{K_{t+1}}{K_t}\right)^2\right).
\end{aligned}$$

2.5 Market Clearing Conditions

All intermediaries are demanded from either domestic or foreign final goods producers

$$Y_t = Q_t^d + Q_d^x. \tag{2.34}$$

In the final goods market equilibrium requires

$$Z_t = C_t + I_t, \tag{2.35}$$

and equilibria in the factor markets require

$$L_t = \int L_t(s)\, ds,\ K_t = \int K_t(s)\, ds.$$

2.6 The Household Budget Constraint and Net Foreign Assets

Manipulating the household budget constraint (2.25) and using the final-good market equilibrium (2.35) yields the following equation which simply states that the net foreign assets position (NFA) changes with

accruing interest and the net export.

$$\begin{aligned}
e_t B_{t+1} + P_t (C_t + I_t) &= e_t B_t \left(1 + i_{t-1}^f\right) + R_t K_t + W_t L_t \\
&\quad + P_t^d Q_t^d + e_t P_t^x Q_t^x - (R_t K_t + W_t L_t) \Rightarrow \\
e_t B_{t+1} &= e_t B_t \left(1 + i_{t-1}^f\right) - P_t Z_t + P_t^d Q_t^d + e_t P_t^x Q_t^x \\
&= e_t B_t \left(1 + i_{t-1}^f\right) + e_t P_t^x Q_t^x - P_t^m Q_t^m \Rightarrow \\
B_{t+1} &= B_t \left(1 + i_{t-1}^f\right) + P_t^x Q_t^x - \frac{P_t^m}{e_t} Q_t^m
\end{aligned}$$

2.7 Monetary Policy

We postulate an imperfect peg against the euro as the monetary policy; in our model the interest rate is the instrument, which is thus used to keep e_t constant up to an exogenous policy shock ξ_t with unity mean;

$$e_t = e\xi_t. \tag{2.36}$$

Log-linearizing equations (2.23) and (2.24) yields the following relation between the internal foreign interest rate and that paid to domestic holders of foreign bonds;

$$\hat{\imath}_t^f = \hat{\imath}_t^* + \hat{\upsilon}_t - \lambda \hat{B}_t.$$

Combining this relation with log-linearised versions of equations (2.29) and (2.30) yields

$$\begin{aligned}
E_t \Delta \hat{e}_{t+1} &= \hat{\imath}_t - \hat{\imath}_t^f = \hat{\imath}_t - \hat{\imath}_t^* + \left(\lambda \hat{B}_t - \hat{\upsilon}_t\right), \\
\hat{\imath}_t &\equiv \log\left(\frac{1 + i_t}{1 + \bar{\imath}}\right), \ \hat{\upsilon}_t \equiv \log\left(\upsilon_t / \upsilon\right), \ \hat{B}_t \equiv \log \frac{\left(B_{t+1}/P_t^*\right)}{\Xi},
\end{aligned}$$

which we can combine with (2.36) to obtain

$$\hat{\imath}_t = \hat{\imath}_t^* + \left(\hat{\upsilon}_t - \lambda \hat{B}_t\right) + E_t \Delta \hat{\xi}_{t+1} \tag{2.37}$$

that is, the interest rate responds (virtually) one-to-one with the foreign interest rate and the UIP shock and is additionally skewed by the spread and the past policy shock.

3 Summarising and Solving the Model

Recapitulating the model, we have formed equations for two intermediary aggregates in the domestic market

$$Q_t^d = \alpha^d \frac{P_t Z_t}{P_t^d}, \tag{3.1}$$

$$Q_t^m = \left(1 - \alpha^d\right) \frac{P_t Z_t}{P_t^m}, \tag{3.2}$$

one intermediary aggregate in the foreign market

$$Q_t^x = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta} Y_t^*, \tag{3.3}$$

and the aggregate price level

$$P_t = \left(P_t^d\right)^{\alpha^d} \left(P_t^m\right)^{1-\alpha^d}. \tag{3.4}$$

Cost minimization implies the aggregate $L-K$ relationship

$$L_t = \frac{1-\psi}{\psi}\frac{R_t}{W_t}K_t, \tag{3.5}$$

and the aggregate $K-Y$ relationship

$$K_t = \frac{1}{\theta_t}\left(\frac{1-\psi}{\psi}\frac{R_t}{W_t}\right)^{-(1-\psi)} Y_t, \tag{3.6}$$

while the marginal cost of producing intermediate goods is

$$MC_t = \frac{1}{\theta_t} W_t^{1-\psi} R_t^{\psi} \psi^{-\psi} (1-\psi)^{-(1-\psi)}. \tag{3.7}$$

We define a utility-based pricing kernel

$$\rho_{t,t+\tau} \equiv \beta^{\tau} \left(U_{C,t+\tau}/U_{C,t}\right)\left(P_t/P_{t+\tau}\right), \tag{3.8}$$

and derive three optimal pricing equations for intermediate goods

$$\sum_{\tau=0}^{\infty} d^{\tau} E_t \left[\left(p_{t,t}^d - (1+\nu_{t+\tau}) MC_{t+\tau}\right)\rho_{t,t+\tau}\left(\frac{p_{t,t}^d}{P_{t+\tau}^d}\right)^{-\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} \frac{Q_{t+\tau}^d}{p_{t,t}^d \nu_{t+\tau}}\right] = 0, \tag{3.9}$$

$$\sum_{\tau=0}^{\infty} d^{\tau} E_t \left[\left(e_{t+\tau} p_{t,t}^x - (1+\nu_{t+\tau}) MC_{t+\tau}\right)\rho_{t,t+\tau}\left(\frac{p_{t,t}^x}{P_{t+\tau}^x}\right)^{-\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} \frac{Q_{t+\tau}^x}{p_{t,t}^x \nu_{t+\tau}}\right] = 0, \tag{3.10}$$

$$\sum_{\tau=0}^{\infty} d^{\tau} E_t \left[R_{t,t+\tau}\left(p_{t,t}^m/e_{t+\tau} - P_{t+\tau}^*\right)\left(p_{t,t}^m\right)^{-\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}}\left(P_{t+\tau}^m\right)^{\frac{1+\nu_{t+\tau}}{\nu_{t+\tau}}} Q_{t+\tau}^m\right] = 0, \tag{3.11}$$

where we use the foreign discount factor

$$R_{t,t+\tau} \equiv \Pi_{s=t}^{t+\tau}\left(1+i_s^*\right)^{-1}, \tag{3.12}$$

and three equations for the aggregate prices of intermediaries

$$\left(P_t^d\right)^{-\frac{1}{\nu_t}} = d\left(P_{t-1}^d\right)^{-\frac{1}{\nu_t}} + (1-d)\left(p_{t,t}^d\right)^{-\frac{1}{\nu_t}}, \tag{3.13}$$

$$\left(P_t^x\right)^{-\frac{1}{\nu_t}} = d\left(P_{t-1}^x\right)^{-\frac{1}{\nu_t}} + (1-d)\left(p_{t,t}^x\right)^{-\frac{1}{\nu_t}}, \tag{3.14}$$

$$\left(P_t^m\right)^{-\frac{1}{\nu_t}} = d\left(P_{t-1}^m\right)^{-\frac{1}{\nu_t}} + (1-d)\left(p_{t,t}^m\right)^{-\frac{1}{\nu_t}}. \tag{3.15}$$

From the household decision problem we use a capital accumulation equation

$$K_{t+1} = K_t(1-\delta) + I_t - \frac{1}{2}\frac{\Phi\left(K_{t+1}-K_t\right)^2}{K_t},\ 0<\delta<1, \tag{3.16}$$

a NFA accumulation equation

$$B_{t+1} = B_t\left(1+i_{t-1}^f\right) + P_t^x Q_t^x - \frac{P_t^m}{e_t} Q_t^m. \tag{3.17}$$

definitions of marginal utility of consumption and disutility of labour

$$U_{C,t} = \zeta_t^b \left(C_t - hC_{t-1}\right)^{-\sigma_C}, \tag{3.18}$$

$$U_{L,t} = \zeta_t^b \zeta_t^L L_t^{\sigma_L}, \tag{3.19}$$

three first-order conditions with respect to capital and bonds

$$E_t \left[\rho_{t,t+1} \frac{P_{t+1}}{P_t} \frac{\left(\frac{R_{t+1}}{P_{t+1}^d} + (1-\delta) - \frac{\Phi}{2} \left(1 - \left(\frac{K_{t+2}}{K_{t+1}} \right)^2 \right) \right)}{\left[1 + \Phi \frac{K_{t+1} - K_t}{K_t} \right]} \right] = 1, \tag{3.20}$$

$$(1 + i_t) E_t \left[\rho_{t,t+1} \right] = 1, \tag{3.21}$$

$$\left(1 + i_t^f\right) E_t \left[\rho_{t,t+1} \frac{e_{t+1}}{e_t} \right] = 1, \tag{3.22}$$

three equations relating to optimal and aggregate wages

$$\chi_t = \frac{1-\psi}{\psi} R_t K_t W_t^{\frac{1}{\gamma}}, \tag{3.23}$$

$$w_{t,t} = (1 + \gamma_t) \frac{\sum_{\tau=t}^{\infty} (\beta D)^{\tau - t} E_t \left[U_{L,\tau} \chi_\tau \right]}{\sum_{\tau=t}^{\infty} (\beta D)^{\tau - t} E_t \left[\left(U_{C,\tau} / P_\tau \right) \chi_\tau \right]}, \tag{3.24}$$

$$W_t^{-\frac{1}{\gamma_t}} = D \left(W_{t-1} \right)^{-\frac{1}{\gamma_t}} + (1 - D) \left(w_{t,t} \right)^{-\frac{1}{\gamma_t}}, \tag{3.25}$$

and two equations for the foreign interest rate and the imperfectly pegged exchange rate;

$$\left(1 + i_t^f\right) = \left(1 + i_t^*\right) \upsilon_t \exp \left\{ -\lambda \frac{e_t B_{t+1}}{P_t \Xi} \right\}, \ \Xi = \frac{e P^x Q^x}{P}, \tag{3.26}$$

$$e_t = e \xi_t. \tag{3.27}$$

The market clearing conditions are

$$Y_t = Q_t^d + Q_t^x, \tag{3.28}$$

$$Z_t = C_t + I_t. \tag{3.29}$$

We log-linearise the model around its deterministic steady state and solve the resulting linear rational expectation system with the method suggested in Sims (2002). The derivation of the steady state is summarised in Appendix B, and the log-linearised system and its solution are described in Appendix C.

4 Estimation

In this section we consider the results and underlying assumptions of our estimation . Before we list our specific assumptions and report our estimation results, however, we briefly motivate the Bayesian methodology that we utilise.

4.1 Estimation Methodology

We seek suitable econometric tools to quantify and evaluate our postulated structural model of the Danish economy given our set of observed time series. Building on the seminal analysis in Smets and Wouters (2003), we follow what **?** styled the strong econometric interpretation of our DSGE model. This implies that we postulate a full probabilistic characterisation of our observed data which allows us to estimate the structural parameters through classical maximum-likelihood methods; or alternatively–following Bayesian methodology–through combining the likelihood function with prior distributions on the structural parameters and maximise the resulting posterior density.

In this paper we follow the Bayesian approach which allows us to formalise the use of any prior knowledge we may have on the structural parameters. On a more practical level it also helps stabilise the nonlinear minimization algorithm which we use for the estimation. Given the limited length of our sample, reasonable assumptions for the prior distributions (including restrictions on the support of certain parameters such as, e.g., standard deviations) are likely to be essential for obtaining plausible estimates. On the other hand, we utilise prior distributions we believe to be broad enough in order for the data to inform us on the structural parameters of the theoretical model.

We fix a subset of key parameters which are likely to be poorly determined in a model that only considers deviations from the steady state. These parameters include $\beta, \delta, \psi, \nu, \alpha^d, \lambda$ which are all assigned values which should be uncontroversial [to be added]. We also fix η at 1 which corresponds to a foreign technology equal to that assumed for the home country.

Our model includes ten structural shocks and nine observed variables. Thus, we can proceed on the assumption that there is no measurement error in the data set without facing the problem of stochastic singularity. In other words, we attribute all stochastic volatility to identified structural shock processes. This approach was succesfully carried out in the Smets and Wouters (2003) analysis of a close variant of the CEE closed-economy model. We should stress, however, that since our open-economy model does not include a number of the empirically motivated frictions of the CEE model, we leave a larger amount of the dynamics to be explained by the exogenous processes; hence, we should be more cautious when we interpret these processes as the true structural shocks.

4.2 Data

We treat Denmark as the home country and a weigthed average of a Germany, France and the Netherlands as the foreign country. For Denmark we include observations of real GDP, total real consumption, the GDP deflator, total employment adjusted for variations in hours worked, and a three-month money-market interest rate, corresponding to the theoretical variables Y, C, P, L and i.

Since our model assumes that the home country is pegging the foreign country, and the Danish krone was effectively pegged to the D-mark before the current peg on the euro, our foreign aggregate should at the same time be broad enough to cover as much as possible of the Danish trade and narrow enough that we can plausibly claim that the relevant exchange rate for the foreign area was historically the D-mark. We settled on Germany, France and the Netherlands which constituted 28 percent of Danish exports in 2003. We used their relative weights from the current effective exchange rate for the Danish krone as calculated by Danmarks Nationalbank which are 69, 17 and 14 percent for Germany, France and the Netherlands, respectively. For this EU aggregate we include observations of geometric averages of real GDP and the GDP deflator, and of the D-mark/euro exchange rate vis-à-vis the Danish krone and a German three-month money-market interest rate, matching the theoretical variables Y^*, P^*, e and i^*.

Since our log-linearized model describes stationary deviations from a steady state, we detrend the log of our GDP, consumption and labour supply series. We further adjust the price series for a nominal trend in inflation and remove the same trend from the interest rates.

Table 1: Parameter Estimates

Parameter		Type	Mean	Std. dev.[1]	Post. Mode	Std. error[2]
d	Calvo, intermediaries	Beta	0.70	0.03	0.941	0.003
D	Calvo, wages	Beta	0.75	0.03	0.767	0.012
h	Habit persistence	Beta	0.70	0.10	0.424	0.012
σ_C	Household IES	Gamma	1	10	1.669	0.020
σ_L	Work effort elasticity	Gamma	2	10	1.007	0.021
Φ	Capital adj. cost	Gamma	15	4	15.007	0.001
Shocks, persistence						
ϱ^b	Discount rate	Beta	0.85	0.06	0.835	0.014
ϱ^l	Labor supply	Beta	0.85	0.06	0.966	0.008
ϱ^t	Technology	Beta	0.85	0.06	0.825	0.017
ϱ^m	Peg	Beta	0.85	0.06	0.899	0.013
ϱ^i	Foreign interest rate	Beta	0.85	0.06	0.878	0.010
ϱ^P	Foreign price level	Beta	0.85	0.06	0.926	0.031
ϱ^Y	Foreign GDP	Beta	0.85	0.06	0.914	0.015
Shocks, volatility						
σ^b	Discount rate	Inv. gamma	0.2	6	0.039	0.003
σ^l	Labor supply	Inv. gamma	1	6	0.158	0.016
σ^t	Technology	Inv. gamma	0.1	6	0.010	0.001
σ^U	UIP	Inv. gamma	0.03	6	0.003	0.000
σ^{pm}	Price markup	Inv. gamma	0.03	6	2.055	0.014
σ^{wm}	Wage markup	Inv. gamma	0.15	6	0.004	0.001
σ^m	Peg	Inv. gamma	0.03	6	0.007	0.001
σ^i	Foreign interest rate	Inv. gamma	0.004	6	0.001	0.000
σ^P	Foreign price level	Inv. gamma	0.005	6	0.003	0.000
σ^Y	Foreign GDP	Inv. gamma	0.01	6	0.007	0.001

Footnote: [1]For the gamma and the inverse gamma distributions, we have cited the shape parameter α rather than the standard deviation. [2]The standard errors of the posterior mode estimates are based on the numerically calculated Hessian matrix.

4.3 Prior Distributions and Posterior Estimates

Description to be added...

4.4 Impulse responses

Description to be added...

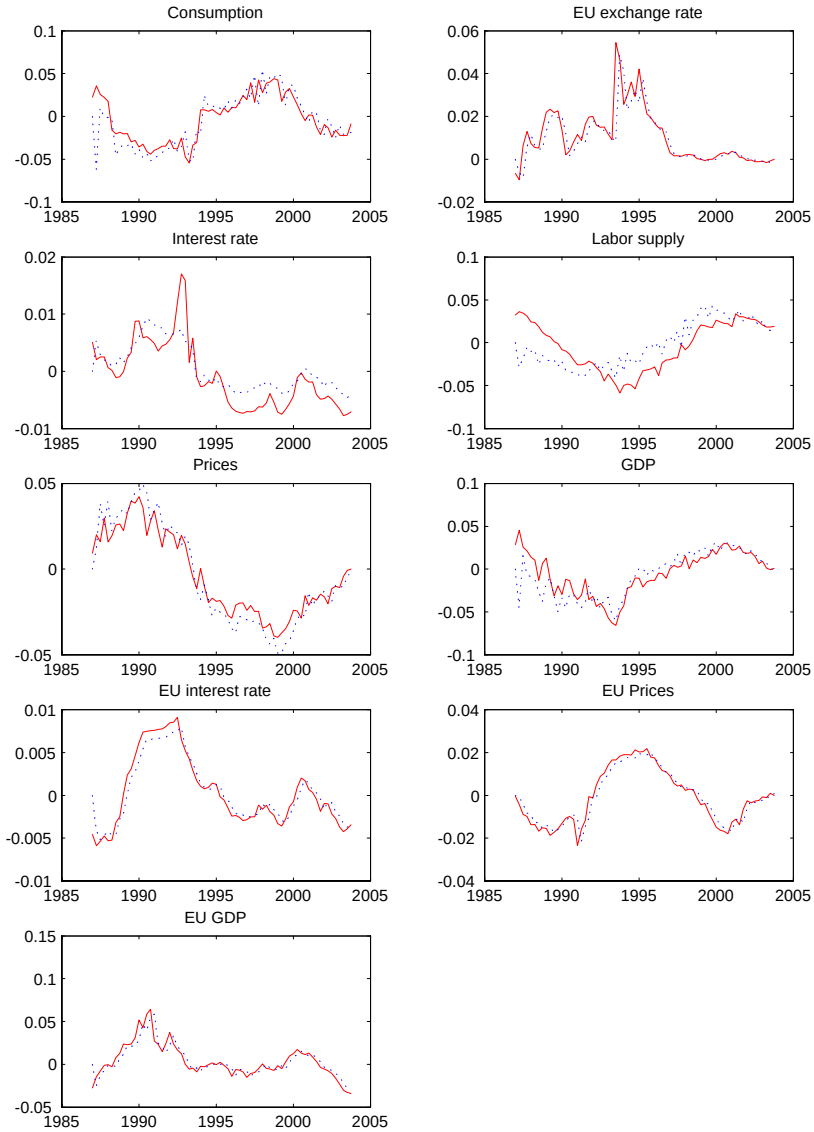
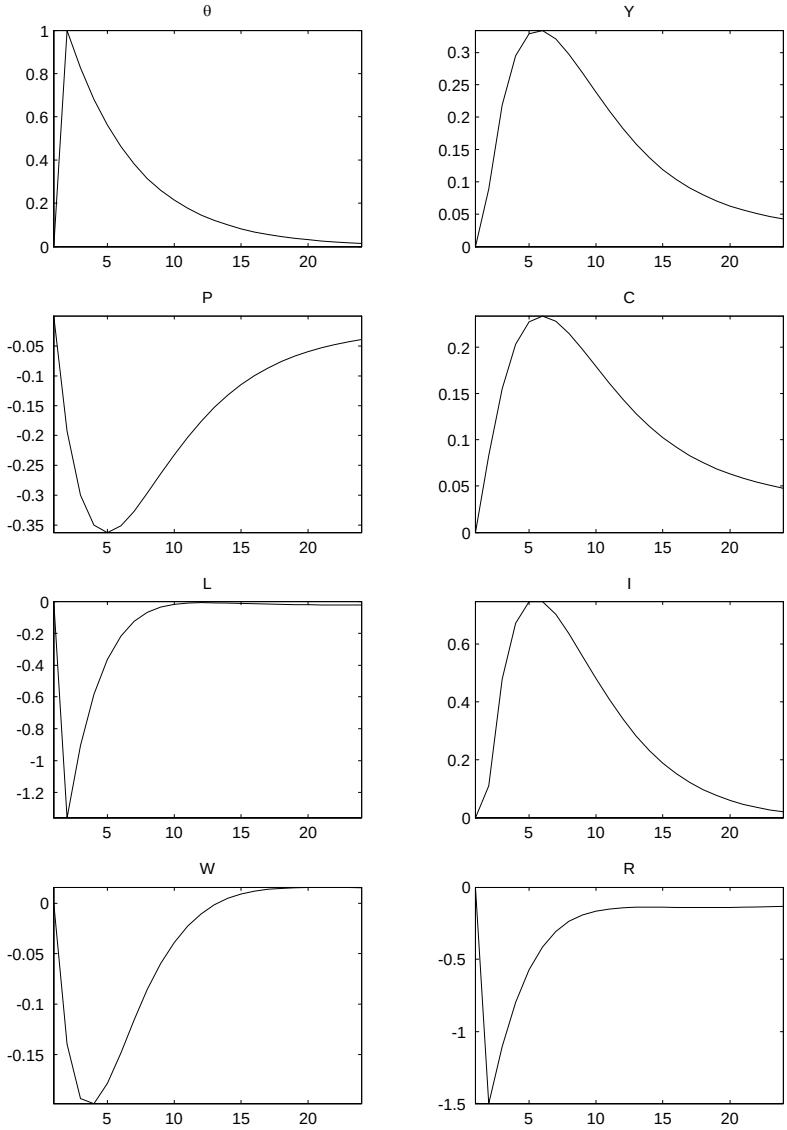


Figure 1: Observed and predicted time series



Response to a technology shock

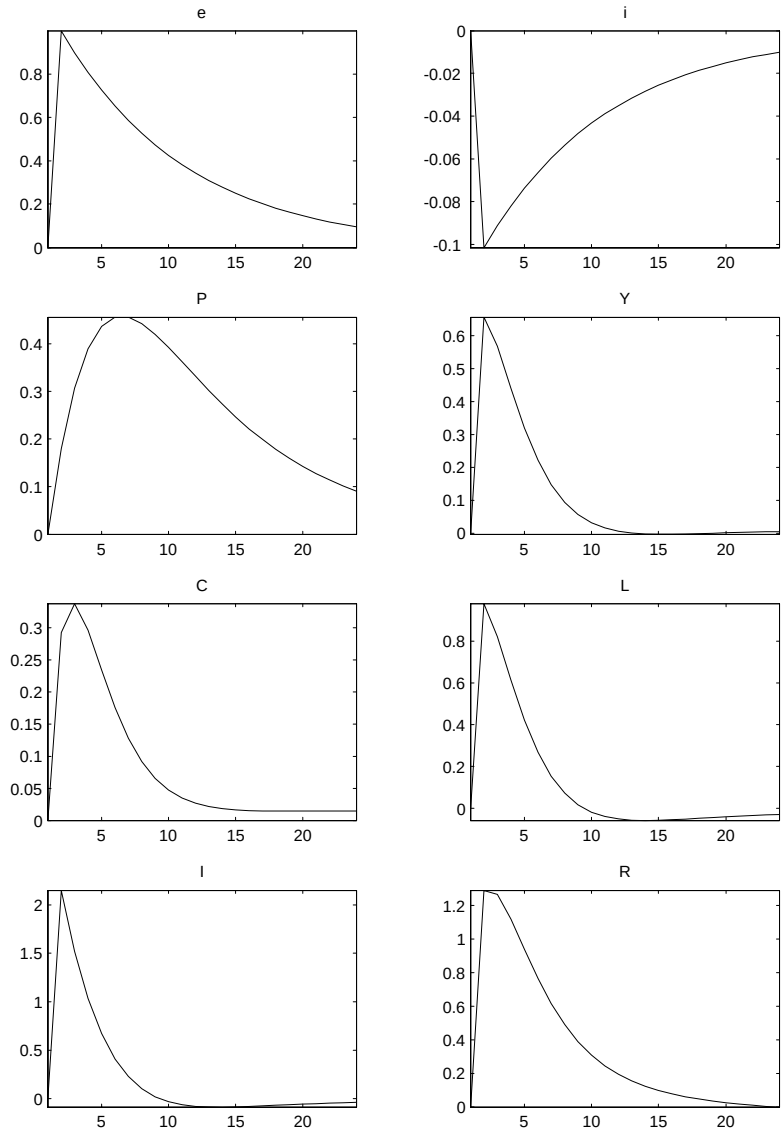


Figure 2: Response to a monetary policy shock

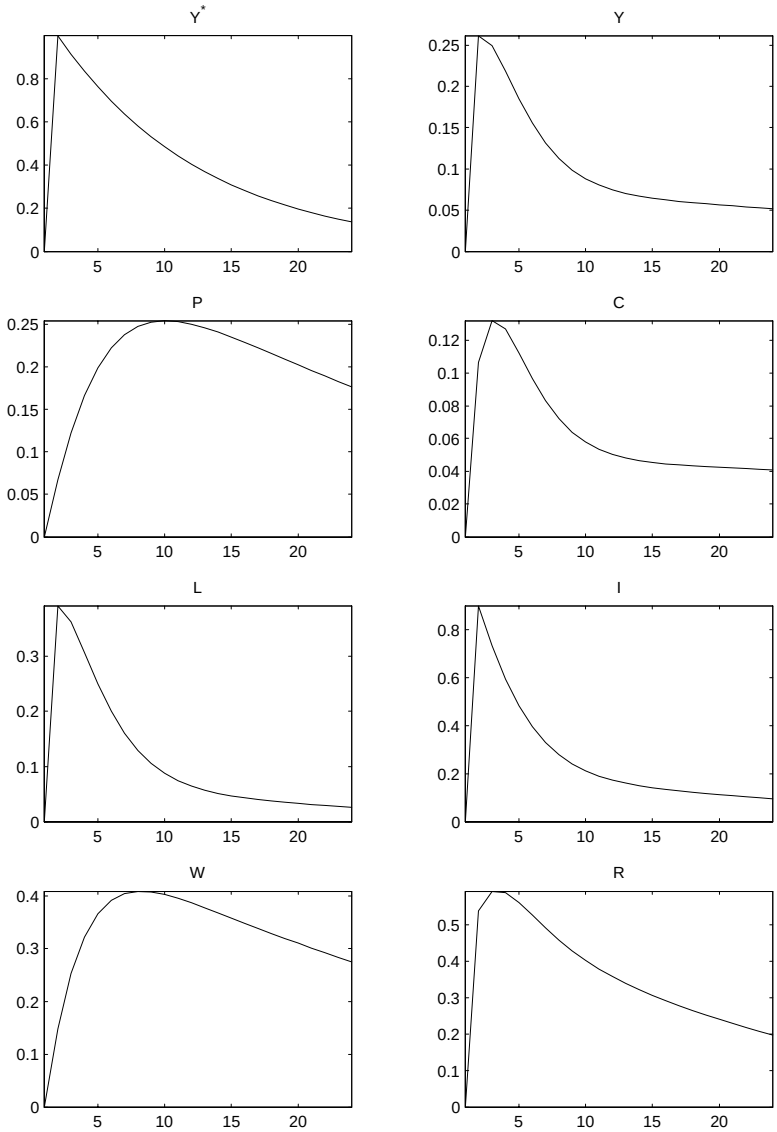


Figure 3: Response to a foreign demand shock

A List of Variables and Parameters

Variables

Z_t	Final goods
P_t	Price of Z
Q_t^i	Intermediate goods
P_t^i	Price of Q^i
$p_{t,\tau}^i$	Intermediary price optimized in period τ
Y_t	GDP $\left(Q^d + Q^x\right)$
R_t	Rental rate of capital
MC_t	Marginal cost in intermediary sector
e_t	Exchange rate
$\rho_{t,\tau}$	Discount factor between periods t and τ
$R_{t,\tau}$	Foreign discount factor
C_t	Final consumption
L_t	Aggregate labor supply
$w_{t,\tau}$	Wage level optimized in period τ
W_t	Aggregate wage level
$l_t(s,j)$	Labor of type j supplied to firm s
K_t	Capital stock
I_t	Investment
A_t	Domestic bonds (0 in eqlm.)
B_t	Foreign bonds in foreign currency
i_t	Domestic interest rate
i_t^f	Return on B_t to domestic agents
Ω_t	Wedge between i_t^* and i_t^f
χ_t	Compound variable in wage eqtn.
$U_{C,t}$	Marginal utility of consumption
$U_{L,t}$	Marginal disutility of labor

Exogenous Variables

ν_t	Net price markup (intermediaries)
θ_t	Technology level in intermediary sector
ζ_t^b	Preference discount rate shock
ζ_t^l	Labor supply shock
υ_t	UIP shock
γ_t	Net wage markup
ξ_t	Exchange-rate policy (peg) shock
Y_t^*	Foreign GDP
P_t^*	Foreign price level
i_t^*	Foreign interest rate

Parameters (time invariant)

α^d	Share of Q^d in final output
ψ	Capital share in intermediate goods
d	Calvo parameter, intermedaries
β	Utility discount factor
h	Habit persistence
σ_C^{-1}	Household IES
σ_L^{-1}	Work effort elasticity
δ	Capital depreciation rate
Φ	Capital adjustment cost
Ξ	SS export in units of Z
λ	Debt premium on foreign bonds
D	Calvo parameter, wages

B Steady State

In the steady state we have

$$P^d = p^d = (1+\nu)\,MC \Longrightarrow \frac{P^d}{P} = (1+\nu)\,mc \tag{B.1}$$

$$P^x = p^x = (1+\nu)\,\frac{MC}{e} \Longrightarrow P^x = (1+\nu)\,mc\left(\frac{P}{e}\right) \tag{B.2}$$

$$P^m = p^m = (1+\nu)\,eP^* \Longrightarrow \frac{P^m}{P} = (1+\nu)\,\frac{e}{P},\ P^* = 1 \tag{B.3}$$

$$W = w = (1+\gamma)\,\frac{U_L}{U_C}P \Longrightarrow \frac{W}{P} = (1+\gamma)\,[(1-h)\,C]^{\sigma_C}\,L^{\sigma_L} \tag{B.4}$$

From (3.8) we have that in the steady state

$$\rho = \beta \tag{B.5}$$

that, combined with (3.21) and (3.22), leads to

$$i = i^f = \frac{1}{\beta} - 1, \tag{B.6}$$

and from (3.20) we get

$$\begin{aligned}
\beta\left(\frac{R}{P}+(1-\delta)\right) &= 1 \Longleftrightarrow \\
\frac{R}{P} &= \frac{1}{\beta}-(1-\delta) \\
&= i+\delta. \qquad \text{(B.7)}
\end{aligned}$$

Hence, marginal costs can be written as

$$\begin{aligned}
mc &= \frac{MC}{P}=\left(\frac{W}{P}\right)^{1-\psi}\left(\frac{R}{P}\right)^{\psi}\psi^{-\psi}(1-\psi)^{-(1-\psi)} \\
&= \left[(1+\gamma)\left[(1-h)C\right]^{\sigma_C}L^{\sigma_L}\right]^{1-\psi}(i+\delta)^{\psi}\psi^{-\psi}(1-\psi)^{-(1-\psi)} \qquad \text{(B.8)} \\
&= (1+\gamma)^{(1-\psi)}\left[(1-h)C\right]^{(1-\psi)\sigma_C}L^{(1-\psi)\sigma_L}(i+\delta)^{\psi}\psi^{-\psi}(1-\psi)^{-(1-\psi)},
\end{aligned}$$

from which we can get the following expression for consumption;

$$\begin{aligned}
\left[(1-h)C\right]^{(1-\psi)\sigma_C} &= (1+\gamma)^{-(1-\psi)}L^{-(1-\psi)\sigma_L}(i+\delta)^{-\psi}\psi^{\psi}(1-\psi)^{1-\psi}mc \Longleftrightarrow \\
C &= \frac{1}{1-h}(1+\gamma)^{-\frac{1}{\sigma_C}}L^{-\frac{\sigma_L}{\sigma_C}}(i+\delta)^{-\frac{\psi}{(1-\psi)\sigma_C}}\psi^{\frac{\psi}{(1-\psi)\sigma_C}}(1-\psi)^{\frac{1}{\sigma_C}}mc^{\frac{1}{(1-\psi)\sigma_C}}. \qquad \text{(B.9)}
\end{aligned}$$

Note also the following implication of equation (B.8) which will prove useful below;

$$(i+\delta)^{-\psi}\psi^{\psi}(1-\psi)^{(1-\psi)}mc=\left[(1+\gamma)\left[(1-h)C\right]^{\sigma_C}L^{\sigma_L}\right]^{1-\psi}.$$

Now, we can determine the relative factor prices as follows;

$$\begin{aligned}
\frac{\frac{R}{P}}{\frac{W}{P}} &= \frac{R}{W}=\frac{i+\delta}{(1+\gamma)\left[(1-h)C\right]^{\sigma_C}L^{\sigma_L}} \\
&= (1+\gamma)^{-1}(i+\delta)\left[(1-h)C\right]^{-\sigma_C}L^{-\sigma_L}. \qquad \text{(B.10)}
\end{aligned}$$

Since

$$P=\left(P^d\right)^{\alpha^d}\left(P^m\right)^{1-\alpha^d},$$

we have from equation (3.4) that

$$\begin{aligned}
1 &= \left(\frac{P^d}{P}\right)^{\alpha^d}\left(\frac{P^m}{P}\right)^{1-\alpha^d} \\
&= \left[(1+\nu)mc\right]^{\alpha^d}\left((1+\nu)\frac{e}{P}\right)^{1-\alpha^d} \Longleftrightarrow \\
(1+\nu)\frac{e}{P} &= (1+\nu)^{-\frac{\alpha^d}{1-\alpha^d}}mc^{-\frac{\alpha^d}{1-\alpha^d}} \Longleftrightarrow \\
\frac{e}{P} &= (1+\nu)^{-\frac{1}{1-\alpha^d}}mc^{-\frac{\alpha^d}{1-\alpha^d}}, \qquad \text{(B.11)}
\end{aligned}$$

and we can now normalize P.

Using this, $(B.2)$ and $(B.3)$ we then have

$$\begin{aligned} P^x &= (1+\nu)\, mc \left((1+\nu)^{-\frac{1}{1-\alpha^d}}\, mc^{-\frac{\alpha^d}{1-\alpha^d}} \right)^{-1} \\ &= (1+\nu)^{\frac{2-\alpha^d}{1-\alpha^d}}\, mc^{\frac{1}{1-\alpha^d}}, \qquad \text{(B.12)} \\ \frac{P^m}{P} &= (1+\nu)^{-\frac{\alpha^d}{1-\alpha^d}}\, mc^{-\frac{\alpha^d}{1-\alpha^d}}. \qquad \text{(B.13)} \end{aligned}$$

In the steady state (3.17) looks like

$$P^x Q^x = \frac{P^m}{e} Q^m,$$

and foreign demands is expressed, cf. (**??**), as

$$\begin{aligned} Q^x &= \left(\frac{P^x}{P^*} \right)^{-\eta} Y^* \\ &= (P^x)^{-\eta}, \end{aligned}$$

since in the steady state we assume that $P^* = Y^* = 1$, implying

$$\begin{aligned} P^x (P^x)^{-\eta} &= \frac{P^m}{e} Q^m \Longleftrightarrow \\ Q^m &= (P^x)^{1-\eta} \frac{e}{P^m} = \frac{1}{1+\nu} (P^x)^{1-\eta}, \end{aligned}$$

where we used that

$$\frac{e}{P^m} = \frac{\frac{e}{P}}{\frac{P^m}{P}} = \frac{(1+\nu)^{-\frac{1}{1-\alpha^d}}\, mc^{-\frac{\alpha^d}{1-\alpha^d}}}{(1+\nu)^{-\frac{\alpha^d}{1-\alpha^d}}\, mc^{-\frac{\alpha^d}{1-\alpha^d}}} = \frac{1}{1+\nu}.$$

Now use $(B.12)$ to obtain

$$\begin{aligned} Q^x &= (1+\nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}}\, mc^{\frac{-\eta}{1-\alpha^d}}, \qquad \text{(B.14)} \\ Q^m &= \frac{1}{1+\nu} (1+\nu)^{\frac{(1-\eta)\left(2-\alpha^d\right)}{1-\alpha^d}}\, mc^{\frac{1-\eta}{1-\alpha^d}} \\ &= (1+\nu)^{\frac{(1-\eta)\left(2-\alpha^d\right)-1+\alpha^d}{1-\alpha^d}}\, mc^{\frac{1-\eta}{1-\alpha^d}} \\ &= (1+\nu)^{\frac{1-\eta\left(2-\alpha^d\right)}{1-\alpha^d}}\, mc^{\frac{1-\eta}{1-\alpha^d}} \qquad \text{(B.15)} \\ &= (1+\nu)^{\frac{1-\eta}{1-\alpha^d}-\eta}\, mc^{\frac{1-\eta}{1-\alpha^d}}. \end{aligned}$$

From (3.1) and (3.2) we obtain

$$Q^d = \frac{\alpha^d}{1-\alpha^d} \left(\frac{P^d}{P} \right)^{-1} \left(\frac{P^m}{P} \right) Q^m.$$

Now, substitute from $(B.1)$, $(B.13)$ and $(B.15)$ and re-arrange to get

$$\begin{aligned}
Q^d &= \frac{\alpha^d}{1-\alpha^d}\left(\frac{P^d}{P}\right)^{-1}\left(\frac{P^m}{P}\right)Q^m \\
&= \frac{\alpha^d}{1-\alpha^d}\left[(1+\nu)\,mc\right]^{-1}\left[(1+\nu)^{-\frac{\alpha^d}{1-\alpha^d}}\,mc^{-\frac{\alpha^d}{1-\alpha^d}}\right](1+\nu)^{\frac{1-\eta\left(2-\alpha^d\right)}{1-\alpha^d}}\,mc^{\frac{1-\eta}{1-\alpha^d}} \\
&= \frac{\alpha^d}{1-\alpha^d}(1+\nu)^{\frac{1-\eta\left(2-\alpha^d\right)}{1-\alpha^d}-1-\frac{\alpha^d}{1-\alpha^d}}\,mc^{\frac{1-\eta}{1-\alpha^d}-\frac{\alpha^d}{1-\alpha^d}-1} \\
&= \frac{\alpha^d}{1-\alpha^d}(1+\nu)^{\frac{1-\eta\left(2-\alpha^d\right)-\left(1-\alpha^d\right)-\alpha^d}{1-\alpha^d}}\,mc^{\frac{1-\eta-\alpha^d-\left(1-\alpha^d\right)}{1-\alpha^d}} \\
&= \frac{\alpha^d}{1-\alpha^d}(1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}}\,mc^{\frac{-\eta}{1-\alpha^d}}.
\end{aligned}$$

Furthermore,

$$\begin{aligned}
Q^m &= \left(1-\alpha^d\right)\frac{PZ}{P^m} \Longleftrightarrow \\
Z &= \frac{1}{1-\alpha^d}\frac{P^m}{P}Q^m \\
&= \frac{1}{1-\alpha^d}(1+\nu)^{-\frac{\alpha^d}{1-\alpha^d}}\,mc^{-\frac{\alpha^d}{1-\alpha^d}}(1+\nu)^{\frac{1-\eta\left(2-\alpha^d\right)}{1-\alpha^d}}\,mc^{\frac{1-\eta}{1-\alpha^d}} \\
&= \frac{1}{1-\alpha^d}(1+\nu)^{\frac{1-\eta\left(2-\alpha^d\right)-\alpha^d}{1-\alpha^d}}\,mc^{\frac{1-\eta-\alpha^d}{1-\alpha^d}} \qquad (B.16) \\
&= \frac{1}{1-\alpha^d}(1+\nu)^{1-\eta\frac{2-\alpha^d}{1-\alpha^d}}\,mc^{1-\frac{\eta}{1-\alpha^d}}.
\end{aligned}$$

Given the various quantities of intermediaries we can obtain a steady-state expression for real GDP;

$$\begin{aligned}
Y &= Q^d + Q^x \\
&= \frac{\alpha^d}{1-\alpha^d}(1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}}\,mc^{\frac{-\eta}{1-\alpha^d}} + (1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}}\,mc^{\frac{-\eta}{1-\alpha^d}} \\
&= (1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}}\,mc^{\frac{-\eta}{1-\alpha^d}}\left[\frac{\alpha^d}{1-\alpha^d}+1\right] \\
&= \frac{1}{1-\alpha^d}(1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}}\,mc^{\frac{-\eta}{1-\alpha^d}}. \qquad (B.17)
\end{aligned}$$

Turning to labour and capital, we have from (3.6) that

$$\begin{aligned}
K &= Y\left(\frac{1-\psi}{\psi}\frac{R}{W}\right)^{-(1-\psi)} \\
&= \frac{1}{1-\alpha^d}\left(1+\nu\right)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}}\left[\frac{1-\psi}{\psi}\frac{i+\delta}{(1+\gamma)\left[(1-h)\,C\right]^{\sigma_C}L^{\sigma_L}}\right]^{-(1-\psi)} \\
&= \frac{1}{1-\alpha^d}\left(1+\nu\right)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}}\left(\frac{\psi}{1-\psi}\right)^{(1-\psi)}(i+\delta)^{-(1-\psi)}\left[(1+\gamma)\left[(1-h)\,C\right]^{\sigma_C}L^{\sigma_L}\right]^{1-\psi} \\
&= \frac{1}{1-\alpha^d}\left(1+\nu\right)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}}\left(\frac{\psi}{1-\psi}\right)^{(1-\psi)}(i+\delta)^{-(1-\psi)}(i+\delta)^{-\psi}\,\psi^{\psi}\,(1-\psi)^{(1-\psi)}\,mc \\
&= \frac{\psi}{(1-\alpha^d)\,(i+\delta)}\left(1+\nu\right)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{1-\frac{\eta}{1-\alpha^d}}, \qquad \text{(B.18)}
\end{aligned}$$

where we made use of (B.8). Thus, using (2.9) we can solve for labour as follows;

$$\begin{aligned}
L &= \frac{1-\psi}{\psi}\frac{R}{W}K \\
&= \frac{1-\psi}{\psi}\frac{i+\delta}{(1+\gamma)\left[(1-h)\,C\right]^{\sigma_C}L^{\sigma_L}}\frac{\psi}{(1-\alpha^d)\,(i+\delta)}\left(1+\nu\right)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{1-\frac{\eta}{1-\alpha^d}} \\
&= \frac{1-\psi}{1-\alpha^d}\left(i+\delta\right)\left[(i+\delta)^{\frac{-\psi}{1-\psi}}\,\psi^{\frac{\psi}{1-\psi}}\,(1-\psi)\,mc^{\frac{1}{1-\psi}}\right]^{-1}\left(1+\nu\right)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{1-\frac{\eta}{1-\alpha^d}} \\
&= \frac{1}{(1-\alpha^d)}\psi^{-\frac{\psi}{1-\psi}}\,(i+\delta)^{\frac{\psi}{1-\psi}}\left(1+\nu\right)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{1-\frac{\eta}{1-\alpha^d}-\frac{1}{1-\psi}} \\
&= \frac{1}{(1-\alpha^d)}\psi^{-\frac{\psi}{1-\psi}}\,(i+\delta)^{\frac{\psi}{1-\psi}}\left(1+\nu\right)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{1-\frac{\eta}{1-\alpha^d}-\frac{1}{1-\psi}}. \qquad \text{(B.19)}
\end{aligned}$$

Substituting (B.19) into (B.9) yields

$$\begin{aligned}
C &= \frac{1}{1-h}\left(1+\gamma\right)^{-\frac{1}{\sigma_C}} L^{-\frac{\sigma_L}{\sigma_C}}\,(i+\delta)^{-\frac{\psi}{(1-\psi)\sigma_C}}\,\psi^{\frac{\psi}{(1-\psi)\sigma_C}}\,(1-\psi)^{\frac{1}{\sigma_C}}\,mc^{\frac{1}{(1-\psi)\sigma_C}} \\
&= \frac{1}{1-h}\left(1+\gamma\right)^{-\frac{1}{\sigma_C}}\left(1-\alpha^d\right)^{\frac{\sigma_L}{\sigma_C}}\psi^{\frac{\sigma_L}{\sigma_C}\frac{\psi}{1-\psi}+\frac{\psi}{(1-\psi)\sigma_C}}\,(1-\psi)^{\frac{1}{\sigma_C}}\,(i+\delta)^{-\frac{\sigma_L}{\sigma_C}\frac{\psi}{1-\psi}-\frac{\psi}{(1-\psi)\sigma_C}}\,(1+\nu)^{\eta\frac{\sigma_L}{\sigma_C}\frac{2-\alpha^d}{1-\alpha^d}} \\
&\quad\times mc^{\frac{1}{(1-\psi)\sigma_C}-\frac{\sigma_L}{\sigma_C}\left(1-\frac{\eta}{1-\alpha^d}-\frac{1}{1-\psi}\right)} \\
&= \frac{\left(1+\gamma\right)^{-\frac{1}{\sigma_C}}}{1-h}\left(1-\alpha^d\right)^{\frac{\sigma_L}{\sigma_C}}\left(\frac{\psi}{i+\delta}\right)^{\frac{(1+\sigma_L)}{\sigma_C}\frac{\psi}{1-\psi}}(1-\psi)^{\frac{1}{\sigma_C}}\,(1+\nu)^{\eta\frac{\sigma_L}{\sigma_C}\frac{2-\alpha^d}{1-\alpha^d}} \\
&\quad\times mc^{\frac{1}{(1-\psi)\sigma_C}-\frac{\sigma_L}{\sigma_C}\left(1-\frac{\eta}{1-\alpha^d}-\frac{1}{1-\psi}\right)},
\end{aligned}$$

so defining

$$\Lambda^C \equiv \frac{\left(1+\gamma\right)^{-\frac{1}{\sigma_C}}}{1-h}\left(1-\alpha^d\right)^{\frac{\sigma_L}{\sigma_C}}\left(\frac{\psi}{i+\delta}\right)^{\frac{(1+\sigma_L)}{\sigma_C}\frac{\psi}{1-\psi}}(1-\psi)^{\frac{1}{\sigma_C}}\,(1+\nu)^{\eta\frac{\sigma_L}{\sigma_C}\frac{2-\alpha^d}{1-\alpha^d}},$$

we can write

$$C = \Lambda^C mc^{\frac{1}{(1-\psi)\sigma_C}-\frac{\sigma_L}{\sigma_C}\left(1-\frac{\eta}{1-\alpha^d}-\frac{1}{1-\psi}\right)}. \qquad \text{(B.20)}$$

Now, combine equations (2.35) and (2.22) evaulated at the steady state in order to obtain

$$\begin{aligned} Z &= C + I = C + \delta K \Rightarrow \\ \Lambda^Z mc^{1-\frac{\eta}{1-\alpha^d}} &= \Lambda^C mc^{\frac{1}{(1-\psi)\sigma_C} - \frac{\sigma_L}{\sigma_C}\left(1-\frac{\eta}{1-\alpha^d}-\frac{1}{1-\psi}\right)} + \delta\Lambda^K mc^{1-\frac{\eta}{1-\alpha^d}}, \end{aligned} \tag{B.21}$$

where we have defined

$$\Lambda^Z \equiv \frac{1}{1-\alpha^d}(1+\nu)^{1-\eta\frac{2-\alpha^d}{1-\alpha^d}}, \ \Lambda^K \equiv \frac{\psi}{(1-\alpha^d)(i+\delta)}(1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}}. \tag{B.22}$$

Hence, we can now obtain a closed-form solution for the real marginal cost from (B.21);

$$\begin{aligned} &\left(\Lambda^Z - \delta\Lambda^K\right) mc^{1-\frac{\eta}{1-\alpha^d}} = \Lambda^C mc^{\frac{1}{(1-\psi)\sigma_C} - \frac{\sigma_L}{\sigma_C}\left(1-\frac{\eta}{1-\alpha^d}-\frac{1}{1-\psi}\right)} \iff \\ &mc^{\frac{1}{(1-\psi)\sigma_C} - \frac{\sigma_L}{\sigma_C}\left(1-\frac{\eta}{1-\alpha^d}-\frac{1}{1-\psi}\right)-\left(1-\frac{\eta}{1-\alpha^d}\right)} = \frac{\Lambda^Z - \delta\Lambda^K}{\Lambda^C} \iff \\ mc &= \left(\frac{\Lambda^Z - \delta\Lambda^K}{\Lambda^C}\right)^{\left[\frac{1}{(1-\psi)\sigma_C} - \frac{\sigma_L}{\sigma_C}\left(1-\frac{\eta}{1-\alpha^d}-\frac{1}{1-\psi}\right)-\left(1-\frac{\eta}{1-\alpha^d}\right)\right]^{-1}} \\ &= \left(\frac{\Lambda^Z - \delta\Lambda^K}{\Lambda^C}\right)^{\left[\frac{1+\sigma_L}{(1-\psi)\sigma_C} - \frac{\sigma_C+\sigma_L}{\sigma_C}\frac{1-\alpha^d-\eta}{1-\alpha^d}\right]^{-1}}. \end{aligned}$$

We note that

$$\begin{aligned} \frac{\Lambda^Z - \delta\Lambda^K}{\Lambda^C} &= \Lambda^C\left[\frac{1}{1-\alpha^d}(1+\nu)^{1-\eta\frac{2-\alpha^d}{1-\alpha^d}} - \frac{\psi}{(1-\alpha^d)(i+\delta)}(1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}}\right] \\ &= \Lambda^C\left[(1+\nu) - \frac{\psi}{(i+\delta)}\right]\frac{(1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}}}{1-\alpha^d}. \end{aligned}$$

B.1 Various steady-state ratios

Using the results above, we can derive the following steady-state ratios for use in the log-linearised system below;

$$\begin{aligned} \frac{Q^d}{Y} &= \frac{\frac{\alpha^d}{1-\alpha^d}(1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}}}{\frac{1}{1-\alpha^d}(1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}}} = \alpha^d, \\ \frac{Q^x}{Y} &= 1-\alpha^d, \end{aligned}$$

$$\left.\begin{aligned} Z &= \tfrac{1}{1-\alpha^d}(1+\nu)^{1-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{1-\frac{\eta}{1-\alpha^d}} = \Lambda^Z mc^{1-\frac{\eta}{1-\alpha^d}} \\ K &= \tfrac{\psi}{(1-\alpha^d)(i+\delta)}(1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{1-\frac{\eta}{1-\alpha^d}} = \Lambda^K mc^{1-\frac{\eta}{1-\alpha^d}} \end{aligned}\right\} \Rightarrow$$

$$\begin{aligned} \frac{K}{Z} &= \frac{\Lambda^K}{\Lambda^Z} = \frac{\psi}{(1-\alpha^d)(i+\delta)}(1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}}\left[\frac{1}{1-\alpha^d}(1+\nu)^{1-\eta\frac{2-\alpha^d}{1-\alpha^d}}\right]^{-1} \\ &= \frac{\psi}{(i+\delta)(1+\nu)}, \end{aligned}$$

$$
\begin{aligned}
\frac{I}{Z} &= \delta\frac{K}{Z} = \frac{\delta\psi}{(i+\delta)(1+\nu)}, \\
\frac{C}{Z} &= 1 - \frac{\delta\psi}{(i+\delta)(1+\nu)}.
\end{aligned}
$$

C Log-linearised System

$$\hat{Q}_t^d = \hat{P}_t + \hat{Z}_t - \hat{P}_t^d, \tag{C.1}$$

$$\hat{Q}_t^m = \hat{P}_t + \hat{Z}_t - \hat{P}_t^m, \tag{C.2}$$

$$\hat{Q}_t^x = -\eta \hat{P}_t^x + \eta \hat{P}_t^* + \hat{Y}_t^*, \tag{C.3}$$

$$\hat{P}_t = \alpha^d \hat{P}_t^d + \left(1 - \alpha^d\right) \hat{P}_t^m, \tag{C.4}$$

$$\hat{L}_t = \hat{R}_t - \hat{W}_t + \hat{K}_t, \tag{C.5}$$

$$\hat{K}_t = -\hat{\theta}_t - (1-\psi)\,\hat{R}_t + (1-\psi)\,\hat{W}_t + \hat{Y}_t, \tag{C.6}$$

$$\widehat{MC}_t = -\hat{\theta}_t + (1-\psi)\,\hat{W}_t + \psi \hat{R}_t, \tag{C.7}$$

$$\hat{\rho}_{t+1} = \hat{U}_{C_{t+1}} - \hat{U}_{C_t} + \hat{P}_t - \hat{P}_{t+1}, \tag{C.8}$$

$$\hat{P}_t^d - d\hat{P}_{t-1}^d = (1-d)(1-d\beta)\left[\widehat{MC}_t + \hat{\nu}_t\right] + d\beta E_t\left[P_{t+1}^d - dP_t^d\right], \tag{C.9}$$

$$\hat{P}_t^x - d\hat{P}_{t-1}^x = (1-d)(1-d\beta)\left(\widehat{MC}_t - \hat{e}_t + \hat{\nu}_t\right) + d\beta E_t\left[\hat{P}_{t+1}^x - d\hat{P}_t^x\right], \tag{C.10}$$

$$\hat{P}_t^m - d\hat{P}_{t-1}^m = (1-d)(1-d\beta)\left(\hat{e}_t + \hat{P}_t^* + \hat{\nu}_t\right) + d\beta E_t\left[\hat{P}_{t+1}^m - d\hat{P}_t^m\right], \tag{C.11}$$

$$\hat{W}_t - D\hat{W}_{t-1} = (1-D)(1-D\beta)\left(\hat{P}_t + \hat{U}_{L,t} - \hat{U}_{C,t} + \hat{\gamma}_t\right) + D\beta E_t\left[\hat{W}_{t+1} - D\hat{W}_t\right], \tag{C.12}$$

$$\hat{K}_{t+1} = (1-\delta)\,\hat{K}_t + \delta \hat{I}_t, \tag{C.13}$$

$$\hat{B}_{t+1} = (1+\bar{\imath})\,\hat{B}_t + \hat{P}_t^x + \hat{Q}_t^x - \hat{P}_t^m + \hat{e}_t - \hat{Q}_t^m,\ \hat{B}_t \equiv \frac{B_t}{P^x Q^x}, \tag{C.14}$$

$$\hat{U}_{C,t} = \hat{\zeta}_t^b - \frac{\sigma_C}{(1-h)}\hat{C}_t + \frac{h\sigma_C}{(1-h)}\hat{C}_{t-1}, \tag{C.15}$$

$$\hat{U}_{L,t} = \hat{\zeta}_t^b + \hat{\zeta}_t^L + \sigma_L \hat{L}_t, \tag{C.16}$$

$$\Phi(1+\beta)\,\hat{K}_{t+1} = E_t\hat{\rho}_{t+1} - \hat{P}_t + \beta(1-\delta)\,E_t\hat{P}_{t+1} + [1-\beta(1-\delta)]\,E_t\hat{R}_{t+1} + \Phi\hat{K}_t + \beta\Phi E_t\hat{K}_{t+2}, \tag{C.17}$$

$$\hat{\imath}_t = -E_t\hat{\rho}_{t+1}, \tag{C.18}$$

$$\hat{\imath}_t^f = -E_t\hat{\rho}_{t+1} - E_t\hat{e}_{t+1} + \hat{e}_t, \tag{C.19}$$

$$\hat{\imath}_t^f = \hat{\imath}_t^* + \hat{\upsilon}_t - \lambda\hat{B}_{t+1}, \tag{C.20}$$

$$\hat{e}_t = \hat{\xi}_t, \tag{C.21}$$

$$\hat{Y}_t = \alpha^d \hat{Q}_t^d + \left(1-\alpha^d\right)\hat{Q}_t^m, \tag{C.22}$$

$$\hat{Z}_t = \frac{C}{Z}\hat{C}_t + \frac{I}{Z}\hat{I}_t. \tag{C.23}$$

The system has 24 endogenous and 10 exogenous variables. Of the latter we assume that the markup shocks and the UIP shock $(\nu_t, \gamma_t, \upsilon_t)$ are i.i.d. and the remaining seven are AR(1) processes;

$$\hat{\zeta}_t^b = \varrho^b \hat{\zeta}_{t-1}^b + \varepsilon_t^b, \tag{C.24}$$
$$\hat{\zeta}_t^l = \varrho^l \hat{\zeta}_{t-1}^l + \varepsilon_t^l, \tag{C.25}$$
$$\hat{\theta}_t = \varrho^t \hat{\theta}_{t-1} + \varepsilon_t^t, \tag{C.26}$$
$$\hat{\xi}_t = \varrho^m \hat{\xi}_{t-1} + \varepsilon_t^m, \tag{C.27}$$
$$\hat{\imath}_t^* = \varrho^i \hat{\imath}_{t-1}^* + \varepsilon_t^i, \tag{C.28}$$
$$\hat{P}_t^* = \varrho^P \hat{P}_{t-1}^* + \varepsilon_t^P, \tag{C.29}$$
$$\hat{Y}_t^* = \varrho^Y \hat{Y}_{t-1}^* + \varepsilon_t^Y. \tag{C.30}$$

C.1 Solving the model with gensys

We solve the log-linearised system (C.1)-(C.30) with the gensys method developed by Sims (2002). For this purpose we collect the 23 endogenous variables with 6 lagged variables and 9 exogenous processes (excluding the policy shock ξ_t) in the (38×1) vector Υ_t;[2]

$$\begin{aligned}\Upsilon_t \quad : \quad & \hat{B}_t, \hat{C}_t, \hat{e}_t, \hat{\imath}_t, \hat{\imath}_t^f, \hat{I}_t, \hat{K}_t, \hat{L}_t, \widehat{MC}_t, \hat{P}_t, \hat{P}_t^d, \hat{P}_t^x, \hat{P}_t^m, \hat{Q}_t^d, \hat{Q}_t^x, \hat{Q}_t^m, \hat{R}_t, \hat{\rho}_{t+1}, \hat{U}_{C,t}, \hat{U}_{L,t}, \hat{W}_t, \hat{Y}_t, \hat{Z}_t, \\ & \hat{K}_{t-1}, \hat{P}_{t-1}^d, \hat{P}_{t-1}^x, \hat{P}_{t-1}^m, \hat{W}_{t-1}, \\ & \hat{\zeta}_t^b, \hat{\zeta}_t^l, \hat{\theta}_t, \hat{\nu}_t, \hat{\gamma}_t, \hat{\xi}_t, \hat{\imath}_t^*, \hat{P}_t^*, \hat{Y}_t^*.\end{aligned}$$

The i.i.d. shocks are included in the vector $\varepsilon_t \equiv \left(\varepsilon_t^b, \varepsilon_t^l, \varepsilon_t^t, \hat{\upsilon}_t, \varepsilon_t^m, \varepsilon_t^{pm}, \varepsilon_t^{wm}, \varepsilon_t^i, \varepsilon_t^P, \varepsilon_t^Y\right)$ includes the set of i.i.d. shocks, and the seven expectational errors are included in the vector $\eta_t = \left(\eta_t^d, \eta_t^x, \eta_t^m, \eta_t^W, \eta_t^K, \eta_t^A, \eta_t^B\right)$ so that we can write the model in the canonical VAR(1) gensys form;$\Gamma_0 \Upsilon_t = \Gamma_1 \Upsilon_{t-1} + \Psi \varepsilon_t + \Pi \eta_t$.

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[2] Hence, we add six identity equations to the system (C.1)-(C.30), corresponding to the six lagged endogenous variables included in Υ_t, and two definitions of the mark-up shocks $\left(\hat{\nu}_t = \varepsilon_t^{mp}, \hat{\gamma}_t = \varepsilon_t^{mw}\right)$.

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