The Impact of Model Periodicity on Inflation Persistence in Sticky Price and Sticky Information Models

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We show that the choice of model periodicity can be of crucial importance for the degree of inflation persistence in the sticky price model of Gali and Gertler and the sticky information model of Mankiw and Reis. The periodicity of the model (passing for example from a semi-annual to a quarterly model) conditions the dynamic response of inflation to a change in monetary growth. The sign of the relationship periodicity-persistence depends on the model considered.

INTRODUCTION

The model of Calvo (1983) has become a standard framework for monetary policy analysis. The main hypothesis of this model is that each firm is able to change its price in any given period with a given, constant probability. With the complementary probability, it is forced to maintain unchanged the price it charged the previous period. Despite price rigidity, this model has been challenged for its inability to capture correctly inflation persistence [Mankiw and Reis (2002)]. Two approaches have been followed in order to introduce persistence in this framework. The first approach [Gali and Gertler (1999)] keeps the hypothesis of sticky prices, but assumes that a fraction of the firms is backward looking. The second approach [Mankiw and Reis (2002)] replaces sticky prices with sticky information. Firms update infrequently their information about current macroeconomic conditions. Information arrivals follow the probabilistic structure of the Calvo model, but information is updated rather than prices.

In both models, the dynamics properties are dependent on the assumptions on the value of the probability of price (or information) updating. This parameter is usually inferred from the empirical price duration estimated from micro and macro studies. An average duration of one year has become the standard hypothesis [Taylor (1999)]. Given the average price duration and the periodicity of the model, the probability of a price change is directly calculated. For example, a price duration of one year is related to a probability of price change of 0.25 each quarter in a quarterly model, of 0.5 each semester in a semi-annual model. In order to insure comparability with sticky prices models, the standard assumption in sticky information models is that information is also updated every year on the average [Mankiw and Reis (2002), Trabandt (2003)].

We argue that when the probability of price/information updating is calibrated in this way, the dynamic properties of the model about inflation persistence are unacceptably sensitive to the periodicity chosen for the model. Moreover, for a given average contract length between revisions, the sign of the relationship periodicity-persistence depends upon the chosen specification. Passing e.g. from a semi-annual model to a quarterly model lowers inflation persistence in the hybrid sticky price model of Gali and Gertler and raises it in the sticky information model of Mankiw and Reis. The latter model always displays considerable persistence, while the former may show very low persistence. This directly results from the use of the probabilistic structure of random updating derived from Calvo (1983). Updating prices with the probability 0.5 each semester or with the probability 0.25 each quarter results in different aggregated prices distributions, which in turn alters dynamics.

In Section I, we present the Phillips curves implied by the hybrid sticky price model and the sticky information model. In Section II, we show why with the probabilistic structure of adjustment used in both models, the Phillips curves are affected by the model periodicity. With illustrate this point by a numerical example in section III. The final section highlights the wider relevance of the analysis and formulates some concluding remarks.

I. THE THEORETICAL FRAMEWORK

We first determine the price that firms would charge each period when prices are entirely flexible. In presence of imperfect competition on the goods market, the price setting behaviour of firms can be represented by:

$$p_t^* = p_t + \phi y_t, \tag{1}$$

where all variables are expressed in logs [see Romer (1996)]. Equation (1) implies that the desired price p_t^* depends on the aggregate price level p_t and the output gap y_t. Potential output is assumed to be constant and is normalized to 0. The parameter ϕ represents the degree of real rigidity. A strong real rigidity (a low ϕ) means the firms are more sensitive to relative prices than to macroeconomic conditions.

Aggregate demand is given by a simple quantity theory equation where log velocity is normalized to zero:

$$y_t = m_t - p_t. \tag{2}$$

The second step consists in specifying the nature of the dynamic constraints placed on the price revisions by the firms. As mentioned earlier, two approaches are considered to give empirically plausible results and are principally used in recent research: the sticky price and the sticky information approaches.

(a) Sticky prices

In the sticky prices model of Gali and Gertler (1999), the starting assumption is that in each period each firm is able to revise its price with probability λ. Conversely, it must keep its price unchanged from one period to the next with probability $1-\lambda$. There are two categories of firms, depending upon the way prices are revised when possible. The ω firms of the first category use a forward looking rule when choosing their new price[1] p_t^f:

$$p_t^f = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_t p_{t+j}^*. \tag{3}$$

The $1-\omega$ firms of the second category use a backward looking rule when setting their new prices p_t^b:

$$p_t^b = x_{t-1} + \pi_{t-1}, \tag{4}$$

where x_t, a price index for the prices modified in $t-1$, is given by:

$$x_t = (1-\omega)\, p_t^f + \omega p_t^b. \tag{5}$$

Thus, the overall price level p_t is given by:

$$p_t = \lambda x_t + (1-\lambda)\, p_{t-1}. \tag{6}$$

The Phillips curve implied by this specification is:

$$\pi_t = \gamma^f E_t \pi_{t+1} + \gamma^b \pi_{t-1} + \chi y_t, \tag{7}$$

with $\gamma^f = \dfrac{(1-\lambda)}{1-\lambda+\omega}$, $\gamma^b = \dfrac{\omega}{1-\lambda+\omega}$ and $\chi = \dfrac{\lambda^2 \phi (1-\omega)}{1-\lambda+\omega}$. This Phillips curve is widely used in papers studying optimal monetary policy rules.

(b) Sticky information

In response to the lack of persistence of the New Keynesian Phillips Curve, Mankiw and Reis (2002) have recently proposed to replace sticky prices by sticky information. All firms are forward looking. They are assumed to choose an entire path of future prices rather than a single constant price. Prices are perfectly flexible, that is, a firm can chose a different price for each of the future periods. Let $x_{t,t+i}$ be the price chosen in t for $t+i$. In each period, a fraction λ of the firms chooses its optimal price path $\{x_{t,t+j}\}_{\, j\in[0;+\infty[}$ in the following way:

$$x_{t,t+j} = E_t p_{t+j}^*. \tag{8}$$

[1] The coefficient of actualization is one. The results presented are not affected by values of this coefficient less than one.

The current aggregate price level p_t is given by the weighted average of all prices at time t:

$$p_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \, x_{t-j,t}. \tag{9}$$

The resulting sticky information Phillips curve is:

$$\pi_t = \kappa y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \, E_{t-1-j} \left(\pi_t + \phi \Delta y_t \right), \tag{10}$$

where $\kappa = (\lambda\phi) / (1-\lambda)$ and $\Delta y_t = y_t - y_{t-1}$. The inflation dynamics depend on expectation errors about the behaviour of the driving variable.

II. PERSISTENCE AND PERIODICITY

Both in the sticky price and the sticky information cases, one must distinguish between (a) the average price duration D, expressed e.g. in years; and (b) the model periodicity, that is, the number of model periods in one year (N): N will be 4 in a quarterly, 2 in a semi-annual model, etc.

The duration D is an empirical variable. There is compelling evidence for setting $D = 1$ (we shall do in the following, see Taylor [1999]). By contrast, there is no reason to favour a particular value of N other than the researcher's choice.[2] Thus, it is natural to require that the choice of N has no impact on the model dynamics. As we shall see, however, this is not the case. In both models, changes in the periodicity imply variations in the contract length distribution, as they modify the probability for some contracts to remain unchanged in future periods. This is reflected in Table 1, that shows as a function of N the percentage of firms that do not update over 1, 2, 3, or 5 years[3].

N	1 year	2 years	3 years	5 years
2	0.25	0.063	0.016	0.001
3	0.296	0.078	0.026	0.002
4	0.316	0.1	0.032	0.003
6	0.335	0.112	0.038	0.004
12	0.352	0.124	0.044	0.005

Table 1

However, this dependency translates into very different implications of a change in monetary policy. We assume for example a one time change in the

[2] It is however possible to impose some restrictions on N. For example, in the model of Gali and Gertler, the agents adjust their price taking into account the inflation in the previous period. Because inflation statistics are at best monthly, it is plausible to assume $N \leq 12$.

[3] The fraction of firms that do not update over j model periods is equal to $\left(\frac{N-1}{N}\right)^j$. The probability of not adjusting during one year is equal to $\left(\frac{N-1}{N}\right)^N$.

money growth rate. In the sticky information model, the fraction of firms which are still using outdated information to set their prices $k > 0$ years after the change in monetary policy increases with N. At the same time, a rise in N implies that the remaining firms update their information more frequently. Thus, an increase in N raises the persistence because (a) fewer firms are aware of the change in monetary policy; and (b) the remaining firms, although they adjust their price path more often, have no impact since prices are flexible. Indeed, when a firm correctly predicts the monetary path, it does not matter how often it revises its price path. The firms with obsolete information have an important impact, because they continue to raise their price at the previous rate of inflation. Since the number of firms with obsolete information rises with N, inflation persistence rises with N.

In the hybrid sticky price model, changes in N have the opposite effect. A rise in N lowers the persistence of inflation. Table 2 shows, as a function of N, the values taken by the parameters of the hybrid Phillips curve (7) in the case $\phi = 0.1$; $\omega = 0.5$:

N	γ_f	γ_b	β
2	0.5	0.5	0.0125
3	0.5714	0.4286	0.0048
4	0.6	0.4	0.0025
6	0.625	0.375	0.001
12	0.6471	0.3529	0.0002
$\to\infty$	$\to 2/3$	$\to 1/3$	$\to 0$

Table 2

One recognizes that the weight γ_f placed on forward inflation in the Phillips curve (7) rises and the weight γ_b placed on backward inflation decreases with N, lowering the persistence. As in the sticky information model, this results from the impact of N on the probability that some contracts remain unchanged in future periods, see Table 1. However, the consequences are very different from those in the sticky information model. Now, the fact that more prices are kept at their old values has little impact, because inflation directly depends on the newly set prices. If the new prices converge faster to their new equilibrium value, inflation persistence is lower. If the aggregate price level tends to rise over time, the fact that a firm is more likely to retain the same price increases the weight on future optimal prices in equation (3). This results in low inflation persistence. Thus, although the structure of adjustment is the same in the sticky price and in the sticky information models, it has very different implications.

III. A NUMERICAL ILLUSTRATION

We illustrate the inflation dynamics implied by the two Phillips curves (7) respectively (10) using a numerical example. The annual money growth Δm is exogenous and chosen in order to reach a given inflation target π^*, i.e. $\Delta m = \pi^*$ in each year. The target π^* is assumed to be initially equal to 5% per year. At time $t = 0$, it is lowered to 0%. This policy change is permanent, fully credible, and unexpected. The prices and prices paths are assumed to be adjusted every year on the average. We compare the inflation dynamics of both models for $N = 2$, $N = 3$, $N = 4$, $N = 6$, and $N = 12$. The results are presented in the Figures 1 and 2. The time unit on the horizontal $t-$axis is one month. Accordingly, the values on the vertical axis are monthly inflation rates.[4]

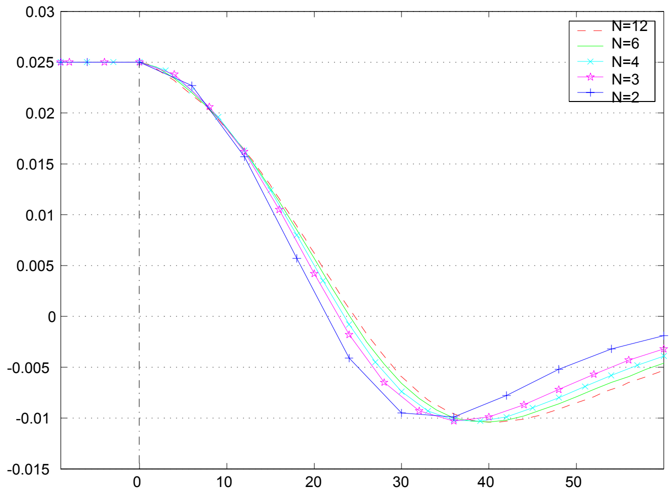

Figure 1: Inflation Dynamics in the Sticky Information Model

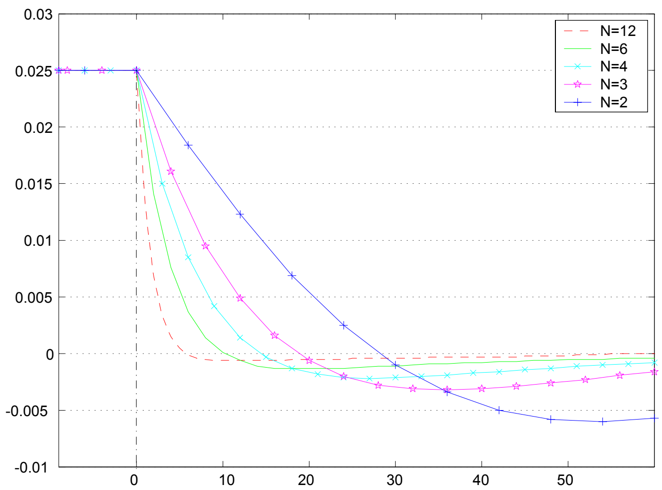

Figure 2: Inflation Dynamics in the Sticky Prices Model

[4]When the inflation values where computed over time intervals greater than one month, we interpolated linearly between the calculated values.

Persistence rises with N in the sticky information model (Figure 1). It decreases with N in the sticky prices model (Figure 2). In the sticky prices monthly model, inflation reaches its new target in less than six months. Differences in inflation persistence are not innocuous. They have an important impact on real variables. Among others, they are determinant for the real cost of disinflation. In Figure 3, we show the average annual output cost of disinflation over the five years following the target's change.

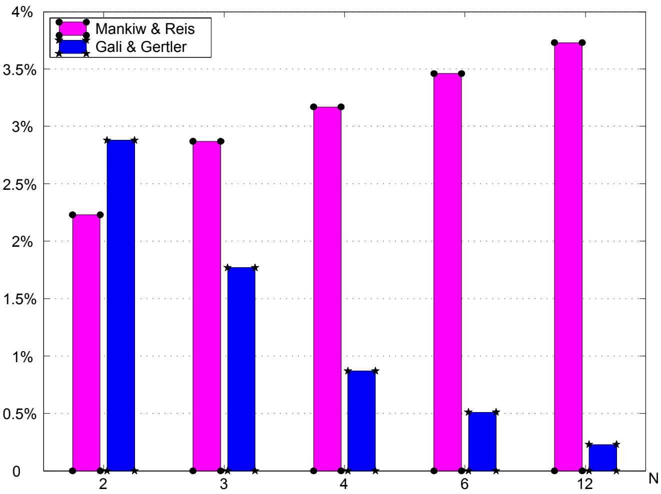

Figure 3: The Cost of Disinflation

These results vividly illustrate that the implications of the sticky price and sticky information models crucially differ depending upon the chosen periodicity. These differences can have important consequences for the optimal monetary policy.

In Figure 4, we indicate the autocorrelation of inflation in the sticky price and the sticky information models for $N = 2, 3, 4, 6, 12$, in the five years after the shock. The inflation values are no longer monthly ones but depend on the periodicity of the model. For example, they are semi-annual values when $N = 2$.

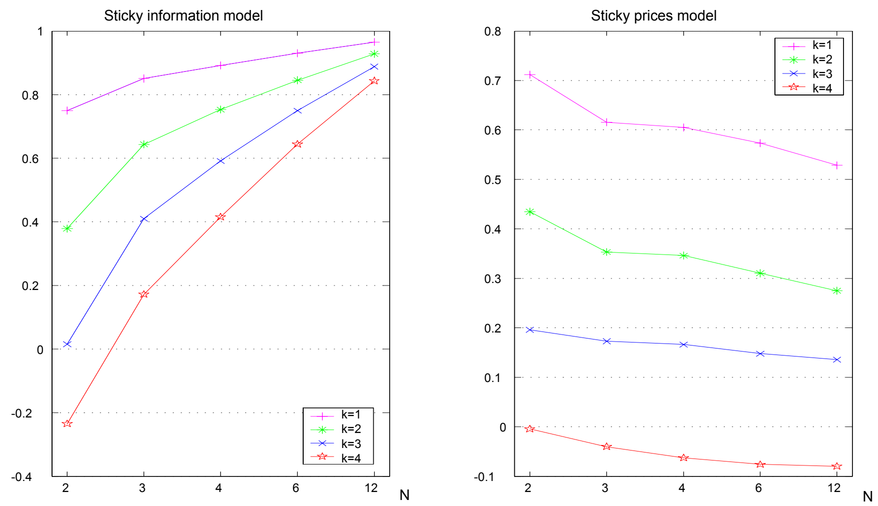

Figure 4: Autocorrelations

In the hybrid sticky prices model, the inflation persistence falls so quickly with N that the autocorrelation of inflation also falls with N. The correlation between two values of inflation six months apart when $N = 2$ is higher than the correlation between two values one month apart when $N = 12$. This is clearly an undesirable feature of the model. Here, the properties of the sticky information model are more satisfactory.

IV. CONCLUSION

Recent models about inflation dynamics are partly derived from optimizing behaviour. However, they have been developed with the explicit goal of reproducing the empirical behaviour of inflation. Their parameters are calibrated in order to reproduce this behaviour. Little attention has been devoted to the impact of the choice of periodicity. In fact, this choice is implicitly considered of no importance. However, the examples developed in this paper show that it can play an important role and that researchers could be careful about it. Moreover, it seems that the impact of model periodicity is endogenous to the structure of each model. The model of Gali and Gertler (1999) seems to be unacceptably sensitive to the periodicity chosen. The sticky information structure of Mankiw and Reis (2002) is more robust in the sense that even if inflation dynamics implied by this model are altered by the periodicity chosen, it does not produce incoherent results.

ACKNOWLEDGMENTS

We are indebted to David de la Croix, Christophe Deissenberg and Eric Girardin for constructive comments.

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